Journal of Applied Fluid Mechanics, Vol. 8, No. 4, pp. 753-765, 2015.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.18869/acadpub.jafm.73.238.22916

Entropy Generation Analysis for Stagnation Point Flow in a Porous Medium over a Permeable Stretching Surface

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(Received April 18, 2014; accepted August 1, 2014)

ABSTRACT

This paper presents entropy generation analysis for stagnation point flow in a porous medium over a permeable stretching surface with heat generation/absorption and convective boundary condition. We have used Von Karman transformations to transform the governing equations into ordinary differential equations.Thevelocity, temperature and concentration profiles obtained using the Homotopy Analysis Method. The HAM is a valid mathematical tool for most of non-linear problems in science and engineering. Finally we have computed the entropy generation number. The effect of the Prandtl number, Brinkman number, Reynolds number, suction/injection parameter, Biot number, Lewis number, Brownian motion parameter, thermophoresisparameterand constant parameters on velocity, concentration and temperature profiles are analyzed. Moreover the influences of the Reynolds number and Brinkman number on the entropy generation are presented.The entropy generation number increases with increasing the Brinkman and Reynolds number.

Keywords: Stagnation point flow; Stretching surface; Convective boundary Conditions; Heat generation/absorption; Entropy analysis.

NOMENCLATURE

A	ration of rates to free stream velocity and stretching velocity
Br	Brinkman number
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
f_w	suction/injection parameter
h	heat transfer coefficient
k	thermal conductivity
K_1	permeability parameter
Le	Lewis number
Nb	Brownian motion parameter
N_G	Entropy generation number
Nt	thermophoresis parameter
Pr	Prandtl number
Q_0	dimensional heat generation/ absorption coefficient
u , v	dimensionless velocity in x,y
u_e	free stream velocity
subscripts	
w	conditions on the wall
$v_w > 0$	suction velocity
$v_w < 0$	injection velocity
γ	Biot number
ρ	density
ν	Kinematic viscosity
$\lambda > 0$ $\lambda < 0$	heat / sink source parameter
Ω	dimensionless temperature difference
α	thermal diffusivity
λ and ς	constant parameter

Re	Reynolds number	∞	free stream conditions
u_w	stretching velocity		

1. INTROUDUCTION

Flow and heat transfer cause to a stretching surface in a motionless or moving fluid is significantly important in number of industrial manufacturing processes that contains both metal and polymer sheets. An interesting fluid mechanical application has been seen in the processes of the polymer extrusion, where the object on passing between two closely placed vertical solid blocks is stretched in a region of fluid-saturated porous medium. The liquid is meant to cool the stretching sheet whose property depends greatly on the rate at which it is cooled and stretched in porous medium. The fluid mechanical properties desired for the outcome of such a process depends mainly on the rate of cooling and the stretching rate. It is important that proper cooling fluid is selected and flow of the cooling liquid due to the stretching sheet can be controlled so as to achieve to the desired properties for the output. The quality of the last product depends on the heat transfer rate in the stretching surface. The temperature distribution, thickness and width reduction are function of draw ratio and stretching distance.

Detailed flow and thermal analyses are frequently needed in many engineering applications, containing process developments and optimizations. The engineering utilization of the second law, known as exergy analysis, has been used for the minimization of entropy generation to find optimal engineering system designs. Mahian et al.(2012) and had been a main part of the rmodynamic analysis of industrial equipments especially in power industries. Entropy generation define the level of irreversibilities accumulating during a process. As a result, entropy production can be used as a criterion to determine the performance of engineering devices. Cengel and Boles (2006). Entropy analysis of flow and heat transfer systems is important as it identifies the factors which are responsible for the loss of useful energy. This energy loss can affect the efficiency of the thermally designed system. By reducing the factors that create entropy, the performance of the system can be enhanced Buttand Ali (2014)**.**For example, a review of entropy generation in natural and mixed convection for energy systems was presented by Oztop and Al-Salem(2012). Several sources are responsible for entropy generation such as heat transfer and viscous dissipation (Bejan 1982, 1996). More recently the second law analyses emphasizing on the entropy generation and its minimization have been used to variant transport process Bejan (1982). [Bejan1996, 1980] was the first person who formulated the analysis of entropy generation ,has found a variety of applications such as the estimation of heat exchangers, two-phase flows, fuel cells and among many others. In during the last 30 years entropy generation minimization (EGM) has been the subject of several investigations(Nag and Mukherjee 1987, 1989; Bejan and Ledezma 1996; Lin and Lee 1997; Sasikumar and Balaji 2002a,b).Rashidiet al.(2014) studied the analysis of entropy generation in an MHD flow over a rotating porous disk with variable physical properties. In another research Rashidiet al.(2013)investigated the entropy generation in steady MHD flow due to a rotating porous disk in a nanofluid. Recently, second-law analysis of fluid flow and heat transfer across a flat plate has been conducted by Malvandi et al.(2012). Although some increasing/decreasing trends for governing parameters were observed, they could not find any optimum case in which the entropy generation is minimized. Then, the effects of adding nanoparticles to the fluid on entropy generation were investigated analytically by Malvandi et al.(2013). Their results showed that more entropy generates in boundary layer with increasing the solid volume fraction. In another studySelame and Vedat(1990)studied the entropy productionin boundary layers. Malvandi et al. (2013)considered the influences of velocity ratio – which represents the ratio of the wall velocity to the free stream fluid velocity on a moving plate. Their outcomes indicated that focusing on the velocity ratio as a pivotal parameter, entropy generation can be minimized. Butt et al. (2012)investigated the Entropy generation in hydrodynamic slip flow over a vertical plate with convective boundary. Makinde (2011)studied second law analysis for variable viscosity hydromagnetic boundary layer flow with thermal radiation and newtonian heat.

The homotopy analysis method (HAM) is used to obtain approximate analytical solutions for the transformed nonlinear equations under the prescribed boundary conditions. The HAM solutions, in comparison with numerical solutions (fourth-order Runge-Kutta shooting quadrature), admit excellent accuracy by Rashidi et al. (2012).

