Unsteady MHD Radiative and Chemically Reactive Free Convection Flow near a Moving Vertical Plate in Porous Medium

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ABSTRACT

We have investigated an unsteady flow of a viscous, incompressible electrically conducting, laminar free convection boundary layer flow of a moving infinite vertical plate in a radiative and chemically reactive medium in the presence of a transverse magnetic field. The equations governing the flow are solved by Laplace transform technique. The expressions for velocity, temperature, concentration are derived and based on these quantities the expressions for skin friction; rate of heat transfer and the rate mass transfer near the plate are derived. The effects of various physical parameters on flow quantities, magnetic parameter, Grashof number, modified Grashof number, heat source parameter, the chemical reaction parameter, Schmidt number and radiation parameter are studied numerically and the results are discussed with the help of graphs. Some important applications of physical interest for different type motion of the plate like case (i) when the plate is moving with uniform velocity, case (ii) when the plate is moving with single acceleration and case (iii) when the plate is moving with periodic acceleration, are discussed.

Keywords: Heat source or sink, Chemical reaction, MHD, Radiation, Porous medium, Heat and mass transfer.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>B₀</td>
<td>external magnetic field</td>
</tr>
<tr>
<td>C</td>
<td>dimensionless concentration</td>
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<tr>
<td>Cₛ</td>
<td>specifies concentration in the fluid</td>
</tr>
<tr>
<td>Cᵤ</td>
<td>specific heat at constant pressure</td>
</tr>
<tr>
<td>Cᵤₛ</td>
<td>fluid concentration near the plate</td>
</tr>
<tr>
<td>Cₓ infinity</td>
<td>fluid concentration far away from the plate</td>
</tr>
<tr>
<td>D</td>
<td>chemical molecular diffusivity</td>
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<tr>
<td>erf</td>
<td>error function</td>
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<tr>
<td>erf c</td>
<td>complementary error function</td>
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<tr>
<td>F</td>
<td>radiation parameter</td>
</tr>
<tr>
<td>Gₘ</td>
<td>mass Grashof number</td>
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<td>Gₜ</td>
<td>thermal Grashof number</td>
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<tr>
<td>g</td>
<td>acceleration due to gravity</td>
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<td>H</td>
<td>heat absorption parameter</td>
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<tr>
<td>k</td>
<td>non-dimensional permeability coefficient of a porous medium</td>
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<tr>
<td>kₛ</td>
<td>non-dimensional rate of chemical reaction</td>
</tr>
<tr>
<td>kₚ</td>
<td>permeability of porous medium</td>
</tr>
<tr>
<td>Kₛ</td>
<td>rate of chemical reaction</td>
</tr>
<tr>
<td>Sₛ</td>
<td>Schmidt number</td>
</tr>
<tr>
<td>Sₒ</td>
<td>Soret number</td>
</tr>
<tr>
<td>T</td>
<td>fluid temperature</td>
</tr>
<tr>
<td>Tᵤ</td>
<td>temperature of the plate</td>
</tr>
<tr>
<td>Tᵤₛ</td>
<td>temperature of the fluid far away from the plate</td>
</tr>
<tr>
<td>t</td>
<td>dimensionless time</td>
</tr>
<tr>
<td>tₛ</td>
<td>time</td>
</tr>
<tr>
<td>u, v</td>
<td>non-dimensional velocity</td>
</tr>
<tr>
<td>y</td>
<td>dimensionless co-ordinate axis normal to the plate</td>
</tr>
<tr>
<td>yₛ</td>
<td>co-ordinate axis normal to the plate</td>
</tr>
<tr>
<td>βₗ</td>
<td>coefficient of volume expansion</td>
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<tr>
<td>βₛ</td>
<td>coefficient of volume expansion with concentration</td>
</tr>
<tr>
<td>K</td>
<td>thermal conductivity of the fluid</td>
</tr>
<tr>
<td>μ</td>
<td>dynamic viscosity</td>
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<tr>
<td>ν</td>
<td>kinematic viscosity</td>
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1. INTRODUCTION

