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Fluid Simulation of the Ion Temperature Effects on a Collisional Magnetized Sheath of a Dusty Plasma

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ABSTRACT

The properties of magnetized dusty plasma sheath with finite ion temperature are studied using a fluid model. Hot electrons, fluid ions, neutral particles and cold fluid dust grains are taken into account in this system. Considering the cross section for collisions between the dust and neutrals has a power law dependence on the dust flow velocity, the fluid model is then solved numerically to obtain detailed sheath information under different ion temperatures. A significant change is observed in the quantities characterizing the sheath with respect to the cold ion assumption. In addition, the result reveals that the effect of ion temperature is more obvious on the dust dynamics in collisional sheath with constant cross section.

Keywords: Fluid model, Ion temperature, Plasma sheath, Dust grains, Magnetic field.

NOMENCLATURE

m_d	mass of dust grains
c_d	dust sound speed
λ_D	electron Debye length
γ	dimensionless parameter corresponding to dust grain gyro radius
f_g	dimensional parameter corresponding to gravitational force
$\nu(v_d)$	dust collision frequency
δ	density ratio of ions to electrons at the sheath edge.
$\sigma(v_d)$	momentum transfer cross section
c_i	ion sound speed
α	parameter of collision
z_d	dust grain charge number
T_e	electron temperature
T_i	ion temperature
T_{ie}	temperature ratio of ions to electrons
M_d	dust Mach number
M_i	ion Mach number

1. INTRODUCTION

A plasma sheath is an ion reach layer that separates plasma from a material boundary. Its function is to form a potential barrier which confines the more mobile species, i.e. electrons in the plasma and accelerates the less mobile species, i.e. charged positive ions out of the plasma and towards the walls. The basic problem of plasma flowing into a wall is important in many aspects of plasma physics and understanding sheaths is of great interest to several fields ranging from material processing to magnetic confinement fusion. The subject of ordinary plasma sheath (electrons- ions) and dusty plasma sheath (electrons- ions and dust) is not well understood and thus a variety of studies have been put forward to describe the characteristics and aspects of plasma sheath such as (the electron, ion and dust density distribution, ion flow velocity , dynamic of dust and electric potential).

The most of these studies have focused on the effect of parameters that modify the plasma sheath characteristics, for example the action of an external magnetic field and collisions force.

The problem of sheath in a magnetized two components plasma has been previously examined (Chodura 1982; Riemann 1994; Kim *et al*. 1995; Zou *et al*. 2004; Brinkmann 2011). For example, the initial work on the plasma sheath in the presence of oblique magnetic field suggested the existence of magnetic presheath (Chodura layer) near the wall (Chodura 1982). However this model has further been generalized to include the effect of collisions (Riemann 1994) and it is shown that the collisions diminish the effect of magnetic field. The study of the plasma sheath in the presence of dust becomes an important research area due to its common observance in laboratory and space plasmas (Nitter 1996; Vladimirov 1998; Bouchoule 1999; Arnas *et al.* 2000; Yu *et al.* 1992). Two important factors which affect the structure of the dust plasma sheath are the external magnetic field and the collision force. Only few recent studies have considered the external magnetic field (Liu *et al.* 2004; Xiu *et al.* 2006). Liu *et al.* (2004) studied the characteristics of dust in a magnetized plasma sheath, assuming that the electrons are in thermal equilibrium and their density is given by the Boltzmann relation, while the ions and dust grains are considered as a cold fluid. But collisional effects were not important and thus not taken into account in their model. So far, the study of magnetized dusty plasma sheath in the presence of collisions has been investigated by several authors (Yu *et al.* 1992; Davoudabadi *et al.* 2005; Pandey *et al.* 2007; Masoudi *et al.* 2009; Mehdipour *et al.* 2010). In these studies the collisions of dust with neutrals in the sheath region can have significant effect. Masoudi *et al.* (2009), based on a previous work (Liu *et al.* 2004), investigated the problem by including the effects of collisions in a sheath dominated by dust-neutral collisions, but they do not include the pressure gradient of the ion in their model. The effect of ion temperature on the plasma sheath structure has been investigated without dust (Minghao *et al.* 2006; El Kaouini *et al.* 2010; Khoramabadi *et al.* 2011). Minghao *et al.* (2006) studied the effects of ion temperature on collisionless and collisional RF sheath in absence of oblique magnetic field and dust particles. Wang *et al.* (2004) studied the effects of ion temperature on dust charging in the sheath of dusty plasma, by considering a single dust particle model and do not include the collision force in their model. In the present study, the main goal is to examine the modification created by the presence of the thermal ions on the characteristics of dusty sheath of collisional magnetized plasma where the dust-neutral collision frequency depend to dust flow velocity. Based on some earlier studies (Liu *et al.* 2004; Masoudi *et al.* 2009), the fluid model is used for a dusty plasma sheath. We simultaneously consider the effects of the electrostatic, gravitational, Lorentz and the collision forces on dust characteristics, by taking into consideration the effect of the ion temperature. To do so that we consider the near wall region of a magnetized dusty plasma which consists of electrons in thermal equilibrium, fluid ions and cold fluid charged dust grains. The neutral particles are assumed to be immobile. We assume that the collisions between ions and neutrals can be neglected. This work is organized as follows. In Section 2 the analytical model of the sheath structure is presented. In Section 3 the numerical results are presented and discussed. Finally, a conclusion is given in Section 4.

