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Dissipation Effects on MHD Nonlinear Flow and Heat Transfer Past a Porous Surface with Prescribed Heat Flux

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ABSTRACT

Viscous and Joule dissipation effects are considered on MHD nonlinear flow and heat transfer past a stretching porous surface embedded in a porous medium under a transverse magnetic field. Analytical solutions of highly nonlinear momentum equation and confluent hypergeometric similarity solution of heat transfer equations in the case when the plate stretches with velocity varying linearly with distance are obtained. The effect of various parameters like suction parameter, Prandtl number, Magnetic parameter, and Eckert number entering into the velocity field, temperature distribution and skin friction co-efficient at the wall are discussed with the aid of graphs.

Keywords: MHD, Viscous dissipation, Joules dissipation, Heat flux, Porous surface.

NOMENCLATURE

C_p	specific heat at constant pressure	u,v	velocity components
Ec	Eckert number	v_w	suction velocity
j	electric current density	x, y	Cartesian coordinates
K	thermal conductivity	ν	kinematic viscosity
M^2	Magnetic parameter	σ	electrical conductivity
n	heat flux parameter	ρ	density of the fluid
Pr	Prandtl number	μ	coefficient of viscosity
R_1	permeability parameter	τ^*	skin friction coefficient
T	temperature of the fluid	k_p	permeability of the porous medium

1. INTRODUCTION

The study of magnetohydrodynamic flows with viscous and Joules dissipation has many important industrial, technological and geothermal applications such as high-temperature plasmas, cooling of nuclear reactors, liquid metal fluids, MHD accelerators and power generation systems.

Javeri and Berlin (1975) dealt with the effect of viscous dissipation and Joule heating on the fully developed MHD flow with heat transfer in a channel. The exact solution of the energy equation was derived for constant heat flux with small magnetic Reynolds number. Hossain (1992) considered the MHD free convection flow with viscous and Joules heating effects past a semi infinite plate with variable plate temperature. Analytical results were obtained by Vajravelu and Hadjinicolaou (1993) for the heat transfer in viscous fluid flow over a stretching sheet with viscous dissipation and internal heat generation. The solutions of the energy equation for the boundary layer flow of an electrically conducting fluid under the influence of a constant transverse magnetic field over a linearly stretching non-isothermal flat sheet was carried out by Chaim (1997). Effects due to dissipation, stress work and internal heat generation are considered.

Combined effect of viscous and Joules dissipation on MHD forced convection over a non-isothermal horizontal cylinder embedded in a fluid saturated porous medium have been studied by Amin (2003). Lahjomri et.al. (2003) analytically studied thermally developing laminar Hartmann flow through a parallel plate channel with prescribed transverse uniform magnetic field, including viscous dissipation, Joule heating and axial heat conduction with uniform heat flux. Viscous and Joule heating effect on forced convection flow of ionized gases adjacent to isothermal porous surfaces is analyzed numerically by Duwairi

(2005). He found that heat transfer rate is decreased due to viscous dissipation effect in both the cases of suction or injection in the fluid. Abo-Eldahab and Aziz (2005) analyzed the effects of viscous and Joules heating on MHD free convection flow past a semi-infinite vertical flat plate in the presence of the combined Hall and ion-slip currents for the case of power-law variation of the wall temperature. In (2005), Tak and Arty Lodha analysed the flow and heat transfer due to a stretching porous surface in the presence of transverse magnetic field including heat due to viscous dissipation.

Very recently authors (2007 & 2008) made an attempt to analyze the nonlinear Hydromagnetic flow with heat and mass transfer of a fluid through a porous medium over a stretching porous surface with constant heat and mass flux. In order to analyse the effects of viscous and Joules dissipation on nonlinear MHD flow with heat transfer over a stretching porous surface embedded in a porous medium, the present investigation is made.

