Unsteady Hydromagnetic Natural Convection Flow past an Impulsively Moving Vertical Plate with Newtonian Heating in a Rotating System

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Abstract

An investigation of unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and heat absorbing fluid past an impulsively moving vertical plate with Newtonian heating embedded in a porous medium in a rotating system is carried out. The governing partial differential equations are first subjected to Laplace transformation and then inverted numerically using INVLAP routine of Matlab. The governing partial differential equations are also solved numerically by Crank-Nicolson implicit finite difference scheme and a comparison has been provided between the two solutions. The numerical solution for fluid velocity and fluid temperature are depicted graphically whereas the numerical values of skin friction and Nusselt number are presented in tabular form for various values of pertinent flow parameters. Present solution in special case is compared with previously obtained solution and is found to be in excellent agreement.

Keywords: Unsteady hydromagnetic natural convection; Coriolis force; Newtonian heating; Heat absorption; Porous medium.

1. Introduction

Theoretical/experimental investigation of natural convection flow past bodies with different geometries embedded in a fluid saturated porous medium has received considerable attention during the past several decades due to its varied and wide industrial applications. Significant applications include chemical catalytic reactors, porous insulation, nuclear waste disposal, use of porous conical bearings in lubrication technology, fibrous and granular insulation systems, grain storage, food processing, energy efficient drying processes, enhanced recovery of oil and gas, coal combustors, underground energy transport etc. The basic problem of natural convection in porous medium is

Nomenclature

\begin{align*}
B_0 & \quad \text{uniform magnetic field} \\
c_p & \quad \text{specific heat at constant pressure} \\
G & \quad \text{thermal Grashof number} \\
g & \quad \text{acceleration due to gravity} \\
h & \quad \text{heat transfer coefficient} \\
K & \quad \text{permeability parameter} \\
K_r & \quad \text{rotation parameter} \\
k & \quad \text{thermal conductivity} \\
M & \quad \text{Magnetic parameter} \\
P & \quad \text{Prandtl number} \\
Q & \quad \text{heat absorption coefficient} \\
T & \quad \text{fluid temperature} \\
\nu & \quad \text{primary fluid velocity} \\
\nu' & \quad \text{Secondary fluid velocity} \\
\beta & \quad \text{coefficient of thermal expansion} \\
\nu & \quad \text{kinematic coefficient of velocity} \\
\rho & \quad \text{fluid density} \\
\tau_x & \quad \text{primary skin friction} \\
\tau_y & \quad \text{secondary skin friction} \\
\sigma & \quad \text{electrical conductivity} \\
\phi & \quad \text{heat absorption parameter} \\
\Omega & \quad \text{uniform angular velocity}
\end{align*}

It is well known that the characteristics of heat transfer are dependent on the thermal boundary conditions. Here a conjugate convective type flow or Newtonian heating is considered. Newtonian heating is a kind of wall-to-ambient heating process where the rate of heat transfer from the bounding surface with a finite heat capacity is proportional to the local surface temperature. This type of situation occurs in many important engineering devices such as in heat exchangers, gas turbines and also in seasonal thermal energy storage systems. Therefore, the interaction of conduction-convection coupled effects is of much significance from practical point of view and it must be considered when evaluating the conjugate heat transfer processes in many engineering applications. Merkin (1994) initiated the study of free convection boundary layer flow over a vertical surface with Newtonian heating while Lesnic et al. (1999, 2000) analyzed free convection boundary layer flow past vertical and horizontal surfaces in a porous medium generated by Newtonian heating. Chaudhary and Jain (2006) investigated unsteady free convection flow past an impulsively started vertical plate with Newtonian heating. Salleh et al. (2009) discussed forced convection boundary layer flow at a forward stagnation point with Newtonian heating. Narahari and Ishak (2011) investigated the effects of thermal radiation on unsteady free convection flow of an optically thick fluid past a moving vertical plate with Newtonian heating. They considered three cases of interest, namely, (i) impulsive movement of the plate; (ii) uniformly accelerated movement of the plate and (iii) exponentially accelerated movement of the plate. Olanrewaju and Makinde (2013) investigated boundary layer stagnation point flow of a nanofluid over a permeable flat surface with Newtonian Heating. Recently, Das et al. (2014a) studied unsteady mixed convection flow past a vertical plate with Newtonian heating.

