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Effects of Rotation and Magnetic Field on Unsteady Couette Flow in a Porous Channel

G.S. Seth[1†], Md. S. Ansari[2] and R. Nandkeolyar[3]

[1] *Department of Applied Mathematics, Indian School of Mines, Dhanbad-826 004, India*

[2] *School of Petroleum Technology, Pandit Deendayal Petroleum University, Gandhinagar-382 007, India*

[3] *School of Applied Sciences, KIIT University, Bhubaneswar-751 024, India*

†*Corresponding Author Email:* gsseth_ism@yahoo.com

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ABSTRACT

Unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system in the presence of a uniform transverse magnetic field is studied. The plates of the channel are considered porous and fluid flow within the channel is induced due to the impulsive movement of the upper plate of the channel. General solution of the governing equations is obtained which is valid for every value of time t. For small values of time t, the solution of the governing equations is obtained by Laplace transform technique. The expression for the shear stress at the stationary plate due to the primary and secondary flows is obtained in both the cases. It is found that the solution obtained by Laplace transform technique converges more rapidly than the general solution when time t is very small. Magnetic field retards the fluid flow in both the primary and secondary flow directions. Rotation retards primary flow whereas it accelerates secondary flow. There exists incipient flow reversal near the stationary plate on increasing rotation parameter K^2. Suction accelerates primary flow whereas it retards secondary flow. Injection retards both the primary and secondary flows.

Keywords: Magnetohydrodynamic Couette flow, Primary and secondary flow, Rotation, Suction/injection.

1. INTRODUCTION

The theory of rotating fluids (Greenspan 1969) is highly important due to its occurrence in various natural phenomena and for its applications in various technological situations which are directly governed by the action of Coriolis force. The broad subjects of Oceanography, Meteorology, Atmospheric Science and Limnology all contain some important and essential features of rotating fluids. Several investigations are carried out on the problem of hydrodynamic flow of a viscous incompressible fluid in rotating medium considering various variations in the problem. Mention may be made of the studies of Greenspan and Howard (1963), Holton (1965), Walin (1969), Siegman (1971), Puri and Kulshrestha (1974), Mazumder (1991), Ganapathy (1994), Hayat and Hutter (2004), Singh et al (2005) and Guria *et al*. (2006). The problem of magnetohydrodynamic flow of a viscous incompressible electrically conducting fluid in a rotating medium is studied by many researchers viz. Seth and Jana (1980), Seth and Maiti (1982), Prasad Rao *et al*. (1982), (Seth *et al*. 1982, 1988, 2009), (Ghosh 1991, 1993, 1996, 2001), Chandran *et al*. (1993), Singh *et al*. (1994), Singh (2000), Hossain *et al*. (2001), Ghosh and Pop (2002), (Hayat *et al*. 2001, 2002, 2004, 2008), Hayat and Abelman (2007), Abelman *et al*. (2009), Wang and Hayat (2004), Seth and Singh (2008), Seth and Ansari (2009), Das *et al*. (2009) and Guria *et al*. (2009) under different conditions and configurations to analyze various aspects of the problem and to find its application in Science and Engineering. Seth *et al*. (1988) and Singh (2000) considered oscillatory hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system under different conditions. Guria *et al*. (2009) investigated oscillatory MHD Couette flow of electrically conducting fluid between two parallel plates in a rotating system in the presence of an inclined magnetic field when the upper plate is held at rest and the lower plate oscillates non-torsionally. Chandran *et al*. (1993) and Das *et al*. (2009) studied unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system when the fluid flow within the channel is induced due to impulsive movement of one of the plates of the channel whereas Singh et al (1994) considered this problem when one of the plates of the channel is set into uniformly accelerated motion. Seth *et al*. (1982) analyzed this problem when the lower plate of the channel moves with time dependent velocity $U(t)$ and the upper plate is kept fixed. They considered two particular cases of interest of the problem, namely, (i) impulsive movement of the plate and (ii) uniformly accelerated movement of the plate. In all these investigations, the channel walls are considered non-porous. However, the study of such fluid flow problem in porous channel may find applications in petroleum, mineral and metallurgical industries, designing of cooling systems with the liquid metals, MHD generators, MHD pumps, MHD accelerators and flow meters, geothermal reservoirs and underground energy transport etc. Taking into account this fact Muhuri (1963), Prasad Rao *et al.* (1982), Bhaskara Reddy and Bathaiah (1982), Singh (2004), Abbas *et al.* (2006) and (Hayat *et al*. 2007, 2008) considered MHD flow within a parallel plate channel with porous boundaries, under different conditions, in non-rotating/rotating system.

