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A Weakly Non Linear Stability Analysis of Heat Transport in Anisotropic Porous Cavity Under Time PeriodicTemperature Modulation

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ABSTRACT

In this paper, we have analyzed the effect of time periodic temperature modulation on convective stability in anisotropic porous cavity. The cavity is heated from below and cooled from above. A weakly non-linear stability analysis is done to find Nusselt number governing the heat transport. The infinitely small disturbances are expanded in terms of power series of amplitude of modulation. Analytically the non-autonomous Ginzburg- landau amplitude equation is obtained for the stationary mode of convection. The effects of various parameters like Vadasz number, mechanical and thermal anisotropic parameters, amplitude of oscillations, frequency of modulation and aspect ratio of the cavity on heat transport is studied and plotted graphically. It is observed that the heat transport can also be controlled by suitably adjusting the external and internal parameters of the system.

Keywords: Temperature modulation; Anisotropic porous cavity; Ginzburg- landau amplitude equation.

NOMENCLATURE

	Latin Symbols		
Ar	aspect ratio of the porous cavity, H/L	ρ	density
Da	Darcy number	α_T	coefficient of thermal expansion
g	gravitational acceleration, $(0,0,-g)$	ψ	Stream function
H	height of cavity	μ	dynamic viscosity
K	permeability of the porous domain	ν	kinematic viscosity, μ/ρ_0
L	length of porous layer	ε	oorosity
Nu	Nusselt number	η	thermal anisotropic parameter
Pr	Prandtl number, ν/K_z	τ	rescaled time
q	velocity of the fluid (u, v, w)	δ_1	amplitude of temperature modulation
Ra_0	Critical Rayleigh number	Ω	frequency of modulation
Ra	Rayleigh number, $\frac{\alpha_T(\Delta T)gHK_z}{\varepsilon\nu\kappa_{T_z}}$	k_T	thermal diffusivity
T	temperature	ε	perturbation parameter
t	time	ξ	mechanical anisotropic parameter
Va	Vadasz number		**Other symbols**
ΔT	temperature difference between the walls	*	non-dimensional value
p	pressure	b	basic state
ΔT	temperature difference between the walls	'	perturbed state

1. INTRODUCTION

There are many practical applications where one has to deal with the temperature gradients depending on space as well as time. This is known as temperature modulation. Most of the researchers had studied temperature gradient constant across the fluid layer which does not ideally cope with real applications. Temperature modulation gives better mechanism to control convective flows. An excellent review of the problems and their applications related to classical Rayleigh- Benard convection is presented in books of Ingham and Pop (2005), Vafai (2000) and Nield and Bejan (2006). Horton and Rogers (1945) first studied convection currents in porous medium. Venezian (1969) was the first to investigate the effect of modulation on the onset of thermal convection with small amplitudes on convective instability in a viscous fluid layer. He performed linear stability analysis using perturbation expansions series of amplitude of oscillations. He obtained the shift in critical Rayleigh number and showed that convection can be controlled by suitably adjusting the modulation frequency. Gershuni et al. (1970) studied similar problem on temperature profiles following rectangular law. Rosenblat and Herbert (1970) investigated thermal instability for low frequency temperature modulation. Rosenblat and Tanaka (1971) studied the effect of thermal modulation on the onset of Raleigh–Benard convection using Galerkin technique and investigated the stability of the system using Floquet theory. Roppo et al. (1984) first studied the non linear problem of thermal instability under temperature modulation. They noticed that stable hexagons are produced near the critical Rayleigh number due to modulation effect. Vadasz (1998) studied the coriolis effect on gravity driven convection in a rotating porous layer heated from below. Malashetty and Basavraja (2004) studied effect of thermal modulation on the onset of double diffusive convection in a horizontal anisotropic porous layer. Shu et al. (2005) gave the comparison of experimental and numerical simulation for natural convection in a cavity under modulated thermal gradients and gravity. Bhadauria (2006) investigated the effect of temperature modulation in vertical magnetic field by Galerkin technique. Bhadauria (2007) studied temperature modulation for double diffusive convection in porous medium with temperature modulation. Bhadauria and Suthar (2009) studied non linear thermal instability with temperature modulation using Lorenz model. They made a comparative study of critical Rayleigh number for different temperature profiles. Siddheshwar et al. (2012) investigated the heat transport by stationary magneto convection in Newtonian liquid under temperature or gravity modulation using Ginzburg Landau model. Siddheshwar and Bhadauria (2012) analytically studied the non linear double diffusive convection in a porous medium under temperature/gravity modulation. Siddeshwar et al. (2013) studied the synchronous and asynchronous boundary temperature modulations of Benard-Darcy convection. Bhadauria et al. (2013) studied the effect of time periodic thermal boundary condition and internal heating on heat transport in a porous medium. They concluded that the heat transport can be controlled by adjusting different intrinsic and extrinsic parameters of the system.

Anisotropy in porous media is generated due to asymmetric geometry of porous matrix. The phenomenon is observed in industries and nature. It is useful in study of extraction of metals from ores where a mushy layer is formed during solidification of alloys.

