



# Demixing of a Binary Fluid Mixture in Case of MHD Flow with Heat and Mass Transfer due to a Point Sink

B. R. Sharma and K. Nath<sup>†</sup>

*Department of Mathematics, Dibrugarh University, Dibrugarh,  
 Assam-786004, India*

<sup>†</sup>*Corresponding Author Email: kabitanath56@gmail.com*

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## ABSTRACT

The present problem concerns with the effects of the magnetic field, mass flux diffusion and heat transfer on demixing of a binary mixture of incompressible viscous electrically conducting fluids in steady, laminar boundary layer flow in presence of a point sink at the vertex of a cone. The momentum, energy and concentration equations are reduced to non-linear coupled ordinary differential equations by similarity transformations and are solved numerically by using MATLAB's built in solver bvp4c. The local skin friction, the Nusselt number and the Sherwood number are tabulated for various values of the parameters. These numerical results have been demonstrated graphically from which it is observed that the effects of various parameters are to separate the components of the binary mixture by collecting the rarer and lighter component near the surface of the cone and throwing the heavier one away from it.

**Keywords:** Binary fluid mixture; Incompressible; Mass flux diffusion; Magnetic field, Heat transfer.

## NOMENCLATURE

$B_0$	magnetic field	Sh	Sherwood number
$C_f$	local skin friction coefficient	$t_d$	Thermal diffusion number
c and T	concentration and temperature	u and w	velocity components along r and z directions respectively
D	binary diffusion coefficient	U	inviscid flow velocity
$E_c$	Eckert number	$\nu$	kinematic viscosity
$f_w$	mass transfer parameter	w and $\infty$	subscripts at the wall and in the free stream respectively
g and G	dimensionless temperature and concentration respectively	$\phi$	semi-vertical angle of the cone
k	thermal conductivity	$\psi$ and f	dimensional and dimensionless stream functions respectively
m	strength of point sink	$\eta$	similarity variable
M	magnetic parameter	$\rho$	density
$\bar{m}$	mass flux of diffusing species	$\sigma$	electrical conductivity
Nu	Nusselt number	$\alpha$	thermal diffusivity
p	static pressure	$\mu$	viscosity
Pr	Prandtl number	$\tau_w$ and $q_w$	shear stress and heat transfer rate at the wall
R	radius of the cone ( $R = r \sin\phi$ )		
$Re_r$	Local Reynolds number		
r and z	distances along and perpendicular to the cone		
$S_c$	Schmidt number		

## 1. INTRODUCTION

This article deals with steady, laminar, heat and mass transfer MHD flow problem and has

applications in vortex chambers, power generators, nuclear reactions, evolution of rotating magnetic stars, geophysical fluid dynamics etc. This flow and heat transfer situation is of considerable interest

because it can occur in many geothermal, geophysical, technological, and engineering applications such as nuclear reactors, migration of moisture through air contained in fibrous insulations, grain storage, nuclear waste disposal, dispersion of chemical pollutants through water-saturated soil, and others. The study of demixing of the boundary layer flow of an electrically conducting binary mixture of incompressible viscous fluids on a cone due to a point sink with an applied magnetic field is relevant in the study of conical nozzle or diffuser flow problems. The effect of heat transfer in axi-symmetric flow, inside a cone due to a point sink, in absence of magnetic field, has been studied by Rosenhead (1963) using similarity transformations. A series solution for the converging motion of the viscous flow inside a cone under some restricted conditions on the potential flow has been studied by Ackerberg (1965). The steady MHD laminar axi-symmetric boundary layer flow in a cone due to a point sink with an applied magnetic field, heat and mass transfer have been investigated by Takhar (1986). The unsteady MHD forced axi-symmetric flow inside a cone due to a point sink has been studied by Eswara and Roy (2000). Eswara and Bommaiah (2004) investigated the influence of variation of viscosity with temperature on axi-symmetric flow inside a cone due to a point sink. In the previous studies with axi-symmetric flow inside a cone due to a point sink, the thermal conductivity of fluid was assumed to be constant. However, it is known that thermal conductivity of fluid may also be change with temperature. Hence, like other thermo-physical properties, temperature-dependent thermal conductivity also plays a vital role in surface friction and heat transfer rate near the wall. The effect of variable thermal conductivity along a stretching sheet with MHD flow and in the presence of heat source or sink has been studied by Sharma and Singh (2008). Seddeek and Salem (2005) investigated the effects of heat and mass transfer on stretching surface with variable viscosity and variable thermal diffusivity. The effects of variable thermal conductivity and variable viscosity on steady free convective heat transfer flow process along an isothermal vertical plate in the presence of heat sink has been presented by Mahanti and Gaur (2009). The effect of chemical reaction in a heat and mass transfer flow process along a vertical surface has been discussed by Muthucumaraswamy (2002). Ibrahim, Elaiw and Bakr (2008) found analytical solutions for heat and mass transfer flow of Newtonian fluid along a vertical permeable surface in the presence of radiation and also with homogeneous first order chemical reaction. Sharma and Singh (2008, 2009, 2010), Sharma and Nath (2012a, b) and Sharma et al. (2011, 2012) have studied the effect of magnetic field on demixing of a binary fluid mixture. Sharma and Singh (2004, 2007) have studied the effect of temperature gradient on demixing of species in hydromagnetic flow of a binary mixture of incompressible viscous fluids between two parallel plates, first taking the plates horizontal and second by taking the plates vertical. They found that the effect of temperature

