



Effect of Homogeneous and Heterogeneous Chemical Reactions on Peristaltic Transport of a Jeffrey Fluid through a Porous Medium with Slip Condition

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ABSTRACT

In this paper, the dispersion of solute matter in a Jeffrey fluid flow through a porous medium in a peristaltic channel has been investigated under the influence of slip boundary conditions. Long wavelength approximation and Taylor's limiting condition are used to obtain the average effective dispersion coefficient in both the cases of homogeneous and heterogeneous chemical reactions. The effects of various pertinent parameters on the effective dispersion coefficient are discussed. Average effective dispersion coefficient increases with amplitude ratio. That is, more dispersion in the presence of peristalsis. Further, the average effective dispersion coefficient increases with the permeability parameter and the slip parameter; but decreases with the Jeffrey number, homogeneous / heterogeneous chemical reaction rate parameter.

Keywords: Dispersion; Peristalsis; Jeffrey fluid; Porous medium; Chemical reaction; Slip condition.

1. INTRODUCTION

The dispersion of a soluble matter in fluids has many biological applications especially in the study of blood circulation. Dispersion of a solute in a viscous fluid under different conditions was studied by several authors (Taylor 1953; Aris 1956; Padma and Ramana Rao 1976; Gupta and Gupta 1972; Ramana Rao and Padma 1975). Subsequently, Chandra and Agarwal (1983), Philip and Chandra (1993), Alemayehu and Radhakrishnamacharya (2012a, 2012b) and Kumar *et al.* (2012) extended the analysis of Taylor to non-Newtonian fluids. Further, a number of authors have studied the dispersion of a solute through a porous medium under different conditions (Mehta and Tiwari 1988; Pal 1999; Rudraiah and Reddy 2013). Flow through porous media has various physiological applications such as the flow of blood in the micro-vessels of the lungs which may be treated as a channel bounded by two thin porous layers (Misra and Ghosh 1997).

Peristalsis is an important mechanism for fluid transport which can be generated by the propagation of waves along the walls of a flexible channel containing fluid. Physiologically, it is known as an automatic and vital process that drives the urine from kidney to the bladder through the ureter, food through the digestive tract, bile from the gall bladder into the duodenum, vasomotion in small blood vessels and many others. The peristaltic action is also exploited in industrial pumping as it

provides efficient means for sanitary fluid transport, noxious fluid transport in nuclear industries, as well as in roller pumps. Several investigators have analyzed the peristaltic motion of both Newtonian and non-Newtonian fluids in mechanical as well as physiological systems (Fung and Yih 1968; Shehawy and Sebaei 2000; Takagi and Balmforth 2011; Radhakrishnamacharya 1982; Rao and Mishra 2004; Bohme and Muller 2013).

It is realized that the Jeffrey fluid model can be used to represent a physiological fluid. This model is a relatively simpler linear model which uses time derivatives instead of convective derivatives. It represents a rheology different from that of Newtonian fluid. Further, Jeffrey fluid model is significant because Newtonian fluid model can be deduced from this as a special case. Some researchers have studied peristaltic motion of Jeffrey fluid under different conditions. (Hayat *et al.* 2012; Vajravelu *et al.* 2011; Pandey and Tripathi 2010; Kothandapani and Srinivas 2008).

It is realized that fluid slips at the walls in certain physiological and engineering situations. The no slip boundary condition is a core concept in fluid dynamics in which the fluid and the boundary move with same velocity. Beaver and Joseph (1967) were the first to propose slip boundary condition. The boundary condition proposed by Beaver and Joseph was simplified by Saffman (1971). The existence of slip phenomenon at the boundaries and interfaces has been observed in the flows of rarefied gases,

physiological flows, hypersonic flows of chemically reacting binary mixture etc. Also flows with slip occur for certain problems in chemical engineering, for example, flows through pipes in which chemical reactions occur at the walls, certain two-phase flows and flows in porous slider bearings.

