

Rayleigh-Taylor Instability at the Interface of Superposed Couple-Stress Casson Fluids Flow in Porous Medium under the Effect of a Magnetic Field

B. M. Agoor^{1†} and N. T. M. Eldabe²

¹*Department of Mathematics Faculty of Science Fayoum University, Fayoum, Egypt.*

²*Department of Mathematics Faculty of Education, Ain Shams University, Cairo, Egypt.*

†Corresponding Author Email: bma00@fayoum.edu.eg

(Received March 10, 2012; accepted December 08, 2013)

ABSTRACT

The Rayleigh-Taylor instability (RIT) at the interface of two superposed Couple-stress Casson fluids flowing in porous medium and in the presence of a uniform normal magnetic field is studied. The fluids have different densities. For mathematical simplicity, the stability analysis based on fully developed approximations is used. The maximum wave numbers and the corresponding maximum frequency are obtained. The Growth rate of Rayleigh-Taylor instability in the case of non-Newtonian Casson fluid with couple-stress through porous medium is discussed. The effects of physical parameters of the problem such as the permeability parameter, magnetic parameter, non-Newtonian Parameter and couple-stress parameter on the regions of stability are discussed numerically and illustrated graphically through a set of figures.

Keywords: Casson equation, Non-Newtonian fluid, Porous medium, Permeability, Magnetic field, Micropolar fluid, Viscosity.

NOMENCLATURE

G_{\max}	ratio of growth rate	α	non-Newtonian Parameter.
\mathbf{g}	gravitational acceleration	β	bond number
k	permeability parameter	γ	Surface tension
l	wave number	η	elevation of the interface
M	magnetic parameter	λ	couple stress parameter
n	frequency (growth rate)	$\mu\beta$	plastic dynamic viscosity
p	pressure	μ	coefficient of viscosity
p_y	yield stress	ν	kinematic viscosity
q_i	velocity	ρ	density of the fluid
T	characteristic time scale	τ_{ij}	stress tensor

1. INTRODUCTION

Chandrasekhar (1981) has discussed the instability of a plane interface between two incompressible viscous fluids with different densities, when the lighter one is accelerated into the heavier one.

Bhatia (1974) studied the influence of the viscosity on stability of a plane interface separating two incompressible superposed conducting fluids of uniform density, when the whole system is stressed by a uniform Magnetic field. He has carried out the stability analysis for two highly viscous fluids of

equal kinematic Viscosity and different uniform densities. The Rayleigh-Taylor instability of two viscoelastic superposed Fluids has been studied by Sharma and Sharma (1978). Kent (1966) investigated the effect of horizontal Magnetic field, which varies in vertical direction, on the stability of parallel flows. He has shown that the system is unstable under certain conditions, while in the absence of a magnetic field the System is known to be stable. In all above studies, the medium field has been considered non-porous. Sunil (2002) discussed the Rayleigh-Taylor instability of two superposed Couple-stress fluids of uniform Densities in a porous medium in the presence of a uniform horizontal magnetic field. Sunil has carried out the stability analysis for two highly viscous fluids of equal kinematic viscosity and equal Couple-stress Kinematic Viscosity. Rudraiah (2010) shown the effects of couple stress fluid on the control of Rayleigh-Taylor instability at the interface between dense fluids accelerated by a lighter fluid. He used approximations to derive the growth rate of Rayleigh-Taylor instability. Dash (1996) introduced the flow characteristics of Casson fluid in a tube filled with homogeneous porous medium, by employing Brinkman model to account Darcy resistance offered by porous medium. This analysis can model the pathological situation of blood flow when fatty plaques of cholesterol and Artery-clogging blood clots are formed in the lumen of the coronary artery. Two cases of permeability porous medium are considered. He studied the effect of permeability factor and yield stress of the fluid on shear stress distribution, wall shear stress, plug flow radius, flow rates and frictional Resistance . A. Mohamed and ELDabe (1978) discussed the electrohydrodynamic stability of a hollow jet under the influence of axial electric field. They show the relation between the electric field strength and the various parameters of the system. EIDabe (1988) studied the electro- hydrodynamic stability of two stratified power low liquids in couette flow under the influence of normal electric field. EIDabe (1988) investigated the electro hydrodynamic stability of two superposed elastic viscous liquids in plane couette flow. EIDabe (1989) has given the effect of tangential electric field on Rayleigh-Taylor instability. Elhefnawy (2001) investigated the nonlinear analysis of the electrohydrodynamic Rayleigh-Taylor instability of a cylindrical interface separating two conducting fluids of circular cross section in the absence of gravity. The main aim of this study is to generalize the work of Rudraiah (2010) to include the flow of non-Newtonian fluids through porous medium in the presence of external magnetic field and to discuss the effects of couple stress, normal Magnetic field, and Permeability

