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On Exchange of Stabilities in Ferromagnetic Convection in a Rotating Ferrofluid Saturated Porous Layer

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ABSTRACT

In the present paper, first of all, it is proved that the 'principle of the exchange of stabilities' is not, in general valid, for the case of free boundaries and then a sufficient condition is derived for the validity of this principle in ferromagnetic convection, for the case of free boundaries, in a horizontal ferrofluid saturated porous layer in the presence of a uniform vertical magnetic field and uniform rotation about the vertical axis.

Keywords: Ferrofluids, Principle of exchange of stabilities, Porous medium, Rotation, Viscosity ratio.

NOMENCLATURE

a	overall horizontal wave number	α	thermal expansion coefficient
d	thickness of the porous layer		
D=d/dz	differential operator	$\beta = \Delta T/d$	temperature gradient
D_a= k/d^2	Darcy number	$\chi = (\partial M/\partial H)H0,T0$	magnetic susceptibility
g	acceleration due to gravity	$\Delta T = T_0 - T_1$	constant temperature difference between the boundaries
H	magnetic field intensity	ϵ	porosity of porous medium
H_0	imposed uniform vertical magnetic field	$\kappa = k_1/\rho_0 C$	effective thermal diffusivity
k	permeability of the porous medium	$\vec{\Omega} = \Omega\hat{k}$	constant angular velocity
$\hat{k}$	unit vector in the vertical direction	ω	complex growth rate
$K = -\left(\frac{\partial M}{\partial H}\right)_{H_0,T_0}$	pyromagnetic coefficient	$\Lambda = \widetilde{\mu_f}/\mu_f$	ratio of viscosities
$M = \lvert\vec{M}\rvert$	Magnitude of the magnetization $\vec{M}$	μ_f	dynamic viscosity
$\vec{M}M_0 = M(H_0, T_0)$	constant mean value of magnetization	$\widetilde{\mu_f}$	effective viscosity
$M_1 = \mu_0 K^2\beta/(1+\chi)\alpha_t\rho_0 g$	magnetic number	μ_0	free space magnetic permeability of vacuum
$M_3 = (1+M_0/H_0)/(1+\chi)$	nonlinearity of magnetization parameter	$\nu = \mu_f/\rho_0$	kinematic viscosity
$P_r = \frac{\nu\epsilon(\rho_0 C)_1}{k_1}$	modified Prandtl number	$\emptyset$	amplitude of perturbed magnetic potentialρFluid density
R=g$\alpha\beta d^4/\nu\kappa$	Rayleigh number	θ	amplitude of perturbed temperature
T	temperature	ζ	amplitude of vertical component of vorticity
T_0	temperature of the lower boundary	**Superscripts**	
T_1	temperature of the upper boundary	*	complex conjugation
$T_a = 4\Omega^2 d^4/\nu^2\epsilon^2$	Taylor number	**Subscripts**	
w	amplitude of vertical component of perturbed velocity	f	fluid
(x , y , z)	Cartesian coordinates	s	solid

1. NTRODUCTION

A Ferrofluid is a colloid suspension containing magnetic nanoparticles covered by a surfactant for preventing their aggregation and suspended in a nonconducting fluid. In recent years Ferrofluids have attracted many researchers due to their practical applications in various fields like viscous damping system, medical sciences (drug targeting, endoscopic analysis, magnetic separation of cells and Magnetic Resonance Imaging (MRI), noiseless printing system etc.(Rosenweig 1985). The magnetization of ferrofluids depends on the magnetic field, temperature and density. Any change in these parameters effects the body force distribution in the fluid layer and give rise to convection in ferrofluids. The magnetic properties of such fluids have been investigated for a considerable time since the 1930s (Elmore 1938). There after there have been numerous studies on convection in ferrofluids with different physical configurations and different boundary conditions. Finlayson (1970) investigated convective instability of a ferromagnetic fluid layer heated from below in the presence of a uniform vertical magnetic field by using linear stability theory and predicted the critical temperature gradient for the onset of convection when both buoyancy and magnetic forces are considered. An exact solution for the case of free boundaries and approximate solutions (for stationary convection) of rigid boundaries have been derived by him. Thermoconvective stability of ferrofluids without considering buoyancy effects has been studied by Lalas and Carmi (1971), whereas Shliomis (1974) studied the linear relation for magnetized perturbed quantities at the limit of instability. Polevikov (1997) studied the stability of a static magnetic fluid under the action of an external pressure drop. Schwab *et al.* (1983) experimentally investigated the problem of Finlayson in the case of a strong magnetic field and detected the onset of convection by plotting the Nusselt number verses the Rayleigh number. Later, Stiles and Kagan (1990) extended the problem to allow for the dependence of effective shear viscosity on temperature and colloid concentration.