However, in all these investigations, the effects of magnetic field are ignored. The interaction between electrically conducting fluid and a magnetic field has profound applications in various technical systems employing liquid metal and plasma flows (Liron and Wilhelm, 1974). Therefore, the study of unsteady hydromagnetic convective boundary layer flow of electrically conducting fluids in porous and non-porous media has become a subject of great interest and is widely investigated due to its significant applications in boundary layer flow control, plasma studies, geothermal energy extraction, solar energy collection, cooling of an infinite metallic plate in a cooling bath, magnetohydrodynamic (MHD) stirring of molten metal, magnetic levitation and casting, MHD marine propulsion and on the performance of many engineering devices, namely, MHD power generators, MHD pumps, MHD accelerators, MHD flow-meters, controlled thermonuclear reactors etc. Keeping in view the importance of such study, Raptis (1986) investigated unsteady two-dimensional natural convection flow of an electrically conducting, viscous and incompressible fluid along an infinite vertical plate embedded in a porous medium in the presence of magnetic field. Jha (1991) considered unsteady hydromagnetic free convection and mass transfer flow past a uniformly accelerated moving vertical plate through a porous medium when magnetic field is fixed with the moving plate. Chamkha (1997) analyzed unsteady MHD free convection flow through a porous medium supported by a surface. Kim (2000) studied unsteady MHD free convection flow past a moving semi-infinite vertical porous plate embedded in a porous medium with variable suction. Hayat et al. (2008) investigated the effects of magnetic field and porous medium on some unidirectional flows of a second grade fluid. In their study MHD flows are induced by the application of periodic pressure gradient or by the impulsive motion of one or two boundaries or by an oscillating plate. Ogulu and Makinde (2008) considered unsteady hydromagnetic free convection flow of a dissipative and radiative fluid past a vertical plate with constant heat flux. Recently, Seth and Sarkar (2014) investigated unsteady hydromagnetic free convection flow of a viscous, incompressible and electrically conducting fluid past an impulsively moving vertical plate with Newtonian surface heating, embedded in a uniform porous medium.

It is noticed that there may be significant temperature difference between ambient fluid and surface of the solid in a number of fluid flow problems of physical interest. Therefore, it is appropriate to consider temperature dependent heat source and/or sink which may have strong influence on heat transfer characteristics. Sparrow and Cess (1961) were one of the initial investigators to study

Investigation of hydromagnetic natural convection flow in a rotating medium is of considerable importance due to its application in various areas of geophysics, astrophysics and fluid engineering viz. maintenance and secular variations of Earth’s magnetic field due to motion of Earth’s liquid core, internal rotation rate of the Sun, structure of the magnetic stars, solar and planetary dynamo problems, turbo machines, rotating MHD generators, rotating drum type separators for liquid metal MHD applications etc. It may be noted that Coriolis and magnetic forces are comparable in magnitude and Coriolis force induces secondary flow in the flow-field. Taking into consideration the importance of such study, unsteady hydromagnetic natural convection flow past an infinite moving plate in a rotating medium has been studied by a number of researchers. Mention may be made of research studies of Singh (1984), Raptis and Singh (1985), Kythe and Puri (1987), Singh et al. (2010) and Seth et al. (2011). Seth et al. (2013) considered effects of rotation on unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and heat radiating fluid past an impulsively moving vertical plate with ramped temperature in a porous medium. Recently, Seth et al. (2015) investigated effects of Hall current and rotation on hydromagnetic natural convection flow with heat and mass transfer of a heat absorbing fluid past an impulsively moving vertical plate with ramped temperature. To the best of our knowledge no researcher has yet considered the effects of rotation and heat absorption on unsteady hydromagnetic natural convection flow past a flat plate embedded in a porous medium when natural convection is induced due to Newtonian heating of the plate.