The quantity and structure of resulting solid can be controlled by influencing the heat and mass transport process. Process such as sedimentation, compaction, frost action and reorientation of the solid matrix are responsible for creation of anisotropic natural porous medium. Fiber materials and insulating materials are some examples of artificial anisotropic porous medium. Ephere (1975) was the first to study the onset of convection in a horizontal porous layer with anisotropic thermal conductivity. Kvernvold and Tyvand (1979) studied the non linear thermal convection in anisotropic porous media. Malashetty and Basavaraj (2002) studied Rayleigh Benard convection with temperature/ gravity modulation in a fluid saturated anisotropic porous medium. Malashetty and Swamy (2007) studied the effect of rotation on the onset of convection in a horizontal anisotropic porous layer. Vanishree (2010) studied the combined effect of temperature and gravity modulation on onset of convection in anisotropic porous medium. Vanishree and Siddheshwar (2010) studied the effect of rotation on thermal convection in anisotropic porous medium with temperature dependent viscosity. Other researchers who studied thermal convection in anisotropic porous medium are Nilsen and Storesletten (1990), Tyvand and Storesletten (1991), Degan et al. (1995), Govinder (2006), Malashetty and Heera (2006), Malashetty and Begum (2011). Sarvanam and Sivkumar (2011) studied thermo-vibration instability in fluid saturated anisotropic porous medium. Bhadauria et al. (2011) natural convection in a rotating anisotropic porous layer with internal heat generation and Om et al. (2011) studied rotating Brinkman-Lapwood convection with modulation effect. Bhadauria (2012) studied double diffusive convection in a saturated anisotropic porous layer with internal heat source. Mishra and Kumar (2013) studied the weakly non linear stability analysis of heat transport in anisotropic porous cavity under G-jitter and concluded that heat transport can be controlled by suitably adjusting the parameter of the system. Mishra and Kumar (2014) investigated the chaotic convection in couple stress liquid saturated in porous cavity and found that their exist a proportional behavior between Rayleigh number and couple stress parameter.

In this paper, we have investigated the effect of time periodic temperature modulation on convective instability in anisotropic porous cavity. The cavity is heated from below and cooled from above. The amplitude of temperature modulation is taken to be very small. A weakly non-linear stability analysis is done to find Nusselt number governing the heat transport. Analytically the Ginzburg- landau amplitude equation is obtained for the stationary mode of convection.

The effects of various parameters like Vadasz number, mechanical and thermal anisotropic parameters, amplitude of oscillations, frequency of modulation and aspect ratio of the cavity on heat transport is studied and analyzed graphically. It is observed that the heat transport can also be controlled by suitably adjusting the external and internal parameters of the system.

2 PROBLEM FORMULATIONS

We consider an anisotropic porous cavity of depth H and width L with stress free boundaries which is heated from below and cooled from above. The X-axis is taken along the lower boundary and the Z-axis is vertically upward. The lower surface is held at temperature $T_0 + \Delta T$ while the upper surface is taken at T_0. A uniform positive adverse temperature gradient ΔT is maintained between the lower and upper surfaces. The extended Darcy model which includes the time derivative term is employed in the momentum equation. The continuity and momentum equations governing the motion of an incompressible fluid are given by

$$\nabla . q = 0, \tag{1}$$

$$\frac{1}{\varepsilon}\left(\frac{\partial q}{\partial t} + \frac{1}{\varepsilon}(q.\nabla)q\right) = -\frac{1}{\rho_o}\nabla p + \frac{\rho}{\rho_o} g - \nu K . q \tag{2}$$

$$\frac{\partial T}{\partial t} + (q.\nabla)T = \nabla.(k_T \nabla T), \tag{3}$$

$$\rho = \rho_0[1 - \alpha_T(T - T_0)], \tag{4}$$

where q is the velocity of fluid in porous medium, p is the fluid pressure, ε is the porosity, K is the permeability tensor $K_x^{-1}(\hat{\imath}\hat{\imath} + \hat{\jmath}\hat{\jmath}) + K_z^{-1}\left((\hat{k}\hat{k})\right)$, k_T is the thermal diffusivity tensor $k_{Tx}(\hat{\imath}\hat{\imath} + \hat{\jmath}\hat{\jmath}) + k_{Tz}(\hat{k}\hat{k})$, T is the temperature, ν is the kinematic viscosity.

Time-periodic boundary temperature

We assume that the externally imposed boundary temperatures oscillates with time. According to the relations used by Venezian (1969),

$$T = T_0 + \frac{\Delta T}{2}\,[1 + \varepsilon^2\delta_1\cos(\Omega t)]\ at\ z = 0,$$
$$T = T_0 - \frac{\Delta T}{2}\,[1 - \varepsilon^2\delta_1\cos(\Omega t + \emptyset)]\ at\ z. = H \tag{5}$$

Here Ω is the modulation frequency and $\emptyset$ is phase angle . The quantity $\varepsilon^2\delta_1$ is the amplitude of modulation, where ε and δ_1 both are small resulting in the modulation to be of small amplitudes. Assuming the basic state to be quiescent. The quantities at the basic state are given by

$$q_b = (0,0,0),\quad p = p_b(z,t),\qquad T = T_b(z,t) and\ \rho = \rho_b(z,t), \tag{6}$$

which satisfy the following equations,

$$\frac{\partial \rho_b}{\partial z} = -\rho_b g, \tag{7}$$

$$\frac{\partial T_b}{\partial t} = k_{Tz}\frac{\partial^2 T_b}{\partial z^2}, \tag{8}$$

$$\rho_b = \rho_0[1 - \alpha_T(T - T_0)]\,. \tag{9}$$

According to the Venezian, we can write the non-dimensionlized basic temperature as,

$$T_b(z,t) = T_0 + 1 - z + \varepsilon^2\delta_1 F(z,t) \tag{10}$$

Where

$$F(z,t) = Re\left[\left\{A(\lambda)e^{\lambda z} + A(-\lambda)e^{-\lambda z}\right\}e^{-i\Omega t}\right],$$

$$A(\lambda) = \frac{1}{2}\frac{(e^{-i\emptyset} - e^{-\lambda})}{(e^{\lambda} - e^{-\lambda})};\quad \lambda = (1 - i)\sqrt{\frac{\Omega}{2}}. \tag{11}$$

