Journal of Applied Fluid Mechanics, Vol. 8, No. 4, pp. 921-929, 2015.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.18869/acadpub.jafm.73.238.23949

Investigation of Transient MHD Couette flow and Heat Transfer of Dusty Fluid with Temperature-Dependent Oroperties

S. Mosayebidorcheh[1†], M. Hatami[2], D.D. Ganji[3], T. Mosayebidorcheh[1]and S. M. Mirmohammadsadeghi[4]

[1] *Young Researchers and Elite Club, Najafabad Branch, Islamic Azad University, Najafabad, Iran*
[2] *Esfarayen University of Technology, Engineering and Technical College, Mechanical Engineering Department, Esfarayen, North Khorasan, Iran*
[3] *Babol University of Technology, Department of Mechanical Engineering, Babol, Iran*
[4] *Manufacturing System Integration (MSI), Department of Mechanical Engineering, Faculty of Engineering, University of Malaya, 50603 Kuala Lumpur, Malaysia*

†*Corresponding Author Email: sobhanmosayebi@yahoo.com*

(Received July 19, 2014; accepted September 7, 2014)

ABSTRACT

In the present study, transient MHD Couette flow and heat transfer of dusty fluid between two parallel plates and the effect of the temperature dependent properties has been investigated. The thermal conductivity and viscosity of the fluid are assumed as linear and exponential functions of temperature, respectively. A constant pressure gradient and an external uniform magnetic field are considered in the main flow direction and perpendicular to the plates, respectively. A hybrid treatment based on finite difference method (FDM) and differential transform method (DTM) is used to solve the coupled flow and heat transfer equations. The effects of the variable properties, Hartman number, Hall current, Reynolds number and suction velocity on the Nusselt number and skin friction factor have been discussed. It is found that when Hartman number increases, skin friction of the upper and lower plates increases.

Keywords: MHD couette flow; Temperature dependent property; Magnetic field; Hybrid DTM-FDM; Nusselt number; Skin friction.

1. INTRODUCTION

The phenomenon of the heat and mass transfer of the fluid and dust particles through a channel has some applications in industries such as polymer technology, electrostatic precipitation, petroleum industry, fluid droplet sprays, application of dust in gas cooling systems, purification of crude oil, fluidization, centrifugal separation of matter from fluid and combustion. The hydrodynamic flow and heat transfer of a dusty fluid has been analyzed by some researchers Saffman (1962), Gupta and Gupta (1976), Prasad, and Ramacharyulu (1979) and Dixit (1980). The MHD flow and heat transfer of a dusty fluid in a channel is important in analysis of the flow-meters, accelerators, pumps and magneto-hydrodynamic generators. Some of the pioneer works which are done about this subject can be found in Singh (1976), Mitra, and Bhattacharyya (1981), Borkakotia and Bharali (1983) and Megahed *et al*. (1988). In these researches, the physical properties of the fluid such as viscosity and thermal conductivity considered to be constant. To reach a more accurate result, it is necessary to use temperature-dependent properties in modeling for them.

Effect of temperature-dependent viscosity on the flow and heat transfer of the fluid in a channel has been investigated by Klemp *et al.* (1990). Attia and Kotb (1996) analyzed the steady MHD fully developed flow between two parallel plates by considering a temperature-dependent function for viscosity. Recently, Attia (1999) expanded the same problem in the transient state. Also, the effects of viscous dissipation and thermal dispersion of a viscous dusty fluid under the influence of a magnetic field have been studied by Sharma and Varshney (2003). Makinde and Chinyoka (2010) investigated the transient MHD flow and heat transfer in a channel. In their work, the properties of the fluid assumed to be variable with temperature. Chinyoka and Makinde (2011) analyzed the unsteady Couette flow for a non-Newtonian fluid under the influence of asymmetric convective cooling process. The unsteady MHD Couette flow and heat transfer of a dusty fluid with temperature dependent properties have been studied using Crank-Nicolson implicit and network simulation method by Attia (2006) and Attia (2008) and Eguia *et al.* (2011), respectively. In other work, Gedik *et al.* (2012) have studied the unsteady viscous incompressible and electrically conducting of two-phase fluid flow in circular pipes with external magnetic and electrical field. They investigated the Effects of both uniform transverse external magnetic and electrical fields applied perpendicular to the fluid and each other on the two-phase (solid/liquid) unsteady flow. Also, Khan *et al.* (2011), have presented analytical solutions for some MHD flows of the Oldroyd-B fluid considering a uniform magnetic field and occupies the space over a flat plate between two sidewalls perpendicular to the plate. Chamkha and Ahmed (2011), have obtained the similarity solution for unsteady MHD flow near a stagnation point of a three-dimensional porous body with heat and mass transfer considering heat generation/absorption and chemical reaction.

