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Inertia Effects in Rheodynamic Lubrication of an Externally Pressurized Thrust Bearing Using Bingham Lubricant with Sinusoidal Injection

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Abstract

In the present theoretical investigation, the combined effects of fluid inertia forces and sinusoidal injection of the Bingham lubricant, on the performance of an externally pressurized thrust bearing with circular geometry are studied. Using the conventional two-constant Bingham model and by adopting the method of averaging inertia terms, the reduced Navier-Stokes equations are modified and numerical solutions have been obtained for the bearing performances such as the pressure distribution and the load carrying capacity for different values of Bingham number, Reynolds number, time and amplitude. The effects of fluid inertia forces and the non-Newtonian characteristics of the Bingham lubricant on the bearing performances for different sinusoidal conditions are discussed.

Keywords: Bingham fluids, Externally pressurized thrust bearing, Sinusoidal injection.

1. Introduction

The recent trend in mechanical industry shows that it designs machines with high operating speeds and heavy load carrying capacity. Therefore, lubrication of modern machines has been a great challenge and an emerging area of interest for the researchers around the world. In recent times, experimental researchers have shown clear evidence that the use of lubricants with variable viscosity can improve the lubricant properties relative to that of lubricants with constant viscosity. Therefore, the interest has been increasing to use the lubricants with variable viscosity called non-Newtonian lubricants, in particular, characterized by an yield value known as visco-plastic lubricants. Hence, visco-plastic lubricants are gaining importance in lubrication analysis. Some of the important visco-plastic models are Bingham plastics, Casson fluids and Herschel-Bulkley fluids. Examples of such fluids are greases, colloidal suspensions, polymeric fluids, starch pastes and blood flow through narrow tubes.

The effects of Bingham lubricant on slider, thrust, journal and squeeze film bearings have been analyzed by many researchers. Covey and Stanmore (1981) obtained theoretical and experimental results for the flow of Bingham fluid in a parallel-plate plasto-meter and discussed the presence and formations of yield surface. Papanastasiou (1987) analyzed the steady, two-dimensional flow of Bingham fluid by considering modified constitutive equation. Kandasamy (1995) analyzed theoretically and obtained numerical solutions for the bearing performances of an externally pressurized thrust bearing lubricated with non-Newtonian fluids. Greztos *et al.* (2008) developed 3-dimensional CFD model to study the behavior and performance characteristics of a journal bearing lubricated with Bingham fluid.

The continuing trend in lubrication industry emphasize that, in order to analyze the performances of the bearing, it is necessary to take into account the combined effects of fluid inertia and viscous forces of the lubricant. Hence, the study of lubricant inertia is assuming greater importance. The influence of fluid inertia forces on the pressure distribution of a step thrust bearing has been analyzed by Hashimoto *et al.* (1985) by using the method of averaging inertia terms. The effects of inertia forces on the bearing performances of journal bearing have been investigated by You and Lu (1988). Roy *et al.* (1993) have studied the inertia effects in an externally pressurized thrust bearing with converging and diverging film using visco-elastic fluid as lubricant. They observed that inertia forces increase the load capacity and pressure for uniform film bearing whereas it decreases in the case of converging and diverging film bearings. Tichy (1986) has studied the inertia effects in hydrodynamic lubrication by a linearization method and applied it to squeeze film damper bearing. Jayakran Amalraj *et al.*

(2010) have investigated the effects of inertia on the film pressure and load carrying capacity of an externally pressurized thrust bearing lubricated with Bingham fluid.

To analyze the effects of sinusoidal motion of the bearing, Usha and vimala (2002) have applied the energy integral approach to find the behaviour of curved squeeze film bearing using Newtonian lubricant. Kandasamy and Vishwanath (2007) have discussed the effects of fluid inertia forces, non-Newtonian characteristics and the amplitudes of sinusoidal squeeze motion on the bearing performances using Bingham fluid. Even though Jayakaran Amalraj *et al.* (2012) has analyzed the performance of an externally pressurized thrust bearing using Herschel - Bulkley fluids with sinusoidal injection recently, they have not considered the inertial effects in their study. Hence, there is a need to analyze the combined effects of fluid inertia and sinusoidal flow rate in an externally pressurized thrust bearing environment. As a first step in this direction, we consider the Bingham fluid which is a particular case of Herschel-Bulkley model in our study.