The effect of homogeneous and heterogeneous chemical reactions on dispersion of a solute in peristaltic motion of a Jeffrey fluid through a porous medium with slip condition has not received any attention. It is realized that porosity and peristalsis may have significant effect on the dispersion of a solute in the fluid flow and this may lead to better understanding of the flow situation in physiological systems. Hence, in this paper the effect of homogeneous and heterogeneous chemical reactions on peristaltic transport of a Jeffrey fluid through a porous medium with slip condition is investigated. Long wavelength approximation and Taylor's approach are used in both the cases of homogeneous and heterogeneous irreversible chemical reactions to obtain an analytical solution for the average effective dispersion coefficient and the effects of various relevant parameters on it are studied.

2. FORMULATION OF THE PROBLEM

Consider the dispersion of a solute in peristaltic flow of a Jeffrey fluid in an infinite uniform channel of width $2d$ and with flexible walls on which are imposed traveling sinusoidal waves of long wavelength. Cartesian coordinate system (x, y) is chosen with x -axis aligned with the center line of the channel. The traveling waves are represented by

$$y = \pm h = \pm \left[d + a \sin \frac{2\pi}{\lambda} (x - ct) \right] \quad (1)$$

where a is the amplitude, c is the speed and λ is the wavelength of the peristaltic wave (Fig. 1).

The equations governing two-dimensional motion of an incompressible Jeffrey fluid for the present problem are given by (Kothandapani and Srinivas 2008)

$$\rho \left[\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right] u = - \frac{\partial p}{\partial x} + \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \frac{\mu u}{k_0} \quad (2)$$

$$\rho \left[\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right] v = - \frac{\partial p}{\partial y} + \frac{\partial S_{yy}}{\partial y} + \frac{\partial S_{yx}}{\partial x} \quad (3)$$

and the equation of continuity is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4)$$

where u, v are the velocity components in the x and y directions respectively, p is the pressure, ρ is the density, $S_{xx}, S_{xy}, S_{yx}, S_{yy}$ are extra stress components, μ is the viscosity coefficient and k_0 is permeability of the medium.

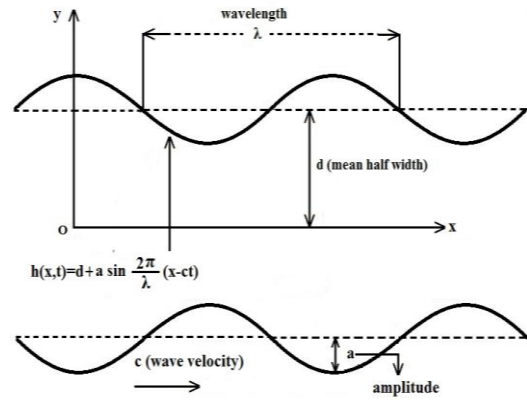


Fig. 1. Geometry of the problem

Under long wavelength approximation, the governing equations for the present problem reduce to,

$$\frac{\partial p}{\partial x} = \frac{\mu}{1 + \lambda_1} \frac{\partial^2 u}{\partial y^2} - \frac{\mu}{k_0} u \quad (5)$$

$$\frac{\partial p}{\partial y} = 0 \quad (6)$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (7)$$

Following Alemayehu and Radhakrishnamacharya (2012a, 2012b), it is assumed that the walls are inextensible so that only lateral motion takes place and the horizontal displacement of the wall is zero.

The relevant slip boundary conditions are given by (Bhatt and Sacheti 1979)

$$u = - \frac{d\sqrt{Da}}{\gamma} \frac{\partial u}{\partial y} \quad \text{at } y = \pm h \quad (8)$$

where Eqs. (8) are Saffman's slip boundary conditions, Da is the permeability parameter (or Darcy number) and γ is the slip parameter.