parameter on the Rayleigh-Taylor instability of the Casson fluid flow in Porous medium.

2. MATHEMATICAL FORMULATION

Consider two fluids one above the other, the lower consists of thin film of an unperturbed thickness $2a$ filled with an incompressible non-Newtonian couple stress fluid of constant density ρ_1 bounded below by a rigid surface and above by another incompressible non-Newtonian couple stress fluid of constant density ρ_2 (where $\rho_1 \ll \rho_2$) with thickness d (where $2a \ll d$), the interface between them is at $y = a$. A magnetic field of strength B is applied perpendicular to the interface (in y -direction) see Fig.1. The basic equations governing the motion of an incompressible, non-Newtonian couple stress fluid through porous medium are given by the following equations:

The continuity equation

$$\nabla \cdot \underline{q} = 0 \tag{1}$$

The momentum equation

$$\rho \left[\frac{\partial \underline{q}}{\partial t} + (\underline{q} \cdot \nabla) \underline{q} \right] = -\nabla p + \mu \nabla^2 \underline{q} + \nabla \cdot \underline{\tau} - \lambda^* \nabla^4 \underline{q} - \frac{\mu}{k^*} \underline{q} + (\underline{J} \wedge \underline{B}) \tag{2}$$

where

$$\left. \begin{aligned} \underline{J} &= \sigma [\underline{E} + \underline{q} \wedge \underline{B}] + \rho_e \underline{q}, \\ \underline{B} &= B \underline{j}, \quad \underline{q} = (u(y), v, 0) \end{aligned} \right\} \tag{3}$$

$\underline{\tau}(\tau_{ij})$ is the stress tensor containing the bi-viscosity casson effect, Nakamura (1988). The bi-viscosity model is constitutive equation of blood. It may be present as follows:

$$\tau_{ij} = \begin{cases} 2(\mu_\beta + \frac{p_y}{\sqrt{2\pi}}) e_{ij}, \pi \geq \pi_c \\ 2(\mu_\beta + \frac{p_y}{\sqrt{2\pi_c}}) e_{ij}, \pi \leq \pi_c \end{cases} \tag{4}$$

where $\pi = e_{ij} e_{ji}$ and e_{ij} is the (i, j)th.

Components of the deformation rate given by,

$$e_{ij} = \frac{1}{2} \left(\frac{\partial q_i}{\partial x_j} + \frac{\partial q_j}{\partial x_i} \right), \tag{5}$$

π_c is a critical value of π based on the Nakamura-Sawada model (1988), μ_β is plastic dynamic viscosity of the Non-Newtonian fluid, p_y is yield

stress of Slurry fluid. This model allows a relatively easy incorporation into the Navier-Stokes framework, as does the porous media formulation in Equation. (2). \underline{q} is the velocity, p is the pressure, ρ is the density of the fluid, t is the time, μ is the coefficient of viscosity, λ^* is the couple stress coefficient, k^* is the permeability of porous medium, \underline{B} is the magnetic field, σ is the electrical conductivity, \underline{E} is the electric field strength, \underline{J} is the electric current density and ρ_e is the density of charge. According to the simplification assumptions in Rudraiah (2010) we shall assume that the Strouhal number, S , which is the measure of local acceleration to the inertial acceleration, is $S = \frac{L^*}{UT} \ll 1$, where $U = \frac{v}{L^*}$ is the