In the present work, the combined effects of fluid inertia and viscous forces on the bearing performances of an externally pressurized circular thrust bearing using Bingham fluids under sinusoidal flow rate have been analyzed. Numerical solutions are obtained for the film pressure and the load capacity for various values of Bingham number, Reynolds number and different amplitudes of Sinusoidal feeding. The effects of fluid inertia forces and the non-Newtonian characteristics of the Bingham lubricant on the bearing performance for different sinusoidal conditions have been discussed.

2. Formulation of the Problem

Geometry of the problem is shown in Fig. 1.

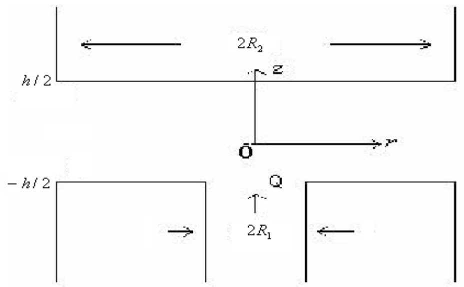

Fig. 1. Geometry of an externally pressurized thrust bearing

Consider an isothermal incompressible steady flow of the time-independent Bingham fluid between two circular plates, separated by a distance 'h'. Let R_1 be the radius of the film inlet and R_2 be the radius of the film outlet. Let P denote the pressure of the film, P_a the atmospheric pressure and ρ the density of the fluid. The cylindrical polar co-ordinates $(r,\ \theta\ z)$ with axial symmetry have been considered. The origin is fixed at the centre of the plate, r measuring the distance along the radial direction and z along the axis normal to the bearing. Let v_r and v_z be velocity components along r and z directions respectively.

The constitutive equation of a Bingham fluid is given by

$$\tau_{ij} = 2\left[\eta_1 + \frac{\eta_2}{I^{1/2}}\right] e_{ij},\quad \left(\frac{1}{2}\tau_{ij}\,\tau_{ij} \ge \eta_2^{\,2}\right) \tag{1}$$

where τ_{ij} are the deviatory stress components, η_1 and η_2 are constants namely the plastic viscosity and yield value respectively, e_{ij} represents the rate of deformation components and $I = 2e_{ij}\,e_{ij}$ is strain invariant. In those regions of the film, where the shear stress is less than the yield value, there will be a core formation which will move with constant velocity, v_c .

Let the boundaries of the core be $z = -\delta(r)\frac{h}{2}$ and $z = \frac{\delta(r)h}{2}$ as shown in Fig. 2.

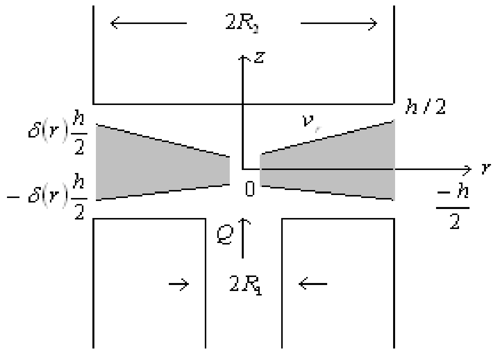

Fig. 2. Shape of the core in an externally pressurized thrust bearing.

Making the usual assumptions of lubrication theory, the equations governing the flow of Bingham material, including inertia forces, can be expressed as

$$\rho\left[v_r\frac{\partial v_r}{\partial r} + v_r\frac{\partial v_r}{\partial z}\right] + \frac{\partial P}{\partial r} = \frac{-\partial \tau_{rz}}{\partial z} \tag{2}$$

$$\frac{\partial P}{\partial z} = 0 \tag{3}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{\partial v_z}{\partial z} = 0 \tag{4}$$

$$\tau_{rz} = \pm\eta_2 + \eta_1\left(\frac{\partial v_r}{\partial z}\right) \tag{5}$$

Equations (1) – (4) are to be solved under the following boundary conditions

$$v_r = 0 \text{ at } z = \pm\frac{h}{2} \tag{6}$$

$$v_r = v_c,\ \text{ a constant at } z = \pm\delta(r)\frac{h}{2} \tag{7}$$

v_r and $\dfrac{\partial v_r}{\partial z}$ are continuous at $\tau_{rz} = \eta_2$ (8)

$P = P_a$ at $r = R_2$ (9)