This paper studies the unsteady MHD Couette flow and heat transfer of an incompressible viscous dusty fluid with temperature dependent thermal conductivity and viscosity. The fluid is between two parallel infinite plates kept at constant and unequal temperatures. The lower plate is maintained stationary while the upper plate is moving with a constant velocity. The fluid is acted upon by an external uniform magnetic field and a constant pressure gradient. The main goal of the present paper is investigation of the effects of the physical parameters such as Reynolds number, Hall parameter, Hartman number and Eckert number on the Nusselt and skin friction of both plates.In this work, the differential transform method (DTM) is used to solve the coupled nonlinear partial differential equation of the described problem. DTM was firstly presented by Zhou (1986) to solve the initial value problems in the analysis of electrical circuit. This method has been applied to some computational and engineering problems such as boundary value problems, Mosayebidorcheh (2013) and Mosayebidorcheh (2014), advective-dispersive transport problem Chen and Ju (2004), nonlinear heat transfer of fins Joneidi *et al.* (2009) and Mosayebidorcheh and Mosayebidorcheh (2012), Strum-Liouville equation Chen and Ho (1996) and vibration problems Ho and Chen (1998). Here, a hybrid numerical algorithm which combines the differential transform and finite difference methods is utilized to study the present problem. This technique has been used for solving the transient heat transfer equation of the fins by Mosayebidorcheh *et al.* (2014). The influence of temperature dependent physical properties and the dimensionless parameters on the Nusselt number and skin friction factor also are discussed.

2. DIFFERENTIAL TRANSFORM METHOD

The differential transform is defined as follows Zhou (1986):

$$X(k)=\frac{H^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_0} \tag{1}$$

where x(t) is an analytical function in the time domain, H is the time interval and X(k) is the transformed function. The inverse transformation is as follows

$$x(t)=\sum_{k=0}^{\infty}X(k)\left(\frac{t-t_0}{H}\right)^k \tag{2}$$

By substituting Eq. (1) into Eq. (2), we can obtain the Taylor series expansion of the x(t) at t_0

$$x(t)=\sum_{k=0}^{\infty}\frac{(t-t_0)^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_0} \tag{3}$$

The function x(t) is usually considered as a series with limited terms and Eq. (2) can be rewritten as:

$$x(t)\approx\sum_{k=0}^{m}X(k)\left(\frac{t-t_0}{H}\right)^k \tag{4}$$

where, *m* represents the number of Taylor series components. Usually, through elevating this value, we can increase the accuracy of the solution.

Some of the properties of DTM are shown in Table 1. These properties are extracted from Eqs.(1) and (4).

Table 1 The properties of the DTM.

Original function	DTM
$f(t)=g(t)\pm s(t)$	$F(k)=G(k)\pm S(k)$
$f(t)=cg(t)$	$F(k)=cG(k)$
$f(t)=\frac{d^n g(t)}{dt^n}$	$F(k)=\frac{(k+n)!}{H^n k!}G(k+n)$
$f(t)=g(t)s(t)$	$F(k)=\sum_{r=0}^{k}G(r)S(k-r)$

3. DESCRIPTION OF THE PROBLEM

Consider the transient flow of an incompressible viscous dusty fluid between two parallel infinite plates at $Y=\pm h$, as shown in Fig. 1.