characteristic velocity, $v = \frac{\mu}{\rho}$ is the kinematic viscosity, $L^* = \sqrt{\frac{\gamma}{\delta}}$ is the characteristic length, γ is the surface tension, $\delta = g(\rho_1 - \rho_2)$, g is the gravitational acceleration and $T = \frac{\mu\gamma}{a^3\delta^2}$ is the

characteristic time scale. The assumption $S \ll 1$ enabled us to neglect the local acceleration term $\frac{\partial u}{\partial t}$ also we can let $\frac{\partial v}{\partial t} = 0$, also we can neglect the inertial acceleration term $(v \frac{\partial u}{\partial y})$ Comparing with the high viscous couple stress fluid. The interface elevation, η , is assumed to be small compared with film thickness $2a$ that is $\frac{\eta}{a} \ll 1$,

These assumptions enabled us to use the creeping flow approximation, which allows us to neglect certain terms in the perturbation equation to obtain linear equations for the interface elevation. Under these assumptions Eq. (1) and Eq. (2), using Eq. (3), Eq. (4) and Eq. (5) become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

$$0 = -\frac{\partial p}{\partial x} + (\mu + \mu\beta) \frac{d^2 u}{dy^2} \tag{7}$$

$$-\lambda^* \frac{d^4 u}{dy^4} - \frac{\mu}{k^*} u - \sigma B^2 u \tag{8}$$

$$0 = \frac{\partial p}{\partial y} - \frac{\mu}{k^*} v \tag{8}$$

According to Rudraiah (2010) the boundary conditions appropriate for the problem under study is the no-slip condition at the rigid surface:

$$u, v = 0 \text{ at } y = -a \tag{9}$$

$$\frac{\partial u}{\partial y} = 0 \text{ at } y = a \tag{10}$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \text{ at } y = \pm a \tag{11}$$

$$p = -\delta\eta - \gamma \frac{\partial^2 \eta}{\partial x^2}, \text{ at } y = a, \tag{12}$$

$$v = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x}, \text{ at } y = a \tag{13'}$$

For linear case Eq. (13') reduces to

$$v = \frac{\partial \eta}{\partial t}, \text{ at } y = a \tag{13}$$

Introducing the non-dimensional variables

$$\left. \begin{aligned} v &= \frac{\delta a^2}{\mu} v', u = \frac{\delta a^2}{\mu} u', p = \delta a p', \eta = a \eta', \\ t &= \frac{\mu}{\delta a} t', x = a x', y = a y', M = \frac{\sigma B^2 a^2}{\mu}, \\ k &= a^2 k', \lambda = \mu a^2 \lambda', \alpha = \frac{\mu \beta}{\mu} \end{aligned} \right\} \tag{14}$$

Equation. (7) and Eq. (8) in dimensionless form become:

$$\lambda D^4 u - (1 + \alpha) D^2 u + (M + \frac{1}{k}) u = -\frac{\partial p}{\partial x} \tag{15}$$

And

$$\frac{\partial p}{\partial y} = \frac{1}{k} v \tag{16}$$

With the boundary conditions:

$$\left. \begin{aligned} u, v &= 0 \text{ at } y = -1, \\ \frac{du}{dy} &= 0 \text{ at } y = 1, \\ \frac{d^2 u}{dy^2} &= 0 \text{ at } y = \pm 1, \\ p &= -\eta - \frac{1}{\beta} \frac{\partial^2 \eta}{\partial x^2} \text{ at } y = 1, \\ v &= \frac{\partial \eta}{\partial t} \text{ at } y = 1 \end{aligned} \right\} \tag{17}$$

Where λ (couple stress coefficient), k (permeability of the porous medium), M (Magnetic parameter), $\beta = \frac{\delta a^2}{\gamma}$ is the Bond

number and α is the non-Newtonian Parameter. Solutions of Eq. (15) and Eq. (16), using boundary conditions Eq. (17) is:

$$u(y) = \left[\left(\frac{-k}{kM+1} \right) \left(\frac{\partial p}{\partial x} \right) \right] + c_1 e^{Ay} + c_2 e^{-Ay} + c_3 e^{by} + c_4 e^{-by} \tag{18}$$

