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Melting with Viscous Dissipation on MHD Radiative Flow from a Vertical Plate Embedded in Non-Darcy Porous Medium

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ABSTRACT

We analyzed in this paper the effect of radiation on melting from a vertical plate embedded in porous medium in MHD mixed convection with viscous dissipation under non-Darcy (Forchheimer) conditions. Both aiding and opposing flows are considered in the study. The numerical results were obtained for velocity and temperature variation in melting region and presented in a graphical form. Heat transfer in the melting region has also been studied and the effect of melting parameter and radiation on Nusselt number are presented in graphical form.

Keywords: Porous medium; Non-Newtonian fluid; Melting; Thermal dispersion; Radiation.

NOMENCLATURE

a:	mean absorption coefficient
B_0:	magnetic field strength
C:	inertial coefficient
c_f:	specific heat of convective fluid
c_r:	temperature ratio $= \frac{T_m}{T_\infty - T_m}$
c_s:	specific heat of solid phase
d:	mean particle diameter
F:	dimensionless Inertia parameter$= \frac{2C\sqrt{K}\,u_\infty}{\upsilon}$
f:	dimensionless stream function
g:	acceleration due to gravity
Ge:	Gebhart number $= \frac{g\beta x}{C_f}$
h:	local heat transfer coefficient
h_{sf}:	latent heat of melting of solid
K:	permeability of the porous medium
k:	thermal conductivity
M:	square of the Hartmann number $= \frac{\sigma B_0^2 K}{\rho\upsilon}$
m:	melting parameter $= \frac{C_f(T_\infty - T_m)}{h_{sf} + C_s(T_m - T_0)}$
Nu:	local Nusselt number $= \frac{hx}{k}$
Pe:	local Pecklet number $= \frac{u_\infty x}{\alpha_m}$
q:	radiative heat flux $= -\frac{4\sigma_R}{3a}\frac{\partial T^4}{\partial y}$
q_w:	wall heat flux
R:	radiation parameter $= \frac{4\sigma_R (T_\infty - T_m)^3}{ka}$
Ra_x:	local Rayleigh number $= \frac{xg\beta k(T_\infty - T_m)}{\nu\alpha_m}$
T:	temperature in thermal boundary layer
T_0:	temperature at the solid region
u:	velocity in x-direction
u_∞:	external flow velocity
v:	velocity in y- direction
x:	coordinate along the melting plate
y:	coordinate normal to melting plate
α:	thermal diffusivity
β:	coefficient of thermal expansion
ε:	mixed convection parameter $= \frac{Ra_x}{Pe_x}$
η:	dimensionless similarity variable
μ:	viscosity
υ:	kinematic viscosity
ρ:	density, kg / m^3
ψ:	Stream function
σ:	electrical conductivity
σ_R:	Stefan – Boltzmann constant
θ:	dimensionless temperature $= \frac{T - T_m}{T_\infty - T_m}$

Subscripts

m:	melting point
∞:	condition at infinity

1. INTRODUCTION

The effect of viscous dissipation has received little attention in the past. In most of the circumstances it is not necessary to allow for viscous dissipation when modeling convection in saturated porous media. Never the less despite the fact that the effect is generally negligible recently several authors have explicitly considered the effects of viscous dissipation in problems involving convection in porous media, and they have disagreed on the way to model this effect. The modeling of viscous dissipation in a porous medium saturated by an incompressible fluid is discussed, for the case of Darcy, Forchheimer and Brinkman models by Nield (2000). Gebhart (1962), Gebhart and Mollendrof (1969) studied the effect of viscous dissipation in natural convection in clear fluids. Fand et al. (1986) investigated experimentally and analytically the effects of viscous dissipation on Darcy free convection heat transfer from a horizontal cylinder embedded in a saturated porous medium. Tashtoush and Kodah (1998) introduced analytical solution for no slip boundary effects in non-Darcian mixed convection from a vertical wall in saturated porous media. Murthy and Singh (1997) studied the effects of viscous dissipation on non-Darcy natural convection in porous medium.