2. THE MODEL FOR THE PLASMA SHEATH

We consider a magnetized stationary (∂t = 0) dusty plasma sheath in contact with a planar wall. The geometrical configuration is described in Fig. 1; the sheath is studied in a one-dimensional coordinate space system and a three-dimensional speed space system. The external magnetic field, which is spatially uniform and constant in time, lies in the x - z plane and makes an angle θ with the x-axis, which is the direction of gravitational force. The region of interest lies between $x = 0$ (the sheath edge), and the wall which can be located anywhere in the region $x > 0$. We assume that the physical parameters change only along the x (normal to the wall, see Fig.1.) direction in the sheath region. At the sheath edge $x = 0$, the potential is assumed to be zero, and the electron, ion and dust densities n_{e0}, n_{i0}, n_{d0} satisfy the quasi-neutrality condition $e(n_{i0} - n_{e0}) + q_{d0} n_{d0} = 0$, where e is the electron charge and q_{d0} is the dust charge at the sheath edge.

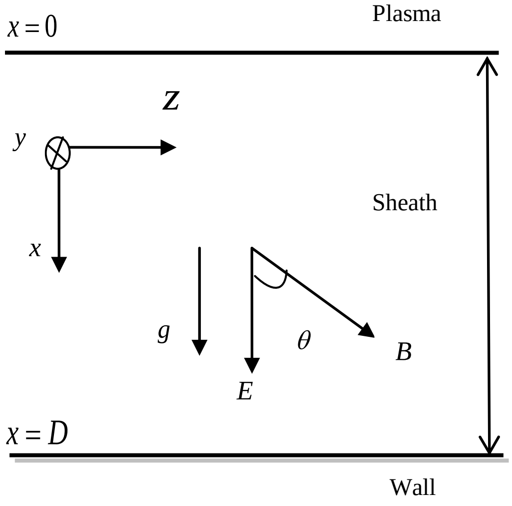

Fig. 1. The geometry of sheath model

The electrons are assumed to be in thermal equilibrium state, thus the density n_e is given by the Maxwell-Boltzmann distribution.

$$n_e = n_{e0} \exp\left(\frac{e\phi}{T_e}\right) \qquad (1)$$

where ϕ is the spatial electric potential in the sheath, T_e is the electron temperature and n_{e0} is the electron density at the sheath edge ($x = 0$).

The ions are treated as a fluid, governed by the continuity and the momentum conservation equations.

$$\frac{\partial}{\partial x}(n_i v_{ix}) = 0 \qquad (2)$$

$$m_i v_{ix} \frac{\partial v_{ix}}{\partial x} = -e\frac{\partial \phi}{\partial x} - \frac{\partial P_i}{n_i \partial x} \qquad (3)$$

where v_{ix}, n_i and m_i are the x-component of ion velocity, density and mass in the sheath respectively. P_i is the ion pressure given by the isothermal equation of state:

$$P_i = n_i T_i \tag{4}$$

where T_i is the ion temperature in eV.