Objective of the present investigation is to study unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and heat absorbing fluid past an impulsively moving infinite vertical plate embedded in a uniform porous medium in a rotating system when the natural convection is induced due to Newtonian heating of the plate. According to the best of authors’ knowledge this problem has not yet received attention of researchers although being significantly important in science and engineering.

2. FORMULATION OF THE PROBLEM AND ITS SOLUTION

Consider unsteady natural convection flow of a viscous, incompressible, electrically conducting and heat absorbing fluid past an infinite vertical plate embedded in a uniform porous medium with Newtonian heating at the surface of the plate. Coordinate system is chosen in such a way that \( x' \)-axis is taken along the plate in the upward direction, \( y' \)-axis is taken normal to the plane of plate in the fluid and \( z' \)-axis is taken normal to the \( x'y' \) plane. Fluid is permeated by a uniform transverse magnetic field \( B_0 \) which is applied in a direction parallel to \( y' \)-axis. The fluid and plate rotate in unison with uniform angular velocity \( \Omega \) about \( y' \)-axis. Initially, i.e. at time \( t' \leq 0 \), both the fluid and plate are at rest and at a uniform temperature \( T_0 \). At time \( t' > 0 \), the plate is given an impulsive motion in \( x' \)-direction against the gravitational field such that it attains a uniform velocity \( U_u \). It is assumed that natural convection is generated by Newtonian heating i.e. rate of heat transfer from the plate is proportional to the local surface temperature. The geometry of the problem is presented in Fig. 1. Since plate is of infinite extent in \( x' \) and \( z' \) directions and is electrically non-conducting, all physical quantities depend on \( y' \) and \( t' \) only. Also no applied or polarized voltages exist so the effect of polarization of fluid is negligible. This corresponds to the case where no energy is added or extracted from the fluid by electrical means (Cramer and Pai, 1973). It is assumed that the induced magnetic field generated by fluid motion is negligible in comparison to the applied one. This assumption is valid because magnetic Reynolds number is very small for liquid metals and partially ionized fluids which are commonly used in industrial applications (Cramer and Pai, 1973).
Taking into consideration the assumptions made above, the governing equations for unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and heat absorbing fluid through a uniform porous medium in a rotating frame of reference, under Boussinesq approximation, are given by

\[
\frac{\partial u'}{\partial t'} + 2\Omega w' = \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} \left( \frac{u'}{K_1} + \frac{v}{K_1'} \right) + g\beta \left( T' - T'_a \right),
\]

where \( u', w', \sigma, \rho, K_1, g, \beta, T', K_1, c_p \) and \( B_0 \) are, respectively, fluid velocity in \( x' \)-direction, fluid velocity in \( y' \)-direction, kinematic coefficient of viscosity, electrical conductivity, fluid density, permeability of porous medium, acceleration due to gravity, coefficient of thermal expansion, fluid temperature, thermal conductivity, specific heat at constant pressure and heat absorption coefficient.

The initial and boundary conditions for the problem are specified as

\[
t' \leq 0 : \quad u' = 0, w' = 0, \quad T' = T'_a \quad \text{for all} \quad y' \geq 0,
\]

\[
t' > 0 : \quad u' = U_o, \quad w' = 0, \quad \frac{\partial T'}{\partial y'} = -\frac{h}{K_1} T' \quad \text{at} \quad y' = 0,
\]

\[
u \rightarrow 0, \quad w' \rightarrow 0, \quad T' \rightarrow T'_a \quad \text{as} \quad y' \rightarrow \infty.
\]

where \( U_o \) is heat transfer coefficient and \( U_o = \frac{h v}{K_1} \).