Now superimpose the small perturbations at the basic state as

$$q = q_b + q',\qquad T = T_b + T',\qquad p = p_b + p',\qquad \rho = \rho_b + \rho', \tag{12}$$

where primes denote the quantities at the perturbed state and subscript “b” refers to the basic state.

Putting Eq. (12) in Eqs. (1)−(4) and using solution of basic state Eq. (6), the perturbed equations are obtained as

$$\nabla . q' = 0, \tag{13}$$

$$\frac{1}{\varepsilon}\left(\frac{\partial q'}{\partial t} + \frac{1}{\varepsilon}(q'.\nabla)q'\right) = -\frac{1}{\rho_0}\nabla p' + g\alpha_T T' - \nu K . q', \tag{14}$$

$$\frac{\partial T'}{\partial t} + (q'.\nabla)T' + w'\frac{\partial T_b}{\partial z} = k_{Tx}{\nabla_1}^2 T' + k_{Tz}\frac{\partial^2 T'}{\partial z^2}. \tag{15}$$

Now performing the non-dimensionalisation in Eqs. (14) – (15) using the transformations

$$q' = \frac{k_{Tz}}{H}q^*,\ p' = \frac{\mu k_{Tz}}{K_z}p^*,\ T' = (\Delta T)T^*,$$
$$(x,y,z) = H(x^*,y^*,z^*),$$
$$t = \frac{H^2}{K_{Tz}}t^* \quad \Omega = \frac{k_{Tz}}{H^2}\Omega^*$$

Eliminating pressure term by taking curl of Eq. (14) and introducing the stream function defined as $(u,v,w) = (\frac{\partial\Psi}{\partial z}, 0, -\frac{\partial\Psi}{\partial x})$, we get

$$\left[\frac{1}{Va}\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) + \left(\frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\right]\Psi + Ra\frac{\partial T}{\partial x} = \frac{1}{Va}\,\frac{\partial(\Psi,\ \nabla^2\Psi)}{\partial(x,z)}, \tag{16}$$

$$\left[\frac{\partial}{\partial t} - \left(\eta\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\right]T = \frac{\partial\Psi}{\partial x}\frac{\partial T_b}{\partial z} + \frac{\partial(\Psi,T)}{\partial(x,z)}\,. \tag{17}$$

Where $Va = \varepsilon Pr/Da$ is Vadsaz number, $Ra = \frac{\alpha_T(\Delta T)gHK_Z}{\varepsilon\nu\kappa_{Tz}}$ is the Rayleigh number and $Da = K_z/H^2$ is the Darcy number. Assuming boundaries are stress free and isothermal, therefore the non-dimensionalised boundary conditions for the perturbed quantities are given by

$$\Psi = \frac{\partial^2\Psi}{\partial z^2} = 0\ at\ z = 0\ and\ z = 1, \tag{18}$$

$$T = 1\ at\ z = 0\ and\ T = 0\ at\ z = 1, \tag{19}$$

$$\frac{\partial T}{\partial x} = 0\ at\ x = 0\ and\ = Ar. \tag{20}$$

Now rescaling time $\tau = \varepsilon^2 t$ to keep the time variation slow. Let $\Omega = \frac{\omega}{\varepsilon^2}$. The Eqs. (16) and (17) can be written as

$$\left(\frac{\varepsilon^2}{Va}\frac{\partial}{\partial\tau}\nabla^2 + {\nabla_\xi}^2\right)\Psi + Ra\frac{\partial T}{\partial x} = \frac{1}{Va}\,\frac{\partial(\Psi,\ \nabla^2\Psi)}{\partial(x,z)}\,, \tag{21}$$

$$\left[\varepsilon^2\frac{\partial}{\partial\tau}-\nabla_\eta{}^2\right]T$$
$$=\{-1+\varepsilon^2\delta_1F(z,\tau)\}\frac{\partial\Psi}{\partial x}$$
$$+\frac{\partial(\Psi,T)}{\partial(x,z)}. \quad (22)$$

Where $\nabla_\xi{}^2=\frac{\partial^2}{\partial x^2}+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}$, $\quad\nabla_\eta{}^2=\eta\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}$.

3. AMPLITUDE EQUATION (GINZBURG-LANDAU EQUATION) AND HEAT TRANSPORT

Introduce the following asymptotic equations in the Eqs. (21) and (22)

$$\left.\begin{aligned}Ra&=Ra_0+\varepsilon^2Ra_2+\cdots\\ \Psi&=\varepsilon\Psi_1+\varepsilon^2\Psi_2+\cdots\\ T&=\varepsilon T_1+\varepsilon^2T_2+\cdots\end{aligned}\right\} \quad (23)$$

where Ra_0 is the critical Rayleigh number at which convection sets in without modulation. Put Eq. (23) in Eqs. (21) and (22).