The dust grains are treated as cold fluid obeys the sources free, the electric, magnetic, gravitational and collision dust-neutral forces are assumed as the dominant forces but the effects of ion drag and thermophoretic forces are neglected (Foroutan *et al.* 2009). The dust fluid equations are written as follows:

$$\frac{\partial(n_d v_{dx})}{\partial x} = 0 \tag{5}$$

$$m_d v_{dx} \frac{\partial \vec{v}_d}{\partial x} = -q_d \frac{\partial \varphi}{\partial x}\vec{x} + q_d \vec{v}_d \wedge \vec{B} - m_d \vec{g} - \vec{F}_{cd} \tag{6}$$

where $q_d = ez_d$, z_d, m_d, n_d and $\vec{v}_d$ are respectively the charge, the charge number, the mass, the density and velocity of the dust grains. $\vec{F}_{cd}$ is the drag force on the dust charged grains due to their collisions with neutrals. It is given by Mahanta *et al.* (2001).

$$\vec{F}_{cd} = m_d \nu(v_d)\vec{v}_d \tag{7}$$

The dust collision frequency can be expressed as a function of neutral gas density n_n, the momentum transfer cross section for collisions between dust and neutrals σ and the total drift velocity v_d as follows:

$$\nu(v_d) = n_n \sigma(v_d) v_d \tag{8}$$

The dependence between the momentum transfer cross section and the dust velocity is of the form (Mahanta *et al.* 2001).

$$\sigma(v_d) = \sigma_s (v_d / c_d)^p \tag{9}$$

Where $c_d = \left(\frac{T_e}{m_d}\right)^{1/2}$ is the dust acoustic speed, $\sigma_s = \pi r_d^2$ is the collision cross-section for collisions of the dust and neutral at dust acoustic speed. In the existing literature there are usually two special cases: constant dust mean-free path (constant cross section) and constant dust mobility corresponding, respectively, to the two parameter values $p = 0$ and $p = -1$. So for generalization, we consider that the cross section has a power law dependence on dust velocity. Therefore the collision force can be considered as follows:

$$\vec{F}_{cd} = m_d n_n \sigma_s \frac{v_d^{p+1}}{c_d^p} \vec{v}_d \tag{10}$$

Where p is a dimensionless parameter which can take various values ranging from 0 to -1.

The system of equations is completed by the Poisson equation which relates the electric potential to the density of dust, ions and electrons as follows:

$$\frac{d^2\phi}{dx^2} = -4\pi(e(n_i - n_e) + q_d n_d) \tag{11}$$

In order to solve the system of Eq. (1) – Eq. (11), we introduce new dimensionless variables as $\eta = -e\phi / T_e$ which is the normalized sheath potential, the density ratio of ion to electron densities $\delta = n_{i0} / n_{e0}$, as well as a normalized coordinate $\xi = x / \lambda_D$ where $\lambda_D = (T_e / 4\pi n_{e0} e^2)$ is the electron Debye length. The collision degree in the sheath α is given by the number of collisions in a Debye length: $\alpha = \lambda_D / \lambda = \lambda_D n_n \sigma_s$, where λ is the mean free path. We also normalized dust and ions velocities as $u_d = v_d / c_d$ and $u_i = v_i / c_i$ respectively. Here $c_d = (T_e / m_d)^{0.5}$, is the dust-acoustic speed, $c_i = (T_e / m_i)^{0.5}$ is the ion–acoustic speed. Substituting these dimensionless variables into Eq. (1) – Eq. (11), we obtain:

$$N_e = \exp(-\eta) \tag{12}$$

$$Nd = \frac{M_d}{u_{dx}} \tag{13}$$

$$N_i = \frac{M_i}{u_{ix}} \tag{14}$$

$$\frac{\partial u_{iX}}{\partial \xi} = \frac{1}{u_{iX} - T_{ie} / u_{iX}} \frac{\partial \eta}{\partial \xi} \tag{15}$$

$$u_{dx} \frac{\partial u_{dx}}{\partial \xi} = z_d \frac{\partial \eta}{\partial \xi} + \gamma z_d \sin\theta u_{dy} + f_g \\ - \alpha u_{dx} u_d^{1+p} \tag{16}$$

$$u_{dx} \frac{\partial u_{dy}}{\partial \xi} = \gamma z_d (\cos\theta u_{dz} - \sin\theta u_{dx}) \\ - \alpha u_{dy} u_d^{1+p} \tag{17}$$

$$u_{dx} \frac{\partial u_{dz}}{\partial \xi} = -\gamma z_d u_{dy} \cos\theta - \alpha u_{dz} u_d^{1+p} \tag{18}$$

$$\frac{\partial^2 \eta}{\partial \xi^2} = \delta \frac{M_i}{u_i} - \exp(-\eta) - (\delta - 1) \frac{M_d}{u_{dx}} \tag{19}$$