2. Mathematical Analysis

Consider the steady two-dimensional hydromagnetic laminar boundary layer flow of an incompressible, viscous and electrically conducting fluid past a stretching porous surface embedded in a porous medium with viscous and Joule dissipation. Let the x and y axes be taken parallel and normal to the wall in the direction of motion of the flow and u and v are velocity components in the x and y directions respectively. A non-uniform magnetic field $\vec{B}(x)$ is applied in the transverse direction such that $\vec{B}(x) = B(x)\,\hat{j}$. Neglecting the induced magnetic field (which is allowed at small magnetic Reynolds numbers) the problem is now governed by the following boundary layer equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)\,u}{\rho} - \frac{\nu}{k_p}u \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = K\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{j^2}{\sigma} \tag{3}$$

The corresponding boundary conditions are

$$\begin{aligned} & y=0,\ u=U(x),\ v=-v_w(x), \\ & \quad -K\frac{\partial T}{\partial y} = q_w = E_0 x^n \\ & y\to\infty, u\to 0, T=T_\infty \end{aligned} \tag{4}$$

where q_w is the rate of heat transfer, E_0 is the positive constant, n is the heat flux parameter, v_w is the suction velocity and T_∞ is the temperature far away from the surface. Following Chaim (1995) and Afzal (1993), it is assumed that

$$U = ax^m, \quad B(x) = B_0 x^{(m-1)/2} \tag{5}$$

where a and B_0 are dimensional constants, m is a power-law exponent $(m \neq -1)$.

Equations (1) to (3) subjected to boundary condition (4) admit self-similar solution in terms of the similarity function $F(\eta)$ and the similarity variables are defined by

$$\begin{aligned} \psi(x,y) &= \left[\frac{2\,\nu\,U(x)}{1+m}\right]^{1/2} F(\eta) \\ \eta(x,y) &= \left[\frac{(1+m)\,U(x)}{2\,\nu\,x}\right]^{1/2} y \\ T - T_\infty &= \frac{E_0 x^n}{K}\sqrt{\frac{\nu}{a}}\,\theta(\eta) \\ v_w(x) &= \lambda\sqrt{\frac{\nu\,a\,(m+1)}{2}}\,x^{(m-1)/2} \\ n &= 2m \end{aligned} \tag{6}$$

where, $\lambda > 0$ for suction at the stretching plate and Ψ is the stream function such that

$$u = \frac{\partial\psi}{\partial y} = ax^m F'(\eta), \quad v = -\frac{\partial\psi}{\partial x} \tag{7}$$

Clearly u and v in equation (7) satisfy the equation of continuity (1). Employing the transformations (6) utilizing (5) the nonlinear partial differential equations (2) to (3) with boundary conditions (4) are reduced to the following nonlinear ordinary differential equations

$$F''' + FF'' - F'^2 - \left(\frac{1}{R_1} + M^2\right)F' = 0 \tag{8}$$

$$\theta'' + \Pr F\theta' - 2\Pr F'\theta = -Ec\Pr\left[F''^2 + M^2(F')^2\right] \tag{9}$$

with boundary conditions

$$\begin{aligned} & F(0)=\lambda, \quad F'(0)=1, \quad F'(\infty)=0, \\ & \theta(\infty)=0,\ \theta'(0)=-1, \end{aligned} \tag{10}$$

where $R_1 = \dfrac{k_p\,a}{\nu}$ is the permeability parameter,

$M^2 = \dfrac{2\,\sigma\,B_0^2}{\rho\,a\,(1+m)}$ is the Magnetic parameter,

$\Pr = \dfrac{\mu C_p}{K}$ is the Prandtl number and

$Ec = \dfrac{a^2}{C_p\left(\dfrac{E_0}{K}\sqrt{\dfrac{\gamma}{a}}\right)}$ is the Eckert number.

It may be noted that, when m = 1, the velocity of the stretching plate is a x, i.e. the plate stretches with a velocity varying linearly with distance.

Equation (8) with boundary conditions (10), is independent of (9) and admit a solution of the form

$$F(\eta) = A + Be^{-\alpha\eta}$$

Where

$$A = \frac{\alpha^2 - \left(M^2 + R_1^{-1}\right)}{\alpha} \quad , \qquad B = \frac{-1}{\alpha}$$

where $\alpha > 0$.