At lowest order, ε equations are

$$\begin{bmatrix}\nabla_\xi^2 & Ra_0\frac{\partial}{\partial x}\\ \frac{\partial}{\partial x} & -\nabla_\eta^2\end{bmatrix}\begin{bmatrix}\Psi_1\\ T_1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix} \quad (24)$$

This is corresponding to linear stability equations for stationary anisotropic porous convection and solution of the above equations can be written as

$$\left.\begin{aligned}\Psi_1(x,z,\tau)&=A(\tau)\sin\left(\frac{\pi x}{Ar}\right)sin(\pi z)\\ T_1(x,z,\tau)&=\frac{-Ar}{\pi(\eta+Ar^2)}A(\tau)\cos\left(\frac{\pi x}{Ar}\right)\sin(\pi z)\end{aligned}\right\} \quad (25)$$

where critical Rayleigh number for anisotropic porous convection in the absence of temperature modulation is given by

$$Ra_0=\frac{\pi^2(\xi+Ar^2)(\eta+Ar^2)}{\xi Ar^2}. \quad (26)$$

At the second order, we have,

$$\begin{bmatrix}\nabla_\xi^2 & Ra_0\frac{\partial}{\partial x}\\ \frac{\partial}{\partial x} & -\nabla_\eta^2\end{bmatrix}\begin{bmatrix}\Psi_2\\ T_2\end{bmatrix}=\begin{bmatrix}R_{21}\\ R_{22}\end{bmatrix} \quad (27)$$

$$R_{21}=\frac{1}{Va}\frac{(\partial\Psi_1,\nabla^2\Psi_1)}{\partial(x,z)}=0, \quad (28)$$

$$R_{22}=\frac{\partial(\Psi_1,T_1)}{\partial(x,z)}=\frac{-\pi}{2(\eta+Ar^2)}[A(\tau)]^2\sin(2\pi z). \quad (29)$$

We obtain second order solution as

$$\left.\begin{aligned}\Psi_2&=0\\ T_2&=-\frac{[A(\tau)]^2}{8\pi(\eta+Ar^2)}\sin(2\pi z)\end{aligned}\right\} \quad (30)$$

The horizontally-averaged Nusselt number $Nu(\tau)$ for the anisotropic porous convection is given by

$$Nu(\tau)=\frac{\left[2Ar\int_{x=0}^{Ar}(1-z+T_2)_z dx\right]_{z=0}}{\left[2Ar\int_{x=0}^{Ar}(1-z)_z dx\right]_{z=0}}. \quad (31)$$

Now substituting Eq. (27) in Eq. (28) and solving the integration, we get

$$\mathrm{Nu}(\tau)=1+\frac{[A(\tau)]^2}{4(\eta+Ar^2)}.$$

At the third order solution, we have

$$\begin{bmatrix}\nabla_\xi^2 & Ra_0\frac{\partial}{\partial x}\\ \frac{\partial}{\partial x} & -\nabla_\eta^2\end{bmatrix}\begin{bmatrix}\Psi_3\\ T_3\end{bmatrix}=\begin{bmatrix}R_{31}\\ R_{32}\end{bmatrix} \quad (32)$$

$$R_{31}=-Ra_2\frac{\partial T_1}{\partial x}-\frac{1}{Va}\frac{\partial(\nabla^2\Psi_1)}{\partial\tau}, \quad (33)$$

$$R_{32}=\frac{\partial\Psi_1}{\partial x}\frac{\partial T_2}{\partial z}-\frac{\partial T_1}{\partial\tau}+\delta_1F(z,\tau)\frac{\partial\Psi_1}{\partial x}. \quad (34)$$

Substituting Ψ_1, T_1, T_2 from Eqs. (25) and (30) in Eqs. (33)-(34), we get

$$R_{31}=\left[\frac{-Ra_2}{\eta+Ar^2}A(\tau)+\frac{\pi^2(Ar^2+1)}{Va\,Ar^2}\frac{dA}{d\tau}\right], \quad (35)$$

$$R_{32}$$
$$=\left[\frac{Ar}{\pi(\eta+Ar^2)}\frac{dA}{d\tau}\right.$$
$$-\frac{\pi}{4Ar(\eta+Ar^2)}A^3(\tau)\cos(2\pi z)$$
$$\left.+\frac{\pi}{Ar}\delta_1F(z,\tau)A(\tau)\right]\cos\left(\frac{\pi x}{Ar}\right)\sin(\pi z). \quad (36)$$

Adjoin of Eq. (21) is obtained. The solutions of adjoin so obtained are as

$$\left.\begin{aligned}\hat{\Psi}_1(x,z,\tau)&=-A(\tau)\sin\left(\frac{\pi x}{Ar}\right)\sin(\pi z)\\ \hat{T}_1(x,z,\tau)&=-\frac{Ar}{\pi(\eta+Ar^2)}A(\tau)\cos\left(\frac{\pi x}{Ar}\right)\sin(\pi z)\end{aligned}\right\} \quad (37)$$

where $\hat{\Psi}_1$, $\hat{T}_1$ denotes solutions of adjoins.