Murthy et al. (2004) have studied the combined effect of radiation and mixed convection from a vertical wall with suction / injection in a non – Darcy porous medium. They have established that Nusselt number increases with the increase in radiation parameter and also with the increase in fluid suction parameter.

Israel – Cookey, et al. (2003) studied the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium. They have found that velocity increases with increase in viscous dissipation parameter and decreases with increase in radiation and magnetic parameters. The same trend is observed in the temperature profile.

Rebhi Damesh (2006) studied MHD mixed convection from radiate vertical isothermal surface embedded in a saturated porous media. They have found that the local Nusselt number for both aiding and opposing flows decreases with increase in radiation conduction parameter.

The effect of increasing the magnetic field parameter is found to decrease the local Nusselt number. The inertial and viscous dissipation effects on mixed convection about a vertical surface have been studied by Ranganathan and Viskanta (1984). They found that the effects of inertia and boundary friction are quite significant and cannot be ignored.

Nakayama and Pop (1991) and Lai and Kulacki (1991) analyzed the inertia effects on mixed convection along a vertical wall using the Forchheimer flow model. A review of both natural and mixed convection boundary-layer flows in Darcian and non-Darcian fluid saturated porous media is given in Nield and Bejan (1992). Raptis (1998) analyzed radiation and free convection flow through a porous medium using Rosseland approximation for the radiative heat flux.

The study of mixed convection under melting along a vertical surface embedded in a porous medium has gained enormous importance in recent years due to its wide range of applications in a variety of areas as mentioned in earlier chapters of this work. The problem of mixed convection on melting from a vertical plate in porous medium has been studied by Gorla et al. (1999). They have obtained the solution by using fourth order Runge-Kutta method and provided the information regarding the distribution of temperature, stream function, velocity and heat transfer rate. They have arrived at a conclusion that melting process is analogous to mass injection or blowing near boundary and, thus, it reduces the heat transfer through solid liquid interface. Bakier (1997) who studied both aiding and opposed convective flow in a porous medium, has established that the melting phenomenon decreases the local Nusselt number at solid liquid interface. They have obtained the solution using analytical homotopy analysis method and satisfactorily verified with numerical results available in the literature.

From a search of the specialized literature, it appears that the only contribution found to melting phenomena accounting for a combination of melting parameter and buoyancy effect under the influence of applied magnetic field is a non – Darcy flow field (Forchheirmer model) adjacent to a vertical impermeable wall embedded in a porous medium investigation by Tashtoush (2005). More recently, Bakier et al. (2009) studied Group method analysis of melting effect on MHD mixed convection flow from a radiative vertical plate embedded in a saturated porous medium. He developed linear transformation group approach to simulate problem of hydro magnetic heat transfer by mixed convection along vertical plate in a liquid saturated porous medium in the presence of melting and thermal radiation effects for opposing external flow. He studied the effects of the pertinent parameters on the rate of the heat transfer in terms of the local Nusselt number at the solid-liquid interface.

The main aim of the present investigation is to illustrate the effects of melting with viscous dissipation on MHD radiative flow from a vertical plate in a non-Darcy porous medium. To the author's knowledge, no studies have thus far been communicated with regard to the boundary layer flow and heat transfer of a mixed convective radiate flow from a vertical plate in a non-Darcy porous medium with melting and viscous dissipation effects.

In the present work we analyzed velocity and temperature profiles of various parameters under the influence of viscous dissipation with melting effect and the results so obtained are compared with relevant results in the existing literature and are found to be in good agreement in the absence of melting effect.

2. MATHEMATICAL FORMULATION

Consider a vertical melting front at the melting point T_m. The co-ordinate system x-y is attached to the melting front as shown in Fig. 1. The porous medium flows to the right with melting velocity across x-axis. The melting front is modeled as a vertical plate. This plate constitutes the inter phase between the liquid phase and the solid phase during melting inside the porous matrix. The temperature of the solid region is considered less than the melting point, i.e.$T_0 < T_m$. On the right hand side of melting front, the liquid is super heated, i.e.$T_\infty > T_m$. A vertical boundary layer flow on the liquid side smoothens the transition from T_m to T_∞. The assisting external flow velocity is taken as u_∞. It is also assumed that the system exists in thermodynamic equilibrium everywhere. The fluid is assumed to be Newtonian and electrically conducting with the constant properties except the density variation in the buoyancy term. Transverse magnetic field is applied to the plate and the effects of flow inertia and the radiation are included. There is no applied electric field and hence the Hall Effect and Joule heating are also neglected.