Solving Eqs. (5) and (6) under the boundary conditions Eq. (8), we get

$$u(y) = \left(\frac{k_0}{\mu} \frac{\partial p}{\partial x} \right) \left[\frac{\cosh(by)}{\cosh(bh) + \frac{d\sqrt{Da}}{\gamma} b \sinh(bh)} - 1 \right] \quad (9)$$

where

$$b^2 = \frac{1 + \lambda_1}{k_0} \quad (10)$$

Further, the mean velocity is defined as

$$\bar{u} = \frac{1}{2h-h} \int u(y)dy \tag{11}$$

Substituting Eq. (9) in Eq. (11), we get

$$\bar{u} = \left(-\frac{k_0}{\mu} \frac{\partial p}{\partial x} \right) \left[1 - \frac{1}{bh} \frac{\sinh(bh)}{\cosh(bh) + \frac{d\sqrt{Da}}{\gamma} b \sinh(bh)} \right] \tag{12}$$

If we now consider convection across a plane moving with the mean speed of the flow, then relative to this plane, the fluid velocity is given by (Gupta and Gupta 1972; Alemayehu and Radhakrishnamacharya 2012a, 2012b)

$$u_x = u - \bar{u} \tag{13}$$

Substituting Eqs. (9) and (12) in Eq. (13), we get

$$u_x = \left(-\frac{k_0}{\mu} \frac{\partial p}{\partial x} \right) \left[\frac{1}{\cosh(bh) + \frac{d\sqrt{Da}}{\gamma} b \sinh(bh)} \right] \left[\frac{\sinh(bh)}{bh} - \cosh(by) \right] \tag{14}$$

2.1 Diffusion with a Homogeneous First order Chemical Reaction

Following Taylor (1953) and Gupta and Gupta (1972), the equation for the concentration of the solute for the present problem under isothermal conditions is given by

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial y^2} - k_1 C \tag{15}$$

where D is the molecular diffusion co-efficient and k_1 is the first order reaction rate constant.

For typical values of physiologically relevant parameters of this problem, it is realized that $\bar{u} = c$ (Alemayehu and Radhakrishnamacharya 2012a, 2012b). Using this condition and making use of the following dimensionless quantities,

$$\theta = \frac{t}{\bar{t}}, \bar{t} = \frac{\lambda}{\bar{u}}, \eta = \frac{y}{d}, \xi = \frac{(x - \bar{u}t)}{\lambda}, H = \frac{h}{d} \tag{16}$$

Eq. (14) reduces to

$$u_x = \left(-\frac{d^2}{\mu} \frac{\partial p}{\partial x} \right) \left[m_1 \cosh\left(\sqrt{\frac{1+\lambda_1}{Da}} \eta \right) + m_2 \right] \tag{17}$$

where

$$m_1 = -\frac{Da}{A}, \quad m_2 = \frac{Da}{A} \frac{\sinh\left(\sqrt{\frac{1+\lambda_1}{Da}} H\right)}{H \sqrt{\frac{1+\lambda_1}{Da}}}, \quad Da = \frac{k_0}{d^2}$$

and

$$A = \cosh\left(\sqrt{\frac{1+\lambda_1}{Da}} H\right) + \frac{\sqrt{1+\lambda_1}}{\gamma} \sinh\left(\sqrt{\frac{1+\lambda_1}{Da}} H\right).$$

Further, Eq. (15) becomes

$$\frac{\partial^2 C}{\partial \eta^2} - \frac{k_1 d^2}{D} C = \frac{d^2}{\lambda D} u_x \frac{\partial C}{\partial \xi} \tag{18}$$

It is assumed that there is no absorption at the walls as in Alemayehu and Radhakrishnamacharya (2012a, 2012b), thus the boundary conditions for the concentration C are:

$$\frac{\partial C}{\partial \eta} = 0 \quad \text{for} \quad \eta = \pm H = \pm [1 + \varepsilon \sin(2\pi\xi)] \tag{19}$$

where $\varepsilon = \frac{a}{d}$ is the amplitude ratio.

Solving Eq. (18) under the boundary conditions Eq.

(19) by assuming $\frac{\partial C}{\partial \xi}$ is independent of η at any

cross section, we get the solution for the concentration of the solute C as

$$C(\eta) = \left[\frac{d^4}{\lambda \mu D} \frac{\partial p}{\partial x} \frac{\partial C}{\partial \xi} \right] \left[l_1 \cosh(\alpha \eta) + l_2 \cosh\left(\sqrt{\frac{1+\lambda_1}{Da}} \eta\right) + l_3 \right] \tag{20}$$

where

$$l_1 = -\frac{Da}{A} \frac{\sqrt{(1+\lambda_1)Da}}{1+\lambda_1 - \alpha^2 Da} \frac{\sinh\left(\sqrt{\frac{1+\lambda_1}{Da}} H\right)}{\alpha \sinh(\alpha H)},$$

$$l_2 = \frac{Da}{A} \frac{Da}{1+\lambda_1 - \alpha^2 Da} \text{ and}$$

$$l_3 = \frac{Da}{A} \frac{\sinh\left(\sqrt{\frac{1+\lambda_1}{Da}} H\right)}{\alpha^2 H \sqrt{\frac{1+\lambda_1}{Da}}}.$$