Where:

$$c_1 = \left(\frac{k}{kM+1} \right) \left(\frac{\partial p}{\partial x} \right) \Delta_1 / \Delta, \quad c_2 = \left(\frac{-k}{kM+1} \right) \left(\frac{\partial p}{\partial x} \right) \Delta_2 / \Delta, \quad c_3 = \left(\frac{k}{kM+1} \right) \left(\frac{\partial p}{\partial x} \right) \Delta_3 / \Delta, \quad c_4 = \left(\frac{-k}{kM+1} \right) \left(\frac{\partial p}{\partial x} \right) \Delta_4 / \Delta,$$

$$\Delta_1 = \begin{vmatrix} -Ae^{-A} & be^b & -be^{-b} \\ A^2e^{-A} & b^2e^b & b^2e^{-b} \\ A^2e^A & b^2e^{-b} & b^2e^b \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} Ae^A & be^b & -be^{-b} \\ A^2e^A & b^2e^b & b^2e^{-b} \\ A^2e^{-A} & b^2e^{-b} & b^2e^b \end{vmatrix}, \quad \Delta_3 = \begin{vmatrix} Ae^A & -Ae^{-A} & -be^{-b} \\ A^2e^A & A^2e^{-A} & b^2e^{-b} \\ A^2e^{-A} & A^2e^A & b^2e^b \end{vmatrix}, \quad \Delta_4 = \begin{vmatrix} Ae^A & -Ae^{-A} & be^b \\ A^2e^A & A^2e^{-A} & b^2e^b \\ A^2e^{-A} & A^2e^A & b^2e^{-b} \end{vmatrix},$$

$$\Delta = \begin{vmatrix} e^{-A} & e^A & e^{-b} & e^b \\ Ae^A & -Ae^{-A} & be^b & -be^{-b} \\ A^2e^A & A^2e^{-A} & b^2e^b & b^2e^{-b} \\ A^2e^{-A} & A^2e^A & b^2e^{-b} & b^2e^b \end{vmatrix},$$

$$A = \frac{1}{\sqrt{2}} \left\{ \frac{1}{\lambda} + \frac{\alpha}{\lambda} - \left[(-4k\lambda(kM+1) + (k+k\alpha)^2 / k\lambda) \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}}$$

$$b = \frac{1}{\sqrt{2}} \left\{ \frac{1}{\lambda} + \frac{\alpha}{\lambda} + \left[(-4k\lambda(kM+1) + (k+k\alpha)^2 / k\lambda) \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}} \tag{19}$$

By using Eq. (18) and Eq. (19), we can write:

$$u(y) = \left[\left(\frac{k}{kM+1} \right) \left(\frac{\partial p}{\partial x} \right) \right] \left\{ -1 + \frac{\Delta_1}{\Delta} e^{Ay} - \frac{\Delta_2}{\Delta} e^{-Ay} + \frac{\Delta_3}{\Delta} e^{by} - \frac{\Delta_4}{\Delta} e^{-by} \right\} \tag{20}$$

3. DISPERSION RELATION:

According to Rudraiah (2010), solving the continuity Eq. (6) by integrate it with respect to y from $y=-1$ to $y=1$ we get:

$$v(1) = \left(\frac{-2k}{kM+1} \right) \left(\frac{\partial^2 p}{\partial x^2} \right) \left\{ -1 + \left(\frac{\Delta_1 - \Delta_2}{A\Delta} \right) \sinh A + \left(\frac{\Delta_3 - \Delta_4}{b\Delta} \right) \sinh b \right\} \tag{21}$$

Using the boundary conditions Eq. (17) and let $\eta = \eta_l e^{ilx + nt}$, hence substituting in Eq. (21) we obtain the following dispersion relation:

$$n = \left(\frac{2kl^2}{kM+1} \right) \left(1 - \frac{l^2}{\beta} \right) \left[1 - \left(\frac{\Delta_1 - \Delta_2}{A\Delta} \right) \sinh A - \left(\frac{\Delta_3 - \Delta_4}{b\Delta} \right) \sinh b \right] \tag{22}$$