3. Solution of the Problem

Averaging the inertia terms in the momentum equation, Eq. (5), Eq. (2) can be expressed as

$$\frac{\rho}{h}\left[\int_{-h/2}^{h/2}\left(v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}\right)dz\right]+\frac{dP}{dr}=\frac{-\partial \tau_{rz}}{\partial z} \tag{10}$$

Using continuity Eq. (4) and boundary conditions Eq. (6) and Eq. (7) in Eq. (10), it becomes

$$\frac{\rho}{h}\left\{\frac{\partial}{\partial r}\int_0^{h/2} v_r^2 dz+\frac{2}{r}\int_0^{h/2} v_r^2 dz\right\}+\frac{dP}{dr}=\frac{-\partial \tau_{rz}}{\partial z} \tag{11}$$

Define the modified pressure gradient f as

$$f=\frac{\rho}{h}\left\{\frac{\partial}{\partial r}\int_0^{h/2} v_r^2 dz+\frac{2}{r}\int_0^{h/2} v_r^2 dz\right\}+\frac{dP}{dr} \tag{12}$$

From Eq. (11) and Eq. (12),

$$f=\frac{-\partial \tau_{rz}}{\partial z} \tag{13}$$

As the modified pressure gradient is independent of z, integrating Eq. (13) results in

$$\tau_{rz} = -fz+\varphi(r) \tag{14}$$

Using Eq. (5) and boundary conditions Eq. (6) and Eq. (7) in Eq. (14), the velocity distribution in the flow region will be

$$v_r=\frac{f}{\eta_1}\left[\frac{h^2}{8}(1-\delta)^2-\frac{\left(z-\frac{\delta h}{2}\right)^2}{2}\right],\ \frac{\delta h}{2}\le z\le\frac{h}{2} \tag{15}$$

and the velocity of the core region will be

$$v_c=\frac{f}{\eta_1}\frac{h^2}{8}(1-\delta)^2,\ 0\le z\le\frac{h}{2} \tag{16}$$

The equation of conservation of mass which depends on the configuration of the bearing is given by

$$Q=4\pi r\int_0^{h/2} v_r dz \text{, where } Q \text{ is the flow rate} \tag{17}$$

Using Eq. (15) and Eq. (16) in Eq. (17) and integrating, the flow rate Q becomes

$$Q=\frac{\pi r f\, h^3(1-\delta)^2(2+\delta)}{12\eta_1} \tag{18}$$

By considering an equilibrium of an element in the yield surface, $-\delta(r)\frac{h}{2}\le z\le\delta(r)\frac{h}{2}$, Eq. (13)becomes

$$-f=-\frac{2\eta_2}{\delta(r)h} \tag{19}$$

Eliminating the pressure gradient from Eq. (18) and Eq. (19), the following algebraic equation is obtained.

$$\frac{6Q\eta_1}{\pi r h^2\eta_2}=\frac{(1-\delta)^2(2+\delta)}{\delta} \tag{20}$$

Substituting Eq. (15) and Eq. (16) into Eq. (12) and using f from Eq. (18), the pressure gradient will be

$$\frac{dP}{dr}=\frac{12Q\eta_1}{\pi r h^3(1-\delta)^2(2+\delta)}+\frac{3\rho Q^2\left[7\delta^2+22\delta+16+r(7\delta+2)\right]d\delta/dr}{20\pi^2h^2r^3(2+\delta)^2} \tag{21}$$

The sinusoidal feeding of the lubricant is defined as

$$Q=Q_0+a\sin(w_f t) \tag{22}$$

which influences the sinusoidal fluid film between the plates, in which Q_0 is mean flow rate, a is the amplitude of the flow rate variation, w_f is the frequency of oscillation and t is the time of oscillation.

The following non-dimensional parameters are introduced;

$$r^*=\frac{r}{R_2};\ \delta^*=\delta(r^*);\ P^*=\frac{P}{\left(\frac{Q_0\eta_1}{\pi h^3}\right)};\ Q^*=\frac{Q}{Q_0};$$

$$A=\frac{a}{Q_0};\ T=w_f t \text{ and } B=\frac{\pi R_2 h^2\eta_2}{2Q_0}$$

Using Eq. (22) and the above non-dimensional parameters, Eq. (20) and Eq. (21) will be transformed as

$$\frac{3\left[1+A\sin(T)\right]}{B\,r^*}=\frac{(1-\delta^*)^2(2+\delta^*)}{\delta^*} \tag{23}$$

$$\frac{dP^*}{dr^*}=\frac{12\left[1+A\sin(T)\right]}{r^*(1+\delta^*)^2(2+\delta^*)}+\left[\frac{3\rho Q_0 h}{20\pi R_2^2\eta_1}\right]\frac{a^2b}{c^3} \tag{24}$$

where $a=1+A\sin(T), c=r^*(2+\delta^*)$ and

$b=7\delta^{*2}+22\delta^*+16+r^*(7\delta^*+2)d\delta^*/dr^*$.