In fluid mechanics problems, stagnation-point is the point in the flow field where local velocity of the fluid becomes zero. Where the fluid is brought to be at rest because of a force exerted by the objectJafar et al.(2012), this point occurs at the surface of the object. (Ariel 1994a,b; and Attia 2003a.b) developed the problems of stagnation point flow. In the last several decades many investigations have been discussed for flow in the close of a stagnation point. The first person who studied two dimensional stagnation point flow over a flat plate was Hiemenz (1911), then this work extended to the axisymmetric cases with heat transfer characteristics by (Homann 1936; Eckert 1942). Rashidi et al. (2011) presented a new analytical solution of stagnation point flow in a porous medium. Chamkha and Issa (2000) studied the hydromagnetic flow over a semi-infinite permeable flat surface with heat generation/ absorption and thermophores is effects. Parametric analysis of entropy generation in off-centered stagnation flow towards a rotating disc was investigated by Rashidiet al. (2011). Mahapatra and Gupta (2001) analyzed two dimensional steady, stagnation point flow of an electrically conducting fluid over a flat stretching sheet. Layeket al. (2007)solved numerically stagnation point flow problem over a heated permeable stretching sheet with heat generation/absorption. Ramachandran et al. (1988) studied two dimensional laminar mixed convection stagnation flows. MHD stagnation point flow and heat transfer through a porous space bounded by a permeable surface was presented by Hayat et al. (2009). Ishak et al. (2008) investigated numerical solution of steady stagnation point flow over a vertical sheet with suction/blowing. Sharma and Singh (2008) analyzed the numerical study of two dimensional magneto hydrodynamic (MHD) steady laminar stagnation point flow of a viscous incompressible electrically conducting fluid of variable thermal conductivity over a stretching surface. Kumaran et al.(2009) was studied two dimensional stagnation point flow in a porous medium. The problem of boundary-layer unsteady mixed convection flow of stagnation point on a heated vertical plate through a porous medium was investigated by Hassanien and Alarabi (2009). Rashidi and Mohimanian Pour (2010) demonstrated the analytic approximate solutions for unsteady boundary-layer flow and heat transfer due to a stretching sheet by the HAM. In another study, Rashidi and Erfani (2011) discussed the modified differential transform method for investigating nano boundary-layers over stretching surfaces. Kechil and Hashim (2009) discussed the problem of two dimensional steady laminar MHD forced flow over a flat plate through porous medium. Mohamed et al. (2013)discussed the numerical investigation of stagnation point flow over stretching sheet with convective boundary condition. Mustafa et al. (2011) discussed the steady two dimensional stagnation point flow towards a stretching sheet in a nanofluid.

In more recent years the international research community has shown great interest in investigating nanofluids to be used as heat transfer fluids or coolants. Several review researches are available in this field with regards to thermal increases(Das et al. 2006; Saidur et al. 2011).Nanofluid is a fluid which the solid nanoparticles with the length scales of nanometers are suspended in a conventional heat transfer fluid. It has been showed that the addition of highly conductive particles can greatly increase the thermal conductivity of the pure ordinary fluid. For instance, it was reported that the effective thermal conductivity of an ethylene–glycol-based nanofluid which includes nano size copper particles with diameters less than 10 nm increased by up to 40 at 0.3 % vol. of dispersed particles Eastman et al. (2001).Alsaedi et al.(2012) investigated the stagnation-point flow of nanofluid near a permeable stretching surface. Hamad and Ferdows (2012) analyzed a stagnation-point flow over a permeable stretching sheet in porous medium with heat generation or absorption effects in a nanofluid. The problem of steady stagnation-point flow and heat transfer of viscous fluid over a shrinking sheet in a porous medium was investigated by Rosaliet al. (2011).

The similarity solution for two-dimensional and axisymmetric bodies in a porous medium was considered by Nakayama and Koyama (1987). Cheng and Minkowycz (1977) solved similarity solutions for free convective heat transfer from a vertical plate in a fluid saturated porous medium. Aldosset al. (1993) considered solutions for mixed convection in porous media. All these studies were concerned with Newtonian fluid flows. Non-Newtonian fluids display a nonlinear relationship between shear stress and shear rate. The analytic approximate solution for steady flow over a rotating disk in porous medium with heat transfer by the HAM was applied by Rashidi et al. (2012). Chen and Chen (1988) obtained similarity solutions for free convection of non-Newtonian fluids over vertical surfaces in porous media. The aim of this paper is study the entropy generation analysis on the stagnation point flow in a porous medium over a permeable stretching surface with convective boundary conditions in the presence of heat.

2. MATHEMATICAL FORMULATION

In two-dimensional Cartesian coordinate system (*x*,*y*), we test the boundary-layer flow of nano fluid near to a stagnation point towards a permeable stretching surface at y = 0. The fluid saturates the porous medium $y > 0$. The *x*-axis is consider in the direction of along the plate and the *y*-axis is normal to the plate with velocity components u, v in these directions, respectively. We define C_w as the value of nanoparticle fraction at the surface. Ambient values of temperature and nanoparticle fraction are considered as T_∞ and c_∞, there is no slip between the ordinary fluid and suspended nanoparticles. Further, the boundary-layer flow is considered in presence of heat source. Fig. 1 presents the geometry of the problem.

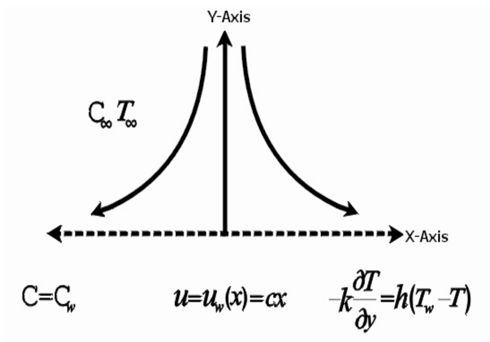

Fig. 1. Problem definition.

Under the boundary-layer approximations, the flow is governed by the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e(x)\frac{du_e(x)}{dx} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{\nu}{K}\left(u_e(x) - u\right), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho c_p}\left(T-T_\infty\right)+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right], \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}. \quad (4)$$

With the following boundary conditions

$$u=u_w(x)=cx,\quad v=v_w,\quad -k\frac{\partial T}{\partial y}=h(T_w-T),\qquad C=C_w \text{ at } y=0,$$
$$u=u_e(x)=ax\qquad v=0\qquad T\to T_\infty\quad C\to C_\infty\ \text{ as }\ y\to\infty, \quad (5)$$

where $u_w(x)$ shows the stretching velocity and $u_e(x)$ is the free stream velocity, ν is the kinematic viscosity, k is the permeability of porous medium, D_B is the Brownian diffusion coefficient, ρ is the density of the ordinary fluid, D_T is the thermophoretic diffusion coefficient, α the thermal diffusivity, τ the ratio of the effective heat capacity of the nanoparticle material and the heat capacity of the ordinary fluid, Q_0 the dimensional heat generation/absorption coefficient, , $v_w>0$ corresponds to suction velocity whereas $v_w<0$ shows the injection or blowing,where h is the heat transfer coefficient and k is the thermal conductivity of the ordinary fluid.