The objective of the present paper is to study unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system in the presence of a uniform transverse magnetic field. The plates of the channel are considered porous and fluid flow within the channel is induced due to the impulsive movement of the upper plate. The general solution of the governing equations is obtained by using the method mentioned by Batchelor (1967). However, the solution obtained by this method converges slowly for small values of time *t*. For small values of time *t*, the solution of the governing equations is obtained by Laplace transform technique (Carslaw and Jaeger 1959). The expression for the shear stress at the stationary plate due to the primary and secondary flows is obtained in both the cases. To study the effects of rotation, magnetic field, time and suction/injection on the flow field, the primary and secondary velocities and shear stress at the stationary plate due to the primary and secondary flows are depicted graphically for various values of K^2, M^2, t and *S*.

2. FORMULATION OF THE PROBLEM

Consider unsteady flow of a viscous incompressible electrically conducting fluid between two parallel porous plates of infinite length distant h apart in the presence of a uniform transverse magnetic field B_0 applied parallel to $z'-$ axis which is normal to the planes of the plates. The fluid as well as plates of the channel are in a state of rigid body rotation about $z'-$ axis with uniform angular velocity Ω. Initially (i.e. when time $t' \le 0$), fluid as well as plates of the channel are assumed to be at rest. When time $t' > 0$ the upper plate $(z' = h)$ starts moving with uniform velocity U_0 along $x'-$ direction in its own plane while the lower plate $(z' = 0)$ is kept fixed. Since plates of the channel are infinite along x' and y' directions and are electrically non-conducting all physical quantities, except pressure, will be functions of z' and t' only. Suction/injection of the fluid takes place through the porous walls of the channel with uniform velocity W_0 which is greater than zero for suction and is less than zero for injection. It is assumed that no applied or polarization voltages exist. This corresponds to the case where no energy is being added or extracted from the fluid by electrical means (Meyer 1958). In general, the electric current flowing in the fluid gives rise to an induced magnetic field which perturbs the applied magnetic field. Since magnetic Reynolds number is very small for metallic liquids and partially ionized fluids so the induced magnetic field may be neglected in comparison to the applied one. This is the well known low magnetic Reynolds number approximation (Cramer and Pai 1973).

Under the above assumptions fluid velocity $\vec{q}$ and magnetic field $\vec{B}$ are given by

$$\vec{q} \equiv (u', v', -W_0), \quad \vec{B} = (0, 0, B_0). \tag{1}$$

Following the studies made by Seth *et al*. (1982, 1988), Chandran *et al*. (1993), Singh *et al*. (1994), Singh (2000) and Hayat *et al* (2004) the governing equations for the flow of a viscous incompressible electrically conducting fluid in a rotating frame of reference are

$$\frac{\partial u'}{\partial t'} - W_0 \frac{\partial u'}{\partial z'} - 2\Omega v' = \upsilon \frac{\partial^2 u'}{\partial z'^2} - \frac{\sigma B_0^{\,2}}{\rho} u', \tag{2}$$

$$\frac{\partial v'}{\partial t'} - W_0 \frac{\partial v'}{\partial z'} + 2\Omega u' = \upsilon \frac{\partial^2 v'}{\partial z'^2} - \frac{\sigma B_0^{\,2}}{\rho} v', \tag{3}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z'}. \tag{4}$$

Equation (4) shows the constancy of pressure along the axis of rotation. The absence of pressure gradient term in Eq. (3) implies that there is a net cross flow in $y'-$ direction. The fluid motion is induced due to the movement of the upper plate in $x'-$ direction, so the pressure gradient term is not taken into account in Eq. (2).

The initial and boundary conditions for the problem are

$$\left.\begin{array}{ll} u' = 0,\ v' = 0\ ; & 0 \le z' \le h \ and\ t' \le 0, \\ u' = 0,\ v' = 0 & at\ z' = 0\ ;\ t' > 0, \\ u' = U_0,\ v' = 0 & at\ z' = h\ ;\ t' > 0. \end{array}\right\} \tag{5}$$

Introducing the non-dimensional variables

$$z = z'/h,\ u = u'/U_0,\ v = v'/U_0 \ and\ t = t'\upsilon/h^2, \tag{6}$$

the Eqs. (2) and (3), in non-dimensional form, become

$$\frac{\partial u}{\partial t} - S\frac{\partial u}{\partial z} - 2K^2 v = \frac{\partial^2 u}{\partial z^2} - M^2 u, \tag{7}$$

$$\frac{\partial v}{\partial t} - S\frac{\partial v}{\partial z} + 2K^2 u = \frac{\partial^2 v}{\partial z^2} - M^2 v, \tag{8}$$

where $S = W_0 h/\upsilon$ is suction/injection parameter $(S > 0$ for suction and $S < 0$ for injection), $M^2 = \sigma B_0^{\,2} h^2/\rho\upsilon$ is magnetic parameter which is the square of Hartmann number and $K^2 = \Omega h^2/\upsilon$ is rotation parameter which is reciprocal of Ekman number.