Integrating Eq. (23), we get

$$P^*-P_a^*=\int_1^{r^*}\left[\frac{12a}{r^*(1+\delta^*)^2(2+\delta^*)}+(\mathrm{Re})\left[\frac{b}{c^3}\right]\right]dr^* \tag{25}$$

where $\mathrm{Re} = \dfrac{3\rho Q_0 h}{20\pi R_2{}^2 \eta_1}$, is the Reynolds Number.

From Eq. (25), the pressure distribution along the fluid film can be obtained numerically for different values of Bingham number and Reynolds number under various sinusoidal conditions.

The load carrying capacity of an externally pressurized circular thrust bearing is given by

$$W = \int_{R_1^*}^{1} \left(P^* - P_a^*\right) r^* \, dr^* \qquad (26)$$

where $R_1{}^* = \dfrac{R_1}{R_2}$, is the ratio of inlet to outlet radius of the bearing.

The above integral Eq. (26) is evaluated numerically for different values of B, Re, A and T to get load carrying capacity of the bearing.

4. Results and Discussion

The Pressure distribution along the fluid film has been obtained numerically for different values of B, Re, A and T are shown in Figs. 3 - 9.

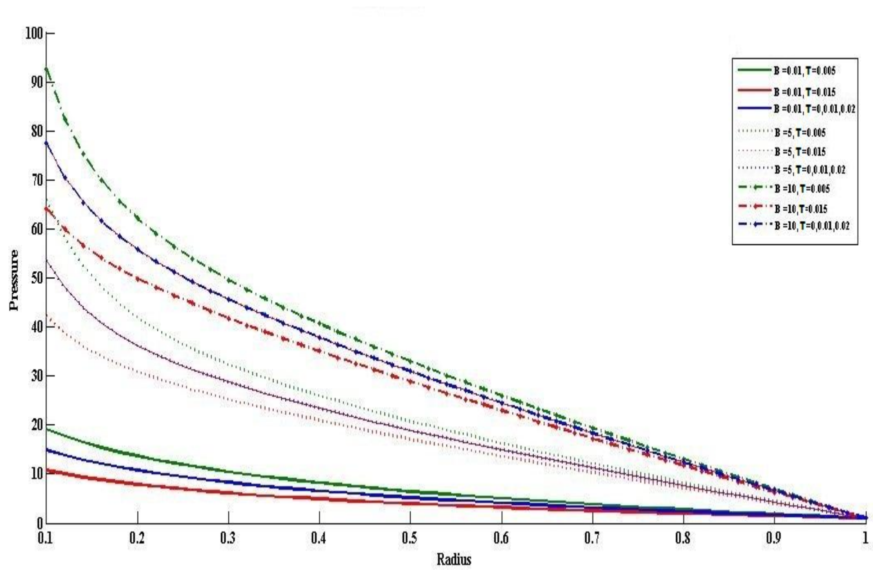

Fig. 3. Pressure distribution along radius for A=0.3; Re =5.