The Benard convection in ferromagnetic fluids has been considered by many authers (Gupta and Gupta 1979, Rudraiah and Shekar 1991, Siddeshwar 1993, Qin and Kaloni 1994, Souhar *et al.* 1999, Aniss *et al.* 2001, Siddheshwar and Abraham 2003). In the Benard convection problem the instability is driven by a density difference caused by a temperature difference between two planes bounding the fluid. If we add some concentration component to this configuration then the resulting phenomenon of convection is known as thermohaline convection or double diffusive convection. Vaidyanathan *et al.* (1997) investigated ferrothermohaline convection in which an incompressible ferromagnetic fluid layer in the presence of transverse magnetic field, heated from below and salted from above is considered and showed that the salinity of a ferromagnetic fluid enables the fluid to get destabilized more when it is salted from above. Sunil *et al.* (2005) studied the effect of magnetic field dependent viscosity on ferrothermohaline convection.

To study the effect of rotation on ferrofluids is an interesting topic. The convective instability analysis for a rotating layer of ferrofluid between free boundaries is studied by Gupta and Gupta (1979) .Sekar *et al.* (2000) studied the effect of rotation on ferrothermohaline convection and derived the conditions for both stationary and oscillatory motions using linear stability theory. Venkatasubramanian and Kaloni (1994) studied the effect of rotation on the Thermoconvective instability of a horizontal ferrofluids layer and discussed the stabilizing effect of rotation. Recently Sunil *et al.* (2011) have studied the effect of rotation on double diffusive convection in a magnetized ferrofluid with internal angular momentum and showed that oscillatory motions are possible due to the presence of rotation, coupling between vorticity and spin, microinertia and solute gradient.

In recent years, many researchers have shown their keen interest in analyzing the onset of convection in a fluid layer subjected to a vertical temperature gradient in a porous medium. The stability of flow of a fluid through porous medium was studied by Lapwood (1948) and Wooding (1960). Taunton and Lightfoot (1972) characterized salt fingers in thermohaline convection in porous medium. Borglin *et al.* (2000) performed experiments to study the nature of flow of ferrofluids in porous media. Ferrothermohaline convection in a porous medium for different configuration is studied by Vaidyanathan *et al.* (1995); Sunil *et al.* (2005) and Vaidyanathan *et al.* (2007). Ferroconvection in an anisotropic porous medium is studied by Sekar et al. (1996) using linear stability analysis and found that the vertical anisotropy stabilizes the system through marginal or stationary mode. Mittal and Rana (2009) studied the effect of dust particles on a layer of micropolar ferromagnetic fluid heated from below saturating a porous medium. The onset of thermomagnetic convection in a ferrofluid saturated porous layer, for a variety of velocity and temperature boundary conditions, is investigated by Shivakumara *et al.* (2009). The effect of rotation on ferromagnetic convection in a ferrofluids saturated porous layer is investigated by Shivakumara *et al.* (2011) and derived the conditions for the occurrence of direct and Hopf bifurcations (oscillatory).The conditions obtained by them for the occurrence of direct bifurcations, for the case of free boundaries, are wave number dependent and do not include magnetic parameters, thus raising doubts about the accurate utilization of the results.

In the present paper it is proved that ‘exchange principle’ is not, in general, valid for the case of free boundaries but a sufficient condition for the validity of this principle can be derived by using Pellew and Southwell technique (1940) for the case of free boundaries. While this case is of little physical interest but it is mathematically important as it enables us to find analytical solutions and to make some qualitative conclusions. For the case of rigid boundaries, to the author's knowledge, it is still an open problem. The results obtained herein are independent of wave number and involve magnetic parameter, thus making the results more realistic.

2. The Physical Configuration and the Governing Equations

A viscous ferrofluid saturated porous layer in the presence of a uniform applied magnetic field H0 acting in the vertical direction is statically confined between two horizontal boundaries z = 0 and z = d which are respectively maintained at constant temperatures T_0 and T_1 ($<T_0$), thus maintaining a constant temperature difference ΔT ($=T_0-T_1$) between the boundaries. A Cartesian coordinate system (x, y, z) is used with z-axis normal to the porous layer. The entire system is rotating with uniform angular velocity $\vec{\Omega}=\Omega\hat{k}$, where $\hat{k}$ is the unit vector in the vertical direction; with the assumption that rotation has no effect on the isotropy of the porous medium. Also the fluid viscosity is taken different from effective viscosity. The governing equations for the flow of an incompressible ferrofluid in a layer of rotating porous medium are (Nanjundappa *et al.* 2010, Venkatasubramanian and Kaloni 1994):