The initial and boundary conditions (5), with the help of (6), yield

$$\left.\begin{array}{ll} u=0,\ v=0\ ; & 0\le z\le 1\ and\ t\le 0,\\ u=0,\ v=0 & at\ z=0\ ;\ t>0,\\ u=1,\ v=0 & at\ z=1\ ;\ t>0.\end{array}\right\}, \tag{9}$$

Combining Eqs. (7) and (8), we obtain

$$\frac{\partial q}{\partial t}-S\frac{\partial q}{\partial z}=\frac{\partial^2 q}{\partial z^2}-\left(M^2+2iK^2\right)q, \tag{10}$$

where $q=u+iv$, and $i=\sqrt{-1}.$

The initial and boundary conditions (9) become

$$\left.\begin{array}{ll} q=0; & 0\le z\le 1,\ t\le 0,\\ q=0, & at\ \ z=0\ ;\ t>0,\\ q=1, & at\ \ z=1\ ;\ t>0.\end{array}\right\} \tag{11}$$

3. SOLUTION OF THE PROBLEM

Following Batchelor (1967) the solution of Eq. (10) subject to the conditions (11) can be written in the form

$$q(z,t)=\frac{\sinh(\alpha+i\beta)z}{\sinh(\alpha+i\beta)}+F_1(z,t), \tag{12}$$

where

$$\alpha=\frac{1}{2\sqrt{2}}\left\{\left[\sqrt{\left(S^2+4M^2\right)^2+64K^4}+\left(S^2+4M^2\right)\right]^{1/2}-\right.$$
$$\left.-\sqrt{2}S\right\}, \tag{13a}$$

$$\beta=\frac{1}{2\sqrt{2}}\left[\sqrt{\left(S^2+4M^2\right)^2+64K^4}-\left(S^2+4M^2\right)\right]^{1/2}. \tag{13b}$$

The first term on the right-hand side of (12) is the steady state solution while $F_1(z,t)$ represents transient solution of Eq. (10).

Now $F_1(z,t)$ satisfies the following differential equation:

$$\frac{\partial F_1}{\partial t}-S\frac{\partial F_1}{\partial z}+\left(M^2+2iK^2\right)F_1=\frac{\partial^2 F_1}{\partial z^2}, \tag{14}$$

with the conditions

$$F_1(0,t)=0,\ F_1(1,t)=0,\ F_1(z,0)=-\frac{\sinh(\alpha+i\beta)z}{\sinh(\alpha+i\beta)}. \tag{15}$$

The solution of Eq. (14) subject to the conditions (15) is given by

$$F_1(z,t)=\sum_{n=1}^{\infty}A_n e^{-\lambda_n^2 t}\sin n\pi z, \tag{16}$$

where

$$\lambda_n^2=n^2\pi^2+(\alpha+i\beta)^2 \tag{17a}$$

and

$$A_n=-2\int_0^1\frac{\sinh(\alpha+i\beta)z}{\sinh(\alpha+i\beta)}\sin n\pi z dz. \tag{17b}$$

Making use of (16) and (17) in (12), the fluid velocity is given by

$$q(z,t)=\frac{\sinh(\alpha+i\beta)z}{\sinh(\alpha+i\beta)}+$$
$$+2\sum_{n=1}^{\infty}\frac{n\pi(-1)^n e^{-\lambda_n^2 t}}{n^2\pi^2+(\alpha+i\beta)^2}\sin n\pi z. \tag{18}$$

Separating real and imaginary parts in Eq. (18), we obtain fluid velocity for the primary and secondary flows as

$$u=\frac{\phi_1(z)\phi_2+\phi_3(z)\phi_4}{\phi_2^2+\phi_4^2}+$$
$$+2\sum_{n=1}^{\infty}\frac{n\pi(-1)^n e^{-\left(n^2\pi^2+\alpha^2-\beta^2\right)t}\sin n\pi z}{\left(n^2\pi^2+\alpha^2-\beta^2\right)^2+4\alpha^2\beta^2}\times$$
$$\times\left\{\left(n^2\pi^2+\alpha^2-\beta^2\right)\cos 2\alpha\beta t-2\alpha\beta\sin 2\alpha\beta t\right\}, \tag{19}$$