Eqs. (1) to (3), in non-dimensional form, assume the following form

\[
\frac{\partial^2 u}{\partial t'^2} + 2K_2 \frac{\partial u}{\partial y'^2} = \frac{\partial^2 u}{\partial y'^2} - M^2 u + \frac{w}{K_1} + G'T,'
\]

\[
\frac{\partial^2 w}{\partial t'^2} - 2K_2 u = \frac{\partial^2 w}{\partial y'^2} - M^2 w + \frac{w}{K_1}.
\]

where

\[
y = y'/U_o, \quad u = u'/U_o, \quad w = w'/U_o, \quad t = t'/U_o^2/\nu.
\]

\[
M^2 = \frac{\sigma B_0^2}{\rho} U_o^2, \quad K^2 = \frac{\nu \Omega^2}{U_o^2}, \quad K_1 = K'_1 U_o^2.
\]

\[
T = (T' - T'_a)/T'_a, \quad G'_T = g\beta \nu T'_a/U_o^3, \quad P_1 = \nu c_p/k_1 \quad \text{and} \quad \phi = \frac{\nu Q_b}{\nu c_p} U_o^2.
\]

\[
M^2, K^2, K_1, G'_T, P_1 \quad \text{and} \quad \phi \quad \text{are, respectively, magnetic parameter, rotation parameter, permeability parameter, Grashof number, Prandtl number and heat absorption parameter.}
\]

The initial and boundary conditions (4), in non-dimensional form, become

\[
t \leq 0 : \quad u = 0, \quad w = 0, \quad T = 0 \quad \text{for all} \quad y,
\]

\[
t > 0 : \quad u = 1, w = 0, \quad \frac{\partial T}{\partial y} = -(1 + T) \quad \text{at} \quad y = 0,
\]

\[
u \rightarrow 0, \quad w \rightarrow 0, \quad T \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty.
\]

Eqs. (5) and (6) are presented, in compact form, as

\[
\frac{\partial F}{\partial t'} = \frac{\partial^2 F}{\partial y'^2} - \lambda F + G'T',
\]

where \( F = u + i w \) and \( \lambda = M^2 + 1/K_1 - 2iK^2 \).

Initial and boundary conditions (8a) to (8c), in compact form, become

\[
F = 0, T = 0 \quad \text{for} \quad y \geq 0 \quad \text{and} \quad t \leq 0,
\]

\[
F = 1, \quad \frac{\partial T}{\partial y} = -(1 + T) \quad \text{at} \quad y = 0 \quad \text{for} \quad t > 0,
\]

\[
F \rightarrow 0, \quad T \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty \quad \text{for} \quad t > 0.
\]

Eqs. (7) and (9), after taking Laplace transform and using initial conditions (10a), reduce to

\[
\frac{d^2 \tilde{T}}{dy'^2} - P_1 (s + \phi) \tilde{T} = 0,
\]

\[
\frac{d^2 \tilde{F}}{dy'^2} - (s + \lambda) \tilde{F} + G_T \tilde{T} = 0,
\]

where \( \tilde{T}(y, s) = \frac{T(y, t)}{T_0} e^{-st} dt \)

\[
\tilde{F}(y, s) = \int_0^s F(y, t) e^{-st} dt \quad \text{and} \quad s > 0 \quad \text{(s being Laplace transform parameter).}
\]

Boundary conditions (10b) and (10c), after taking Laplace transform, become
\[
\bar{r} = \frac{1}{s} \frac{d\bar{T}}{dy} = \left( \frac{1}{s} + \frac{T}{\bar{T}} \right) \quad \text{at } y = 0, 
\]
(13a)

\[
\bar{T} \to 0, \quad \bar{r} \to 0, \quad \text{as } y \to \infty. 
\]
(13b)

Solution of Eqs. (11) and (12) subject to the boundary conditions (13a) and (13b) are given by

\[
\bar{T}(y, s) = \frac{e^{-y \sqrt{s \phi}}}{s \sqrt{s + \phi}} \right] \right), 
\]
(14)

\[
\bar{F}(y, s) = \frac{1}{s} \frac{G_i}{\sqrt{s + \phi}} \right] \right) \left( e^{-y \sqrt{s \phi}} - e^{-y \sqrt{s \phi}} \right), 
\]
(15)

where \( G_i = G_i/(1 - P_i) \) and \( \lambda_i = \lambda_i/(1 - P_i) \).