The solvability condition for the third order solution is given by The solvability condition for the third order solution is given as

$$\int_{z=0}^{1}\int_{x=0}^{Ar}[\hat{\Psi}_1\ R_{31}+Ra_0\,\hat{T}_1R_{32}]dxdz=0. \quad (38)$$

Now substituting Eqs. (35)-(37) into the Eq. (38), we get the autonomous Ginzburg- Landau equation for stationary instability with a time periodic coefficient in the form

$$\left[\frac{\pi^2(Ar^2+1)}{VaAr^2}+Ra_0\frac{Ar^2}{\pi^2(\eta+Ar^2)^2}\right]\frac{dA(\tau)}{d\tau}$$
$$-\frac{\{Ra_2-2Ra_0\delta_1I(\tau)\}}{\eta+Ar^2}A(\tau)+\frac{Ra_0}{8(\eta+Ar^2)^2}A^3(\tau)$$
$$=0, \quad (39)$$

where $I(\tau)=\int_0^1F(z,\tau)\sin^2(\pi z) \quad (40)$

The solution of Eq. (40) is obtained using fourth order Runge-Kutta method numerically subject to the initial condition $A(0)=a_0$, where a_0 is a chosen initial amplitude of convection. Here $Ra_2=Ra_0$ is taken to keep the parameters to be minimum.

4. RESULTS AND DISCUSSIONS

In this paper, we have studied the effect of

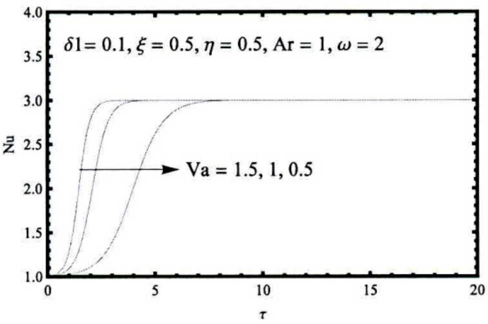

Figure.1(a): Effect of Vadasz number

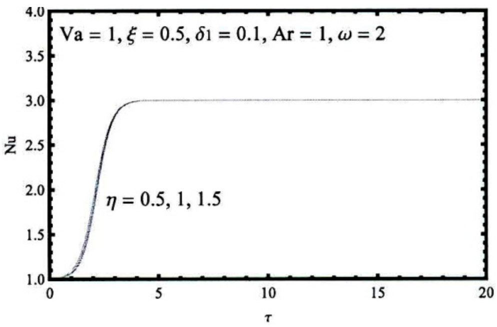

Figure.1(b): Effect of Thermal anisotropic parameter

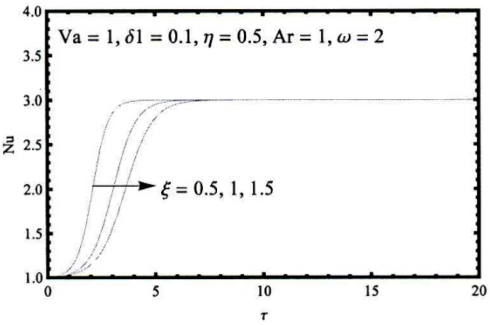

Figure.1(c): Effect of Mechanical anisotropic parameter

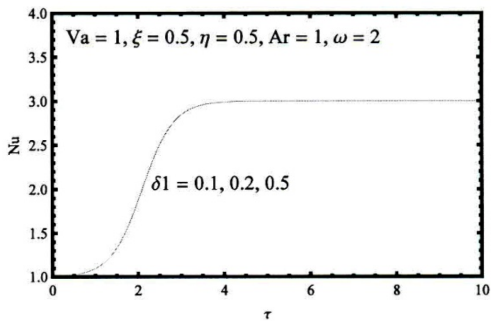

Figure.1(d): Effect of Amplitude of modulation

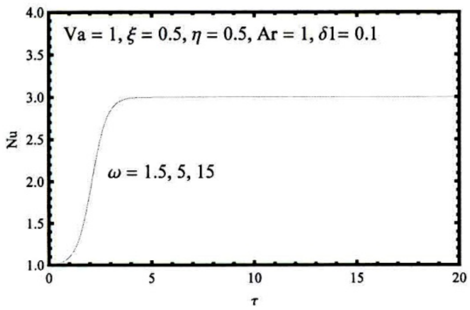

Figure.1(e): Effect of Frequency of modulation

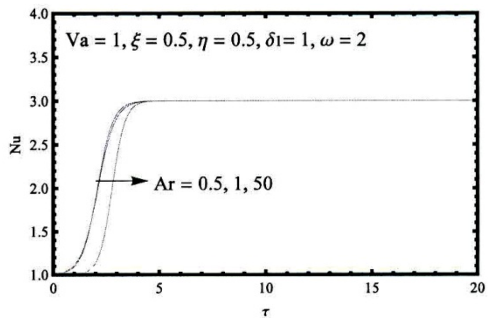

Figure.1(f): Effect of Aspect ratio

Fig. 1. In-phase modulation ($\emptyset = 0$), Graph of Nusselt number (Nu) with respect to rescaled time (τ) for different intrinsic and extrinsic parameters of the system as $\mathbf{Va = 1,\ \eta = 0.5,\ \xi = 0.5,\ \delta_1 = 0.1,\ \omega = 2,\ Ar = 1.}$