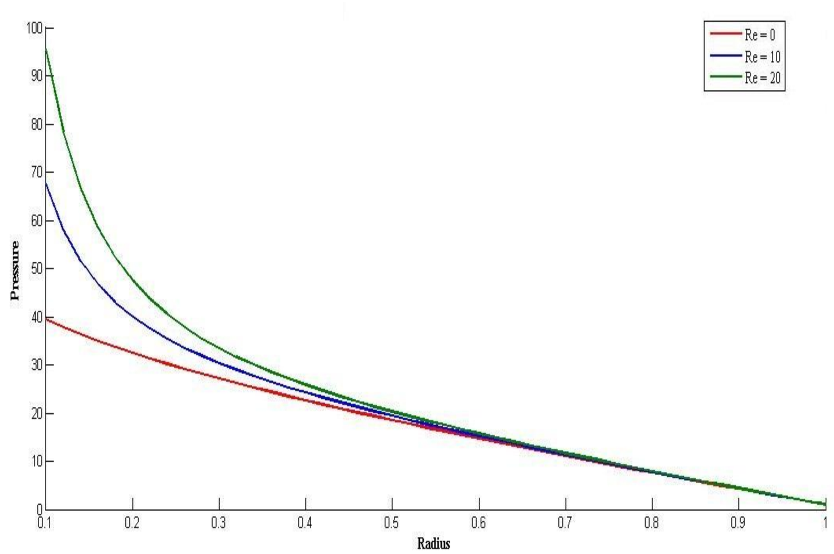

Fig. 4. Pressure distribution along radius for *T*=0, 0.01, 0.02; *A*=0.3; *B*=5.

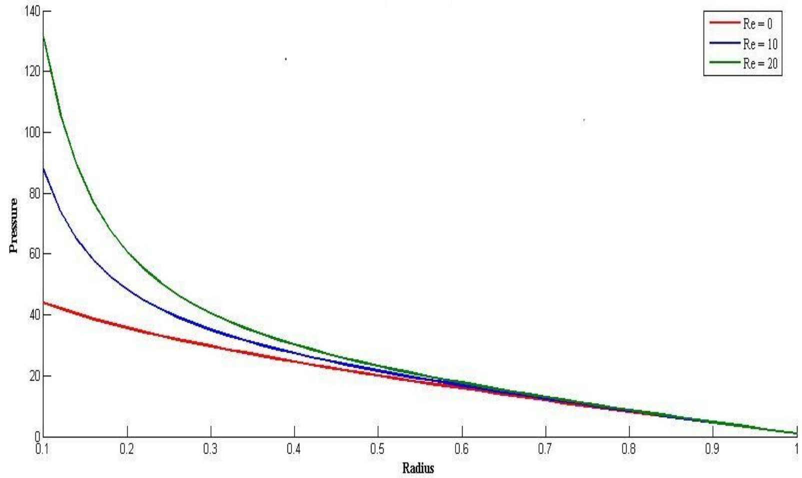

Fig. 5. Pressure distribution along radius for T=0.005; A=0.3; B=5.

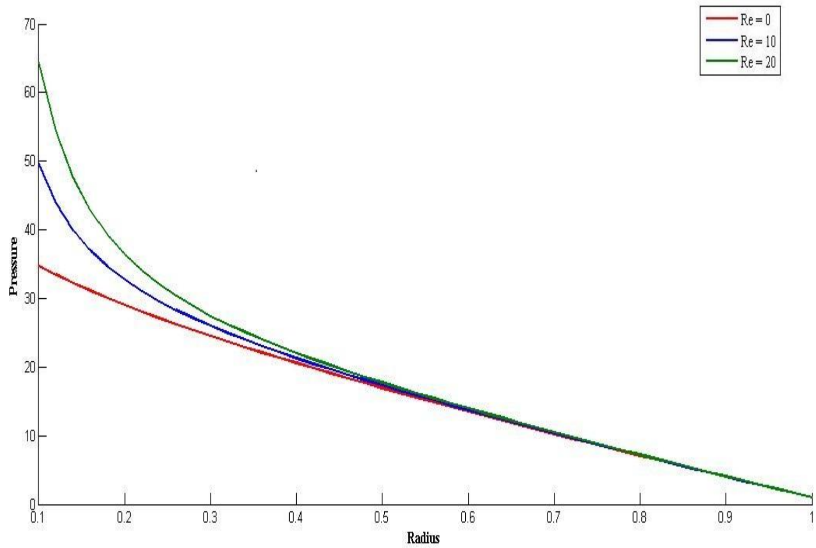

Fig. 6. Pressure distribution along radius for T=0.015; A=0.3; B=5.