MHD free convection flows occur frequently in nature. Flows of fluids through porous medium are of principal interest now a days and have been attracting the attention of many researchers due to their applications in the fast growing fields of science and technology viz. in the fields of agricultural engineering to study the ground water resources, in petroleum technology to study the moment of natural gas, oil, and water through the oil reservoirs. The applications of hydro magnetic incompressible viscous flow in science and engineering involving heat and mass transfer under the influence of chemical reaction is of great importance to many areas of science and engineering. This frequently occurs in petro-chemical industry, power and cooling systems, chemical vapor deposition on surfaces, cooling of nuclear reactors, heat exchanger design, forest fire dynamics and geophysics as well as in magnetohydrodynamic power generation systems. Heat transfer by thermal radiation is becoming of greater importance when we are considered with space applications, higher operating temperature and also power engineering. Moreover, considerable interest has been evinced in radiation interaction with convection and chemical reaction for heat and mass transfer in fluids. This is due to the significant role of thermal radiation in the surface heat transfer when convection heat transfer is small, particularly, in free convection problems involving absorbing emitting fluids. Das and Das (2009) studied MHD free convection near a moving a vertical plate in the presence of thermal radiation. Rebhi et al. (2009) have investigated combined effect of heat generation or absorption and first-order chemical reaction on micro polar fluid flows over a uniformly stretched permeable surface. Ibrahim and Makinde (2010) considered chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction. Rajesh and Varma (2009) investigated radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature. Recently, Chandrakala (2011) studied radiation effects on flow past an impulsively started vertical oscillating plate with uniform heat flux. Deka and Neog (2009) considered unsteady MHD flow past a vertical oscillating plate with thermal radiation and variable mass diffusion. Prasad et al. (2010) studied mass transfer effects on unsteady MHD free convection flow past a vertical porous plate embedded in a porous medium. Srinivas and Muthuraj (2010) considered Effects of thermal radiation and space porosity on MHD mixed convection flow in a vertical channel using homotopy analysis method. Again, Srinivas and Muthuraj (2010) considered a MHD flow with slip effects and temperature-dependent heat source in a vertical wavy porous space. Unsteady MHD free convection flow and mass transfer near a moving a vertical plate in the presence of thermal radiation is investigated by Abzal et al. (2011). Sharma and Singh (2009) have studied the effects of variable thermal conductivity and heat source/sink on MHD flow near a stagnation point on a linearly stretching sheet. Effects of varying viscosity and thermal conductivity on steady free convective flow and heat transfer along an isothermal vertical plate in the presence of heat sink was considered by Mahanti and Gaur (2009). Recently variable heat and mass transfer, radiation and heat source or sink effects are considered by Chamkha and Ahmed (2011), Seth et al. (2011), Das et al. (2011), Singk et al. (2011) and Raju et al. (2011).

The present study is an analytical solution of combined effect of heat generation or absorption and homogeneous chemical reaction on unsteady MHD free convection flow near a moving vertical plate through a porous medium in presence of thermal radiation. A general exact solution for the partial differential equations governing the flow is obtained with the aid of the usual Laplace transform technique. Also the applications of general solution for the important cases of the flow are discussed.

2. FORMULATION OF THE PROBLEM

We consider an unsteady free convection flow of a viscous incompressible electrically conducting and radiating fluid along an infinite non conducting vertical flat plate or (surface) in the presence of Magnetic field $\vec{B}_0$ applied in the direction of the flow. On this plate, an arbitrary point has been chosen as the origin of a co-ordinate system with $\vec{X}$ -axis along the plate in the upward direction and the $\vec{Y}$ -axis normal to the plate. Initially for time $t' \leq 0$, the plate and the fluids are at the same constant temperature $T_\infty$ in a stationary condition. Subsequently ($t' > 0$), the plate is assumed to be accelerating with a velocity $U(t')$ in its own plane along the $\vec{X}$ -axis; instantaneously, the temperature of the plate is raised to $T_\infty'$, which is here after regarded as constant. For free convection flows, here so the physical variables are functions of the space co-ordinate $y$ and time $t'$ only. Under the above assumptions, the fully developed flow of a radiating gas is governed by the following set of equations.