temperature modulation on thermal instability in anisotropic porous cavity. A weakly non-linear instability analysis is done to study the heat transfer under different regimes. We have considered only small amplitude temperature modulation. The effect of different parameters Vadasz number (Va), thermal anisotropic parameter (η), mechanical anisotropic parameter (ξ), amplitude of temperature modulation(δ_1), frequency of modulation (ω) and aspect ratio (Ar) on heat transfer with respect to rescaled time is done. Each parameter is one by one changed keeping others constant and nature of heat transfer is observed. The Values of different intrinsic and extrinsic parameters of the system are taken $\mathrm{Va} = 1,\ \eta = 0.5,\ \xi = 0.5,\ \delta_1 = 0.1,\ \omega = 2,\ \mathrm{Ar} = 1$ from Bhadauria et al. (2013), so that it has physical significance. The numerical values of Nusselt number is obtained from the Eq.(31) by solving the autonomous amplitude Ginzburg-Landau Eq. (40). The graphs of Nusselt number with respect to rescaled time τ is presented in the fig.(1), fig.(2) and fig.(3). The figures show that the temperature modulation destabilizes the onset of convection. The results obtained are in line with Bhadauria et al. (2013).

It is observed that the value of Nusselt number starts with 1 remains constant for some time shows conduction state, then increases with time shows convection state and further increasing time becomes constant, reaches to steady state in general. The temperature modulation has following three cases:

1. In-phase modulation ($\emptyset = 0$)
2. Out-phase modulation ($\emptyset = \pi$)
3. Only lower boundary modulation phase ($\emptyset = -i\infty$)

4.1 In-Phase Modulation

The results are in line with non modulated case. From fig.1 (a), we observe that as the Vadasz number (Va) increases from 0.5 to 1.5, the Nusselt number (Nu) increases in conduction and convection state. After some time the Nusselt

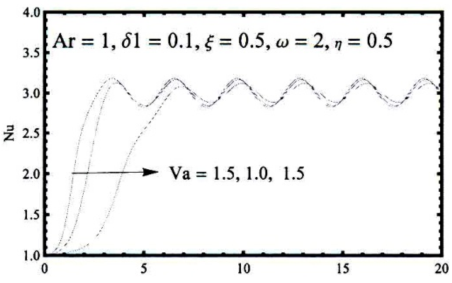

Figure.2(a): Effect of Vadasz number

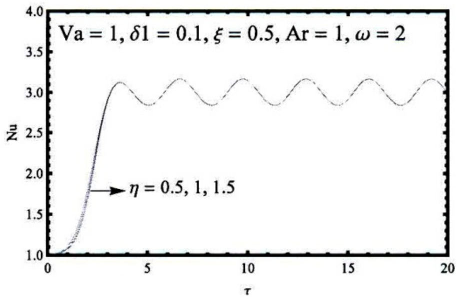

Figure.2(b): Effect of Thermal anisotropic parameter

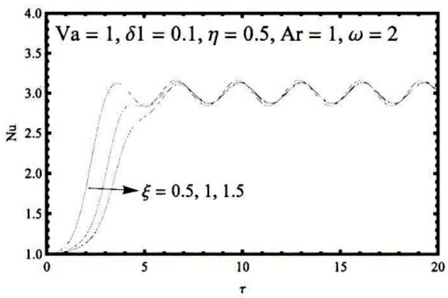

Figure.2(c): Effect of Mechanical anisotropic parameter

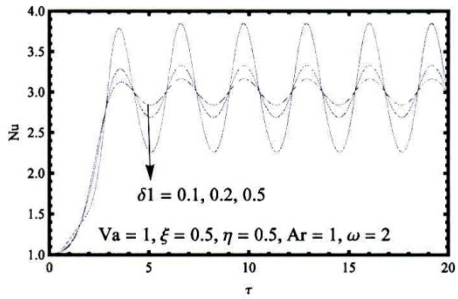

Figure.2(d): Effect of Amplitude of modulation

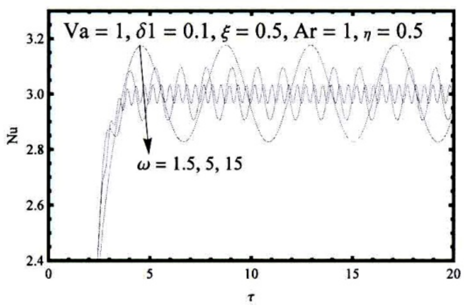

Figure.2(e): Effect of Frequency of modulation

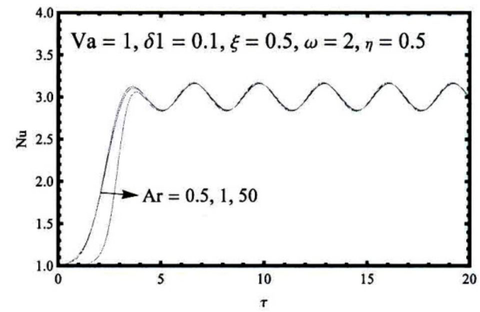

Figure.2(f): Effect of Aspect ratio

Fig. 2. Out-phase modulation ($\emptyset = \pi$), Graph of Nusselt number (Nu) with respect to rescaled time (τ) for different intrinsic and extrinsic parameters of the system as $\mathbf{Va = 1,\ \eta = 0.5,\ \xi = 0.5,\ \delta_1 = 0.1,\ \omega = 2,\ Ar = 1.}$