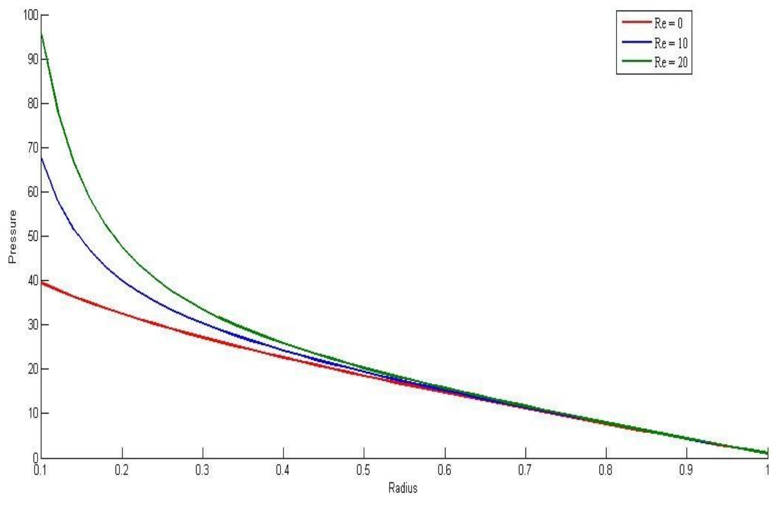

Fig. 7. Pressure distribution along radius for T=0, 0.01, 0.02; A=0.6; B=5.

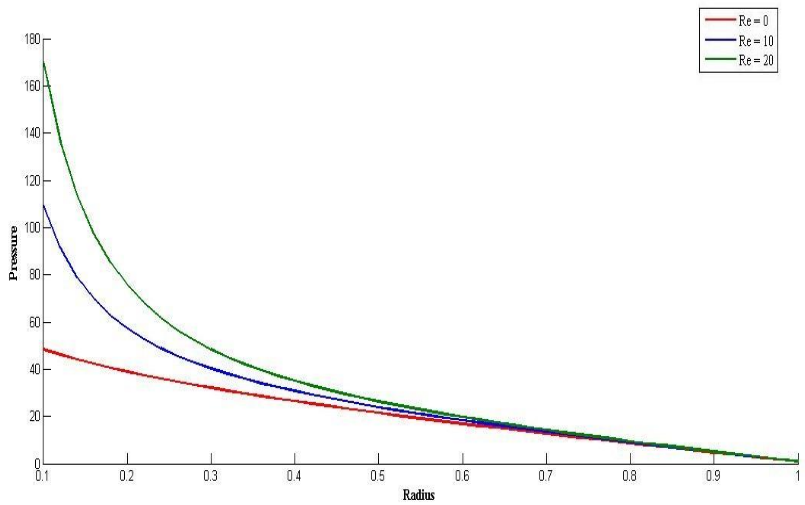

Fig. 8. Pressure distribution along radius for *T*=0.005; *A*=0.6*; B*=5.

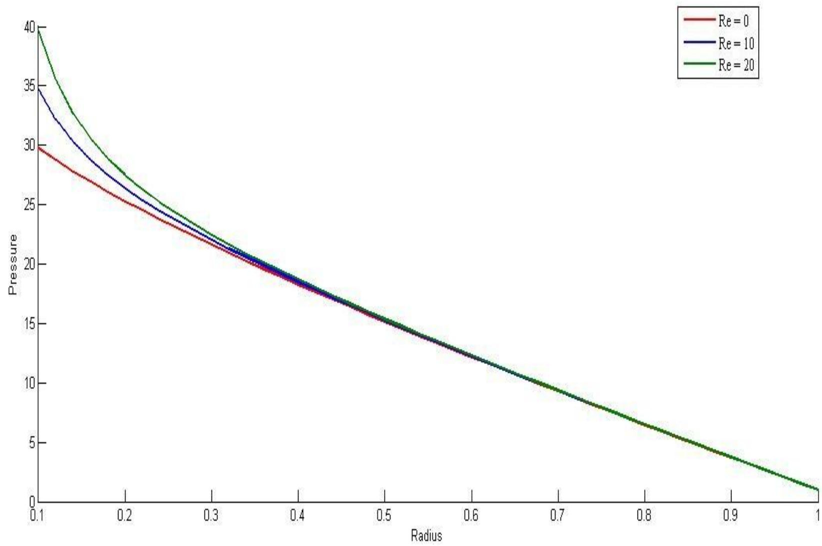

Fig. 9. Pressure distribution along radius for *T*=0.015; *A*=0.6; *B*=5.

It is very much evident that the pressure decreases gradually from centre to periphery. Further, it has been observed that, if the Bingham number increases, then the pressure distribution along the fluid film also increases significantly. Moreover, there is an increase in pressure as Reynolds number increases. In particular, the influence of inertial effects on pressure distribution is found to be appreciable near the centre of the bearing. However, with respect to time, the effects sinusoidal feeding is clearly reflected on the pressure distribution. For a frequency of 50 Hz, the film pressure keeps on increasing and reaches its maximum at T = 0.005.Then the pressure starts decreasing and attains its minimum at T = 0.015 before changing its course. As the amplitude increases, the film pressure increases for 0<T<0.01. However, this trend is reversed for 0.01<T<0.02.