\[
\begin{align*}
\frac{\partial u^*}{\partial t'} &= v \frac{\partial^2 u^*}{\partial y^2} + \frac{\sigma B_0^2}{\rho_0} (\theta^* - T_\infty) \\
&+ g B_0 \left(C_\gamma - C_0\right) - \frac{\sigma B_0^2}{\rho(1 + n^2)} u^* - \frac{\partial}{\partial y} u^* \\
\end{align*}
\]

(1)
\[ \frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} - Gr \theta - Gm C - M U = 0 \]  

(7)

where

\[ M_1 = M + \frac{1}{K} \]

(8)

The corresponding initial and boundary conditions in non dimensional form are

\[ u = 0, \theta = 0, C = 0 \text{ for all } y^* \geq 0, t^* > 0 \]

\[ u = f(\gamma), \theta = 1, C = 1 \text{ at } y = 0, t^* > 0 \]  

(10)

3. **Solution of the Problem**

In order to obtain the analytical solution of the system of differential Eq. (7) to Eq. (9) subject to the initial and boundary conditions Eq. (10), we shall use the Laplace transformation technique. Thus, the general solution of the present problem for the temperature \( \theta(y, t) \), the velocity \( u(y, t) \) and the concentration \( C(y, t) \) for \( t > 0 \) are given by

\[ \theta(y, t) = \frac{1}{2} \left[ e^{-\eta \sqrt{\text{erfc}} \frac{\eta - \sqrt{M_1 - \eta \gamma}}{\sqrt{\eta}} - \eta \sqrt{\text{erfc}} \frac{\eta - \sqrt{M_1 - \eta \gamma}}{\sqrt{\eta}}} \left( \eta - \sqrt{M_1 - \eta \gamma} \right) + e^{-\eta \sqrt{\text{erfc}} \frac{\eta - \sqrt{M_1 - \eta \gamma}}{\sqrt{\eta}}} \right] (11) \]

\[ \phi(y, t) = L^{-1} \left[ f(x) e^{-\eta x} \right] \]

(13)

\[ \psi(y, t) = \frac{A}{c} (A + B - C + D) + \frac{\eta}{c} (K + F - G + H) \]

(14)

where

\[ A = e^{-\eta \sqrt{M_1 - \eta \gamma}} \text{erfc} \left( \eta - \sqrt{M_1 - \eta \gamma} \right) + \eta \sqrt{\text{erfc}} \frac{\eta - \sqrt{M_1 - \eta \gamma}}{\sqrt{\eta}} \]

\[ B = 1 \left[ \eta \sqrt{F} \sqrt{\text{erfc}} \left( \eta - \sqrt{M_1 - \eta \gamma} \right) + e^{\eta \sqrt{\text{erfc}}} \right] \]

(15)
where \( a = F - M_s, b = -Gr \), \( e = a \) \( \frac{h}{h-1}, d = \frac{h-1}{h-1} = \frac{d}{h-1} \) and \( \sigma = \frac{Gr}{h-1} \).

Since non dimensional temperature \( \theta(y,t) \) is clearly described in Eq. (11) and concentration \( C(y,t) \) is clearly described in Eq. (12), so we shall confine ourselves to non-dimensional velocity \( u(y,t) \) for various types of \( f(t) \).

4. APPLICATION IN ENGINEERING

In this section, we now consider some important cases of flow as given below.

Case I: Motion of the plate with uniform velocity

Let \( f(t) = H(t) \), Heaviside unit step function
\[
\dot{f}(s) = L(f(t)) = \frac{1}{s}.
\]

In this case, we observe that the result of Eq. (11) is an affected and Eq. (13) for \( u(y,t) \) becomes
\[
u(y,t) = \frac{1}{2} \left[ e^{-\sqrt{M_f} \text{erfc} \left( \eta + \sqrt{M_f} \right)} + \psi(y,t) \right]
\]
where \( \psi(y,t) \) is given from Eq. (14).