An exact inverse Laplace transform of Eq. (14) can be obtained when \( \phi = 0 \) i.e. in the absence of heat absorption (Chaudhary and Jain, 2006). Moreover, inverse Laplace transform of the second term in Eq. (15) can be obtained only when \( \lambda = 0 \) and \( \phi = 0 \) i.e. in the absence of magnetic field, porous medium, rotation and heat absorption (Chaudhary and Jain, 2006). Therefore, the presence of either magnetic field or Coriolis force or permeability of medium or heat absorption in Eq. (15) and the presence of heat absorption in Eq. (14) causes the task to obtain analytical solution of the governing equations impossible and no researcher has yet obtained a closed form analytical solution taking into account any of the above entities to the best of our knowledge. Thus, the Laplace transform inversion of Eqs. (14) and (15) is obtained numerically using INVLAP routine in Matlab. Exact inversion of Eq. (14) can be obtained in the absence of heat absorption i.e. when \( \phi = 0 \) which agrees with that of Chaudhary and Jain (2006) and is given by

\[
T(y, t) = e^{-y \sqrt{t/\pi}} \text{erfc} \left( \frac{y}{\sqrt{2 \pi}} \right) - \text{erfc} \left( \frac{y}{\sqrt{2 \pi}} \right). 
\]
(16)

### 3. NUMERICAL SOLUTION

Eqs. (5) to (7) subject to the initial and boundary conditions (8) cannot be solved analytically due to the reasons mentioned in the previous section and hence we resorted to INVLAP routine of Matlab. However, Eqs. (5) to (7) under the initial and boundary conditions (8) can be solved numerically using Crank-Nicolson implicit finite difference scheme. Therefore, we have also obtained numerical solution of this problem using Crank-Nicolson implicit finite difference scheme.

For this purpose, the region under consideration is restricted to a rectangle of finite dimensions with \( y_{max} = 6 \) (corresponding to \( y \to \infty \)) and \( t_{max} = 2 \). Assumption of \( y_{max} = 6 \) was finalized when boundary condition (10c) was satisfied within tolerance limit of \( 10^{-4} \). Computational domain is divided into 241 \( \times \) 801 grid points and the grid refinement check is performed by comparing results in this case (with mesh size \( \Delta y \times \Delta t \) where \( \Delta y = 1/40 \) and \( \Delta t = 1/400 \)) with the results obtained when mesh size is reduced to 50\% of the present case and it is noticed that the difference between these two results is less than half a unity in the fourth decimal place. The finite difference equations for each time step constitute a tridiagonal system of equations which are solved by Thomas algorithm as given in Carnahan et al. (1969). Numerical solution for fluid temperature and fluid velocity is obtained corresponding to desired degree of accuracy for required time by performing computations for a number of time steps. It was found that the absolute difference between the numerical values of fluid temperature and fluid velocity obtained for two consecutive time steps is less than \( 10^{-4} \). Hence the scheme designed is stable. Moreover, Crank-Nicolson scheme has local truncation error of \( O((\Delta y)^2 + (\Delta t)^2) \) which tends to zero as \( \Delta y \) and \( \Delta t \) tends to zero which justifies consistency (Antia, 1991, pp. 643-644). Stability and consistency together ensure convergence of the scheme.

Skin friction \( \tau \) and Nusselt number \( N_s \) are given by

\[
\tau = \frac{\partial u}{\partial y} \bigg|_{y=0}, 
\]
(17)

\[
N_s = \frac{\partial T}{\partial y} \bigg|_{y=0}. 
\]
(18)

The numerical values of skin friction and Nusselt number are obtained using computed values of fluid velocity and fluid temperature respectively. It may be noted that the derivatives involved in Eqs. (17) and (18) are evaluated using five point forward difference formula for the first order derivative (Antia, 1991, page 161).