number becomes constant independent of Vadasz number reaching to steady state. So, effect of porosity i.e. Vadasz number and heat transfer are same in nature for the smaller values of time and becomes constant in steady state. From fig.1(b), we observe that as the thermal anisotropic parameter (η) increases from 0.5 to 1.5, the Nusselt number (*Nu*) slightly varies in convection state otherwise remains constant as the time passes. Thus increase in the thermal anisotropic parameter has negligible effect on heat transport. From fig.1(c), we observe that as mechanical anisotropic parameter (ξ) increases from 0.5 to 1.5, the Nusselt number (*Nu*) decreases in conduction and convection state. After some time the Nusselt number becomes constant independent of mechanical anisotropic parameter. So, effect of mechanical anisotropic parameter and heat transport are opposite in nature for the smaller values of time and becomes constant in steady state. From fig.1 (d), we observe that when amplitude of temperature modulation (δ_1) increases from 0.1 to 0.5, magnitude of Nusselt number (*Nu*) remains almost constant. So the effect of increase in amplitude of temperature modulation is negligible on heat transport. From fig.1 (e), it is observed that as the frequency of modulation (ω) increases from 1.5 to 15, the magnitude of Nusselt number (*Nu*) remains almost constant and the effect of modulation on heat transfer is negligible. From fig.1 (f), it is observed that as the aspect ratio (Ar) of the anisotropic porous cavity increases from 0.5 to 50, the Nusselt number (*Nu*) decreases in conduction and convection state. After some time the Nusselt number becomes constant independent of Aspect ratio. So, the heat transport decreases in conduction as well as convection state but after some time becomes constant in steady state.

4.2 Out-Phase Modulation Case

From fig.2 (a), we observe that as the Vadasz number (Va) increases from 0.5 to 1.5, the Nusselt number (*Nu*) increases in conduction and convection state. After some time the Nusselt number becomes oscillatory in nature independent of Vadasz number and reaches to steady state. So, effect of porosity i.e. Vadasz number and heat

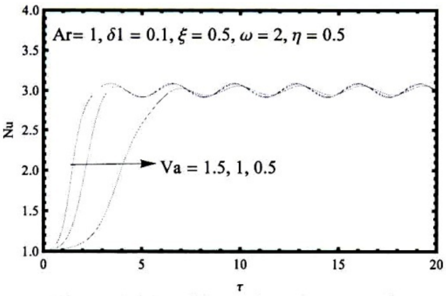

Figure.3(a): Effect of Vadasz number

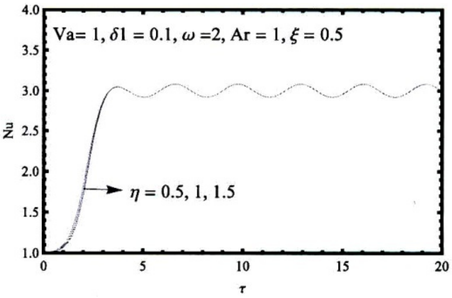

Figure.3(b): Effect of Thermal anisotropic parameter

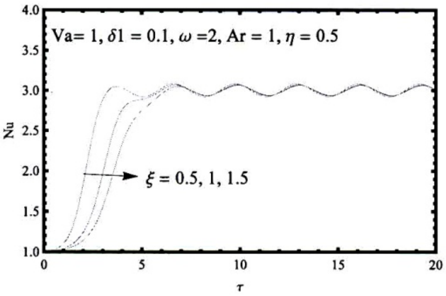

Figure.3(c): Effect of Mechanical anisotropic Parameter

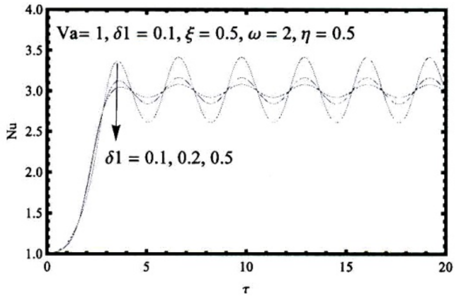

Figure.3(d): Effect of Amplitude of modulation

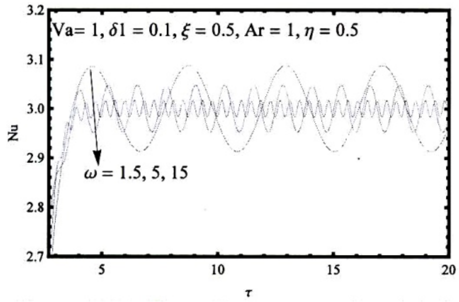

Figure.3(e): Effect of Frequency of modulation

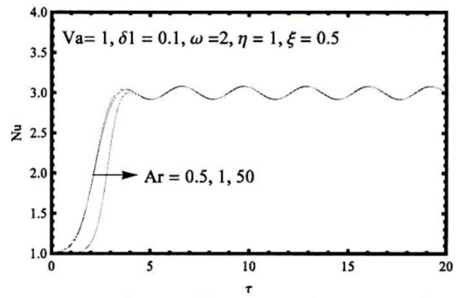

Figure.3(f): Effect of Aspect ratio

Fig. 3. Only lower boundary modulation phase ($\emptyset = -i\infty$), Graph of Nusselt number (Nu) with respect to rescaled time (τ) for different intrinsic and extrinsic parameters of the system as $\mathrm{Va} = 1,\ \eta = 0.5,\ \xi = 0.5,\ \delta_1 = 0.1,\ \omega = 2,\ \mathrm{Ar} = 1.$