The volumetric rate Q at which the solute is transported across a section of the channel of unit breadth is defined by

$$Q = \int_{-H}^H C u_x d\eta \tag{21}$$

Substituting Eqs. (17) and (20) in Eq. (21), we get the volumetric rate Q as

$$Q = -2 \frac{d^6}{\lambda \mu^2 D} \frac{\partial C}{\partial \xi} \left(\frac{\partial p}{\partial x} \right)^2 F(\xi, \alpha, \varepsilon, \lambda_1, Da, \gamma) \quad (22)$$

where

$$F(\xi, \alpha, \varepsilon, \lambda_1, Da, \gamma) = \left[\frac{l_1 m_1 Da}{\alpha^2 Da - (1 + \lambda_1)} \right] \left[\alpha \cosh(H \sqrt{\frac{1 + \lambda_1}{Da}}) \sinh(\alpha H) - \sqrt{\frac{1 + \lambda_1}{Da}} \cosh(\alpha H) \sinh(H \sqrt{\frac{1 + \lambda_1}{Da}}) \right] + l_1 m_2 \left[\frac{\sinh(\alpha H)}{\alpha} \right] + l_2 m_1 \left[\frac{H}{2} + \frac{\sinh(2H \sqrt{\frac{1 + \lambda_1}{Da}})}{4 \sqrt{\frac{1 + \lambda_1}{Da}}} \right] + (l_2 m_2 + l_3 m_1) \left[\frac{\sinh(H \sqrt{\frac{1 + \lambda_1}{Da}})}{\sqrt{\frac{1 + \lambda_1}{Da}}} \right] + l_3 m_2 H. \quad (23)$$

Now comparing the Eq. (22) with Fick's first law of diffusion, we get the equivalent dispersion coefficient D^* with which the solute disperses relative to a plane moving with the mean speed of the flow is obtained as,

$$D^* = 2 \frac{d^6}{\mu^2 D} \left(\frac{\partial p}{\partial x} \right)^2 F(\xi, \alpha, \varepsilon, \lambda_1, Da, \gamma) \quad (24)$$

Let the average of F be \bar{F} , and is defined by

$$\bar{F} = \frac{1}{0} \int F(\xi, \alpha, \varepsilon, \lambda_1, Da, \gamma) d\xi \quad (25)$$

2.2 Diffusion with Combined Homogeneous and Heterogeneous Chemical Reactions

In this case, we assume that in addition to a first order irreversible chemical reaction in the bulk of the fluid (homogeneous), there is also similar one at the walls (heterogeneous) which are assumed to be catalytic. Then under Taylor's limiting condition, Eq. (15) still governs the diffusion of the solute.

The boundary conditions at the walls (Philip and Chandra 1993) are

$$\frac{\partial C}{\partial y} + f C = 0 \text{ at } y = h = [d + a \sin \frac{2\pi}{\lambda} (x - \bar{u}t)] \quad (26)$$

$$\frac{\partial C}{\partial y} - f C = 0 \text{ at } y = -h = -[d + a \sin \frac{2\pi}{\lambda} (x - \bar{u}t)] \quad (27)$$

If we introduce the dimensionless variables Eq. (16), the diffusion equation remains as in Eq. (18) and the boundary conditions become

$$\frac{\partial C}{\partial \eta} + \beta C = 0 \text{ at } \eta = H = [1 + \varepsilon \sin(2\pi\xi)] \quad (28)$$

$$\frac{\partial C}{\partial \eta} - \beta C = 0 \text{ at } \eta = -H = -[1 + \varepsilon \sin(2\pi\xi)] \quad (29)$$

where $\beta = fd$ is the heterogeneous reaction rate parameter corresponding to catalytic reaction at the walls.