$$v=\frac{\phi_3(z)\phi_2-\phi_1(z)\phi_4}{\phi_2^2+\phi_4^2}-$$
$$-2\sum_{n=1}^{\infty}\frac{n\pi(-1)^n e^{-\left(n^2\pi^2+\alpha^2-\beta^2\right)t}\sin n\pi z}{\left(n^2\pi^2+\alpha^2-\beta^2\right)^2+4\alpha^2\beta^2}\times$$
$$\times\left\{2\alpha\beta\cos 2\alpha\beta t+\left(n^2\pi^2+\alpha^2-\beta^2\right)\sin 2\alpha\beta t\right\}, \tag{20}$$

where

$$\phi_1(z)=\sinh\alpha z\cos\beta z,\ \phi_3(z)=\cosh\alpha z\sin\beta z,$$
$$\phi_2=\sinh\alpha\cos\beta,\ \phi_4=\cosh\alpha\sin\beta.$$

It is evident from the solutions (19) and (20) that the transient effects die out as time $t\to\infty$ and the ultimate steady state is reached. The steady state primary and secondary fluid velocities assume the form

$$u=\frac{\phi_1(z)\phi_2+\phi_3(z)\phi_4}{\phi_2^2+\phi_4^2}, \tag{21}$$

$$v=\frac{\phi_3(z)\phi_2-\phi_1(z)\phi_4}{\phi_2^2+\phi_4^2}. \tag{22}$$

For large values of rotation parameter K^2 boundary layer type flow is expected. For the boundary layer flow near the moving plate $z=1$, introducing boundary layer coordinate $\xi=1-z$, we obtain primary and secondary velocities from (21) and (22) as

$$u=e^{-\alpha_1\xi}\cos\beta_1\xi,\ v=e^{-\alpha_1\xi}\sin\beta_1\xi, \tag{23}$$

where

$$\alpha_1=-\frac{S}{2}+K\left\{1+\frac{M^2}{4K^2}+\frac{S^2}{16K^2}\right\}, \tag{24a}$$

$$\beta_1 = K\left\{1-\frac{M^2}{4K^2}-\frac{S^2}{16K^2}\right\}. \quad (24b)$$

Solution (23) reveals that there arises a thin boundary layer of thickness $O\left(\alpha_1^{-1}\right)$ near the moving plate which may be identified as modified Ekman boundary layer and can be viewed as classical Ekman boundary layer modified by magnetic field and suction/injection. The exponential terms in Eq. (23) die out quickly as ξ increases. When $\xi \ge 1/\alpha_1$, i.e. outside the boundary layer region, the primary and secondary velocities assume the form $u \approx 0$ and $v \approx 0$. Thus we conclude that in a rapidly rotating system, the fluid flow is confined to the boundary layer region only.

Like the case of large values of rotation parameter K^2, we can expect boundary layer type flow near the moving plate for large values of magnetic parameter M^2, In this case the velocity distributions are

$$u = e^{-\alpha_2\xi}\cos\beta_2\xi, \quad v = e^{-\alpha_2\xi}\sin\beta_2\xi, \quad (25)$$

Where

$$\alpha_2 = -\frac{S}{2}+M\left\{1+\frac{S^2}{8M^2}\right\}, \;\; \beta_2 = \frac{K^2}{M}. \quad (26)$$

The expression (25) demonstrates the existence of a thin boundary layer of thickness $O\left(\alpha_2^{-1}\right)$ adjacent to the moving plate which may be recognized as modified Hartmann boundary layer and can be viewed as classical Hartmann boundary layer modified by suction/injection. Also in this case fluid flow is confined to the boundary layer region only which extends up to the thickness $O\left(\alpha_2^{-1}\right)$.

3.1 Shear Stress at the Stationary Plate

Non-dimensional shear stress at the stationary plate $z = 0$ due to the primary and secondary flows in the case of general solution is given by

$$\left(\tau_{x0}+i\tau_{y0}\right) = \frac{(\alpha+i\beta)}{\sinh(\alpha+i\beta)} + 2\sum_{n=1}^{\infty}\frac{n^2\pi^2(-1)^n e^{-\left(n^2\pi^2+(\alpha+i\beta)^2\right)t}}{\left(n^2\pi^2+(\alpha+i\beta)^2\right)}, \quad (27)$$