#### 3.1 Validation of Numerical Solution

In order to validate our numerical scheme we have presented in Fig. 2 a comparison between the exact values of fluid temperature computed from exact solution (16) with the numerical values of fluid temperature obtained by Crank-Nicolson implicit finite difference scheme and by INVLAP routine of Matlab for the case when \( \phi = 0 \) (absence of heat absorption). It is seen that there is an excellent agreement between these solutions.

Expression for Nusselt number \( N_s \) when \( \phi = 0 \) is obtained using solution (16) which is given by

\[
N_s = e^{-\frac{t}{\tau}} \text{erfc} \left( \sqrt{\frac{t}{\tau}} \right). 
\]
(19)
We have presented in Table 1 a comparison between the numerical values of Nusselt number obtained using the INVLAP routine of Matlab and finite difference scheme mentioned above with the exact value obtained from expression (19). It is evident from Table 1 that the numerical values of Nusselt number obtained through finite difference scheme are in good agreement with the values of Nusselt number obtained by INVLAP routine of Matlab. Moreover, it is also noticed from Table 1 that numerical values for Nusselt number obtained by INVLAP routine of Matlab are in excellent agreement with the exact values of Nusselt number obtained from (19). This justifies the correctness of the results presented in the manuscript.

<table>
<thead>
<tr>
<th>$P_r$ → $\phi$</th>
<th>0.3</th>
<th>0.5</th>
<th>0.71</th>
<th>0.3</th>
<th>0.5</th>
<th>0.71</th>
<th>0.3</th>
<th>0.5</th>
<th>0.71</th>
</tr>
</thead>
<tbody>
<tr>
<td>t ↓ 0.3</td>
<td>5.0084</td>
<td>3.1459</td>
<td>2.5052</td>
<td>5.009</td>
<td>3.1462</td>
<td>2.5055</td>
<td>5.009</td>
<td>3.1462</td>
<td>2.5055</td>
</tr>
<tr>
<td>0.5</td>
<td>10.2251</td>
<td>5.0084</td>
<td>3.5683</td>
<td>10.2295</td>
<td>5.0090</td>
<td>3.5687</td>
<td>10.2295</td>
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<td>3.5686</td>
</tr>
<tr>
<td>0.7</td>
<td>20.268</td>
<td>7.7264</td>
<td>4.9304</td>
<td>20.3074</td>
<td>7.7281</td>
<td>4.9310</td>
<td>20.3074</td>
<td>7.7281</td>
<td>4.9310</td>
</tr>
</tbody>
</table>

4. RESULTS AND DISCUSSION

In order to analyze the effects of magnetic field, rotation, permeability of the medium, thermal buoyancy force, heat absorption, thermal diffusion and time on the flow-field, the numerical solution of primary fluid velocity $u$ and secondary fluid velocity $w$ is depicted graphically versus boundary layer coordinate $y$ in Figs. 3 to 9 for various values of magnetic parameter $M^2$, rotation parameter $K^2$, permeability parameter $K_t$, Grashof number $G_r$, heat absorption parameter $\phi$, Prandtl number $P_r$ and time $t$. It is revealed from the Figs. 3 to 9 that, secondary fluid velocity attains maximum value near surface of the plate and then decrease properly on increasing boundary layer coordinate $y$ to approach free stream value. This is due to the fact that Coriolis force is dominant in the region near the axis of rotation.

Figure 3 illustrates the influence of magnetic field on the primary fluid velocity $u$ and secondary fluid velocity $w$. It is revealed from Fig. 3 that both $u$ and $w$ decrease on increasing $M^2$. Since $M^2$ signifies the relative strength of magnetic force to viscous force, $M^2$ increases on increasing the strength of magnetic force. This implies that, magnetic field has a tendency to retard fluid flow in both the primary and secondary flow directions throughout the boundary layer region. This phenomenon is attributed to the Lorentz force, induced due to the movement of an electrically conducting fluid in the presence of magnetic field, which has a tendency to resist fluid motion.