Introducing the similarity transformations and dimensionless temperature and concentration

$$\eta=y\sqrt{\frac{c}{\nu}},\psi(x,y)=x\sqrt{c\nu}\,f(\eta),$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\qquad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}. \quad (6)$$

Continuity Equation is automatically satisfied and Eqs. (2)-(5) are reduced in the following forms:

$$f'''(\eta)-f'^2(\eta)+f(\eta)f''(\eta)+A^2-K_1\left(f'(\eta)-A\right)=0 \quad (7)$$

$$\theta''(\eta)+Pr\begin{pmatrix}f(\eta)\theta'(\eta)+\lambda\theta(\eta)\\+Nb\theta'(\eta)\phi'(\eta)+Nt(\theta')^2\end{pmatrix}=0, \quad (8)$$

$$\phi''(\eta)+Le\,f(\eta)\phi'(\eta)+\frac{Nt}{Nb}\theta''(\eta)=0, \quad (9)$$

$$f(0)=f_w,\ f'(0)=1,\ \theta'(0)=-\gamma(1-\theta(0)),\ \phi=1\ \text{ at }\ \eta=0,$$
$$f'=A,\quad \theta=0,\quad \phi=0,\quad \text{as}\quad \eta=\infty, \quad (10)$$

where $Pr=\mu c_p/k$ the Prandtl number, $K_1=\nu/cK$ the permeability parameter of porous medium, $\lambda=Q_0/c\rho c_p$ the heat source $(\lambda>0)$ or sink $(\lambda<0)$ parameter, $A=\frac{a}{c}$ defines the ratio of rates to free stream velocity and stretching velocity, $f_w=-v_w/\sqrt{c\nu}$ the suction/ injection parameter , $\gamma=\frac{h}{k}\sqrt{\frac{\nu}{c}}$ the Biot number , $Le=\nu/D_B$ the Lewis number , the Brownian motion parameter $Nb=\tau D_B(\phi_w-\phi_\infty)/\nu$, $Nt=\tau D_T(T_w-T_\infty)/T_\infty\nu$ the thermophoresis parameter and fw is the suction/injection parameter with $fw>0$ showing a uniform suction through the wedge surface**.**

2.1 Ham Solution

In order to satisfy the mentioned boundary conditions, we select the appropriate initial approximations, as follows:

$$f_0(x)=(A-1)e^{(-x)}+f_w-A+1+Ax, \quad (11)$$

$$\theta_0(x)=\frac{\gamma e^{-x}}{\gamma+1}, \quad (12)$$

$$\phi_0(x)=e^{-x}. \quad (13)$$

The linear operators $\mathcal{L}_f(f)$, $\mathcal{L}_\theta(\theta)$ and $\mathcal{L}_\phi(\phi)$ are defined as:

$$\mathcal{L}_f=f'''+f'', \quad (14)$$

$$\mathcal{L}_\theta=\theta''+\theta', \quad (15)$$

$$\mathcal{L}_\phi=\phi''+\phi', \quad (16)$$

which fulfill the following properties:

$$\mathcal{L}_f(c_1e^{-x}+c_2x+c_3)=0, \quad (17)$$

$$\mathcal{L}_\theta(c_4e^{-x}+c_5)=0, \quad (18)$$

$$\mathcal{L}_\phi(c_6e^{-x}+c_7)=0, \quad (19)$$

where $c_i, i=1-7,$ are the arbitrary constants. According to Eqs. (7)-(9), the nonlinear operators are introduced as

$$\mathcal{N}_f\left[\frac{\partial^3\hat{f}(\eta;q)}{\partial\eta^3}\right]=\frac{\partial^3\hat{f}(\eta;q)}{\partial\eta^3}-\left(\frac{\partial\hat{f}(\eta;q)}{\partial\eta}\right)^2+\hat{f}(\eta;q)\frac{\partial^2\hat{f}(\eta;q)}{\partial\eta^2}+A^2-k_1\left(\frac{\partial\hat{f}(\eta;q)}{\partial\eta}-A\right), \quad (20)$$

$$\mathcal{N}_\phi\left[\hat{f}(\eta;q),\hat{\theta}(\eta;q),\hat{\phi}(\eta;q)\right]=\frac{\partial^2\hat{\phi}(\eta;q)}{\partial\eta^2}+Le\hat{f}(\eta;q)\frac{\partial\hat{\phi}(\eta;q)}{\partial\eta}+\frac{Nt}{Nb}\frac{\partial^2\hat{\theta}(\eta;q)}{\partial\eta^2}. \quad (21)$$

The auxiliary functions become:

$$H_f=e^{-x},H_\theta=1,H_s=1. \quad (23)$$

The symbolic software MATHEMATICA is used to solve the *m*th order deformation equations (24)-(26).

$$\mathcal{L}_f\left[f_m(\eta)-x_mf_{m-1}(\eta)\right]=\hbar\mathcal{H}f(\eta)R_{f,m}(\eta), \quad (24)$$

$$\mathcal{L}_\theta\left[\theta_m(\eta)-x_m\theta_{m-1}(\eta)\right]=\hbar\mathcal{H}\,\theta(\eta)R_{\theta,m}(\eta), \quad (25)$$

$$\mathcal{L}\phi\left[\phi_m(\eta)-x_m\phi_{m-1}(\eta)\right]=\hbar\mathcal{H}\,\phi(\eta)R_{\phi,m}(\eta), \quad (26)$$

where $\hbar$ is the auxiliary nonzero parameter and

$$R_{f,m}(\eta)=\frac{\partial^3f_{m-1}(\eta)}{\partial\eta^3}-\sum_{n=0}^{m-1}\frac{\partial f_n(\eta)}{\partial\eta}\frac{\partial f_{m-1-n}(\eta)}{\partial\eta}+\sum_{n=0}^{m-1}f_n(\eta)\frac{\partial^2f_{m-1-n}(\eta)}{\partial\eta^2}+A^2-k_1\left(\frac{\partial f_{m-1}(\eta)}{\partial\eta}-A\right), \quad (27)$$