On separating real and imaginary parts in Eq. (27), the shear stress components τ_{x0} and τ_{y0} due to the primary and secondary flow respectively, are

$$\tau_{x0} = \frac{2(\alpha\sinh\alpha\cos\beta+\beta\cosh\alpha\sin\beta)}{\cosh 2\alpha-\cos 2\beta} + 2\sum_{n=1}^{\infty}\frac{n^2\pi^2(-1)^n e^{-\left(n^2\pi^2+\alpha^2-\beta^2\right)t}}{\left(n^2\pi^2+\alpha^2-\beta^2\right)^2+4\alpha^2\beta^2}\times \times\left\{\left(n^2\pi^2+\alpha^2-\beta^2\right)\cos 2\alpha\beta t-2\alpha\beta\sin 2\alpha\beta t\right\}, \quad (28)$$

$$\tau_{y0} = \frac{2(\beta\sinh\alpha\cos\beta-\alpha\cosh\alpha\sin\beta)}{\cosh 2\alpha-\cos 2\beta} - 2\sum_{n=1}^{\infty}\frac{n^2\pi^2(-1)^n e^{-\left(n^2\pi^2+\alpha^2-\beta^2\right)t}}{\left(n^2\pi^2+\alpha^2-\beta^2\right)^2+4\alpha^2\beta^2}\times \times\left\{\left(n^2\pi^2+\alpha^2-\beta^2\right)\sin 2\alpha\beta t+2\alpha\beta\cos 2\alpha\beta t\right\}, \quad (29)$$

The general solution, given by (19) and (20) for the fluid velocity is valid for every value of time *t*. But it converges slowly for small values of time $t\,(i.e.\; t \ll 1)$ (Batchelor 1967). In the absence of suction/injection $(i.e.\; S = 0)$ the general solution (19) and (20) reduces to the solution obtained by Das *et al*. (2009).

For small values of time *t*, following Carslaw and Jaegar (1959) the solution of Eq. (10) subject to the initial and boundary conditions (11) is obtained by Laplace transform technique and is represented in the following form

$$q(z,t) = F(z,t)e^{-\left(M^2+2iK^2\right)t}, \quad (30)$$

where

$$F(z,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[\left(M^2+2iK^2\right)^n(4t)^n\times \times\left\{e^{aS/2}\left(i^{2n}erfc\left(\frac{a}{2\sqrt{t}}\right)-\frac{aS^2\sqrt{t}}{4}i^{2n+1}erfc\left(\frac{a}{2\sqrt{t}}\right)\right)- -e^{bS/2}\left(i^{2n}erfc\left(\frac{b}{2\sqrt{t}}\right)-\frac{bS^2\sqrt{t}}{4}i^{2n+1}erfc\left(\frac{b}{2\sqrt{t}}\right)\right)\right\}\right], \quad (31)$$

where

$$a = 2m+1-z, \;\; b = 2m+1+z\,, \quad (32a)$$

$$i^n erfc(x) = \int_x^{\infty} i^{n-1}erfc(\xi)\,d\xi, \quad (32b)$$

$$ierfc(x) = \int_x^{\infty} erfc(\xi)\,d\xi, \quad (32c)$$

$$i^0 erfc(x) = erfc(x). \quad (32d)$$

The solution (30) may be written as

$$q(z,t) = e^{-\left(M^2+2iK^2\right)t}\sum_{n=0}^{\infty}\left(M^2+2iK^2\right)^n(4t)^n T_r, \quad (33)$$

$$r = 0,\; 2,\; 4,\; 6,\ldots,$$

where

$$T_r = \sum_{m=0}^{\infty}\left[e^{aS/2}\left(i^r erfc\left(\frac{a}{2\sqrt{t}}\right)-\frac{aS^2\sqrt{t}}{4}i^{r+1}erfc\left(\frac{a}{2\sqrt{t}}\right)\right)- -e^{bS/2}\left(i^r erfc\left(\frac{b}{2\sqrt{t}}\right)-\frac{bS^2\sqrt{t}}{4}i^{r+1}erfc\left(\frac{b}{2\sqrt{t}}\right)\right)\right]. \quad (34)$$

On separating real and imaginary parts in Eq. (33), we obtain fluid velocity for the primary and secondary flows as

$$u = e^{-M^2 t}\Big\{\Big[T_0 + M^2(4t)T_2 + \left(M^4 - 4K^4\right)(4t)^2 T_4 + \\ + \left(M^6 - 12M^2K^4\right)(4t)^3 T_6 + ...\Big]\cos 2K^2 t + \\ + \Big[\left(2K^2\right)(4t)T_2 + 4M^2K^2(4t)^2 T_4 + \\ + \left(6M^4K^2 - 8K^6\right)(4t)^3 T_6 + ...\Big]\sin 2K^2 t\Big\}, \tag{35}$$

$$v = e^{-M^2 t}\Big\{\Big[\left(2K^2\right)(4t)T_2 + 4M^2K^2(4t)^2 T_4 + \\ + \left(6M^4K^2 - 8K^6\right)(4t)^3 T_6 + ...\Big]\cos 2K^2 t - \\ - \Big[T_0 + M^2(4t)T_2 + \left(M^4 - 4K^4\right)(4t)^2 T_4 + \\ + \left(M^6 - 12M^2K^4\right)(4t)^3 T_6 + ...\Big]\sin 2K^2 t\Big\}. \tag{36}$$

Equations (35) and (36) describe the fluid velocities for small values of time *t*. In the absence of suction/injection $(S = 0)$ Eqs. (35) and (36) are in agreement with the solution obtained by Das *et al.* (2009).