Where l the wave number and n is the frequency represents the growth rate of the interface as a function of couple stress parameters (λ), the non-Newtonian parameter (α), the magnetic Parameter (M) and the permeability of the porous medium (k). In the absence of couple stress parameter (λ) the growth rate Eq. (22) takes the following form:

$$n_b = \left(\frac{2kl^2}{kM+1} \right) \left(1 - \frac{l^2}{\beta} \right) \tag{23}$$

Which depends only on the magnetic parameter (M), the permeability of the porous medium (k), wave number (l) and Bond number (β) which is the reciprocal of surface tension.

4. RESULTS AND DISCUSSION:

The analytical solution obtained in the present investigation by using normal mode technique. The critical cut-off wave number obtained from the dispersion relation Eq. (22) when we equate it by zero as in the following Eq. (24)

$$l_{ct} = \sqrt{\beta} \tag{24}$$

However, at $\frac{\partial n}{\partial l}=0$, the maximum wave number can be written as

$$l_{\max} = \frac{l_{ct}}{\sqrt{2}} \tag{25}$$

We can find the maximum growth rate in the form

$$n_{\max} = \frac{-k\beta}{2(kM+1)} \left[-1 + \left(\frac{\Delta_1 - \Delta_2}{A\Delta} \right) \sinh A + \left(\frac{\Delta_3 - \Delta_4}{b\Delta} \right) \sinh b \right] \tag{26}$$

And

$$n_{b\max} = \frac{k\beta}{2(kM+1)} \tag{27}$$

The ratio of the growth rates G_{\max} , can be obtained from Eq. (26) and Eq. (27) as:

$$G_{\max} = \frac{n_{\max}}{n_{b\max}} = \left[1 - \left(\frac{\Delta_1 - \Delta_2}{A\Delta} \right) \sinh A - \left(\frac{\Delta_3 - \Delta_4}{b\Delta} \right) \sinh b \right] \tag{28}$$

The effects of couple stress parameter λ , magnetic parameter M , permeability parameter k , non-Newtonian parameter α , and Bond number β on the growth rate (frequency) n and the ratio of growth rate G_{\max} are illustrated graphically through a set of Figures. Figure 2 is used to show the influence of the couple-stress parameter λ on the frequency n . It is clear that the frequency parameter n decreases with increasing the value of couple-stress parameter λ . Fig. 3 illustrates the dependence of the frequency parameter n on the magnetic parameter M . It is clear that n decreases when M increases, while in Fig. 4 we can conclude that the frequency n increases when the permeability parameter k increases. Fig. 5 depicts that the frequency parameter n decreases with increasing of the Bond number β . From Figs. 6 and 7 we found that the increase in the non-Newtonian parameter α causes decreasing in the frequency parameter n . Figs. 8 and 9 represent the effect of couple-stress parameter λ on the ratio of growth rate G_{\max} . The ratio of growth rate G_{\max} decreases up to ($\lambda \geq 2.8$) then increases up to ($\lambda \geq 5.5$) with increasing the couple stress parameter λ . A study of Figs. 10 and 11

reveals that the ratio of growth rate G_{\max} increases with the magnetic parameter M increasing. From Figs. 12 and 13 we found that the increases of the permeability parameter k implying increases in the ratio of growth rate G_{\max} . Figs. 14 is used to show the influence of the non-Newtonian parameter α on the ratio of growth rate G_{\max} . The ratio G_{\max} increases with increasing the non-Newtonian parameter α .