gradient is to separate the components of the binary mixture and the magnetic field increases the effect of species demixing.

The objective of this paper is to investigate numerically the demixing of fluid mixture in case of two dimensional steady, laminar, MHD boundary layer flow of an incompressible viscous electrically conducting binary mixture of fluids in a circular cone due to a point sink at the vertex with an applied weak axial magnetic field, mass flux diffusion and heat transfer. The results are presented as concentration profiles for different values of parameters entering in the problem. The effects of the parameters  $f_w$ ,  $t_d$  and  $S_c$  on the local skin friction, the Nusselt number and the Sherwood number are presented numerically in tabular form.

## 2. MATHEMATICAL FORMULATION

We consider 1species demixing due to pressure gradient and temperature gradient in a steady laminar axi-symmetric boundary layer flow of a binary mixture of an electrically conducting incompressible fluid in a circular cone with a hole at the vertex. The hole is considered as three-dimensional sink. A magnetic field  $B_0$  fixed relative to the fluid is applied in the z-direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. The wall and the free stream are maintained at a constant temperature and concentration. The Hall effect term is neglected. The effect of mass transfer (suction and injection) has been included in the analysis. It is assumed that the injected binary fluid mixture possesses the same physical properties as the boundary layer binary fluid mixture and has a static temperature equal to the wall temperature. The boundary layer equations under the foregoing assumptions are:

$$\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(rw) = 0, \tag{1}$$

$$u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho} u, \tag{2}$$

$$u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \alpha \frac{\partial^2 T}{\partial z^2} + \frac{\mu}{\rho c_p} \left( \frac{\partial u}{\partial z} \right)^2, \tag{3}$$

$$u \frac{\partial c}{\partial r} + w \frac{\partial c}{\partial z} = D \left[ \frac{\partial^2 c}{\partial z^2} + S_T \frac{\partial c}{\partial z} \frac{\partial T}{\partial z} + S_T c \frac{\partial^2 T}{\partial z^2} \right], \tag{4}$$

$$\text{where } -\frac{1}{\rho} \frac{\partial p}{\partial r} = U \frac{\partial U}{\partial r} + \frac{1}{\rho} \sigma B_0^2 U, \\ U = -m/r^2, m > 0. \tag{5}$$

The boundary conditions are given by

$$u(r,0) = 0, w(r,0) = -w_w, \\ T(r,0) = T_w, c(r,0) = c_w \\ u(r,\infty) = U, T(r,\infty) = T_\infty, c(r,\infty) = c_\infty \tag{6}$$

Applying the following transformations

$$\eta = m^{\frac{1}{2}} z / (2\nu r^3)^{\frac{1}{2}} \\ ru = \frac{\partial \psi}{\partial z}, rw = -\frac{\partial \psi}{\partial r}, \psi = -(2\nu m r)^{\frac{1}{2}} f(\eta),,$$

$$\begin{aligned}
 u &= Uf'(\eta), w = \left( \frac{mv}{2r^3} \right)^{\frac{1}{2}} (f - 3\eta f'), \\
 g(\eta) &= \frac{T-T_\infty}{T_w-T_\infty}, G(\eta) = \frac{c-c_\infty}{c_w-c_\infty}, \\
 M &= \frac{2\sigma B_0^2 r^3}{(m\rho)}, Pr = \frac{\nu}{\alpha}, Sc = \frac{\nu}{D}, \\
 f_w &= -w_w \left( 2r^3/mv \right)^{\frac{1}{2}}, Ec = U^2/c_p(T_w - T_\infty)
 \end{aligned}
 \tag{7}$$