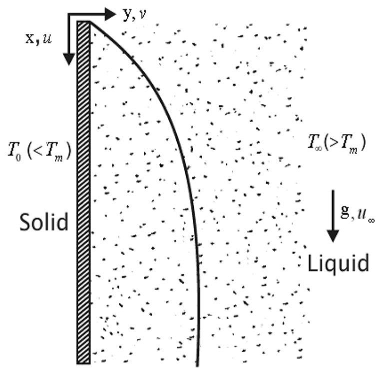

Fig. 1. Schematic diagram of the problem.

Taking into account the effects of viscous dissipation and radiation, the governing equations for steady non-Darcy flow in a non-Newtonian fluid saturated porous medium can be written as follows.

The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

The momentum equation is

$$\frac{\mu}{K}\frac{\partial u}{\partial y} + \frac{\mu}{K}\frac{C\sqrt{K}}{\nu}2u\frac{\partial u}{\partial y} + \sigma B_0^2\frac{\partial u}{\partial y} = -\rho_\infty g\beta\frac{\partial T}{\partial y} \tag{2}$$

The energy equation with viscous dissipation according Bourhan Taushtoush (2000) is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\alpha_m\frac{\partial T}{\partial y}\right) + \frac{\upsilon}{KC_f}u\left[u + \frac{C\sqrt{K}}{\upsilon}u^2\right] - \frac{1}{\rho C_f}\frac{\partial q}{\partial y} \tag{3}$$

The radiative heat flux term q is written using the Rosseland approximation (Sparrow and Cess (1978), Raptis (1998)) as $q = -\frac{4\sigma_R}{3a}\frac{\partial T^4}{\partial y}$ Where σ_R is the Stefan - Boltzmann constant and 'a' is the mean absorption coefficient and α_m is the molecular thermal diffusivity.

The physical boundary conditions for the present problem are

$$y = 0,\ T = T_m,\ k\frac{\partial T}{\partial y} = \rho[h_{sf} + C_s(T_m - T_0)]v \tag{4}$$

$$\text{and}\quad y\to\infty,\ T\to T_\infty,\ u\to u_\infty \tag{5}$$

Introducing the stream function ψ with

$$u = \frac{\partial\psi}{\partial y}, \text{and } v = -\frac{\partial\psi}{\partial x}$$

The continuity Eq. (1) will be satisfied and the Eq. (2) and Eq. (3) transform to

$$\frac{\mu}{K}\frac{\partial^2\psi}{\partial y^2} + 2\frac{\mu}{K}\frac{C\sqrt{K}}{\upsilon}\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y^2} + \sigma B_0^2\frac{\partial^2\psi}{\partial y^2} = -\rho_\infty g\beta\frac{\partial T}{\partial y} \tag{6}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left[\alpha_m\frac{\partial T}{\partial y}\right] + \frac{\upsilon}{K\,C_f}\left(\frac{\partial\psi}{\partial y}\right)^2 + \frac{C}{\sqrt{K}}\frac{1}{C_f}\left(\frac{\partial\psi}{\partial y}\right)^3 + \frac{4\sigma_R}{3\rho C_f a}\frac{\partial}{\partial y}\left[\frac{\partial T^4}{\partial y}\right] \tag{7}$$

Introducing the similarity variables as

$$\psi = f(\eta)(\alpha_m u_\infty x)^{1/2},\ \eta = \left(\frac{u_\infty x}{\alpha_m}\right)^{\frac{1}{2}}\left(\frac{y}{x}\right),\ \theta(\eta) = \frac{T-T_m}{T_\infty - T_m},$$