5. CONCLUSION

A theoretical study of the Rayleigh-Taylor instability of the flow of Casson fluids with couple stresses through porous medium under the effect of a magnetic field is investigated. This problem is modulated mathematically by a system of differential equations which governing the motion of the fluids with appropriate boundary conditions. The system of equations are solved under some assumptions to obtain the dispersion relation. The effects of the physical parameters of the problem such as couple stress parameter λ , magnetic parameter M , non-Newtonian parameter α , permeability parameter k , and Bond number β on the regions of stability are discussed numerically and illustrated graphically through a set of figures. It is clear that the effect of these parameter is to stabilize the interface between the two fluids, while the permeability parameter k have destabilizing effect.

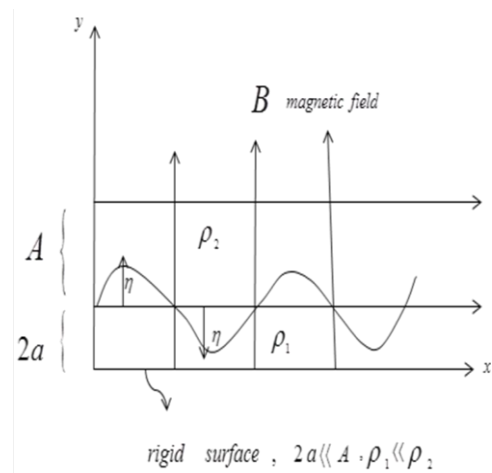


Fig. 1. Magnetic field of strength B applied perpendicular to the interface

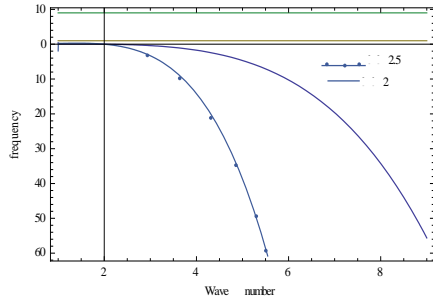


Fig. 2. Frequency is plotted against wave number for different values of λ when $\alpha = 5, k = 2.5, M = 3, \beta = 4$

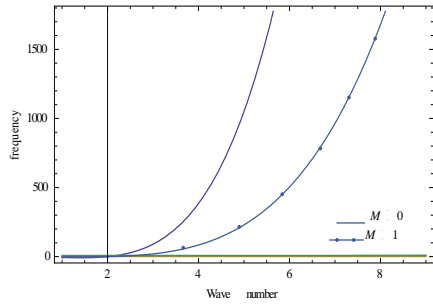


Fig. 3. Frequency is plotted against wave number for different values of M when $\alpha = 5, k = 2.5, \lambda = 2, \beta = 4$

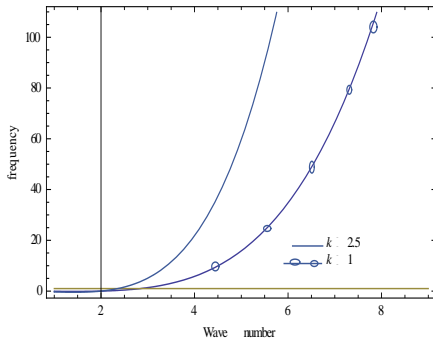


Fig. 4. Frequency is plotted against wave number for different values of k when $\alpha = 5, \lambda = 2, M = 2, \beta = 4$

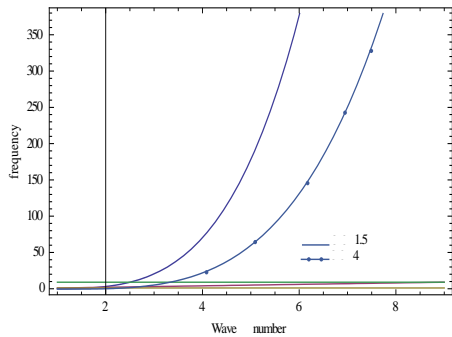


Fig. 5. Frequency is plotted against wave number for different values of β when $\alpha = 5, k = 2.5, M = 2, \lambda = 2$