$$R_{\theta,m}(\eta)=\frac{\partial^2\theta_{m-1}(\eta)}{\partial\eta^2}+\Pr\Big(\sum_{n=0}^{m-1}\big(f_n(\eta)\frac{\partial\theta_{m-1-n}(\eta)}{\partial\eta}\big)+\lambda\theta_n(\eta)$$
$$+Nb\sum_{n=0}^{m-1}\Big(\frac{\partial\phi_n(\eta)}{\partial\eta}\frac{\partial\theta_{m-1-n}(\eta)}{\partial\eta}\Big)$$
$$+Nt\sum_{n=0}^{m-1}\Big(\frac{\partial\theta_n(\eta)}{\partial\eta}\frac{\partial\theta_{m-1-n}(\eta)}{\partial\eta}\Big),\qquad(28)$$

$$R_{\phi,m}(\eta)=\frac{\partial^2\theta_{m-1}(\eta)}{\partial\eta^2}+\Pr\Big(\sum_{n=0}^{m-1}\big(f_n(\eta)\frac{\partial\theta_{m-1-n}(\eta)}{\partial\eta}\big)+\lambda\theta_n(\eta)+$$
$$Nb\sum_{n=0}^{m-1}\Big(\frac{\partial\phi_n(\eta)\partial\theta_{m-1-n}(\eta)}{\partial\eta}\Big)+Nt\sum_{n=0}^{m-1}\Big(\frac{\partial\theta_n(\eta)}{\partial\eta}\frac{\partial\theta_{m-1-n}(\eta)}{\partial\eta}\Big)\Big),\qquad(29)$$

And

$$\chi_m=\begin{cases}0 & m\le 1,\\ 1 & m>1.\end{cases}\qquad(30)$$

Are the involved parameters in the HAM. The HAM has the ability to adjust and control the convergence region of obtained solutions through the convergence control parameter $\hbar$ and an appropriate initial guess Rashidi et al. (2011). It is important to choose a proper value of auxiliary parameter is important to control and speed the convergence of the approximation series by thehelp of the so-called $\hbar$ – curve. Obviously, the valid regions of $\hbar$ correspond to the line segments nearly parallel to the horizontal axis. The $\hbar$ – curves of $f''(0)$, $\theta''(0)$ and $\phi''(0)$ obtained by

The18th order of HAM solution are figured in Fig. 2.The residual errors for the HAM 18th-order approximation solutions are defined:

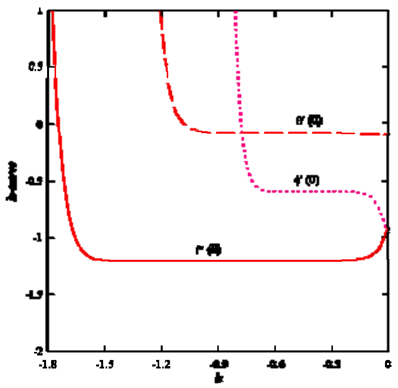

Fig. 2. The $\hbar$ -curves of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ obtained by the 18th-order HAM approximation when.

$$k_1=0.5, A=\lambda=\gamma=Nt=f_w=Nb=0.1,$$
$$Le=1, \Pr=0.71, \mathrm{Re}=5, Br=5.$$

$$\mathrm{Re}\,s_f=\frac{d^3f(\eta)}{d\eta^3}-\left(\frac{df(\eta)}{d\eta}\right)^2+f(\eta)\frac{d^2f(\eta)}{d\eta^2}$$
$$+A^2-k_1\left(\frac{df(\eta)}{d\eta}-A\right),\qquad(31)$$

$$\mathrm{Re}\,s_\theta=\frac{d^2\theta(\eta)}{d\eta^2}+\Pr\left(\begin{array}{l}f(\eta)\dfrac{d\theta(\eta)}{d\eta}+\lambda\theta(\eta)\\ +Nb\dfrac{d\theta(\eta)}{d\eta}\dfrac{d\phi(\eta)}{d\eta}\\ +Nt\left(\dfrac{d\theta(\eta)}{d\eta}\right)^2\end{array}\right),\qquad(32)$$

$$\mathrm{Re}s_\phi=\frac{d^2\phi(\eta)}{d\eta^2}+Lef(\eta)\frac{d\phi(\eta)}{d\eta}+\frac{Nt}{Nb}\left(\frac{d\theta(\eta)}{d\eta}\right)^2.\qquad(33)$$

In order to choose the optimal value of auxiliary parameter $(\hbar)$, the averaged residual error is introduced as (see Liao 2010; Rashidi et al. 2011; Turkyilmazoglu 2010; Rashidi et al. 2014, for more details):

$$\Delta_{f,m}=\frac{1}{K}\sum_{i=0}^{K}\left[\mathrm{Re}s_f\left(\sum_{j=0}^{m}f_j(i\Delta x)\right)\right]^2,\qquad(34)$$

$$\Delta_{\theta,m}=\frac{1}{K}\sum_{i=0}^{K}\left[\mathrm{Re}s_\theta\left(\sum_{j=0}^{m}\theta_j(i\Delta x)\right)\right]^2,\qquad(35)$$

$$\Delta_{\phi,m}=\frac{1}{K}\sum_{i=0}^{K}\left[\mathrm{Re}s_\phi\left(\sum_{j=0}^{m}\phi_j(i\Delta x)\right)\right]^2,\qquad(36)$$

where $\Delta x=10/K$ and $K=20$. For the given order of approximation m, the optimal value of $\hbar$ is given by the minimum values of the $\Delta_{f,m}$, $\Delta_{\theta,m}$ and $\Delta_{\phi,m}$ corresponding to nonlinear algebraic equations:

$$\frac{d\Delta_{f,m}}{d\hbar}=0,\quad\frac{d\Delta_{\theta,m}}{d\hbar}=0,\quad\frac{d\Delta_{\phi,m}}{d\hbar}=0.\qquad(37)$$

For example, in order to find the optimal values of $\hbar$, the residual error which is displayed in Eq. (34), for the HAM 18th-order approximation solutions is presented in Fig. 3.