The momentum Eq. (6) and energy Eq. (7) are reduced to

$$(1+M+Ff^1)\,f^{11} + \epsilon\,\theta^1 = 0 \tag{8}$$

$$\theta^{11} + \frac{1}{2}f\,\theta^1 + \frac{Ge}{\varepsilon}\left(f^{1^2} + \frac{F}{2}f^{1^3}\right) + \frac{4}{3}R\left[\left((\theta + C_r)^3\theta^{11} + 3\theta^{1^2}(\theta + C_r)^2\right)\right] = 0 \tag{9}$$

Where Ra_x is the local Rayleigh number, c_r is the temperature ratio which assumes small values by its definition (taken as 0.1 in the present work), $\varepsilon = \frac{Ra_x}{Pe_x}$ the mixed convection flow governing parameter (positive when the buoyancy is aiding the external flow and is negative when the buoyancy is opposing the external flow), Pe_x is the local Pecklet number, M is the magnetic parameter, Ge_x is the local Gebhart number, F is the flow inertia coefficient and R is the radiation parameter.

The boundary conditions (4) and (5) now become

$$\text{At } \eta=0,\ \theta=0 \text{ and } f(0) + 2m\,\theta^1(0)=0. \tag{10}$$

$$\text{as } \eta\to\infty,\ \theta=1 \text{ and } f^1(\infty) = 1. \tag{11}$$

where m is the dimensionless melting parameter.

3. SOLUTION PROCEDURE

The Eq. (9) together with Eq. (8) is split into system of first order ordinary differential equations. Using boundary conditions (10) and (11) they are solved numerically by means of the fourth order Runge-Kutta method with a shooting technique and by giving appropriate initial guess values for $\theta^1(0)$. The solution, thus, obtained is matched with the given values at $f^1(\infty)$ and $\theta(0)$. An accuracy upto 4^{th} decimal place is considered for convergence.

The heat transfer coefficient in terms of the Nusselt number can be expressed as

$$\frac{Nu_x}{(Pe_x)^{1/2}} = [1 + \frac{4}{3}R(\theta(0) + C_r)^3]\theta^1(0) \tag{12}$$

In addition the boundary condition $\eta \to \infty$ is approximated by $\eta_{max} = 5$ which is found sufficiently large for the velocity and temperature to approach the relevant free stream properties.

The above Eq.s (8), (9) and (12) are solved for different set of conditions by varying the critical parameters and the results thus obtained are presented in graphical form from Fig. 2 to Fig. 9.

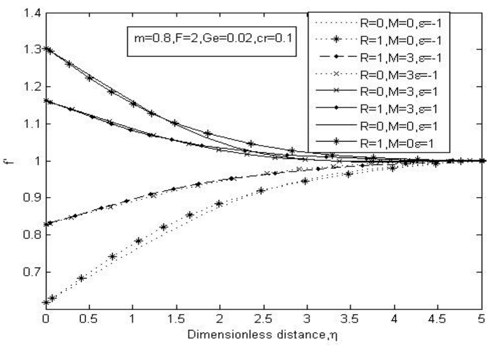

Fig. 2. The effect of radiation and magnetic field on velocity distribution in aiding and opposing flow.

4. RESULTS AND DISCUSSION

Figure 2 shows the effect of radiation and magnetic field on velocity distribution under aiding and opposing flow conditions. Under aiding flow conditions, it is found that the increasing magnetic effect decreases the local velocity and also the rate of change of velocity. Further, increase in radiation increases the local velocity or rate of change in velocity. However, the change in velocity under the influence of magnetic field is less when compared to that without magnetic field. These effects are found inverse in the opposing flow when compared to aiding flow.

Figure 3 shows the combined effect of radiation and melting parameter on velocity distribution under aiding flow conditions. It is found from the figure that with increase in melting parameter, the velocity within the boundary increases. Further, with increase in radiation also at a given location in the boundary, the velocity increases. Also, it is found that increase in melting parameter or radiation increases the boundary layer thickness. However, the effect of radiation on velocity in the presence of melting is found less.