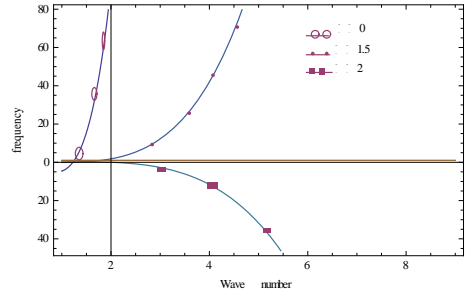


Fig. 6. Frequency is plotted against wave number for different values of α when $\lambda = 2, k = 2.5, M = 2, \beta = 4$

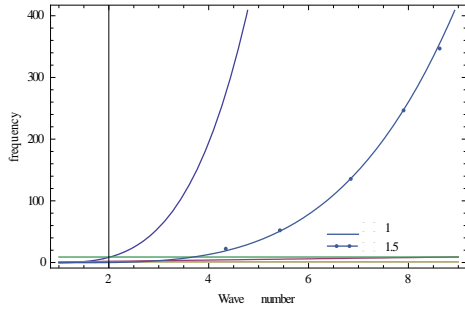


Fig. 7. Frequency is plotted against wave number for different values of α when $\lambda = 2, k = 2.5, M = 2, \beta = 4$

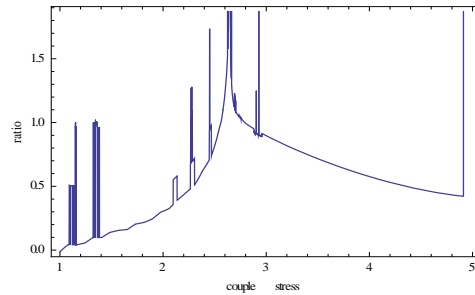


Fig. 8. Ratio of growth rate is plotted against λ when $\alpha = 5, k = 2.5, M = 3, l = 2$

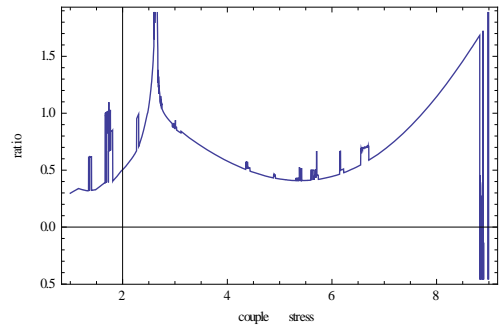


Fig. 9. Ratio of growth rate is plotted against λ when $\alpha = 5, k = 2.5, M = 3, l = 2$

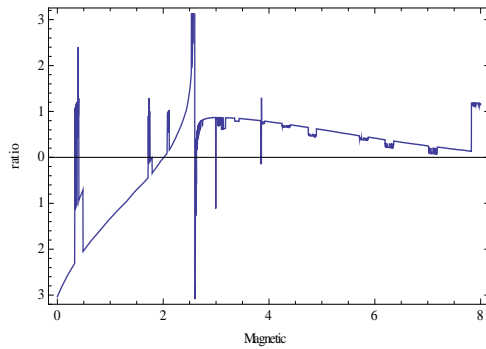


Fig. 10. Ratio of growth rate is plotted against M when $\alpha = 5, k = 2.5, \lambda = 3, l = 2$

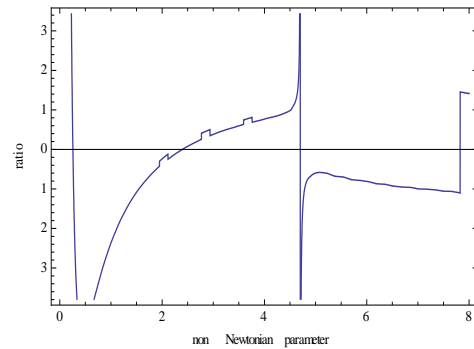


Fig. 14. Ratio of growth rate is plotted against α when $k = 5, M = 2.5, \lambda = 3, l = 2$

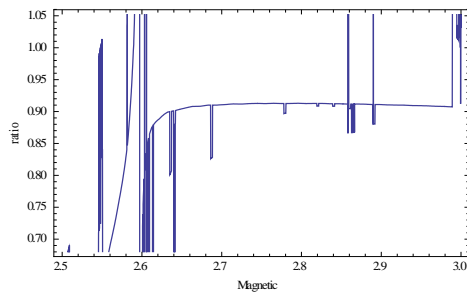


Fig. 11. Ratio of growth rate is plotted against M when $\alpha = 5, k = 2.5, \lambda = 3, l = 2$