The numerically computed load capacity of the bearing for various values of Bingham number, Reynolds number, amplitude and time are shown in Figs.10-12. It has been observed that there is an appreciable increase in the value of the load capacity for the Bingham lubricants relative to that of Newtonian lubricant. The load carrying capacity of the bearing is found to increase with the increase of Bingham number, Reynolds number and amplitude of the sinusoidal feeding. Further, it has been observed that the effects of fluid inertia on the load capacity is appreciable when 0 < T < 0.01 and marginal for 0.01< T < 0.02, for a frequency of 50Hz. Moreover, as the amplitude increases, the influence of inertia forces on the load capacity are significant when 0 < T < 0.01 and it is negligible for 0.01< T < 0.02. However with respect to time, the load capacity of the bearing increases continuously and reaches its maximum at T = 0.005.Then the load capacity starts decreasing and attains its minimum when T = 0.015 before changing its course.

To the best of our knowledge, not many works in the externally pressurized thrust bearing with visco-plastic lubricants are available in the literature. However, our results are compared with our earlier works which are available in the literature. Under the non-sinusoidal condition (T=0) with the inertia less environment (Re=0), our results lead to the results of Kandasamy (1995). When T=0 (non-sinusoidal condition), the present results completely match with the results reported earlier by Jayakaran (2010). Further, for the case of Re=0, the results obtained in our analysis agree with the results given by Jayakaran (2012) for the particular case of flow behavior index being one (which corresponds to Bingam lubricant) in their analysis.