The solution of Eq. (18) under the boundary conditions Eqs. (28) and (29) is

$$C(\eta) = \left[\frac{d^4}{\lambda \mu D} \frac{\partial p}{\partial x} \frac{\partial C}{\partial \xi} \right] \left[n_1 \cosh(\alpha \eta) + n_2 \cosh\left(\sqrt{\frac{1 + \lambda_1}{Da}} \eta\right) + n_3 \right] \quad (30)$$

where

$$n_1 = -\frac{Da}{AL} \left[\frac{\sqrt{(1 + \lambda_1) Da}}{1 + \lambda_1 - \alpha^2 Da} \sinh\left(\sqrt{\frac{1 + \lambda_1}{Da}} H\right) + \beta \left(\frac{\sinh\left(\sqrt{\frac{1 + \lambda_1}{Da}} H\right)}{\alpha^2 H \sqrt{\frac{1 + \lambda_1}{Da}}} + \frac{Da}{1 + \lambda_1 - \alpha^2 Da} \cosh\left(H \sqrt{\frac{1 + \lambda_1}{Da}}\right) \right) \right]$$

$$n_2 = \frac{Da}{A} \frac{Da}{1 + \lambda_1 - \alpha^2 Da},$$

$$n_3 = \frac{Da}{A} \frac{\sinh\left(\sqrt{\frac{1 + \lambda_1}{Da}} H\right)}{\alpha^2 H \sqrt{\frac{1 + \lambda_1}{Da}}}$$

and $L = \alpha \sinh(\alpha H) + \beta \cosh(\alpha H)$.

The volumetric rate Q at which the solute is transported across a section of the channel of unit breadth is defined by

$$Q = \int_{-H}^H C u_x d\eta \quad (31)$$

Substituting Eqs. (17) and (30) in Eq. (31), we get the volumetric rate Q as

$$Q = -2 \frac{d^6}{\lambda \mu^2 D} \frac{\partial C}{\partial \xi} \left(\frac{\partial p}{\partial x} \right)^2 G(\xi, \alpha, \beta, \varepsilon, \lambda_1, Da, \gamma) \quad (32)$$

where

$$\begin{aligned}
 G(\xi, \alpha, \beta, \varepsilon, \lambda_1, Da, \gamma) = & \\
 & \left[\frac{n_1 m_1 Da}{\alpha^2 Da - (1 + \lambda_1)} \right] \left[\alpha \cosh\left(H \sqrt{\frac{1 + \lambda_1}{Da}}\right) \sinh(\alpha H) - \right. \\
 & \left. \sqrt{\frac{1 + \lambda_1}{Da}} \cosh(\alpha H) \sinh\left(H \sqrt{\frac{1 + \lambda_1}{Da}}\right) \right] + \\
 & n_1 m_2 \left[\frac{\sinh(\alpha H)}{\alpha} \right] + n_2 m_1 \left[\frac{H}{2} + \frac{\sinh\left(2H \sqrt{\frac{1 + \lambda_1}{Da}}\right)}{4 \sqrt{\frac{1 + \lambda_1}{Da}}} \right] \\
 & + (n_2 m_2 + n_3 m_1) \left[\frac{\sinh\left(H \sqrt{\frac{1 + \lambda_1}{Da}}\right)}{\sqrt{\frac{1 + \lambda_1}{Da}}} \right] + n_3 m_2 H. \quad (33)
 \end{aligned}$$

Proceeding as in the case of homogeneous chemical reaction, we now have the effective dispersion coefficient D^* given by

$$D^* = 2 \frac{d^6}{\mu^2 D} \left(\frac{\partial p}{\partial x} \right)^2 G(\xi, \alpha, \beta, \varepsilon, \lambda_1, Da, \gamma) \quad (34)$$

Let the average of G be \bar{G} , and is defined by

$$\bar{G} = \int_0^1 G(\xi, \alpha, \beta, \varepsilon, \lambda_1, Da, \gamma) d\xi \quad (35)$$

3. NUMERICAL RESULTS AND DISCUSSION

The Eq. (25) and (35) show the equivalent dispersion coefficient D^* through the functions \bar{F} and \bar{G} respectively. \bar{F} and \bar{G} have been computed numerically using MATHEMATICA software and the results are presented graphically. The dimensionless parameters involved in the discussion are: the amplitude ratio ε , the homogeneous reaction rate α , the heterogeneous reaction rate β , the Jeffrey number λ_1 , the Darcy number Da and the slip parameter γ .