The equation of continuity is

$$\nabla.\vec{q}=0. \tag{1}$$

The equation of motion is

$$\rho_0\left[\frac{1}{\epsilon}\frac{\partial\vec{q}}{\partial t}+\frac{1}{\epsilon^2}(\vec{q}.\nabla)\vec{q}\right]=-\nabla p+\rho\vec{g}-\frac{\mu_f}{k}\vec{q}+\widetilde{\mu_f}\nabla^2\vec{q}+\nabla.(\vec{H}\vec{B})+2\frac{\rho_0}{\epsilon}(\vec{q}\times\vec{\Omega})+\frac{\rho_0}{2}\nabla(|\vec{\Omega}\times\vec{r}|^2) \tag{2}$$

The equation of heat conduction is

$$\epsilon\left[\rho_0 C_{V,H}-\mu_0\vec{H}.\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\right]\frac{DT}{Dt}+(1-\epsilon)(\rho_0 C)_s\frac{\partial T}{\partial t}+\mu_0 T\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}.\frac{D\vec{H}}{Dt}=k_1\nabla^2 T \tag{3}$$

The Maxwell equations in the magnetostatic limit are

$$\nabla.\vec{B}=0,\qquad \nabla\times\vec{H}=0 \ \text{ or } \ \vec{H}=\nabla\varphi \tag{4a,b}$$

Also $\vec{B}$, $\vec{M}$ and $\vec{M}$ are related by

$$\vec{B}=\mu_0(\vec{M}+\vec{H}) \tag{5}$$

The magnetization is assumed to be aligned with the magnetic field, but allow a dependence on the magnitude of magnetic field as well as temperature in the form

$$\vec{M}=\frac{M}{H}(H,T)\vec{M} \tag{6}$$

The magnetic equation of the state is linearized about H_0 and T_0 to take the form

$$M=M_0+\chi(H-H_0)-K(T-T_0). \tag{7}$$

Equation of state for density is

$$\rho=\rho_0[1+\alpha(T_0-T)] \tag{8}$$

where $\vec{q}=(u,v,w)$ is the velocity vector, p is the pressure, $\vec{H}=(H_x,H_y,H_z)$ is the magnetic field intensity, $\vec{B}=(B_x,B_y,B_z)$ is the magnetic induction, $\vec{M}=(M_x,M_y,M_z)$ is the magnetization, μ_f is the dynamic viscosity, $\widetilde{\mu_f}$ is the effective viscosity, C is the specific heat, $C_{V,H}$ is the specific heat at constant volume and magnetic field, φ is the magnetic scalar potential, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is the Laplacian operator, ρ_0 is value of density ρ at some reference temperature T_0 and the subscript s denotes the solid.

The initial stationary state is given by

$\vec{q}=(u,v,w)=(0,0.0)$, $p=p_b$, $\vec{H_b}=[0,0,H_b(z)]$, $\vec{M}=[0,0,M_b(z)]$ and $T=T_b(z)$, where subscript b denotes the basic state. Thus Tb satisfies

$$\nabla^2 T_b=0, \tag{9}$$

together with the already mentioned boundary conditions. Solving Eq. (9), we obtain $T_b=T_0-\beta z$, where $\beta=\frac{\Delta T}{d}$. Clearly Eq. (4b) is identically satisfied and Eq. (4a) yields,

$$H_b(z)+M_b(z)=C_1 \tag{10}$$

where C_1 is a constant. Thus it is clear that there exists the following solutions for the basic state

$$q_b=0,\ T_b=T_0-\beta z\ ,\ \vec{H_b}(z)=\left[H_0-\frac{K\beta z}{1+\chi}\right]\hat{k},\ \vec{M_b}(z)=\left[M_0+\frac{K\beta z}{1+\chi}\right]\hat{k}, \tag{11}$$

where $\hat{k}$ is the unit vector in the vertical direction.