Where $\gamma = \frac{eB\lambda_D}{(m_d T_e)^{1/2}}$, $T_{ie} = T_i / T_e$, $f_g = \frac{g\lambda_D}{C_d^2}$,

$M_i = \frac{v_{ix0}}{c_i}$, $M_d = \frac{v_{dx0}}{c_d}$, $u_d = \left(u_{dx}^2 + u_{dy}^2 + u_{dz}^2\right)^{1/2}$

Here the boundary conditions are as follows: at the wall $\xi = d$, $\eta(d) = \eta_\omega$; at the sheath edge $\xi = 0$, the boundary conditions are $\eta = 0$ and $u_d(0) = M_d$.

3. NUMERICAL RESULTS AND DISCUSSION

In this section we present the numerical results of the structure of dusty plasma sheath, obtained from numerically solved Eq. (12) – Eq. (19) of the model depicted in section 2. For the numerical calculations, we choose the following plasma parameters:

$n_{e0} = 10^9 (cm^{-3})$, $T_e = 2(eV)$, $M_i = 1.1$,
$M_d = 5.0$, $\delta = 1.01$ and $\partial\eta / \partial\xi = 0.01$.

We consider the dust as a spherical particle of 4 μm radius with uniform mass density $\rho_d = 2(g/cm^3)$. This assumption enables us to compare our results with other correlative works like (Driouch *et al*. 2011 and Masoudi *et al*. 2009), in the case of the zero ions temperature and for one species of positive ions.

Figure 2 shows that the ion, electron and dust densities in the sheath vary in the cases of different temperature ratio T_{ie} (temperature ratio of ions to electrons). From the figure we can see that both the electron and ion densities fall slowly. Moreover, the slower the two densities fall, the higher the ion temperature becomes. Also the change of the dust density distribution is not obvious because their dust density is very small. For a certain position where electrons begin to vanish, the sheath becomes a pure ion one.

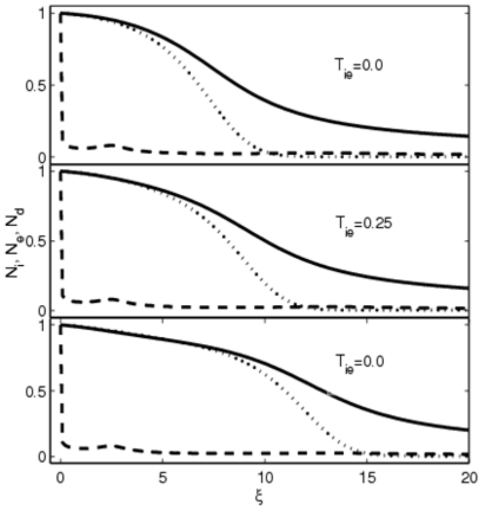

Fig. 2. Normalized density distributions of ions N_i (solid line), electrons N_e (dotted line) and dust grains N_d (dashed line) versus ξ in the plasma sheath with different T_{ie} values and for the parameters: α =0.2, θ =50°, γ =0.1 and z_d =-1000

Figure 3 shows the effect of the ion temperature on the space charge density distribution in a magnetized plasma sheath. The solid line is the space charge density curve in the absence of ion temperature. Because of the characteristic of density distribution of ions, dusts and electrons, the space charge presents a peak (a positive space charge is formed), which indicates that in this region, more positive particles are gathered to shield the negative potential. The increase of ion temperature leads to a decrease of the positive space charge and a moving towards the wall.