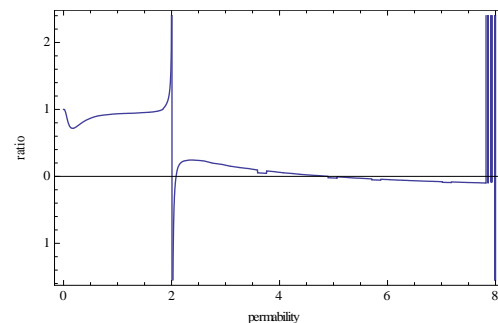


Fig. 12. Ratio of growth rate is plotted against k When $\alpha = 5, M = 2.5, \lambda = 3, l = 2$

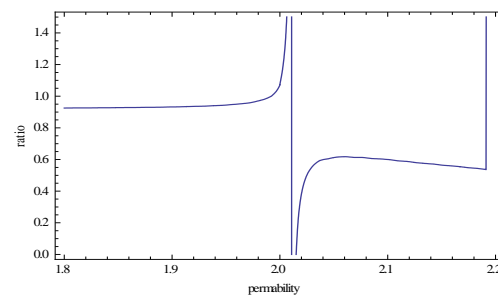


Fig. 13. Ratio of growth rate is plotted against k when $\alpha = 5, M = 2.5, \lambda = 3, l = 2$

REFERENCES

- Abdel Raouf F. Elhefnawy Bothaina, M. H.Agoor and Abd Elmonem Khalil Elcoot (2001). Nonlinear electrohydrodynamic stability of a finitely conducting jet under an axial electric field. *Physica A* 6018,1-21.
- Abou El Magd A. Mohamed and Nabil, T.EIDabe (1978). Electro hydrodynamic stability a hollow jet. *Journal of the mathematical and physical society of Egypt*,45.
- Bhatia, P. K. (1974).Rayleigh-Taylor instability of two viscous superposed conducting fluids. *Nuovo cim.* 19B, 161 .
- Chandrasekhar, S. (1981).hydrodynamic and hydro magnetic stability, Dover Publication, New York.
- Dash, R. K. and K. N. Mehta(1996). Casson fluid flow in a pipe fluid with a homogeneous porous medium. *Journal Engn Sci.* 34(10), 1145-1156.
- Kent, A. (1966).Instability of laminar flow of a perfect magnetofluid. *Phys. Fluids* 9,1286.
- Nabil, T. ElDabe (1988). Electro hydrodynamic stability of two superposed elasticoviscous liquids in plane couette flow. *Journal of mathematical physics*, 29(1),2790-2800.
- Nabil, T. ElDabe (1988). Electro hydrodynamic stability of two stratified power law liquids in couette flow. *Il Nuovo cimento-vol.101B(2)* 221-235.
- Nabil, T. ElDabe (1989).Effect of a tangential electric field on Rayleigh-Taylor instability.

- Journal of the physical society of Japan* 58(1), 115-120.
- Nakamura, M. and T. Sawada (1988). Numerical study on the flow of a non-Newtonian fluid through an axis symmetric stenosis. *Asme J. Biomechanical Engineering*, 110(2), 137-143.
- Rudraiah, N. and G. Chandrashekhara (2010). Effects of couple-stress on the growth rate of Rayleigh-Taylor instability at the interface in a finite thickness couple-stress fluid. *Journal of applied Fluid Mechanics*, 3(1), 83-89.
- Sharma, R. C. and K. C. Sharma (1978). Rayleigh-Taylor instability of two superposed conducting fluids in presence of suspended particles. *Acta physica Hungarica* 45, 213
- Sunil, R. C. Sharma and R. S. Chandel (2002). On superposed couple-stress fluids in porous medium in hydro magnetics. *Z. Naturforsch* 57a, 955-960.