To analyze the stability of the system, we perturb all the variables in the form

$$\vec{q}=\vec{q'},\ p=p_b(z)+p',\quad T=T_b(z)+T',\quad \vec{H}=\vec{H_b}(z)+\vec{H'},\ \vec{M}=\vec{M_b}(z)+\vec{M'}, \tag{12}$$

where $\vec{q'}$, p', T', $\vec{H'}$ and $\vec{M'}$ are perturbed variables and are assumed to be small. Using Eq. (12) in Eq. (5) and Eq. (6) and using Eqs. (4a,b), we get (dropping the primes for convenience)

$$H_x+M_x=\left(1+\frac{M_0}{H_0}\right)H_x,\ H_y+M_y=\left(1+\frac{M_0}{H_0}\right)H_y\ \text{and}\ H_z+M_z=(1+\chi)H_z-KT \tag{13}$$

where $K\Delta T<<(1+\chi)H_0$ is assumed.

Now taking curl of Eq. (2) and linearizing, the z-component of the resulting equation can be written as

$$\frac{\rho_0}{\epsilon}\frac{\partial\zeta}{\partial t}=-\frac{\mu_f}{k}\zeta+\widetilde{\mu_f}\nabla^2\zeta+\frac{2\rho_0}{\epsilon}\Omega\frac{\partial w}{\partial z} \tag{14}$$

which is the vorticity transport equation for the present problem.

Again, substituting Eq. (12) into Eq. (2), linearizing, eliminating the pressure term by operating curl twice and using Eq. (13), together with $\vec{H} = \nabla\varphi'$, the z-component of the resulting equation can be written as (dropping the primes for convenience)

$$\left(\frac{\rho_0}{\epsilon}\frac{\partial}{\partial t} + \frac{\mu_f}{k} - \widetilde{\mu_f}\nabla^2\right)\nabla^2 w = -\mu_0 K\beta\frac{\partial}{\partial z}\left(\nabla_h^2\varphi\right) + \frac{\mu_0 K^2\beta}{1+\chi}\nabla_h^2 T + g\rho_0\alpha_t\nabla_h^2 T - \frac{2\rho_0}{\epsilon}\Omega\frac{\partial\zeta}{\partial z}, \quad (15)$$

where $\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the horizontal Laplacian operator. Eq. (3) on using Eq. (12) and linearizing, becomes (dropping the primes)

$$(\rho_0 C)_1\frac{\partial T}{\partial t} - \mu_0 T_0 K\frac{\partial}{\partial t}\left(\frac{\partial\varphi}{\partial z}\right) = k_1\nabla^2 T + \left[(\rho_0 C)_2 - \frac{\mu_0 T_0 K^2}{1+\chi}\right]\beta w \quad (16)$$

where $(\rho_0 C)_1 = \epsilon\rho_0 C_{V,H} + \epsilon\mu_0 H_0 K + (1-\epsilon)(\rho_0 C)_s$ and $(\rho_0 C)_2 = \epsilon\rho_0 C_{V,H} + \epsilon\mu_0 H_0 K$.

Equations (4a) and (4b), on using Eq. (12) and Eq. (13), may be written as (dropping the primes)

$$(1+\chi)\frac{\partial^2\varphi}{\partial z^2} + \left(1 + \frac{M_0}{H_0}\right)\nabla_h^2\varphi - K\frac{\partial T}{\partial z} = 0 \quad (17)$$

Now we analyze the perturbations w,T, φ, ζ into two-dimensional periodic waves. We assume normal mode expansion of these variables of the form

$$(w, T, \varphi, \zeta)(x,y,z) = [w''(z), \theta''(z), \emptyset''(z), \zeta''(z)]\exp\left[i\left(k_x x + k_y y\right) + nt\right] \quad (18)$$

where k_x and k_y are the wave numbers in the x- and y- directions respectively and $\bar{k} = \sqrt{k_x^2 + k_y^2}$ is the resultant wave number ; n is a constant which can be complex in general.

Substituting Eq. (18) into Eqs. (14)-(17), and nondimensionalizing the variables by setting

$$z^* = \frac{z}{d}, w^* = \frac{d}{\nu}w, \theta^* = \frac{\kappa}{\beta\nu d}\theta, \emptyset^* = \frac{(1+\chi)\kappa}{K\beta\nu d^2}\emptyset, \zeta^* = \frac{d^2}{\nu}\zeta, a = \bar{k}d \text{ and } D^* = d\frac{d}{dz}, \quad (19)$$

where $\nu = \frac{\mu_f}{\rho_0}$ is the kinematic viscosity, $\kappa = \frac{k_1}{(\rho_0 C)_2}$ is the effective thermal diffusivity , we get (dropping the asterisks)

$$[\Lambda(D^2 - a^2) - Da^{-1} - \omega](D^2 - a^2)w = -Ra^2[M_1 D\emptyset - (1+M_1)\theta] + T_a^{1/2}D\zeta, \quad (20)$$