3.2 Shear Stress at the Stationary Plate

Non-dimensional shear stress at the stationary plate $z = 0$ due to the primary and secondary flows in the case of solution for small values of time *t* is given by

$$\left(\tau_{x0} + i\tau_{y0}\right) = e^{-\left(M^2 + 2iK^2\right)t}\sum_{n=0}^{\infty}\left(M^2 + 2iK^2\right)^n (4t)^n Y_{2r-1} \tag{37}$$
$$r = 0,1,2,3,...,$$

where

$$Y_{2r-1} = \sum_{m=0}^{\infty}\Bigg[e^{aS/2}\left\{\frac{1}{2\sqrt{t}} i^{2r-1}erfc\left(\frac{a}{2\sqrt{t}}\right) - \frac{S}{2}\left(1 + \frac{aS}{4}\right)\times \\ \times i^{2r}erfc\left(\frac{a}{2\sqrt{t}}\right) + \frac{S^2\sqrt{t}}{4}\left(1 + \frac{aS}{4}\right) i^{2r+1}erfc\left(\frac{a}{2\sqrt{t}}\right)\right\} + \\ + e^{bS/2}\left\{\frac{1}{2\sqrt{t}} i^{2r-1}erfc\left(\frac{b}{2\sqrt{t}}\right) - \frac{S}{2}\left(1 + \frac{bS}{4}\right)\times \\ \times i^{2r}erfc\left(\frac{b}{2\sqrt{t}}\right) + \frac{S^2\sqrt{t}}{4}\left(1 + \frac{bS}{4}\right) i^{2r+1}erfc\left(\frac{b}{2\sqrt{t}}\right)\right\}\Bigg]. \tag{38}$$

On separating real and imaginary parts in Eq. (37), the shear stress components τ_{x0} and τ_{y0} due to the primary and secondary flow respectively are

$$\tau_{x0} = e^{-M^2 t}\Big\{\Big[Y_{-1} + M^2(4t)Y_1 + \left(M^4 - 4K^4\right)(4t)^2 Y_3 + \\ + \left(M^6 - 12M^2K^4\right)(4t)^3 Y_5 + ...\Big]\cos 2K^2 t + \\ + \Big[\left(2K^2\right)(4t)Y_1 + 4M^2K^2(4t)^2 Y_3 + \\ + \left(6M^4K^2 - 8K^6\right)(4t)^3 Y_5 + ...\Big]\sin 2K^2 t\Big\}, \tag{39}$$

$$\tau_{y0} = e^{-M^2 t}\Big\{\Big[\left(2K^2\right)(4t)Y_1 + 4M^2K^2(4t)^2 Y_3 + \\ + \left(6M^4K^2 - 8K^6\right)(4t)^3 Y_5 + ...\Big]\cos 2K^2 t - \\ - \Big[Y_{-1} + M^2(4t)Y_1 + \left(M^4 - 4K^4\right)(4t)^2 Y_3 + \\ + \left(M^6 - 12M^2K^4\right)(4t)^3 Y_5 + ...\Big]\sin 2K^2 t\Big\}. \tag{40}$$

4. RESULTS AND DISCUSSION

To study the effects of rotation, magnetic field, suction/injection and time on the flow-field velocity profiles are drawn versus *z* for various values of rotation parameter K^2, magnetic parameter M^2, suction/injection parameter *S* and time *t* in Figs. 1 to 6 while numerical values of non-dimensional shear stress components τ_{x0} and τ_{y0} at the stationary plate *z*=0 are depicted graphically for different values of K^2, M^2, *S* and *t* in Figs. 7 to 10. It is revealed from Figs. 1 and 2 that the primary velocity *u* decreases while secondary velocity *v* increases on increasing K^2. This is justified due to the fact that the Coriolis force induces secondary flow. Also there exists incipient flow reversal near the stationary plate in the primary flow direction on increasing K^2. Both the primary velocity *u* and secondary velocity v decrease on increasing M^2. This is expected because magnetic field tends to retard the fluid velocity. It is evident from Figs. 3 and 4 that, on increasing *S*, primary velocity *u* increases while secondary velocity *v* decreases in the case of suction where as both the velocities decrease in the case of injection. It is observed from Figs. 5 and 6 that both the primary and secondary velocities increase on increasing *t* in the case of general solution as well as in the case of solution for small values of time *t*. It is also noticed from Figs. 5 and 6 that when time *t* is very small, the solution for small values of time *t* obtained by Laplace transform technique converges more rapidly than that of the general solution. This is in agreement with the statement made by Batchelor (1967).