Hence the exact solution is

$$F(\eta) = \frac{1}{\alpha}\left[\alpha^2 - \left(R_1^{-1} + M^2\right) - e^{-\alpha\eta}\right] \tag{11}$$

To obtain the solution of equation (9), a new independent variable ξ is introduced

$$\xi = \frac{-\Pr}{\alpha^2} e^{-\alpha\eta} \tag{12}$$

Using (11) and (12), (9) yields

$$\xi\frac{d^2\theta}{d\xi^2} + [(1-K_1) - \xi]\frac{d\theta}{d\xi} + 2\theta = -\frac{Ec\,\alpha^2\left(\alpha^2 + M^2\right)}{\Pr}\xi \tag{13}$$

where

$$K_1 = \Pr\left(1 - \frac{R_1^{-1} + M^2}{\alpha^2}\right)$$

with corresponding boundary conditions

$$\theta(\xi = 0) = 0, \qquad \theta'(\xi = \frac{-\Pr}{\alpha^2}) = -\frac{\alpha}{\Pr} \tag{14}$$

Equation (13) is confluent hypergeometric equation (Abramowitz and Stegun, 1965) with non-homogeneous part, the solution of which may be expressed as follows

$$\theta = C\,{}_1F_1[-2;1-K_1;\xi] + D\xi^{K_1}\,{}_1F_1[K_1-2;1+K_1;\xi] - \frac{Ec\,\alpha^2(\alpha^2+M^2)}{2\Pr(2-K_1)}\xi^2$$

where C and D are constants which can be obtained by using the boundary condition (14). Substituting the values of C and D, the equation is reduced as

$$\theta(\xi) = -\frac{Ec\,\alpha^2(\alpha^2+M^2)}{2\Pr(2-K_1)}\xi^2 - \frac{\xi^{K_1}\,{}_1F_1\left[K_1-2;1+K_1;\xi\right]\left(\frac{\alpha}{\Pr} + \frac{Ec\left(\alpha^2+M^2\right)}{2-K_1}\right)}{\left(\frac{-\Pr}{\alpha^2}\right)^{K_1-1}\left[K_1\,{}_1F_1\left[K_1-2;1+K_1;-\frac{\Pr}{\alpha^2}\right] - \frac{\Pr}{\alpha^2}\left(\frac{K_1-2}{1+K_1}\right){}_1F_1\left[K_1-1;2+K_1;-\frac{\Pr}{\alpha^2}\right]\right]}$$

The function θ interms of variable η can now be expressed as

$$\theta(\eta) = -\frac{Ec(\alpha^2+M^2)}{2\alpha^2(2-K_1)}\Pr e^{-2\alpha\eta} - \frac{\left(\frac{-\Pr}{\alpha^2}\right)e^{-\alpha K_1\eta}\,{}_1F_1\left[K_1-2;1+K_1;\frac{-\Pr}{\alpha^2}e^{-\alpha\eta}\right]\left(\frac{\alpha}{\Pr} + \frac{Ec\left(\alpha^2+M^2\right)}{2-K_1}\right)}{\left[K_1\,{}_1F_1\left(K_1-2;1+K_1;-\frac{\Pr}{\alpha^2}\right) - \frac{\Pr}{\alpha^2}\left(\frac{K_1-2}{1+K_1}\right){}_1F_1\left[K_1-1;2+K_1;\frac{-\Pr}{\alpha^2}\right]\right]}$$

2.1 Skin friction coefficient

The non-dimensional form of skin-friction (or) skin friction coefficient at the wall can be calculated as

$$\tau^* = \frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\mu\,a\,x\left(\frac{a}{\nu}\right)^{1/2}} = F''(0) = -\alpha$$

3. DISCUSSION OF THE RESULTS

A boundary layer problem for momentum, heat transfer over a stretching porous surface with prescribed heat flux in the presence of a transverse variable magnetic field is examined in this work. Porous medium, viscous and Joule's dissipation are taken into consideration in this study. The MHD boundary layer equations of momentum and heat transfer are solved analytically and different analytical expressions are obtained for non-dimensional velocity and temperature profiles for prescribed heat flux case. Computation results are carried out for different values of suction parameter λ, Magnetic parameter M^2, Prandtl number *Pr*, Eckert number *Ec* and permeability parameter R_1.