$$(D^2 - a^2 - P_r\omega)\theta + P_r M_2\omega D\emptyset = -(1 - M_2 A)w, \quad (21)$$

$$(D^2 - a^2 M_3)\emptyset = D\theta, \quad (22)$$

$$[\Lambda(D^2 - a^2) - Da^{-1} - \omega]\zeta = -T_a^{1/2}Dw, \quad (23)$$

where z is the real independent variable such that $0 \le z \le 1$, D is differentiation w.r.t. z, a2 is the square of the wave number, $P_r = \frac{\nu\epsilon(\rho_0 C)_1}{k_1} > 0$ is the modified Prandtl number, $\omega = \frac{nd^2}{\nu\epsilon}$ is complex growth rate, $R = \frac{g\alpha_t\beta d^4}{\kappa\nu} > 0$ is the thermal Rayleigh number, $T_a = \frac{4\Omega^2 d^4}{\nu^2\epsilon^2}$ is the Taylor number, $\Lambda = \frac{\widetilde{\mu_f}}{\mu_f} > 0$ is the ratio of viscosities , $D_a = \frac{k}{d^2} > 0$ is the Darcy number, $M_1 = \mu_0 K^2\beta/(1+\chi)\alpha_t\rho_0 g > 0$ is the magnetic number, $M_3 = (1 + M_0/H_0)/(1+\chi) > 0$ is the measure of nonlinearity of magnetization , $A = \frac{(\rho_0 C)_1}{(\rho_0 C)_2}$ is a positive constant and $M_2 = \frac{\mu_0 K^2 T_0}{(1+\chi)(\rho_0 C)_1}$ is a nondimensional parameter. $\omega = \omega_r + i\omega_i$ is a complex constant in general such that ω_r and ω_i are real constants and as a consequence the dependent variables $w(z) = wr(z) + iwi(z)$, $\theta(z) = \theta_r(z) + i\theta_i(z)$, $\phi(z) = \phi_r(z) + i\phi_i(z)$ and $\zeta(z) = \zeta_r(z) + i\zeta_i(z)$ are complex valued functions of the real variable z such that wr(z), wi(z), $\theta_r(z), \theta_i(z), \phi_r(z), \phi_i(z), \zeta_r(z)$ and $\zeta_i(z)$ are real valued functions of the real variable z.

Since M_2 is of very small order3), it is neglected in the subsequent analysis and thus Eq. (21) takes the form

$$(D^2 - a^2 - P_r\omega)\theta = -w \quad (24)$$

The constant- temperature boundaries are considered to be either free or rigid. Hence the boundary conditions are:

$$w = 0 = \theta = D\emptyset = D\zeta = D^2 w \text{ at } z = 0 \text{ and } z = 1 \quad (25)$$

(both the boundaries free)

$$\text{or } w = 0 = \theta = \emptyset = \zeta = Dw \text{ at } z = 0 \text{ and } z = 1, \quad (26)$$

(both the boundaries rigid)

It may further be noted that Eq. (20) and Eq. (22) to Eq. (26) describe an eigen value problem for ω and govern ferromagnetic convection in a rotating ferrofluid saturated porous layer.

3. Mathematical Analysis

First of all it is shown that 'principle of exchange of stabilities' is not, in general, valid for the case of free boundaries. It is shown as follows:

Multiplying Eq. (20) by w^*(* denotes the complex conjugation) throughout and integrating the resulting equation over the vertical range of z, we get

$$\int_0^1 w^*[\Lambda(D^2 - a^2) - D_a^{-1} - \omega](D^2 - a^2)w dz = -Ra^2 M_1\int_0^1 w^* D\emptyset dz + Ra^2(1+M_1)\int_0^1 w^*\theta dz + T_a^{1/2}\int_0^1 w^* D\zeta dz \quad (27)$$

Using Eqs. (24), (22), and Eq. (23) and the boundary conditions Eq. (25), we can write

$$-Ra^2M_1\int_0^1 w^*D\emptyset dz = Ra^2M_1\int_0^1 D\emptyset(D^2 - a^2 - P_r\omega^*)\theta^*\, dz$$

$$= Ra^2M_1\int_0^1 D\emptyset D^2\theta^* dz - Ra^2M_1(a^2 + P_r\omega^*)\int_0^1 \theta^* D\emptyset dz$$

$$= Ra^2M_1\int_0^1 D\emptyset D^2\theta^* dz + Ra^2M_1(a^2 + P_r\omega^*)\int_0^1 \emptyset D\theta^* dz, \quad (28)$$