3.1 Homogeneous Chemical Reaction

Figures 2 – 6 show the effects of various parameters on the dispersion in the presence of homogeneous chemical reaction in the bulk of the medium. It is noticed that the average effective dispersion coefficient increases with amplitude ratio ε (Figs. 3 and 4). This may mean that peristalsis enhances dispersion.

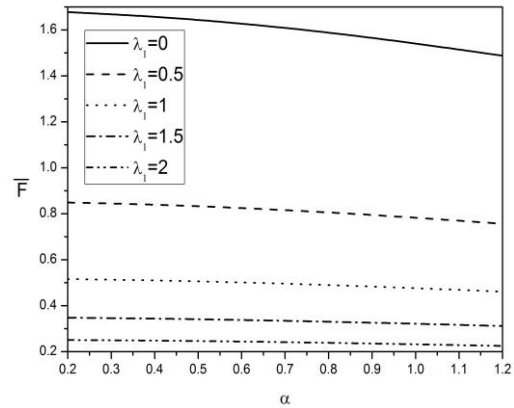


Fig. 2. Effect of λ_1 on \bar{F} ($Da = 0.01$; $\varepsilon = 0.2$; $\gamma = 0.1$)

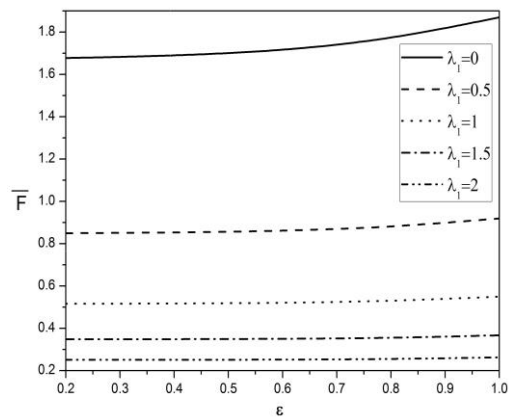


Fig. 3. Effect of λ_1 on \bar{F} ($Da=0.01$; $\alpha = 0.2$; $\gamma = 0.1$)

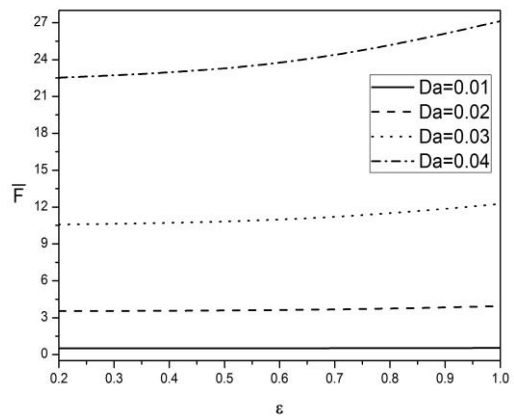


Fig. 4. Effect of Da on \bar{F} ($\lambda_1 = 1$; $\alpha = 0.2$; $\gamma = 0.1$)

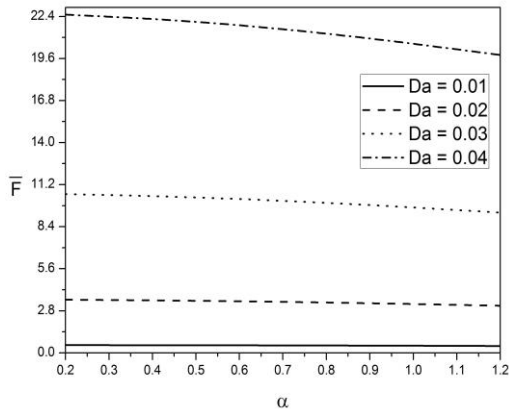


Fig. 5. Effect of Da on \bar{F} ($\lambda_1 = 1$; $\epsilon = 0.2$; $\gamma = 0.1$)