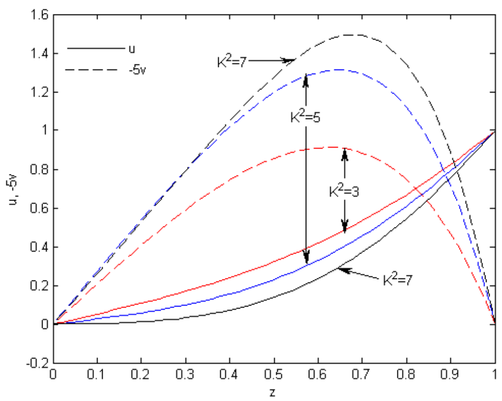

Fig. 1 Primary and secondary velocity profiles when $M^2 = 4,\ S = 1$ and $t = 1$.

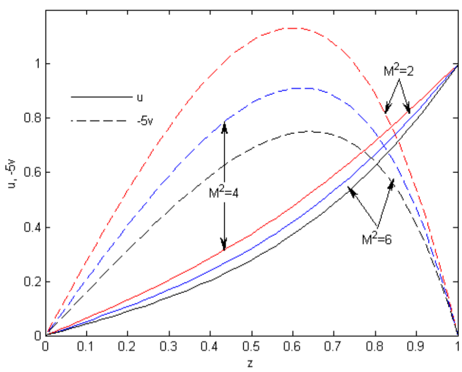

Fig. 2 Primary and secondary velocity profiles when $K^2 = 3,\ S = 1$ and $t = 1$.

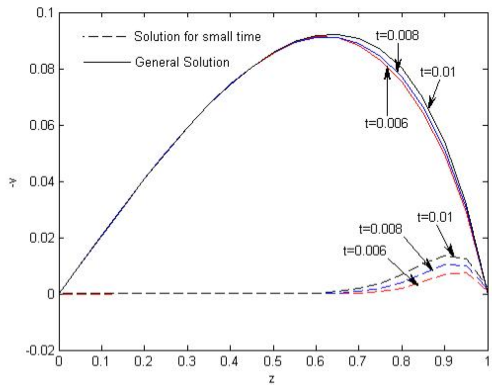

Fig. 6 Secondary velocity profiles when $K^2 = 3,\ M^2 = 4$ and $S = 1$.

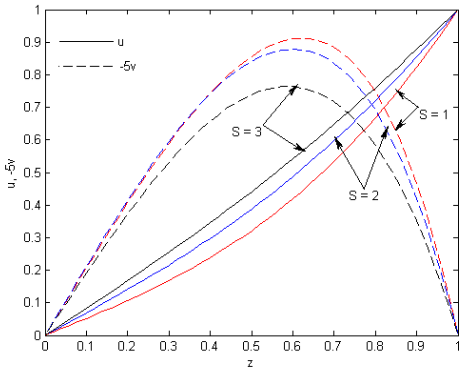

Fig. 3 Primary and secondary velocity profiles when $K^2 = 3,\ M^2 = 4$ and $t = 1$.

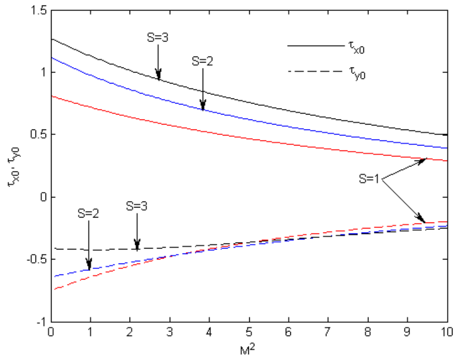

Fig. 7 Profiles of Primary and secondary shear components when $K^2 = 3$ and $t = 1$.

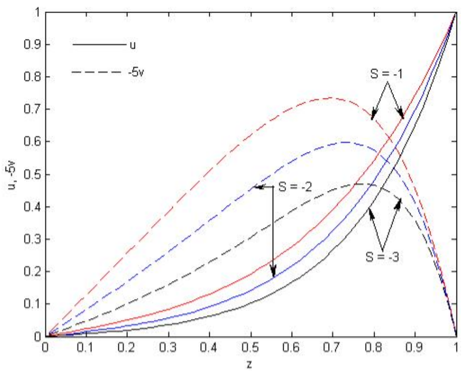

Fig. 4 Primary and secondary velocity profiles when $K^2 = 3,\ M^2 = 4$ and $t = 1$.