transport are same in nature for the smaller values of time and becomes oscillatory in steady state. From fig.2 (b), we observe that as the anisotropic parameter (η) increases from 0.5 to 1.5, the Nusselt number (*Nu*) slightly varies in convection state otherwise remains constant but oscillatory as the time passes. Thus increase in the thermal anisotropic parameter has negligible effect on heat transport. From fig.2(c), we observe that as mechanical anisotropic parameter (ξ) increases from 0.5 to 1.5, the Nusselt number (*Nu*) decreases in conduction and convection state. After some time the Nusselt number becomes oscillatory in nature independent of mechanical anisotropic parameter. So, effect of mechanical anisotropic parameter and heat transport are opposite in nature for the smaller values of time and becomes almost same but oscillatory in steady state. From fig.2 (d), we observe that when amplitude of temperature modulation (δ_1) increases from 0.1 to 0.5, magnitude of Nusselt number (*Nu*) increases in convection state. In steady state the heat transport is oscillatory in nature. Further the amplitude of oscillations increases with increase in amplitude of temperature modulation. From fig.2 (e), it is observed that as the frequency of modulation (ω) increases from 1.5 to 15, the magnitude of Nusselt number (*Nu*) increases in conduction and convection state. Afterwards in steady state the heat transport is oscillatory in nature. Further the amplitude of oscillations increases with increase in frequency of modulation. From fig.2 (f), it is observed that as the aspect ratio (Ar) of the anisotropic porous cavity increases from 0.5 to 50, the Nusselt number (*Nu*) decreases in conduction and convection state. After some time the Nusselt number becomes oscillatory (almost same) independent of aspect ratio (Ar). So, the heat transport decreases in conduction as well as convection state but after some time becomes equal in steady state.

4.3 Only Lower Boundary Modulation Phase

From fig.3 (a), we observe that as the Vadasz number (Va) increases from 0.5 to 1.5, the Nusselt number (*Nu*) increases in conduction and convection state. After some time the Nusselt number becomes oscillatory in nature independent of Vadasz number and reaches to steady state. So, effect of porosity i.e. Vadasz number and heat transport are same in nature for the smaller values of time and becomes oscillatory in steady state. From fig.3 (b), we observe that as the anisotropic parameter (η) increases from 0.5 to 1.5, the Nusselt number (*Nu*) slightly varies in convection state otherwise remains constant but oscillatory as the time passes. Thus increase in the thermal anisotropic parameter has negligible effect on heat transport. From fig.3(c), we observe that as mechanical anisotropic parameter (ξ) increases from 0.5 to 1.5, the Nusselt number (*Nu*) decreases in conduction and convection state. After some time the Nusselt number becomes oscillatory in nature independent of mechanical anisotropic parameter. So, effect of mechanical anisotropic parameter and heat transport is opposite in nature for the smaller values of time and becomes almost same but oscillatory in steady state. From fig.3 (d), we observe that when amplitude of temperature modulation (δ_1) increases from 0.1 to 0.5, magnitude of Nusselt number increases in convection state. In steady state the heat transport is oscillatory in nature. Further the amplitude of oscillations increases with increase in amplitude of temperature modulation. From fig.3 (e), it is observed that as the frequency of modulation (ω) increases from 1.5 to 15, the magnitude of Nusselt number slightly increases in conduction and convection state. Afterwards in steady state the heat transport is oscillatory in nature. Further the amplitude of oscillations increases with increase in frequency of modulation. From fig.3 (f), it is observed that as the aspect ratio (Ar) of the anisotropic porous cavity increases from 0.5 to 50, the Nusselt number (*Nu*) decreases in conduction and convection state. After some time the Nusselt number becomes oscillatory (almost same) independent of aspect ratio. So, the heat transport decreases in conduction as well as convection state but after some time becomes equal in steady state.

5. CONCLUSIONS

We conclude that by properly adjusting the different parameters in model, we can control the heat transfer. The results obtained are in line with Bhadauria et al. (2013).The observations of authors are as follows:

For In Phase Modulation, the heat transfer increases with increases in Va, δ_1, ω while remains almost constant with increase in ξ, η, Ar in conduction as well as convection state. In steady state there is no effect of any of the six parameters on heat transfer. For Out Phase Modulation, the heat transfer increases with decrease in $\mathrm{Va}, \xi, \delta_1, \omega, \mathrm{Ar}$ and remains almost same with increase in η in conduction and convection state. The heat transfer is oscillatory in steady state. For Lower Boundary Modulation the nature of heat transfer is similar to that out phase modulation but varies in magnitude of heat transfer, amplitude of oscillations and wave length of oscillations. In all cases the heat transfer increases from IMP, LBM to OPM. So, authors conclude that heat transfer can be controlled by suitably adjusting the individual parameters as per the requirement.

Nature of heat transfer in steady state as time passes is as

Increase in para.\ moduln.	In phase modulation	Out phase modulation	Lower boundary modulation
Va	Constant	Oscillatory	oscillatory*
η	Constant	Oscillatory	oscillatory*
ξ	Constant	Oscillatory	oscillatory*
ω	Constant	Oscillatory	oscillatory*
δ_1	Constant	Oscillatory	oscillatory*
Ar	Constant	Oscillatory	oscillatory*

*The amplitude of oscillations increases but wave length shortens comparatively.

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