2.2 Entropy Generation Analysis

The volumetric rate of local entropy generation of the nanofluid, is given by

$$\dot S'''_{gen}=\frac{k}{T_\infty^2}\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]$$
$$+\frac{\mu}{T_\infty}\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right\}$$
$$+\frac{RD}{C_\infty}\left[\left(\frac{\partial C}{\partial x}\right)^2+\left(\frac{\partial C}{\partial y}\right)^2\right]$$
$$+\frac{RD}{T_\infty}\left[\left(\frac{\partial T}{\partial x}\right)\left(\frac{\partial C}{\partial x}\right)+\left(\frac{\partial T}{\partial y}\right)\left(\frac{\partial C}{\partial y}\right)\right],\qquad(38)$$

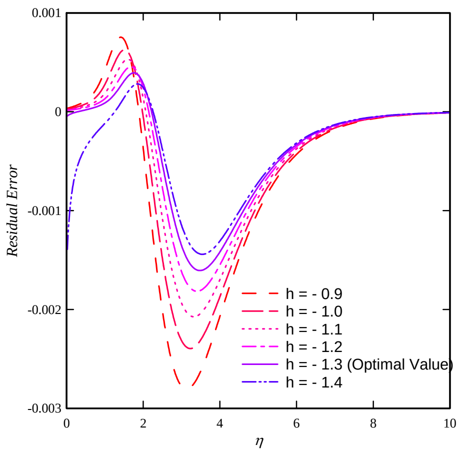

Fig. 3. Residual error of Eq. (34) using 18th-order HAM approximation when.

$k_1 = 0.5, A = \lambda = \gamma = Nt = f_w = Nb = 0.1,$
$Le = 1, \Pr = 0.71, \mathrm{Re} = 5, Br = 5.$

Aboveequationclearly denotes contributions of four sources of entropy generation. The first term on the right-hand side is the entropy generation due to heat transfer across a finite temperature difference; the second term is the local entropy generation due to viscous dissipation.

By using the boundary-layer approximation, the above equation yields to

$$\dot{S}'''_{gen} = \frac{k}{T_\infty^2}\left(\frac{\partial T}{\partial y}\right)^2 + \frac{\mu}{T_\infty}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{RD}{C_\infty}\left(\frac{\partial C}{\partial y}\right)^2 + \frac{RD}{T_\infty}\left(\frac{\partial T}{\partial y}\right)\left(\frac{\partial C}{\partial y}\right). \tag{39}$$

It is appropriate to determine dimensionless number for entropy generation rate N_S. We obtained the entropy generation number by division of the local volumetric entropy generation rate $\dot{S}'''_{gen}$ to a characteristic rate of entropy generation $\dot{s}_o'''$. The characteristic entropy generation rate is

$$\dot{S}_0''' = k\left(\Delta T\right)^2 / l^2 T_\infty^2. \tag{40}$$

Therefore, the entropy generation number is

$$N_G = \frac{\dot{S}'''_{gen}}{\dot{s}_o'''}. \tag{41}$$

Using Eqs. (39), (40) and (41) the entropy generation number is given by

$$N_G = \frac{\dot{S}'''_{gen}}{\dot{S}_0'''} = Re\,\theta'^2(\eta) + Re\frac{Br}{\Omega} f''^2(\eta) + Re\,\lambda\left(\frac{\varsigma}{\Omega}\right)^2 \phi'^2(\eta) + Re\,\lambda\left(\frac{\varsigma}{\Omega}\right)\theta'(\eta)\phi'(\eta), \tag{42}$$

where Re and Br are respectively the Reynolds number and the Brinkman number. Ω is the dimensionless temperature difference. ς and λ are constant parameters.

These number are given by the following relationships

$$Re = c\,l^2/\nu \quad Br = \mu u_w^2/k\Delta T \quad \Omega = \Delta T/T_\infty$$
$$\varsigma = \Delta C/C_\infty \quad \lambda = RDC_\infty/k. \tag{43}$$

3. RESULTS AND DISCUSSION

The nonlinear ordinary differential Eqs.(7)-(9) subject to the boundary conditions (10) are solved numerically for some values of different parameters. The aim of this section is to analyze the effects of various physical parameters on the velocity, temperature, nanoparticle concentration fields and entropy generation. Figs. 4-7 showed that by increasing suction $(f_w > 0)$ the fluid velocity f', concentration profile ϕ and temperature profile θ decreases, respectively, whereas the entropy generation number increases. Fig.8 presents the effects of k_1 on f'.

It is observed that an increases in k_1 yields a decreases in fluid motion. Figs.9-11 presented the effects of k_1 on ϕ, N_G and θ. We observed that an increase in k_1 the concentration profile ϕ, entropy generation number and temperature profile θ increases.

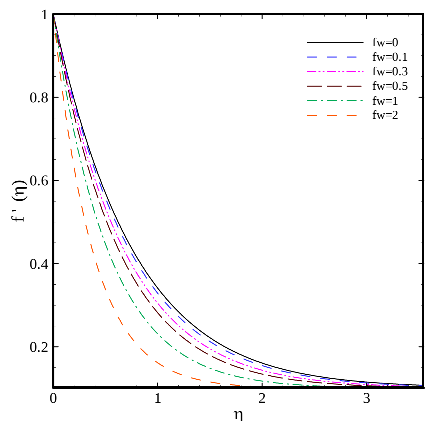

Fig. 4. Influence of f_w on f'.

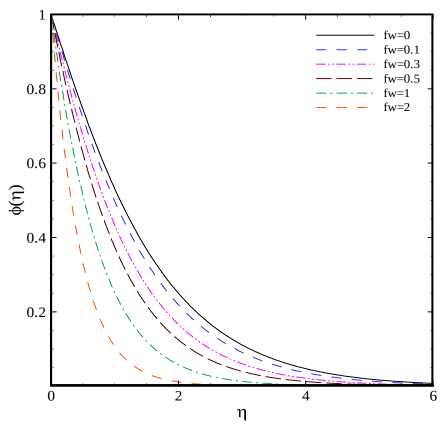

Fig. 5. Influence of f_w on ϕ.

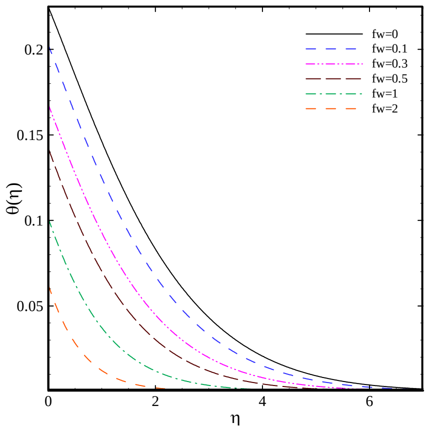

Fig. 6. Influence of f_w on θ.