$$= Ra^2M_1\int_0^1 D\emptyset D^2\theta^* dz + Ra^2M_1(a^2 + P_r\omega^*)\int_0^1 \emptyset(D^2 - a^2M_3)\emptyset^* dz,$$

using Eq. (3):

$$R(1+M_1)a^2\int_0^1 w^*\theta dz = -R(1+M_1)a^2\int_0^1 \theta(D^2 - a^2 - P_r\omega^*)\theta^* dz \quad (29)$$

and

$$T_a^{1/2}\int_0^1 w^* D\zeta dz = -T_a^{1/2}\int_0^1 \zeta D\, w^* dz = \int_0^1 \zeta[\Lambda(D^2 - a^2) - D_a^{-1} - \omega^*]\zeta^* dz. \quad (30)$$

Combining Eq. (27) and Eq. (30), we obtain

$$\int_0^1 w^*[\Lambda(D^2 - a^2) - D_a^{-1} - \omega](D^2 - a^2)wdz = Ra^2M_1\int_0^1 D\emptyset D^2\theta^* dz + Ra^2M_1(a^2 + P_r\omega^*)\int_0^1 \emptyset(D^2 - a^2M_3)\emptyset^* dz - Ra^2(1+M_1)\int_0^1 \theta(D^2 - a^2 - P_r\omega^*)\theta^* dz + \int_0^1 \zeta[\Lambda(D^2 - a^2) - D_a^{-1} - \omega^*]\zeta^* dz \quad (31)$$

Integrating the various terms of Eq. (31) by parts for an appropriate number of times and making use of the boundary conditions Eq. (25) and the equality

$$\int_0^1 \psi^* D^{2n}\psi dz = (-1)^n\int_0^1 |D^n\psi|^2 dz, \quad (32)$$

Where $\psi = w (n = 1,2)$ or $\psi = \theta, \zeta, \emptyset (n = 1)$, we may write Eq. (31) in the form

$$\Lambda\int_0^1(|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2) + (D_a^{-1} + \omega)\int_0^1(|Dw|^2 + a^2|w|^2)dz =$$

$$-Ra^2M_1\int_0^1 D^2\emptyset D\theta^* dz - Ra^2M_1(a^2 + P_r\omega^*)\int_0^1(|D\emptyset|^2 + a^2M_3|\emptyset|^2)dz + \quad (33)$$

$$Ra^2(1+M_1)\int_0^1(|D\theta|^2 + a^2|\theta|^2 + P_r\omega^*|\theta|^2)dz - \int_0^1[\Lambda(|D\zeta|^2 + a^2|\zeta|^2) + D_a^{-1}|\zeta|^2 + \omega^*|\zeta|^2)dz$$

Now multiplying complex conjugate of Eq. (22) by $\emptyset$ and integrating over the vertical range of z , we obtain $\int_0^1(|D\emptyset|^2 + a^2M_3|\emptyset|^2)dz = -\int_0^1 \emptyset D\theta^* dz$, which clearly implies that $\int_0^1 \emptyset D\theta^* dz$ is real.

Now multiplying Eq. (22) by $D\theta^*$ and integrating over the vertical range of z, we obtain

$$\int_0^1 D^2\emptyset D\theta^* dz - a^2M_3\int_0^1 \emptyset D\theta^* dz = \int_0^1 |D\theta|^2 dz,$$

since $\int_0^1 \emptyset D\theta^* dz$ is real, therefore $\int_0^1 D^2\emptyset D\theta^* dz$ is also real as right hand side is real.

Equating imaginary parts on both sides of Eq. (33) and using the fact that $\int_0^1 D^2\emptyset D\theta^* dz$ is real, we get

$$\omega_i\Big[\int_0^1(|Dw|^2 + a^2|w|^2)dz - RM_1P_ra^2\int_0^1(|D\emptyset|^2 + a^2M_3|\emptyset|^2)dz + R(1+M_1)P_ra^2\int_0^1|\theta|^2 dz - \int_0^1|\zeta|^2 dz\Big] = 0 \quad (34)$$

which clearly implies that ω_i , in general, is not equal to zero. Hence the required conclusion directly follows from this.

Now we derive a sufficient condition for the existence of ‘principle of exchange of stabilities’ in the present case.

We now prove the following theorem:

Theorem1: If $(w, \theta, \phi, \zeta, \omega), \omega = \omega_r + i\omega i, \omega r \geq 0$, is a solution of Eqs.(20) and Eq. (22) to Eq. (25) and $\frac{RM_1P_r}{\pi^4} + \frac{T_a}{D_a^{-1}\Lambda^2\pi^2} \leq 1$, then $\omega_i = 0$.