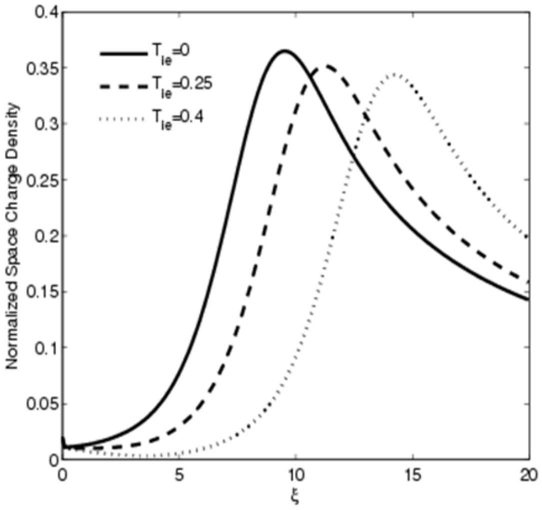

Fig. 3. The normalized distributions of net space charge density versus ξ for the same parameters as Fig. 2 and with different T_{ie} values

From Poisson's equation it is well known that the space charge creates the electric field. The positive space charge in the sheath region creates an electric field toward the wall. According to Fig. 4, an increase in T_{ie} gives rise to an increase in the normalized electric field. Thus, in Fig. 5 we have displays the increase of the normalized electric potential with increase of T_{ie}. Accordingly the sheath thickness (the distance between the sheath edge and the wall with a constant electric potential) decreases with increasing the ion temperature. Indeed reducing the space charge density means to reduce the sheath width.

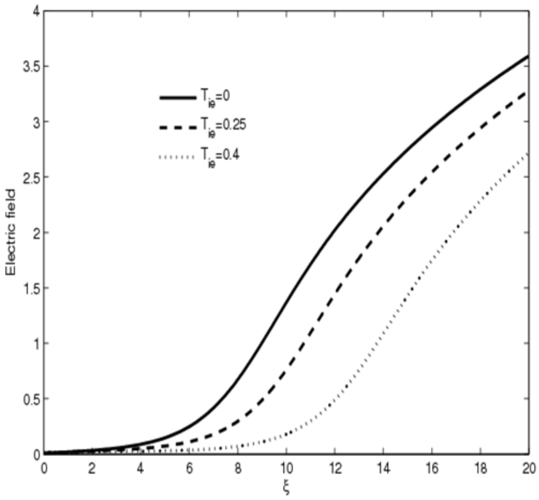

Fig. 4. The normalized distributions of spatial electrostatic field versus ξ with the same parameters as fig. 2 with different T_{ie} values

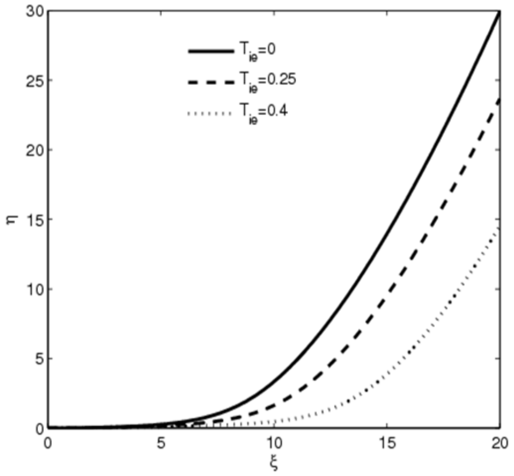

Fig. 5. The normalized distributions of spatial electrostatic field versus ξ with the same parameters as fig.2 with different T_{ie} values

Now we study the effect of the ion temperature on the dust velocity. We assume that the ion temperature cannot affect the dust velocity distribution directly (Eqs. (16)–(18)). The effect of ion temperature brings the increase of the ion velocity distribution (Eq. 15). From the Poisson equation, the increase of the ion density distribution would lead to the decrease of the electrostatic potential (see Fig. 5). And then this would bring modification in x-component of the dust velocity distribution. So we illustrate in Fig.6 the effects of the ion temperature on the distributions of dust velocity in depth direction for two cases; collisional plasma sheath with $p = 0$ (constant mean free path) and $p = -1$ (constant ion mobility), where we take $\alpha = 0.2$.