$$\text{and } t_d = S_T(T_w - T_\infty)$$

it can easily be verified that the balance of mass given by equation (1) is identically satisfied. The set of equations (2) – (4) are coupled non – linear partial differential equations. Introducing the relation (7) into the equations (2) – (4) we obtain the following non-linear coupled ordinary differential equations

$$f''' - ff'' + 4(1 - f'^2) + M(1 - f') = 0, \tag{8}$$

$$\frac{1}{Pr} g'' + Ec f'' - fg' = 0, \tag{9}$$

$$G'' - S_c fG' + t_d(G'g' + Gg'') = 0. \tag{10}$$

The boundary conditions (6) reduce to

$$\begin{aligned}
 f = f_w, f' = 0, g = G = 1 & \quad \text{at } \eta = 0 \\
 f' \rightarrow 1, g \rightarrow 0, G \rightarrow 0 & \quad \text{as } \eta \rightarrow \infty.
 \end{aligned}
 \tag{11}$$

It may be remarked that the boundary layer approximation is not valid in the immediate neighbourhood of the hole. Also the mass transfer parameter  $f_w$  will be a constant if the velocity normal to the wall  $w_w$  varies as  $r^{-\frac{3}{2}}$  as  $mv$  is a constant. Also  $f_w \geq 0$  according to whether it is injection or suction.

The local skin-friction coefficient can be expressed in the form

$$c_f = \frac{2\tau_w}{\rho U^2} = 2^{\frac{1}{2}} Re_r^{-\frac{1}{2}} f_w'' \tag{12}$$

$$\text{where } \tau_w = -\mu(u_z)_w, Re_r = m/(rv)$$

The local heat-transfer coefficient in terms of Nusselt number is given by

$$Nu = rq_w / [k(T_w - T_\infty)] = -2^{-\frac{1}{2}} Re_r^{\frac{1}{2}} g_w' \tag{13}$$

$$\text{where } q_w = -k(T_z)_w.$$

Similarly, the local mass flux of the diffusing species in terms of Sherwood number can be expressed as

$$Sh = r\bar{m}_w / [\rho D(c_w - c_\infty)] = -2^{-\frac{1}{2}} Re_r^{\frac{1}{2}} G_w' \tag{14}$$

$$\text{where } \bar{m}_w = -\rho D(c_z)_w.$$

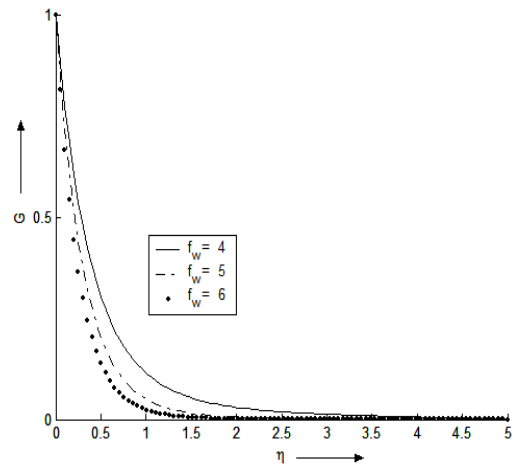
### 3. RESULTS AND DISCUSSION

The set of equations (8) to (10) under boundary conditions (11) are non-linear coupled ordinary differential equations so their solutions cannot be obtained in closed form therefore these equations are solved numerically with MATLAB's built – in solver bvp4c.

Numerical calculations have been carried out for concentration of the rarer component of the binary fluid mixture for various values of the parameters  $f_w$ ,  $t_d$  and  $S_c$  and are plotted against  $\eta$  in Figures 1-3.

Concentration distribution of the rarer and lighter component of the binary fluid mixture is plotted against  $\eta$  in Figure 1 for various values of the mass transfer parameter  $f_w$  ( $= 4, 5, 6$ ) by taking  $Ec=1, M=1, Pr=0.7, S_c=0.7$  and  $t_d=0.001$  to exhibit the effect of the mass transfer represented by  $f_w$  on the species separation. It is found that the concentration of the rarer and lighter component of the binary mixture is more near the surface of the cone and decreases exponentially as  $\eta$  increases to 2.5. Thereafter in the region  $\eta > 2.5$  no variation in  $G$  is observed. Thus we conclude that the separation of the binary mixture takes place mostly in the region  $0 < \eta < 2.5$  and thereafter separation is found to be negligible. It is evident from Fig. 1 that the rate of separation can be enhanced by increasing the values of  $f_w$ .