Figure 1 shows the effect of magnetic parameter M^2 on non-dimensional transverse velocity profile. It is found that the effect of magnetic parameter is to reduce the transverse velocity and also noted that, as we move away from the wall the effect of magnetic field is found to be uniform. Figure 2 is depicted for velocity profile for different values of λ. From this figure, it is apparent that the effect of porosity is to enhance the transverse velocity.
Figure 3 shows the dimensionless longitudinal velocity for different values of M^2. It is found that the effect of magnetic parameter is to decelerate the longitudinal velocity. This is due to the increase of M^2, signifies the increase of Lorentz force that opposes the horizontal flow in the reverse direction. The effect of porosity over dimensionless longitudinal velocity is disclosed and is shown in Fig. 4. Further, it is noted that a decrease in longitudinal velocity accompanies a rise in λ, with all profiles tending asymptotically to the horizontal axis. In all cases, the non-dimensional velocity is observed maximum at the wall. It is also noted that the boundary layer thickness is reduced due to the effect of porosity.

Figure 5 is plotted for temperature distribution for different values of M^2 when *Pr* = 0.71, λ=3, *Ec* = 0.2

and R_1 = 100. It is interesting to note that there is a significant enhancement of temperature on the wall when it is porous. The application of magnetic field introduces additional skin-frictional heating which results in higher temperature on the wall with the increase of thermal boundary layer thickness. The effect of suction parameter λ over the temperature distribution is represented in Fig. 6. It is clear that the temperature is reduced due to the increase in suction parameter.

Figure 7 shows the temperature profile for different values of *Pr*. It reveals that the temperature decreases with increase in *Pr* which implies that thermal boundary layer thickness decreases. Figure 8 demonstrates the temperature distribution for different values of Eckert number *Ec*. It is observed that increasing values of *Ec* is to increase the temperature distribution in flow region. This is due to the heat energy stored in the liquid because of the frictional heating.

Figure 9 illustrates the skin friction against permeability parameter for various values of magnetic parameter. The effect of magnetic parameter is to decrease the skin friction coefficient. Figure 10 represents the skin friction coefficient against permeability parameter for various values of suction parameter. The influence of suction parameter is to suppress the skin friction coefficient.

4. Conclusion

MHD flow with heat transfer in a porous medium over a stretching porous surface with viscous and Joule dissipation effects are analyzed. Analytical results of the transformed MHD boundary layer equations have been obtained. The results obtained were validated against those of Anjali Devi and Ganga (2008). In the absence of the dissipation effects and porous medium the results are in very good agreement when *m*=1. The following specific conclusions were obtained:

- Magnetic parameter decreases both dimensionless longitudinal and transverse velocity significantly.
- The effect of suction parameter is to accelerate the dimensionless transverse velocity where as its effect over nondimensional longitudinal velocity is to decelerate it.
- There is a significant increase in temperature when the wall is porous.
- The temperature decreases with increasing suction parameter.
- It is interesting to note that temperature reduces due to increase in Prandtl number.
- Thermomagnetic layer becomes thick due to increase in Eckert number.
- Magnetic parameter and suction parameter decreases the skin friction coefficient at the wall.

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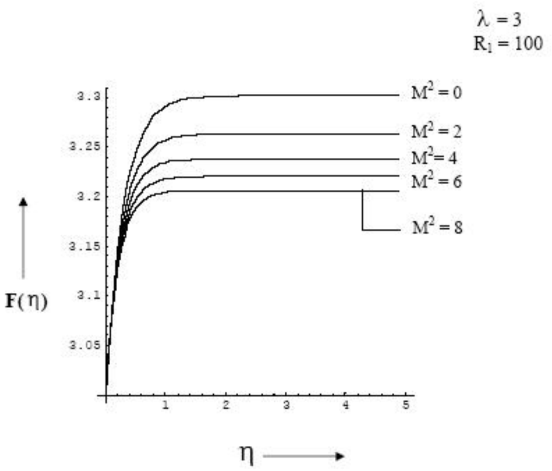