Case II: Motion of the plate with single acceleration

Let \( f(t) = H(t) \), Heaviside unit step function
\[
\dot{f}(s) = L(f(t)) = \frac{1}{s^2}.
\]

In this case, we observe that the result of Eq. (11) is an affected and Eq. (13) for \( u(y,t) \) becomes
\[
u(y,t) = \frac{1}{2} \left[ \left( t - \frac{1}{2} \right) e^{-\sqrt{M_f} \text{erfc} \left( \eta - \sqrt{M_f} \right)} + \psi(y,t) \right]
\]
where \( \psi(y,t) \) is given from Eq. (14).

Case III: Motion of the plate with periodic acceleration

For this case \( f(t) = H(t) \), Heaviside unit step function
\[
\dot{f}(s) = L(f(t)) = \frac{s}{s^2 + w^2}.
\]

In this case, we observe that the result of Eq. (11) is an affected and expression (10) for \( u(y,t) \) becomes
\[
u(y,t) = \frac{1}{2} \left[ e^{-M_f \sqrt{M_f} \text{erfc} \left( \eta + \sqrt{M_f} \right)} + \psi(y,t) \right]
\]

where \( \psi(y,t) \) is given from Eq. (15).

It should be noted that our results of case (i) are identical with those of Mazumdar and Deka (2007).

5. SKIN FRICTION

Knowing the velocity field, we now study the effect of \( t, Pr, F, M, \) etc. on the skin friction. In the non-dimensional form, it is given by
\[
\tau = \left( \frac{\partial u}{\partial y} \right)_{y=0} = \frac{\sqrt{M_f} \text{erfc} \left( \frac{\sqrt{M_f}}{\sqrt{M_f}} \right) + \psi(y,t)}{1}.
\]

Case (I): When the plate is moving with uniform velocity, then
\[
\left( \frac{\partial \phi}{\partial y} \right)_{y=0} = \frac{\sqrt{M_f} \text{erfc} \left( \frac{\sqrt{M_f}}{\sqrt{M_f}} \right) + \psi(y,t)}{1}.
\]
Case (II): When the plate is moving with single acceleration, then
\[
\left( \frac{\partial \theta}{\partial y} \right)_{y=0} = -\sqrt{M_t} \left( t + \frac{1}{2M_t} \right) e^{\left( \frac{1}{\sqrt{M_t}} \right)} \left( -M_t e^{-M_t} \right) \quad (21)
\]

Case (III): When the plate is moving with periodic acceleration, then
\[
\left( \frac{\partial \theta}{\partial y} \right)_{y=0} = -\sqrt{M_t} \left( 2 \frac{1}{\sqrt{M_t}} e^{\left( \frac{1}{\sqrt{M_t}} \right)} \left( -M_t e^{-M_t} \right) \right) \quad (22)
\]

6. NUSSELT NUMBER
An important phenomenon in this study is to understand the effect of t, F and H on the Nusselt Number. In non-dimensional form, the rate of heat transfer is given by
\[
Nu = \frac{\partial \theta}{\partial y} \bigg|_{y=0} = \sqrt{Pr} \epsilon \left( \frac{Pr}{Pr} \right) + \frac{1}{\sqrt{Pr}} \epsilon \frac{Pr}{Pr} \quad (23)
\]

7. SHERWOOD NUMBER
An important phenomenon in this study is to understand the effect of F, t, and H on the Sherwood Number. In non-dimensional form, the rate of mass transfer is given by
\[
Sh = \left( \frac{\partial C}{\partial y} \right)_{y=0} = \sqrt{Sc} \epsilon \left( \frac{Sc}{Pr} \right) + \frac{1}{\sqrt{Sc}} \epsilon \frac{Sc}{Pr} \quad (24)
\]