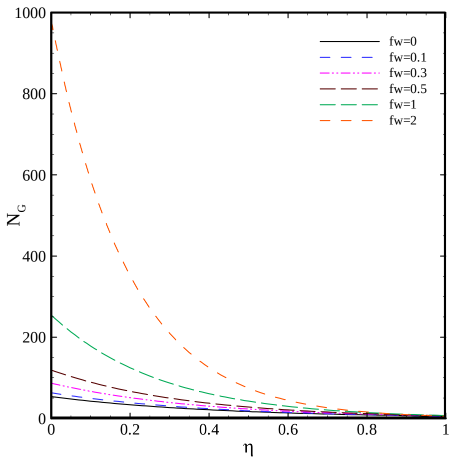

Fig. 7. Influence of f_w on N_G.

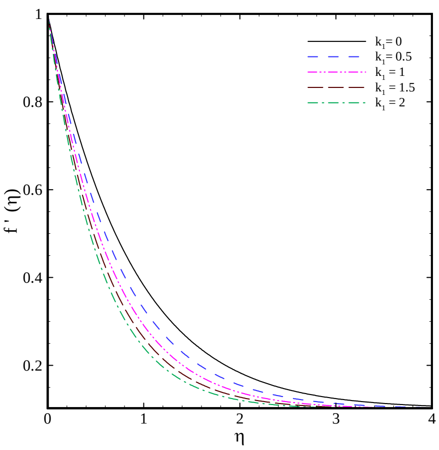

Fig. 8. Influence of k_1 on f'.

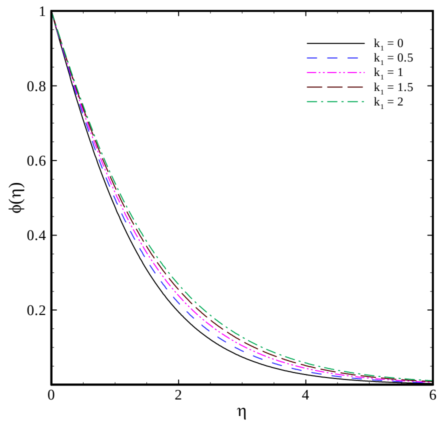

Fig. 9. Influence of k_1 on ϕ.

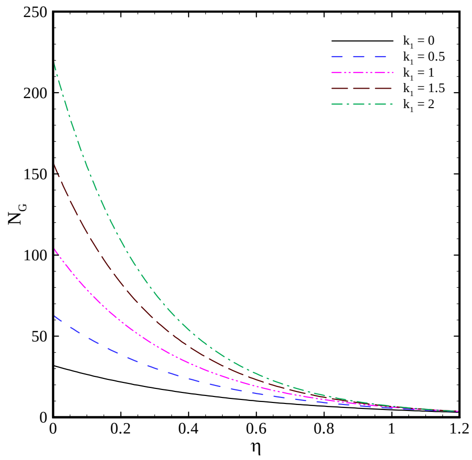

Fig. 10. Influence of k_1 on N_G.

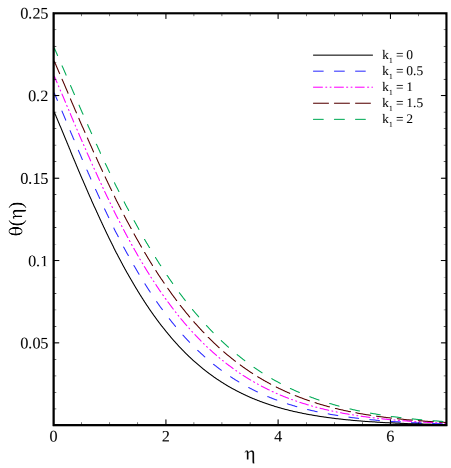

Fig. 11. Influence of k_1 on θ.

Figs.12–15 depict the variation of the A on the velocity, concentration, temperature and entropy generation number. It is observed from Fig. 12 that the f' increases as the A increases. But Figs. 13-15, shows that the ϕ, θ and N_G decreases as A increases. The influence of heat sink parameter $(\lambda < 0)$ on θ is displayed in Fig. 16.It is noted that heat sink gives a decrease in the temperature of the fluid.

Fig. 17 plots the effects of heat source parameter $(\lambda > 0)$. As expected heat source provides an increase in the temperature of the fluid. The effects of Nb on the temperature θ and concentration ϕ are showed in Fig. 18 and Fig 19. When Nb increases, the temperature increases but the concentration decreases.Fig. 20 and Fig21displayed the concentration and temperature profiles increases as Nt increases. It is clear from Fig. 22, with increasing of Le results in a decrease in the concentration profile.

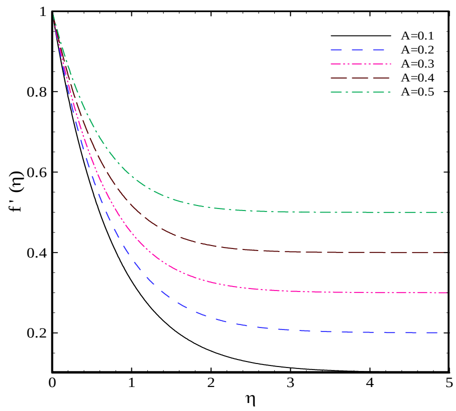

Fig. 12. Influence of A on f'.

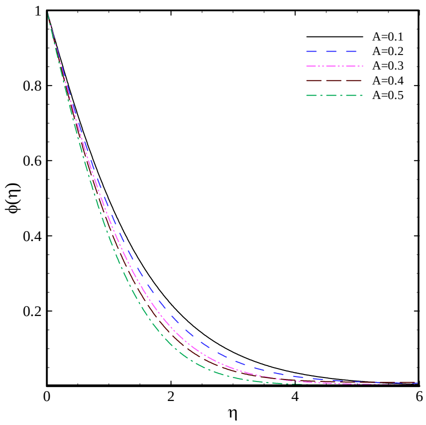

Fig. 13. Influence of A on ϕ.