Fig. 1. Non-dimensional Transverse Velocity profiles for various M^2

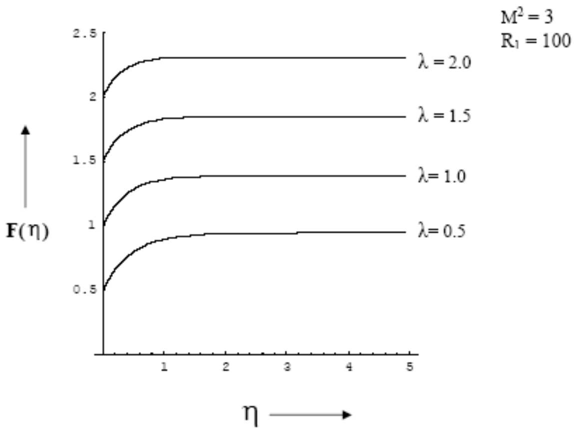

Fig. 2. Effect of λ on dimensionless Transverse Velocity

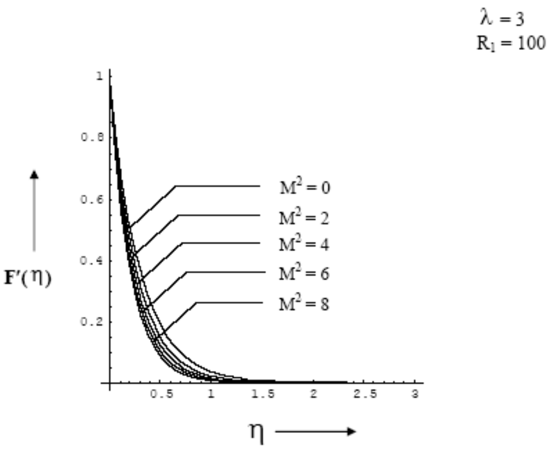

Fig. 3. Dimensionless longitudinal velocity profiles for various M^2

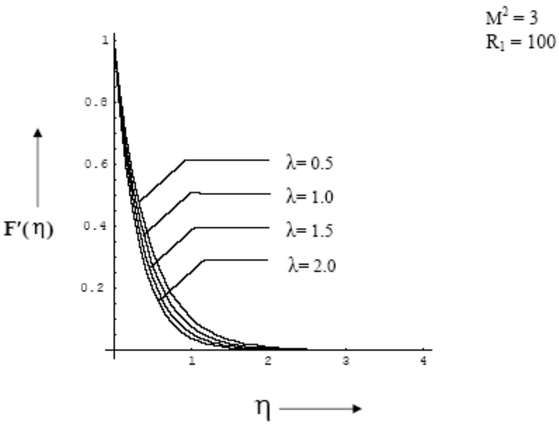

Fig. 4. Non-dimensional Longitudinal Velocity profiles for different λ

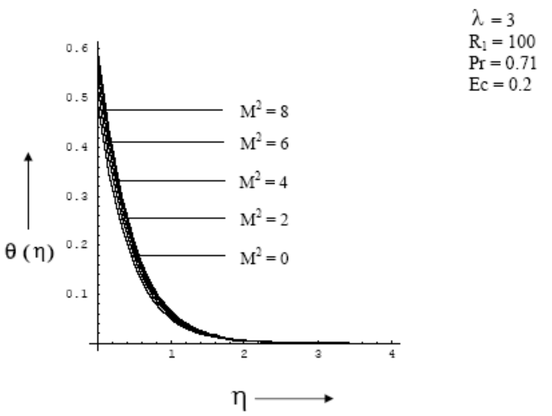

Fig. 5. Temperature Distribution for various values of M^2