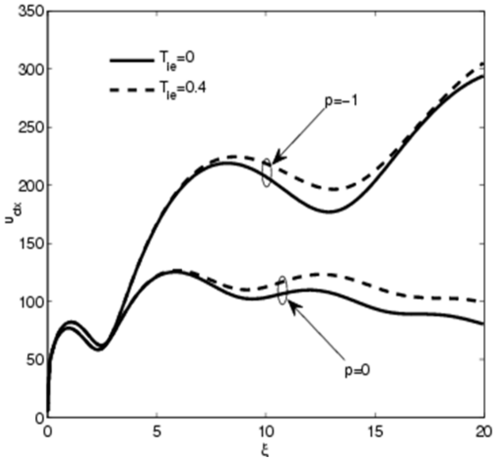

Fig. 6. The normalized distributions of dust velocity in the depth direction with different T_{ie} values and for two cases: $p = 0$; constant mean free path and $p = -1$; constant collision frequency, the other parameters are (α =0.2 , θ =50°, γ =0.1, z_d =-1000)

The figure shows that the variation of the T_{ie} value has no effects on the fluctuation of the dust velocity near the sheath edge. However, by increasing the distance from the sheath edge, the effect of ion temperature is more obvious for the case $p = 0$. In addition the dust velocity decreases by increasing the T_{ie} value, from the collisional force (Eq.15), for a given α value; the increasing of the p value corresponds to the increase in the force of collision, so to the increase in the effect of collision.

Figures 7 and Fig. 8 show the effects of the ion temperature on the distributions of dust velocity in the sheath for the collisional and collisionless cases. One can see from the figures that the ion temperature has strong effect for the collisional case (Fig.7) compared to the collisionless case (Fig.8), also it is seen that in a collisional case the curve reveals the existence of more fluctuations compared to collisionless case. In addition the increase of the ion temperature in a magnetized plasma sheath leads to the increase of the dust velocity distribution.

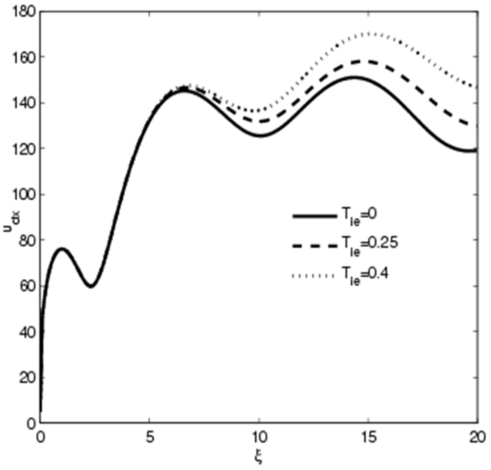

Fig. 7. The normalized distributions of dust velocity versus ξ with the same parameters as fig.7 with different T_{ie} values and for the collision plasma sheath α =10

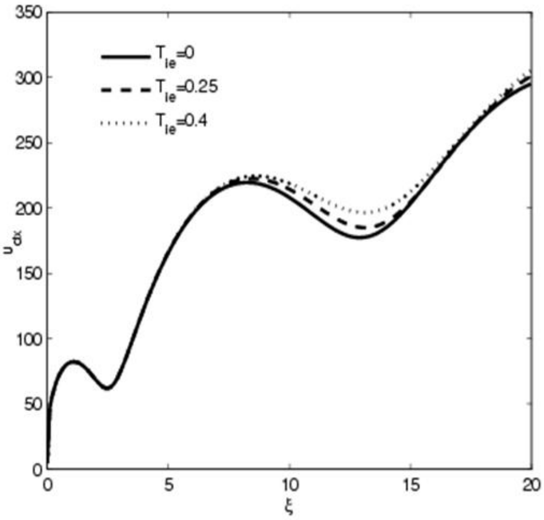

Fig. 8. The normalized distributions of dust velocity versus ξ with different T_{ie} values and for the collisionless plasma sheath α =0, the other parameters are: θ =50°, γ =0.1 and z_d =-1000

4. CONCLUSION

In this work, using fluid model, we have studied numerically the structure of a dusty plasma sheath in an external magnetic field and by taking into account the ion temperature effect. The results have been used to indicate the intricate effects of the ratio of ion to electron temperature on the plasma sheath. By increasing the ion temperature (increasing the ratio T_{ie}), the dust velocity increases, the ion density decreases slowly down, while electron density falls exponentially, the wall potential decreases leading to an increase of the thickness of the sheath. Also, it is found that the effect is more significant on the collisional sheath rather than on the collisionless sheath.

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