Figure 2 displays concentration distribution of the rarer and lighter component of the binary fluid mixture against  $\eta$  for various values of the thermal diffusion number  $t_d$  ( $= 0.001, 0.068, 0.45$ ) by taking  $Ec=1, M=1, Pr=0.7, S_c=0.7$  and  $f_w = 4$ . It is found from the graph that the concentration of the rarer and lighter component of the binary mixture is more near the surface of the cone and decreases exponentially as  $\eta$  increases to 3. Thereafter in the region  $\eta > 3$  no variation in  $G$  is observed. Thus we conclude that the separation of the binary mixture



**Fig. 1. The graph of G against  $\eta$   $Ec=1, M=1, Pr=0.7, S_c = 0.7$  and  $t_d=0.001$  for various values of  $f_w$ .**

takes place mostly in the region  $0 < \eta < 2.5$  and thereafter separation is found to be negligible. It is evident from Figure 1 that the rate of separation can be enhanced by decreasing the values of  $t_d$ .

Figure 3 displaying concentration distribution of the rarer and lighter component of the binary fluid mixture against  $\eta$  for various values of the Schmidt number  $S_c$  ( $= 0.6, 0.7, 0.9$ ) by taking  $Ec=1, Pr=0.7, M=1, t_d=0.001$  and  $f_w=4$  exhibits the effect of the Schmidt number  $S_c$  on the species separation. The effect of the parameter  $S_c$  is found to be similar to the mass transfer parameter  $f_w$ .

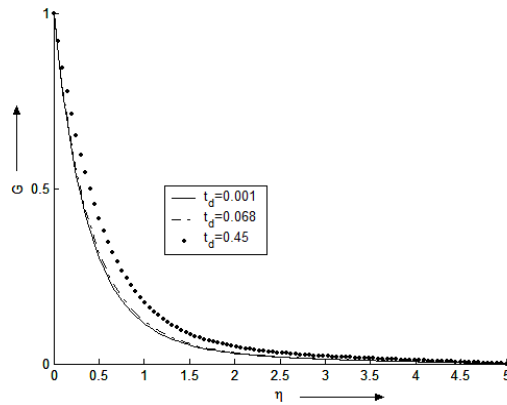


Fig. 2. The graph of  $G$  against  $\eta$   $Ec=1, M=1, Pr = 0.7, S_c = 0.7$  and  $f_w = 4$  for various values of  $t_d$ .

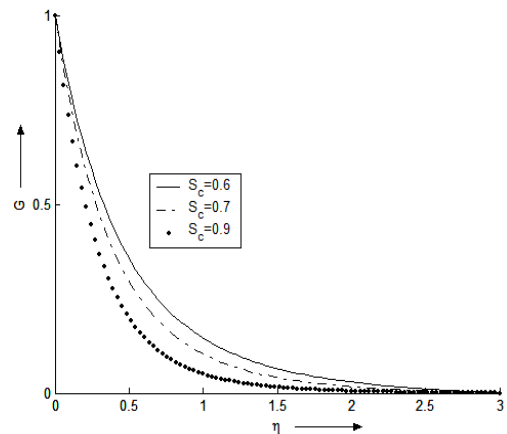


Fig. 3. The graph of  $G$  against  $\eta$   $Ec=1, Pr = 0.7, M = 1, t_d = 0.001$  and  $f_w = 4$  for various values of  $S_c$ .

From the process of numerical computation, the local skin friction, the Nusselt number and the Sherwood number, which are respectively proportional to  $f''(0), -g'(0)$  and  $-G'(0)$ , are also worked out and their numerical values are presented in a tabular form in Table 1.

Table 1 Numerical values of  $f''(0), -g'(0)$  and  $-G'(0)$  for  $Pr = 0.7, Ec=1$  and  $M=1$ .

$f_w$	$t_d$	$S_c$	$f''(0)$	$-g'(0)$	$-G'(0)$
4	0.001	0.7	5.2292	1.8210	2.5631
5	0.001	0.7	6.0634	2.6248	3.3458
6	0.001	0.7	6.9322	3.3687	4.0810
4	0.001	0.7	5.2292	1.8210	2.5631
4	0.068	0.7	5.2292	1.8210	2.4299
4	0.45	0.7	5.2292	1.8210	1.6567
4	0.001	0.6	5.2291	1.8635	2.1911
4	0.001	0.7	5.2291	1.8635	2.5968
4	0.001	0.9	5.2291	1.8635	3.4177

#### 4. CONCLUSION

In the present investigation the effects of all these parameters are to demix the binary mixture by collecting the rarer and lighter component of the

binary fluid mixture near the surface of the cone and throwing the heavier component away from it.

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