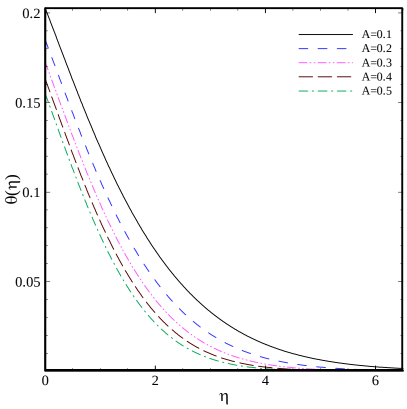

Fig. 14. Influence of A on θ

Fig. 23 demonstrate the effect of the γ on ϕ. An increases in the γ leads to an increases in concentration. Fig. 24 it is observed that for larger amounts of the γ, θ is larger and the γ has a thickening effect on the thermal boundary-layer.

The effect of Prandtl number on the temperature distribution is presented in Fig. 25It is obvious that the thermal boundary-layer thickness is inversely proportional to the square root ofPrandtl number. Thus, as it is shown in Fig. 25, the thermal boundary-layer thicknesses get decreased with increasing the value of the Prandtl number.

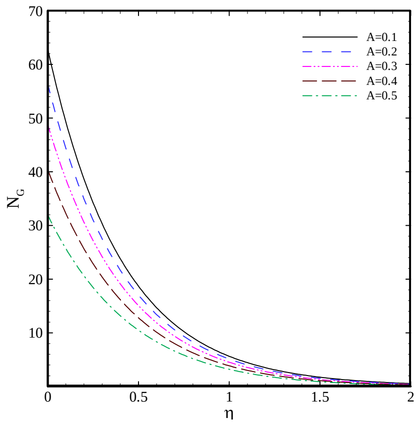

Fig. 15. Influence of A on N_G.

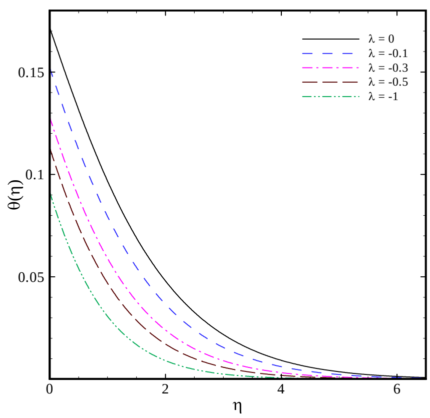

Fig. 16. Influence of λ on θ.

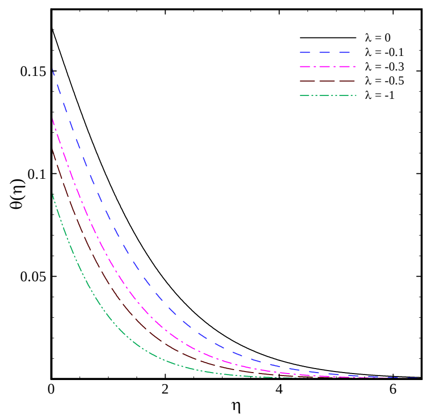

Fig. 16. Influence of λ on θ.

In other word, the flow with large Prandtl number prevents spreading of heat in the fluid.From Fig. 26, we observe that the concentration profiles decreases when Pr increases. The effect of the Re on the entropy generation number is illustrated in Fig. 27. An increase of the Re yields higher entropy generation number (N_G).Fig. 28 shows the influence of the Brinkman number (Br), on the entropy generation number (N_G). The N_G increases with increasing the Brinkman number.Fig. 28demonstrates the effect of Brinkman number on the entropy generation number . An increase in the entropy generation produced by fluid friction and joule dissipation occurs with increasing the value of the Brinkman number.

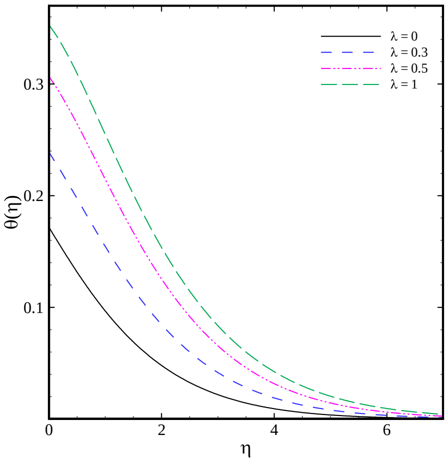

Fig. 17. Influence of λ on θ .

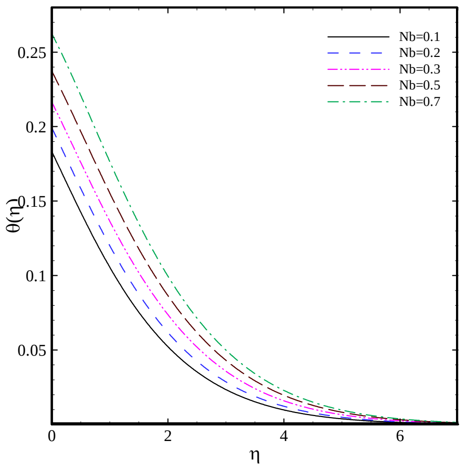

Fig. 18.Influence of Nb on θ .

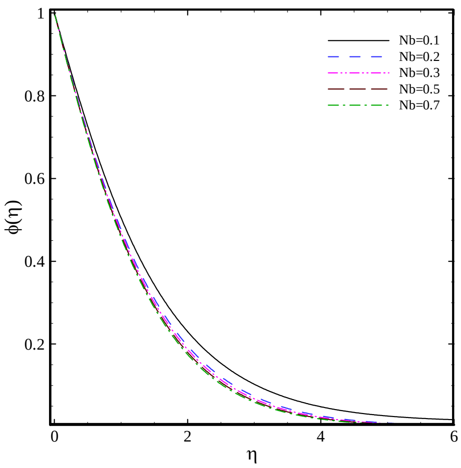

Fig. 19. Influence of Nb on ϕ .

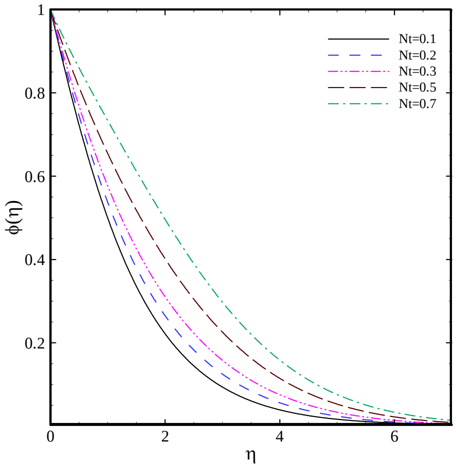

Fig. 20. Influence of Nt on ϕ .