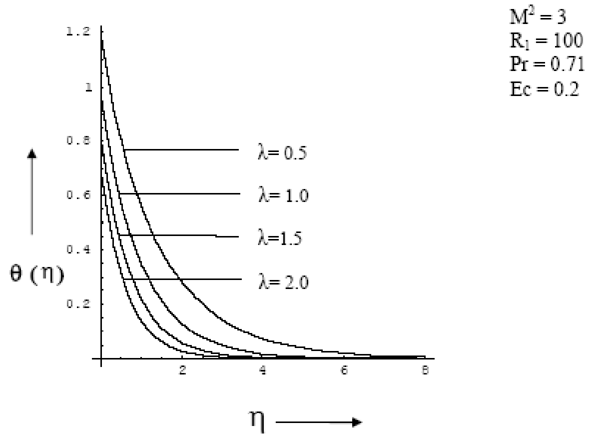

Fig. 6. Temperature Distribution for different λ

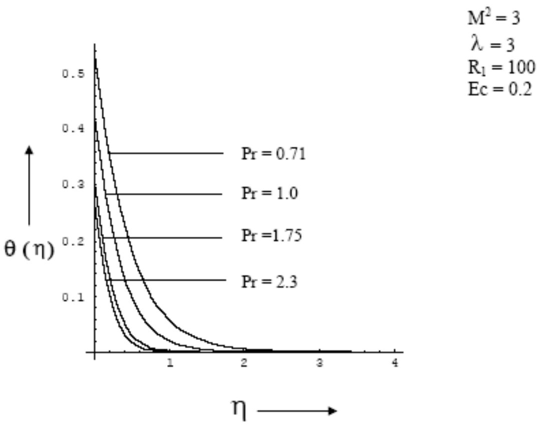

Fig. 7. Effect of Pr over Temperature distribution

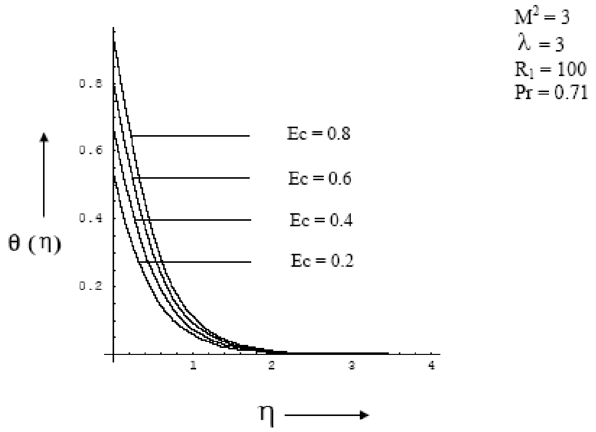

Fig. 8. Temperature profiles for Different Ec

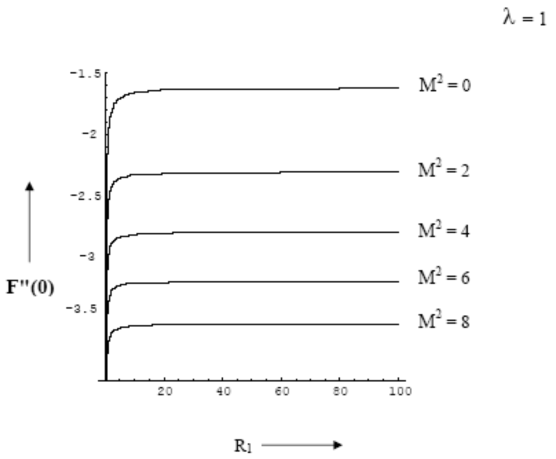

Fig. 9. Skin friction coefficient for various values of M^2

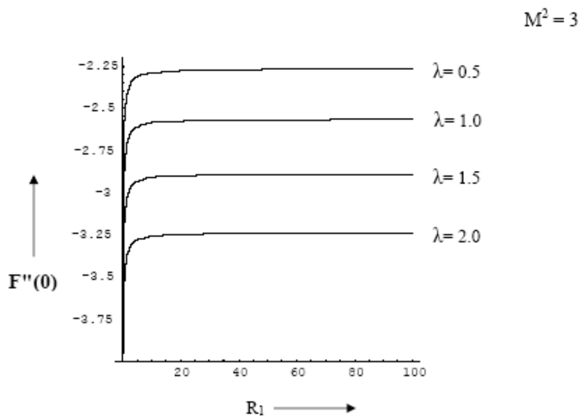

Fig. 10. Skin friction coefficient for various λ