Proof: Equating imaginary parts on both sides of Eq. (33), using the fact that $\int_{-1/2}^{1/2} D^2\emptyset D\theta^* dz$ is real and cancelling$\omega_i (\neq 0)$, we can write

$$\int_0^1(|Dw|^2 + a^2|w|^2)dz = RM_1P_ra^2\int_0^1(|D\emptyset|^2 + a^2M_3|\emptyset|^2)dz - R(1+M_1)P_ra^2\int_0^1|\theta|^2 dz + \int_0^1|\zeta|^2 dz \quad (35)$$

Now multiplying Eq. (22) by ϕ^* (complex conjugate of ϕ) throughout and integrating first term on the left hand side and the right hand side by making use of appropriate boundary conditions on ϕ and θ, we have from the final equation

$$\int_0^1(|D\phi|^2 + a^2M_3|\phi|^2)dz = \int_0^1(D\phi^*)\,\theta dz$$

$$\leq \left|\int_0^1(D\phi^*)\theta dz\right|$$

$$\leq \int_0^1|(D\phi^*)\theta|dz \quad (36)$$

$$\leq \int_0^1|D\phi^*|\,|\theta|dz \leq \int_0^1|D\phi|\,|\theta|dz$$

$$\leq \left(\int_0^1|D\phi|^2 dz\right)^{1/2}\left(\int_0^1|\theta|^2\, dz\right)^{1/2},$$

(utilizing the Schwartz inequality)

which implies that

$$\int_0^1|D\phi|^2 dz \leq \left(\int_0^1|D\phi|^2 dz\right)^2\left(\int_0^1|\theta|^2 dz\right)^{1/2}$$

Thus

$$\left(\int_0^1|D\phi|^2 dz\right)^{1/2} \leq \left(\int_0^1|\theta|^2 dz\right)^{1/2}.$$

Using this inequality in inequality Eq. (36), we obtain

$$\int_0^1 (|D\phi|^2 + a^2 M_3 |\phi|^2) dz \le \int_0^1 |\theta|^2 dz \qquad (37)$$

Now multiplying Eq. (23) by ζ^* (complex conjugate of ζ) and integrating the various terms of the resulting equation by parts for an appropriate number of times by making use of boundary conditions on ζ and w, we have

$$\int_0^1 [\wedge (|D\zeta|^2 + a^2|\zeta|^2) + D_a^{-1}|\zeta|^2 + \omega|\zeta|^2] dz = T_a{}^{1/2} \int_0^1 \zeta^* D\, wdz \quad = -T_a{}^{1/2} \int_0^1 (D\zeta^*)\, wdz$$

Equating the real partson both sides, we get

$$\int_0^1 [\wedge (|D\zeta|^2 + a^2|\zeta|^2) + D_a^{-1}|\zeta|^2 + \omega_r|\zeta|^2] dz = -\text{Realpartof} T_a{}^{1/2} \int_0^1 (D\zeta^*)\, wdz$$

$$\le \left| T_a{}^{1/2} \int_0^1 (D\zeta^*) wdz \right| \qquad (38)$$

$$\le T_a{}^{1/2} \int_0^1 |(D\zeta^*) w|\, dz$$

$$\le T_a{}^{1/2} \int_0^1 |D\zeta^*||w| dz$$

$$\le T_a{}^{1/2} \int_0^1 |D\zeta|\, |w| dz$$

$$\le T_a{}^{1/2} \left(\int_0^1 |D\zeta|^2\, dz\right)^{1/2} \left(\int_0^1 |w|^2\, dz\right)^{1/2}$$

(using the Schwartz inequality) which implies that

$$\wedge \int_0^1 |D\zeta|^2\, dz \le T_a{}^{1/2} \left(\int_0^1 |D\zeta|^2 dz\right)^{1/2} \left(\int_0^1 |w|^2\, dz\right)^{1/2}.$$

Thus

$$\left(\int_0^1 |D\zeta|^2\, dz\right)^{1/2} \le \frac{T_a{}^{1/2}}{\wedge} \int_0^1 |w|^2 dz.$$

Using this inequality in inequality Eq. (38), we have

$$\int_0^1 [\wedge (|D\zeta|^2 + a^2|\zeta|^2) + D_a^{-1}|\zeta|^2 + \omega_r|\zeta|^2]\, dz \le \frac{T_a}{\wedge} \int_0^1 |w|^2 dz \qquad (39)$$