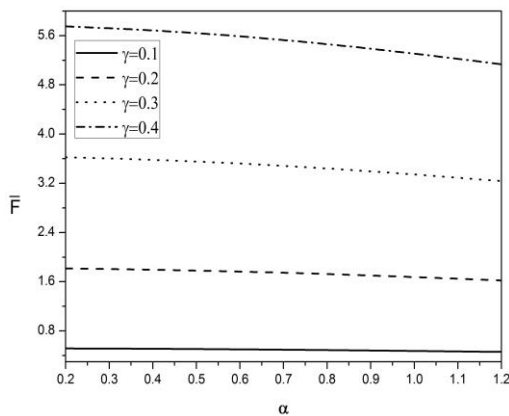


Fig. 6. Effect of γ on \bar{F} ($Da = 0.01$; $\lambda_1 = 1$; $\epsilon = 0.2$)

It is observed that the average effective dispersion coefficient decreases with Jeffrey number λ_1 (Figs. 2 and 3) but increases with Darcy number Da (Figs. 4 and 5) and slip parameter γ (Fig. 6). Further, it is also observed that the average effective dispersion coefficient decreases with homogeneous chemical reaction rate parameter α (Figs. 2, 5 and 6). This result agrees with that of Gupta and Gupta (1972), Ramana Rao and Padma (1975), Padma and Ramana Rao (1976). This result is expected since increase in α leads to an increase in number of moles of solute undergoing chemical reaction, and these results in decrease of the dispersion.

3.2 Combined Homogeneous and Heterogeneous Chemical Reaction

Figures 7 - 12 display the effects of various parameters on the average effective dispersion coefficient \bar{G} for the case of combined first order chemical reaction both in the bulk and at the walls. It is seen that the average effective dispersion coefficient increases with amplitude ratio ϵ (Figs. 8 and 9). This implies that the peristalsis enhances dispersion of a solute in fluid flow.

It is observed that the average effective dispersion coefficient \bar{G} decreases with Jeffrey number λ_1 (Figs. 7 and 8), homogeneous chemical reaction rate α (Fig. 12) and heterogeneous chemical reaction rate β (Figs. 7, 10 and 11).

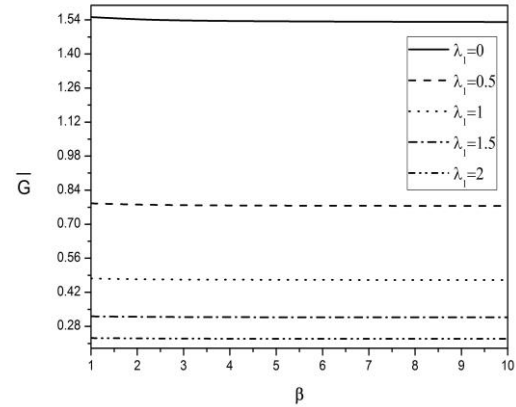


Fig.7. Effect of λ_1 on \bar{G} ($Da = 0.01$; $\alpha = 0.5$; $\epsilon = 0.2$; $\gamma = 0.1$)

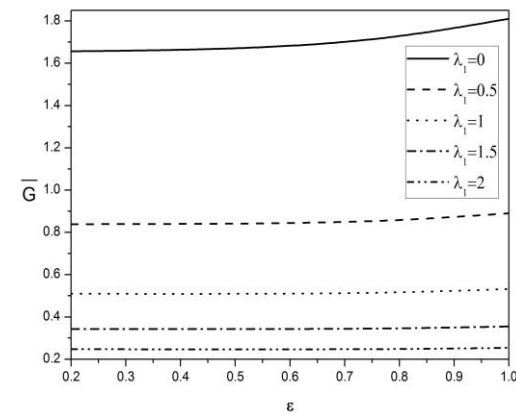


Fig. 8. Effect of λ_1 on \bar{G} ($Da = 0.01$; $\alpha = 0.2$; $\beta = 5$; $\gamma = 0.1$)