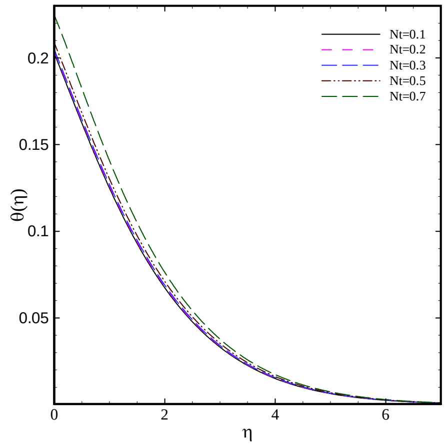

Fig. 21. Influence of Nt on θ .

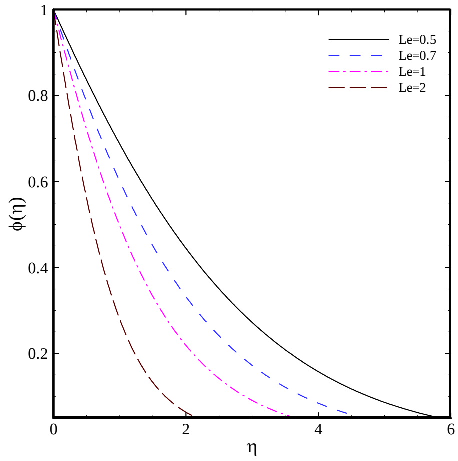

Fig. 22. Influence of Le on ϕ .

4. CONCLUSIONS

In the current perusal, the mathematical formulation has been derived for the entropy generation analysis in stagnation point flow in a porous medium over a permeable stretching surface. HAM is used to solve the system of ordinary differential equations. A good agreement can be seen between the results of this study and the results of previously published data. It is apparently seen that the HAM is very powerful and efficient technique in finding analytical solutions for wide classes of nonlinear partial differential equations.The effects of physical flow parameters such as heat sink /source parameter,Reynoldsnumber,Prandtl number and Brinkman number on the velocity, temperature, nanoparticle concentration fields and entropy generation were shown and discussed. It is found that by increasing Brinkman number and Reynolds number the entropy generation is increases. The equation of entropy generation over a permeable stretching surface was derived.

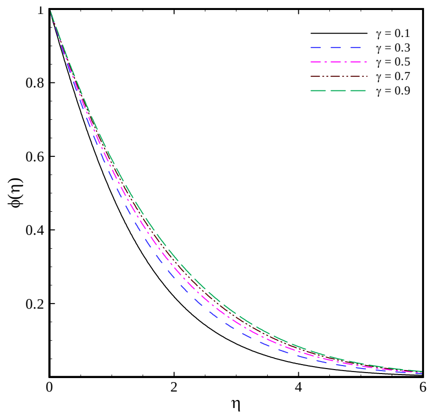

Fig. 23. Influence of γ **on** ϕ .

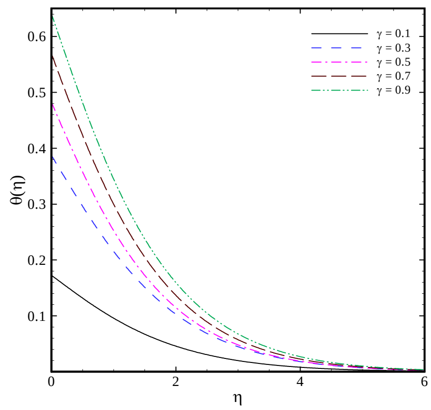

Fig. 24. Influence of γ **on** θ .

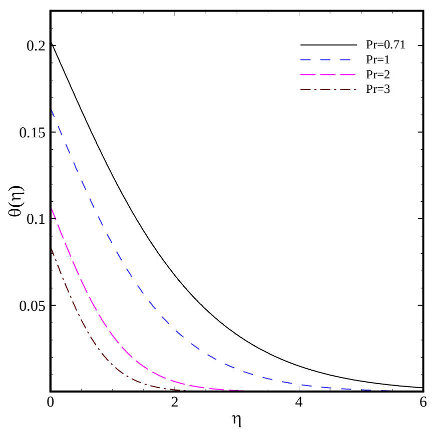

Fig. 25. Influence of $\Pr$ **on** θ .

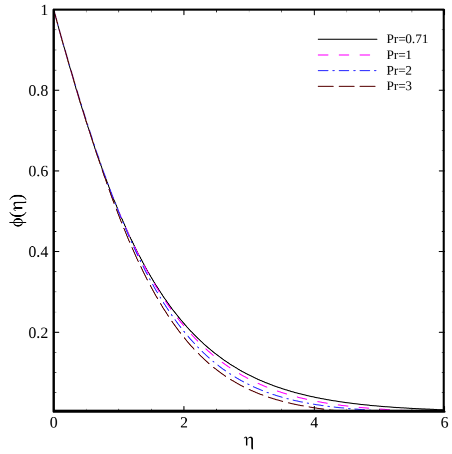

Fig. 26. Influence of $\Pr$ **on** ϕ .

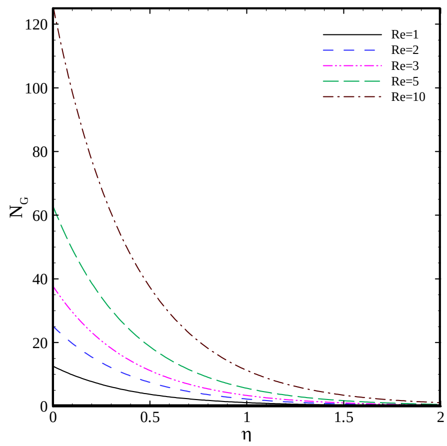

Fig. 27. Influence of Re **on** N_G .

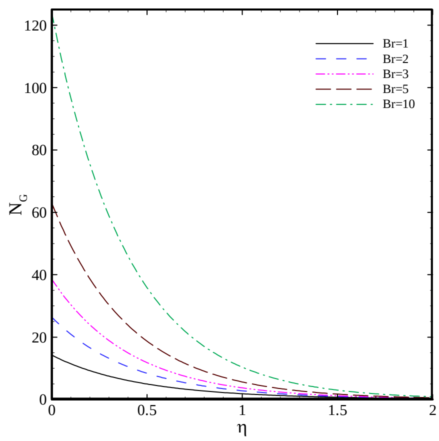

Fig. 28. Influence of Br **on** N_G .

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