8. RESULT AND DISCUSSION
In order to get physical insight into problem, the numerical computations have been carried out and the influence of various physical parameters viz., magnetic parameter M, heat generation / absorption parameter H, time t, Grashof number Gr, modified Grashof number Gm, permeability parameter K, phase angle \( \alpha \), the chemical reaction parameter \( K_c \), Prandtl number Pr, Schmidt number Sc and Radiation parameter F on flow quantities. The values of Prandtl number are chosen Pr = 0.71 (air). The values of Schmidt number are chosen to represent the presence of species by hydrogen (0.22), water vapor (0.60), ammonia (0.78), and Carbon dioxide (1.00). Figure 1(a) depicts the temperature profiles against y for different values of F, t, and H. The magnitude of the temperature is maximum at the plate and then decays to zero asymptotically. From this figure it is noticed that temperature decreases with the increase in F and H where as it shows different effect for t. Figure 1(b) represents the concentration profiles due to the variations in \( K_c \), Sc and t. The concentration decreases with an increase in \( K_c \), Sc and it shows the reverse effect in case of t. further, it is noticed that the concentration is maximum at the surface and falls exponentially.

Figures 2(a) and 2(b) correspond to the plates moving uniform velocity, Figs. 3(a) and 3(b) corresponds to plates moving with single acceleration and Figs. 4(a) and 4(b) correspond to the periodic acceleration. From these figures it is evident that the magnitude of the velocity near the plate exceeds and at the plate the
velocity shoot occurs. An increase in Gm, t, K, Pr and H results in an increase in the velocity. It is due to fact that increases in the values modified Grashof number has the tendency to increase thermal and mass buoyancy effect. This gives rise to an increase in the induced flow. The reverse effect is observed in case of Gr, M and F, it is because of the application transverse magnetic field which results a Lorentz force similar to drag force that tends to resist fluid flow and thus reduces it velocity. Figures 5(a), 5(b) and 5(c) depict skin friction against time t for different values of physical parameters. It is observed that skin friction decreases with an increase in M and F but it shows reverse effect in the case of Gr and Gm. Variations in Nusselt number against t for different values of F and H are presented in Fig. 6. Nusselt number increases with the increase in both F and H. Effect of chemical reaction on Sherwood number is displayed in Fig. 7. From this figure it is observed that Sherwood number increases with the increase in chemical reaction parameter.

Fig. 2. Velocity profiles, when the plate is moving with Uniform velocity in variations of F, t, M, Gr, Gm, K, H and Pr for fixed values of Kc =1, Sc=0.22.

Fig. 3. Velocity profiles, when the plate is moving with Single acceleration in variations of F, t, M, Gr, Gm, K, H, and Pr, for fixed values of Kc =1, Sc=0.22.
Fig. 4. Velocity profiles, when the plate is moving with periodic acceleration in variations of \( \omega t, \ F, \ t, \ M, \ Gm, \ K, \ H, \) and Pr for fixed values of Pr=0.71, Kc =1, Sc=0.22.

Fig. 5. Skin friction for uniform velocity (a),(b) and for single acceleration (c).
In this paper, an analytical solution of combined effect of heat generation or absorption and homogeneous chemical reaction on unsteady MHD free convection flow near a moving vertical plate through a porous medium in presence of thermal radiation. A general exact solution for the partial differential equations governing the flow is obtained with the aid of the usual Laplace transform technique. Also the applications of general solution for the important cases of the flow are discussed. Also a physical example for evolution of numerical values of the velocity, temperature and concentration is considered. From this study the following conclusions are made:

a. The concentration decreases with an increase in \( K_c \) and Sc and it shows the reverse effect in case of \( t \).

b. Temperature decreases with the increase in \( F \) and \( H \) where as it shows different effect for \( t \).

c. Increase in \( G_m \), \( t \), \( K \), \( Pr \) and \( H \) results in an increase in the velocity and the reverse effect is observed in case of \( Gr \), \( M \) and \( F \).

d. Skin friction decreases with an increase in \( M \) and \( F \) but it shows reverse effect in the case of \( G_m \) and \( G_m \).

e. Nusselt number increases with the increase in both \( F \) and \( H \).

f. Sherwood number increases with the increase in chemical reaction parameter.

**REFERENCES**