We note that since w and θ satisfy w(0) = 0 = w(1) and θ (0) = 0 θ (1), we have by Rayleigh-Ritz inequality (Schultz 1973)

$$\int_0^1 |Dw|^2\, dz \ge \pi^2 \int_0^1 |w|^2\, dz \qquad (40)$$

and

$$\int_0^1 |D\theta|^2\, dz \ge \pi^2 \int_0^1 |\theta|^2\, dz \qquad (41)$$

Now upon using Eq. (40) in Eq. (39), we obtain

$$\int_0^1 |\zeta|^2 dz \le \frac{T}{Da^{-1}\wedge} \int_0^1 |w|^2 dz \le \frac{T}{D_a^{-1}\wedge\pi^2} \int_0^1 |Dw|^2\, dz \qquad (42)$$

Now multiplying Eq. (24) by θ^* (complex conjugate of θ) throughout and integrating the first term on the left hand side once by making use of the boundary conditions on θ namely θ(0) = 0 = θ(1), we have from the real part of the final equation

$$\int_0^1 (|D\theta|^2 + a^2|\theta|^2 + P_r\omega_r|\theta|^2) dz = \text{Realpartof} \int_0^1 \theta^* wdz \le \left|\int_0^1 \theta^* wdz\right| \le \int_0^1 |\theta^* w| dz \le \int_0^1 |\theta^*||w|\, dz \le \int_0^1 |\theta||w|\, dz$$

$$\le \left(\int_0^1 |\theta|^2\, dz\right)^{1/2} \left(\int_0^1 |w|^2 dz\right)^{1/2}.$$

(using Schwartz inequality)

Combining this inequality with the inequality (41) and the fact that$\omega_r \ge 0$,we obtain

$$\pi^2 \int_0^1 |\theta|^2\, dz \le \left(\int_0^1 |\theta|^2 dz\right)^{1/2} \left(\int_0^1 |w|^2 dz\right)^{1/2},$$

which implies that

$$\left(\int_0^1 |\theta|^2 dz\right)^{1/2} \le \frac{1}{\pi^2} \left(\int_0^1 |w|^2 dz\right)^{1/2}$$

and thus

$$\int_0^1 (|D\theta|^2 + a^2|\theta|^2 + P_r\omega_r|\theta|^2)\, dz \le \frac{1}{\pi^2} \int_0^1 |w|^2\, dz$$

which upon using inequality (40) gives

$$a^2 \int_0^1 |\theta|^2\, dz \le \frac{1}{\pi^4} \int_0^1 |Dw|^2\, dz \qquad (43)$$

Using inequalities Eq. (37), Eq. (42) and Eq. (43) in Eq. (35), we obtain $\int_0^1 (|Dw|^2 + a^2|w|^2)\, dz \le \frac{RM_1P_r}{\pi^4} \int_0^1 |Dw|^2\, dz - R(1+M_1)a^2 P_r \int_0^1 |\theta|^2\, dz + \frac{T_a}{D_a^{-1}\wedge\pi^2} \int_0^1 |Dw|^2\, dz,$

which gives

$$\left[1 - \left(\frac{RM_1P_r}{\pi^4} + \frac{T_a}{D_a^{-1}\wedge\pi^2}\right)\right] \int_0^1 |Dw|^2\, dz + a^2 \int_0^1 |w|^2\, dz + R(1+M_1)P_r a^2 \int_0^1 |\theta|^2\, dz \le 0$$

and thus we necessarily have

$$\frac{RM_1P_r}{\pi^4} + \frac{T_a}{D_a^{-1}\wedge\pi^2} > 1 \qquad (44)$$

Hence, if $\frac{RM_1P_r}{\pi^4} + \frac{T_a}{D_a^{-1}\wedge\pi^2} \le 1$, we must have $\omega_i = 0$.

This establishes the theorem.

Theorem1 may be stated , from the physical point of view, as: for ferromagnetic convection in a rotating ferrofluid saturated porous layer,for the case of free boundaries, a necessary condition for the occurrence of Hopf bifurcations (oscillatory motions) is that $\frac{RM_1P_r}{\pi^4} + \frac{T_a}{D_a^{-1}\wedge\pi^2} > 1$ and hence a sufficient condition for the occurrence of direct bifurcation (stationary convection) is that $\frac{RM_1P_r}{\pi^4} + \frac{T_a}{D_a^{-1}\wedge\pi^2} \le 1$.

4. Conclusion

Thus it is analytically shown that the 'principle of the exchange of stabilities' is not, in general, valid. A sufficient condition for the validity of this principle is also derived. The sufficient condition obtained herein is independent of wave number and incorporate a magnetic parameter, thus showing a definite improvement over the existing results.

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