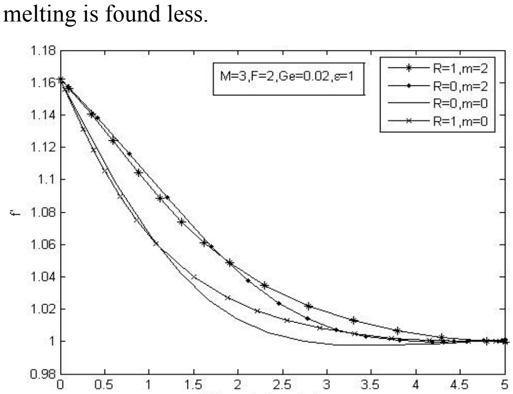

Fig. 3. The effect of radiation and melting parameter on velocity distribution.

Figure 4 shows that the effect of inertia in the presence of magnetic field on velocity distribution in opposing flow conditions. With increase in inertia, the rate of change of velocity within the boundary is found to decrease. Also, at a given location, the inertia tends to increase the velocity under opposing flow conditions. Further, the magnetic field also exhibits the same trend as inertia and is found that for high range magnetic field strength, the effect of inertia on velocity variation is relatively small.

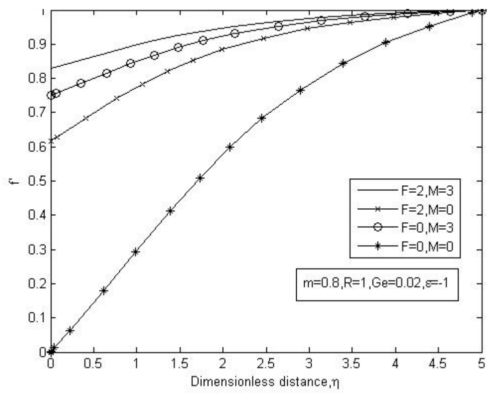

Fig. 4. The effect of inertia on velocity distribution in the presence of magnetic field.

Figure 5 shows the effect of inertia on the velocity profile under different melting conditions. The inertia effect tends to suppress the buoyancy induced flow in the boundary layer and so retards the velocity of the fluid. However, the melting parameter is found to increase the velocity. It is clear that the effect of melting is less under inertia conditions or non-Darcy condition as it suppresses the flow, when compared to Darcy conditions.

Figure 6 shows the effect of melting parameter on temperature distribution under aiding flow conditions. With increase in melting parameter, the temperature is found to decrease at a given location. It is also evident that increase in melting decreases the rate of change of temperature within the boundary.

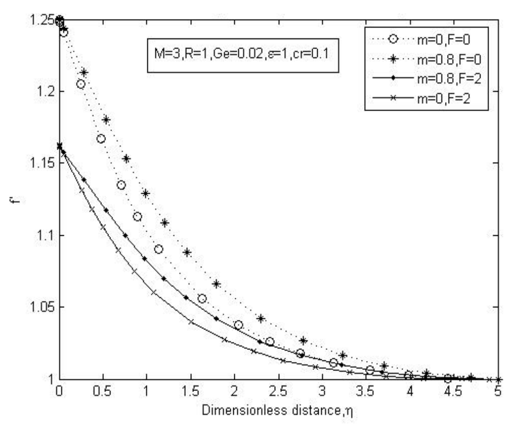

Fig. 5. The effect of inertia and melting on velocity distribution in the presence of radiation.