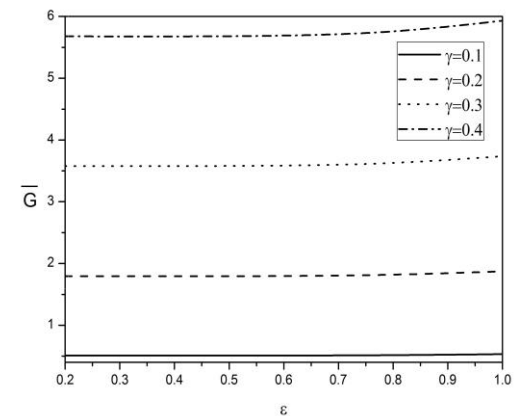


Fig. 9. Effect of γ on \bar{G} ($Da = 0.01$; $\lambda_1 = 1$; $\alpha = 0.2$; $\beta = 5$)

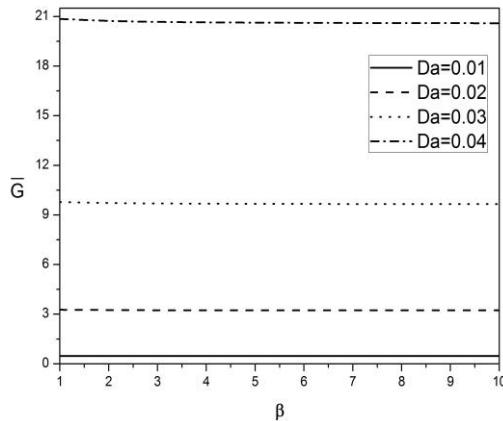


Fig. 10. Effect of Da on \bar{G} ($\alpha = 0.5$; $\lambda_1 = 1$; $\varepsilon = 0.2$; $\gamma = 0.1$)

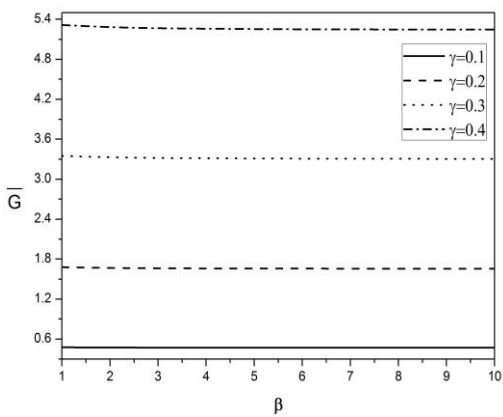


Fig. 11. Effect of γ on \bar{G} ($Da = 0.01$; $\lambda_1 = 1$; $\alpha = 0.5$; $\varepsilon = 0.2$)

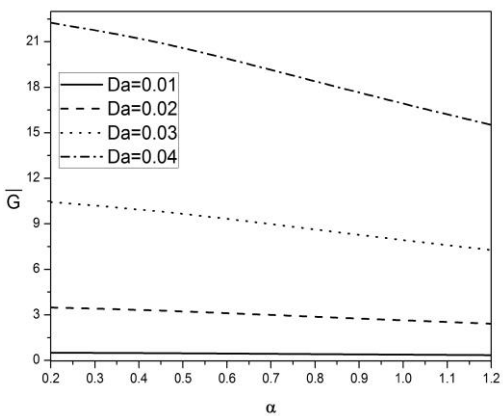


Fig. 12. Effect of Da on \bar{G} ($\beta = 5$; $\lambda_1 = 1$; $\varepsilon = 0.2$; $\gamma = 0.1$)

4. CONCLUSION

The effect of homogeneous and heterogeneous chemical reactions on peristaltic motion of a Jeffrey fluid through a porous medium with slip condition has been studied under long wavelength approximation and Taylor's limiting condition. It is observed that peristaltic motion enhances dispersion

and dispersion decreases with Jeffrey number λ_1 . It is also observed that the effective dispersion coefficient increases with Darcy number Da , and the slip parameter γ in both the cases of homogeneous and combined homogeneous and heterogeneous chemical reactions. Further, average dispersion coefficient decreases with homogeneous reaction rate α and the heterogeneous reaction rate β .

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