![Fig. 3. Primary and Secondary velocity profiles when $K^2=5, K_t=0.4, G_r=4, \phi=2, P_r=0.71$ and $t=0.5$](image1)

![Fig. 4. Primary and Secondary velocity profiles when $M^2=6, K_t=0.4, G_r=4, \phi=2, P_r=0.71$ and $t=0.5$](image2)
Figure 4 demonstrates the effects of rotation on the primary and secondary fluid velocities. It is perceived from Fig. 4 that $u$ decreases on increasing $K^2$ throughout the boundary layer region whereas $w$ increases on increasing $K^2$ in the region near the plate and it decreases on increasing $K^2$ in the region away from the plate. This implies that, rotation tends to retard fluid flow in the primary flow direction throughout the boundary layer region whereas it has a reverse effect on fluid flow in the secondary flow direction in the region near the plate. Although rotation is known to induce secondary flow in the flow-field by suppressing primary flow, its accelerating effect on the fluid flow in secondary flow direction is prevalent only in the region near the plate.

Figure 5 presents the influence of permeability of the medium on the primary and secondary fluid velocities. It is evident from Fig. 5 that both $u$ and $w$ increase on increasing $K_1$. It may be noted that an increase in $K_1$ implies that there is a decrease in the resistance of the porous medium. Due to this reason permeability of the medium tends to accelerate fluid flow in both the primary and secondary flow directions throughout the boundary layer region. Figure 6 depicts the effects of thermal buoyancy force on the primary and secondary fluid velocities. It is noticed from Fig. 6 that both $u$ and $w$ increase on increasing $G_r$. Since $G_r$ presents the relative strength of thermal buoyancy force to viscous force, $G_r$ increases on increasing the strength of thermal buoyancy force. This implies that, thermal buoyancy force tends to accelerate fluid flow in both the primary and secondary flow directions throughout the boundary layer region.

Figure 7 illustrates the influence of heat absorption on the primary and secondary fluid velocities. It is perceived from Fig. 7 that both $u$ and $w$ decrease on increasing $\phi$. This implies that, heat absorption tends to retard fluid flow in both the primary and secondary flow directions throughout the boundary layer region.

Figures 8 and 9 depict the effects of thermal diffusion and time on the primary and secondary fluid velocities. It is noticed from Figs. 8 and 9 that both $u$ and $w$ decrease on increasing $P_r$ whereas both $u$ and $w$ increase on increasing $t$. Since $P_r$ is a measure of relative strength of viscosity to thermal diffusivity of the fluid, $P_r$ decreases on increasing thermal diffusivity. This implies that, thermal diffusion tends to accelerate fluid flow in both the primary and secondary flow directions throughout the boundary layer region. As time progresses, fluid flow is getting accelerated in both the primary and secondary flow directions throughout the boundary layer region.
The numerical solution of fluid temperature $T$ is depicted graphically versus boundary layer coordinate $y$ in Figs. 2, 10 and 11 for various values of $P_r$, $\phi$ and $t$. Figures 2, 10 and 11 reveal that fluid temperature $T$ decreases on increasing $P_r$ and $\phi$ whereas it increases on increasing $t$. This implies that, throughout the boundary layer region, thermal diffusion tends to enhance fluid temperature whereas heat absorption has a reverse effect on it. Fluid temperature is getting enhanced with the progress of time.

The numerical values of primary skin friction $\tau_x$ and secondary skin friction $\tau_z$ are presented in tabular form in Tables 2 to 4 for various values of $M^2$, $K^2$, $G$, $K$, $\phi$ and $t$ taking $P_r = 0.71$ (ionized air).

It is perceived from Table 2 that both $\tau_x$ and $\tau_z$ increase on increasing $K^2$. This implies that rotation tends to enhance both the primary and secondary skin frictions. It is noticed from Tables 2 to 4 that $\tau_x$ increases on increasing $M^2$ and $\phi$ whereas it decreases on increasing $G$, $K$, $t$ and $\tau_z$ decreases on increasing $M^2$ and $\phi$ whereas it increases on increasing $G$, $K$, $t$. This implies that magnetic field and heat absorption tend to enhance primary skin friction whereas these agencies have reverse effect on secondary skin friction. Thermal buoyancy force and permeability of the medium tend to reduce primary skin friction whereas these agencies have reverse effect on secondary skin friction. As time progresses, primary skin friction is getting reduced whereas secondary skin friction is getting enhanced.