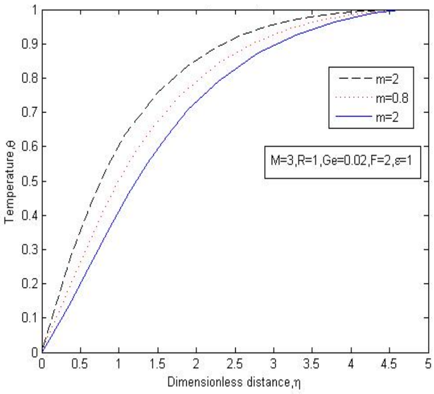

Fig. 6. The effect of melting on Temperature distribution

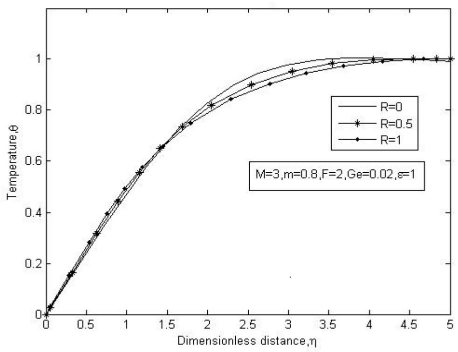

Fig. 7. The effect of radiation on temperature distribution

Figure 7 shows the effect of radiation on temperature under aiding flow conditions. The temperature within the boundary at a given location is found decreasing with increase in radiation parameter. The effect is found dominant at the center of boundary and is negligible near the wall and the edge. Further, it is observed that gradual variation of temperature is found within boundary with increase in radiation parameter.

Figure 8 shows the effect of Gebhart number on the velocity profile. The velocity decreases with Gebhart number. However, the increase in melting parameter increases the velocity within the boundary and thus, decreases the boundary larger thickness. The effect of melting is found similar irrespective of the influence of Gibhart number.

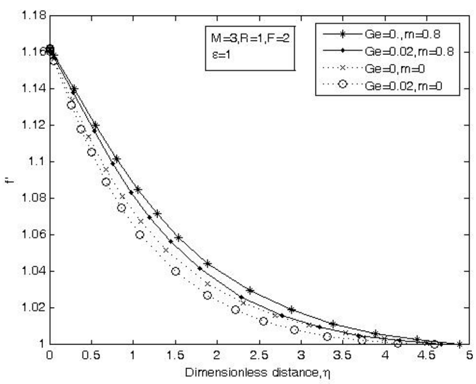

Fig. 8. The effect of Ge on Velocity distribution.

The effect of melting in case of aided flow is shown in Fig. 9. The heat transfer rate as a function of mixed convective parameter under different conditions is evaluated and presented in Table 1 and Table 2. With increase in the buoyancy parameter, the rate of heat transfer increases. Further, the rate of increase is found less under high melting conditions.

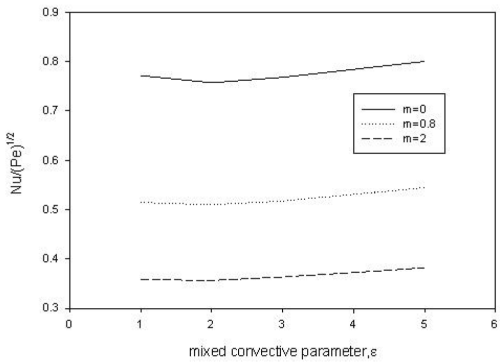

Fig. 9.The effect of melting on heat transfer

Table 1 The effect of radiation on Nusselt number (M =3, m = 0.8, Ge = 0.02, F = 2, c_r = 0.1) Nu /$(Pe)^{1/2}$

R	ϵ = 1	ϵ = - 1
0.0	0.7990	0.5180
0.25	0.8125	0.5300
0.50	0.8256	0.5423
0.75	0.8383	0.5527
1	0.8513	0.5674

Table 2 The effect of magnetic field on Nusselt number(ε=1, R = 1, Ge = 0.02, c_r = 0.1, F = 2) Nu /$(Pe)^{1/2}$

M	m = 0	m = 0.8
0	1.1221	0.9240
1	1.0874	0.8882
2	1.0640	0.8666
3	1.0494	0.8514
4	1.0386	0.8394

5. CONCLUSION

The melting phenomenon has been analyzed with mixed convection flow and heat transfer in a fluid saturated porous medium considering the effects of applied magnetic field, viscous dissipation and radiation by taking Forchheimer extension. These effects are analyzed for both the aiding and opposing flows. With increase in radiation, the velocity increases in aiding flow whereas, it decreases in opposing flow. The increase in melting parameter increases the velocity within the boundary and thus, decreases the boundary layer thickness. The velocity decreases with Gebhart number in aiding flow. The Nusselt number decreases with increase in both melting parameter and magnetic field. The effect of radiation is to enhance the heat transfer coefficient in both aiding and opposing flows. Further this work can be extended to a mixed convection flow from a vertical plate embedded in a Non-Newtonian fluid saturated non-Darcy porous medium with melting effect.

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