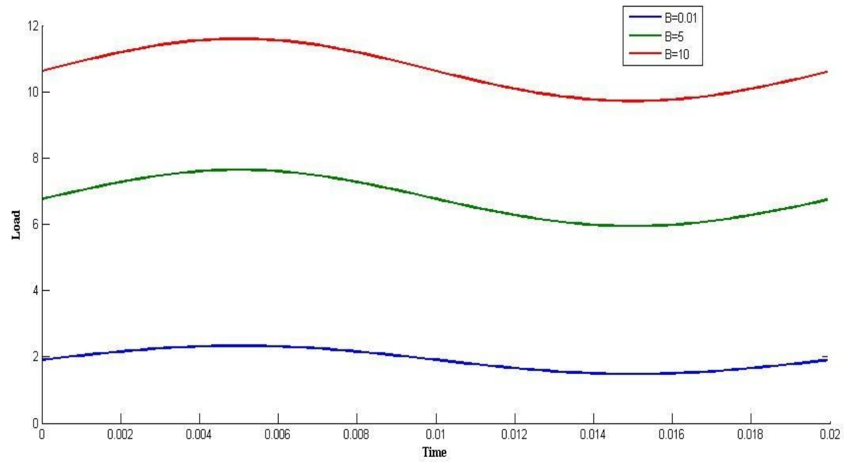

Fig. 10. The Load capacity variation with time, *Re = 0.*

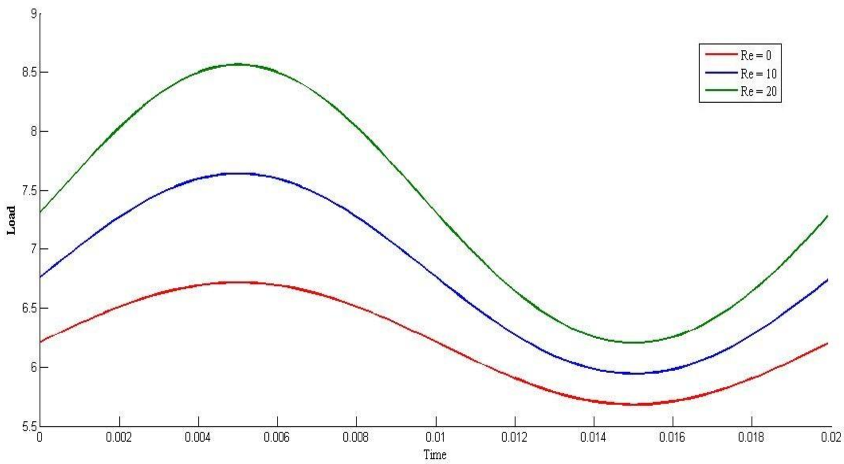

Fig. 11. The Load capacity variation with time, *A= 3 & B = 5.*

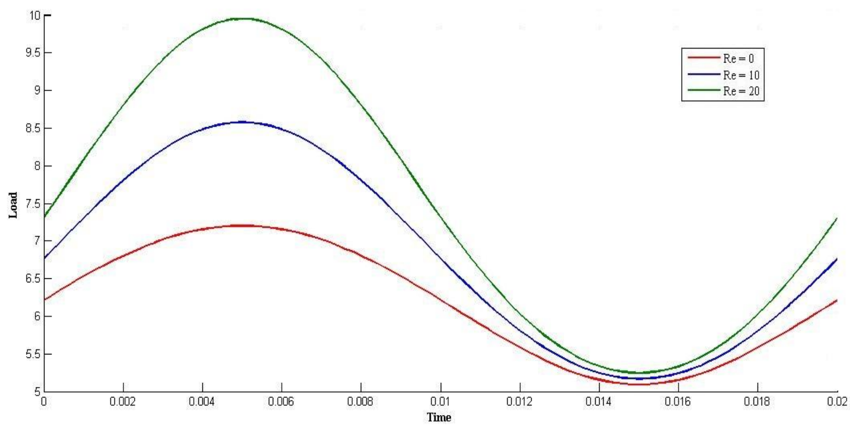

Fig. 12. The Load capacity variation with time, *A =6 & B = 5.*

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REFERENCES

Covey, G.H. and B.R. Stanmore (1981). Use of Parallel Plate Plastometer for the Characterization of Viscous Study with a Yield Stress, *J. Non-Newtonian Fluid Mechanics*, 8, 249 – 260.

Gertzon, K.P., P.G. Nikolakopoulos and C.A. Papadopoulos (2008). CFD Analysis of Journal Bearing Hydrodynamic Lubrication by Bingham Lubricant, *Tribology International*, 41, 1190 – 1204.

Hashimoto, H, S. Wada and N. Kiyokawa (1985). An Influence of Fluid Inertia Forces on the Pressure Distributions of Step Thrust Bearing, *Bull. JSME*, 28(243), 2091-2098.

Jayakaran Amalraj, I., S. Narasimman and A. Kandasamy (2010). Inertia Effects in Rheodynamic Lubrication of an Externally Pressurized Thrust Bearing using Bingham fluid, *Proc.8th Asian Computational Fluid Dynamics Conference, Hong Kong.*

Jayakaran Amalraj, I., S. Narasimman and A. Kandasamy (2012). Rheodynamic Lubrication of an Externally Pressurized Thrust Bearing using Herschel–Bulkley Fluid with Sinusoidal Injection" *Journal of Applied Fluid Mechanics*, 5,71-79.

Kandasamy, A. (1995). Lubrication of an Externally Pressurized Thrust Bearing with non- Newtonian Lubricants, *Proceedings of International Conference on Advances in Mechanical Engineering*, Bangalore, India, 1175-1183.

Kandasamy, A. and K.P. Vishwanath (2007). Rheodynamic Lubrication of a Squeeze Film Bearing with Inertia Effects under Sinusoidal Squeeze Motion using Bingham Fluids, *Computational and Applied Mathematics*, 26, 381-396.

Papanastasiou, T.C. (1987). Flows of Materials with Yield, *Journal of Rheology*, 31, 385-404.

Roy, J. S., S. Padhy and L. K. Bhopa (1993). Inertia Effects in Externally Pressurized Thrust Bearing with Converging and Diverging Film using visco-elastic Lubricant, *Acta Mechnica*, 96, 1 – 12.

Tichy, J.A. (1986). The Effect of Fluid Inertia in Hydrodynamic Lubrication studied by a Linearization Method: Two Useful Applications, *Int. J. of Engineering Sciences*, 24(10), 1607-1613.

Usha.R. and P. Vimala (2002). Curved Squeeze Film with Inertial Effects-Energy Integral Approach, *Fluid Dynamic Research*, 30, 139-153.

You, H.I. and S.S. Lu (1988). The Effect of Fluid Inertia on the Operating Characteristics of a Journal bearing, *Journal of Tribology*, 10(3), 499-506.