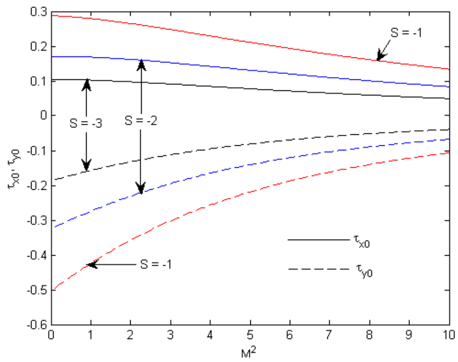

Fig. 8 Profiles of Primary and secondary shear components when $K^2 = 3$ and $t = 1$.

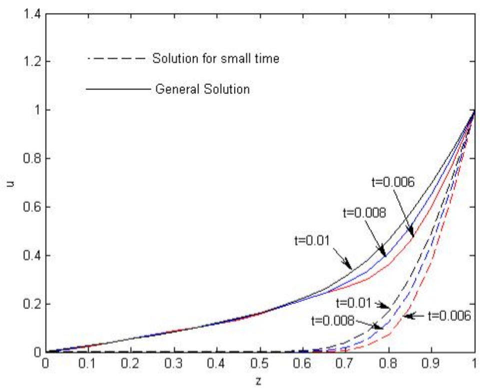

Fig 5. Primary velocity profiles when $K^2 = 3,\ M^2 = 4$ and $S = 1$.

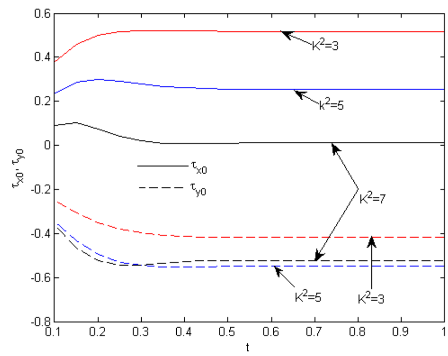

Fig. 9 Profiles of Primary and secondary shear components when $M^2 = 4$ and S=1.

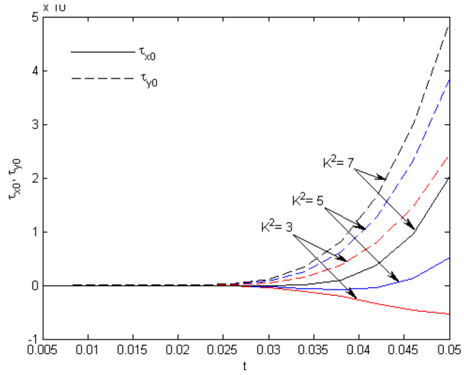

Fig. 10 Profiles of Primary and secondary shear components when $M^2 = 4$ and $S = 1$.

It is observed from Figs. 7 and 8 that the primary shear stress τ_{x0} decreases whereas secondary shear stress τ_{y0} increases on increasing M^2 in the case of both suction/injection. On increasing *S*, the primary shear stress τ_{x0} increases in the case of suction whereas it decreases in the case of injection. The secondary shear stress τ_{y0} decreases with the increase in *S* throughout the channel in the case of suction whereas in the case of injection it decreases with the increase in *S* when $M^2<3$ and its characteristics are changed on increasing *S* when $M^2>3$. It is noticed from Figs. 9 and 10 that τ_{x0} decreases with the increase in K^2 when $t<0.1$ or $t\geq0.1$ whereas τ_{y0} increases with the increase in K^2 when $t<0.3$ and thereafter its characteristics are changed. When $t<0.1$, τ_{y0} increases on increasing t whereas τ_{x0} increases on increasing t except when $K^2=3$. This is due to the fact that there exists incipient flow reversal near the stationary plate on increasing K^2. When $t\geq0.1$, both the primary and secondary shear stress components first increase, attain a maximum and then decrease with the increase in time *t*.

5. CONCLUSION

The effects of rotation and magnetic field on unsteady Couette flow of a viscous incompressible electrically conducting fluid between two horizontal parallel porous plates in a rotating medium is investigated. It is found that magnetic field has tendency to retard the fluid flow in both the primary and secondary flow directions. Rotation retards primary flow whereas it accelerates secondary flow. Also there exists incipient flow reversal near the stationary plate in primary flow direction on increasing rotation parameter K^2. Suction accelerates primary flow whereas it retards secondary flow. Injection retards both the primary and secondary flows. Fluid flow in both the primary and secondary flow directions increases on increasing time *t* and the solution for small values of time *t*, obtained by Laplace transform technique, converges more rapidly than that of general solution when time *t* is very small.

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