### Table 2 Primary and Secondary Skin Frictions when $G = 4$, $K = 0.4$, $\phi = 2$ and $t = 0.5$

<table>
<thead>
<tr>
<th>$K^2$</th>
<th>$\tau_x$</th>
<th>$\tau_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rightarrow M^2 )</td>
<td>( \rightarrow 5 )</td>
<td>( 7 )</td>
</tr>
<tr>
<td>6</td>
<td>2.1031</td>
<td>2.4341</td>
</tr>
<tr>
<td>9</td>
<td>2.5266</td>
<td>2.7891</td>
</tr>
<tr>
<td>12</td>
<td>2.9278</td>
<td>3.1382</td>
</tr>
</tbody>
</table>

### Table 3 Primary and Secondary Skin Frictions when $M^2 = 6$, $K^2 = 5$, $\phi = 2$ and $t = 0.5$

<table>
<thead>
<tr>
<th>$K$</th>
<th>$\tau_x$</th>
<th>$\tau_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rightarrow G, \phi )</td>
<td>( \rightarrow 0.2 )</td>
<td>( 0.4 )</td>
</tr>
<tr>
<td>2</td>
<td>3.0256</td>
<td>2.6954</td>
</tr>
<tr>
<td>4</td>
<td>2.4574</td>
<td>2.1031</td>
</tr>
<tr>
<td>6</td>
<td>1.8892</td>
<td>1.5107</td>
</tr>
</tbody>
</table>

### Table 4 Primary and Secondary Skin Frictions when $M^2 = 6$, $K^2 = 5$, $G = 4$ and $K = 0.4$

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\tau_x$</th>
<th>$\tau_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rightarrow \phi )</td>
<td>( \rightarrow 0.3 )</td>
<td>( 0.5 )</td>
</tr>
<tr>
<td>2</td>
<td>2.4968</td>
<td>2.1031</td>
</tr>
<tr>
<td>4</td>
<td>2.6602</td>
<td>2.4789</td>
</tr>
<tr>
<td>6</td>
<td>2.7773</td>
<td>2.6958</td>
</tr>
</tbody>
</table>
5. CONCLUSIONS

An investigation of unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and heat absorbing fluid past an impulsively moving infinite vertical plate embedded in a uniform porous medium in a rotating system when the natural convection is induced due to Newtonian heating of the plate is carried out. Significant findings are as follows:

(i) Magnetic field has a tendency to retard fluid flow in both the primary and secondary flow directions throughout the boundary layer region. Rotation tends to retard fluid flow in the primary flow direction throughout the boundary layer region whereas it has a reverse effect on fluid flow in the secondary flow direction in the region near the plate. Permeability of the medium, thermal buoyancy force and thermal diffusion tend to accelerate fluid flow in both the primary and secondary flow directions throughout the boundary layer region whereas heat absorption has a reverse effect on it. As time progresses, fluid flow is getting accelerated in both the primary and secondary flow directions throughout the boundary layer region.

(ii) Thermal diffusion tends to enhance fluid temperature whereas heat absorption has a reverse effect on it.

(iii) Rotation tends to enhance both the primary and secondary skin frictions. Magnetic field and heat absorption tend to enhance primary skin friction whereas these agencies have reverse effect on secondary skin friction. Thermal buoyancy force and permeability of the medium tend to reduce primary skin friction whereas these agencies have reverse effect on secondary skin friction. As time progresses, primary skin friction is getting reduced whereas secondary skin friction is getting enhanced.

REFERENCES


subjected to power law heat flux. J. Appl. Fluid Mech. 6(4), 563-569.