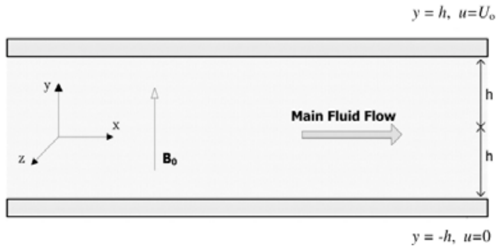

Fig. 1. The geometry of the problem.

The dusty particles are assumed to be dispensed through the fluid. The two infinite plates are considered to be electrically non-conducting and maintained at constant and different temperatures. The temperatures of the lower plate and upper plate are T_1 and T_2 respectively. The motion is produced by a constant pressure gradient in the x-direction and the lower plate is kept stationary while the upper plate has a constant velocity U_0. A uniform suction is applied to the y-direction and the velocity of the fluid in this direction is constant and denoted by v_0. The constant magnetic field with value B_0 is affected in the positive y-direction. Regarding a very small magnetic Reynolds number, the induced magnetic field can be neglected. The flow starts from rest at $\tau = 0$ and no-slip condition is considered for the flow at $y = \pm h$. The initial temperatures of the dust particles and fluid are T_1. The viscosity of the fluid is varied exponentially with temperature and electrical conductivity is a linear function of temperature. The physical parameters of the problem are constant in directions of the x and z, because the plates are assumed infinite in these directions. The fluid and dust particles velocities are as follow

$$\vec{v}_f = U(y,t)\vec{i} + v_0\vec{j} + V(y,t)\vec{k}$$
$$\vec{v}_p = W(y,t)\vec{i} + Q(y,t)\vec{k} \qquad (5)$$

The governing equations of the present problem are based on the conversation laws of momentum and energy for both fluid and dust particles. The Navier-Stokes equations are as follow:

$$\rho\frac{\partial U}{\partial \tau} + \rho v_0\frac{\partial U}{\partial Y} = -\frac{dP}{dX} + \frac{\partial}{\partial Y}\left(\mu\frac{\partial U}{\partial Y}\right) - \frac{\sigma B_0^2}{1+m^2}(U+mV) - KN(U-W) \qquad (6)$$

$$\rho\frac{\partial V}{\partial \tau} + \rho v_0\frac{\partial V}{\partial Y} = \frac{\partial}{\partial Y}\left(\mu\frac{\partial V}{\partial Y}\right) - \frac{\sigma B_0^2}{1+m^2}(V-mU) - KN(V-Q) \qquad (7)$$

where μ is the viscosity of the clean fluid, ρ is the density of the clean fluid, *dp/dX* is the pressure gradient in x-direction, *K* is the Stokes constant, *N* is the number of dust particles per unit volume, σ is the electric conductivity, m is the Hall parameter given by $m = \sigma\beta B_0$ and β is the Hall factor. The motion of the dust particles can be obtained by Newton's second law

$$m_p\frac{\partial W}{\partial \tau} = KN(U-W) \qquad (8)$$

$$m_p\frac{\partial Q}{\partial \tau} = KN(V-Q) \qquad (9)$$

The initial conditions are given by:

$$m_p\frac{\partial Q}{\partial \tau} = KN(V-Q) \qquad (10)$$

For $\tau > 0$ the no-slip condition implies that

$$U = V = 0; \quad at\ Y = -h$$
$$U = U_0; V = 0 \quad at\ Y = +h \qquad (11)$$

The energy equations for both the clean fluid and dust particles are given by

$$\rho c_p\frac{\partial T}{\partial \tau} + \rho c_p v_0\frac{\partial T}{\partial Y} = \frac{\partial}{\partial Y}\left(k\frac{\partial T}{\partial Y}\right) + \mu\left[\left(\frac{\partial U}{\partial Y}\right)^2 + \left(\frac{\partial V}{\partial Y}\right)^2\right] + \frac{\sigma\beta_0^2}{1+m^2}\left(U^2+V^2\right) + \frac{\rho_p C_s}{\gamma_T}(\Psi - T) \qquad (12)$$

$$\frac{\partial \Psi}{\partial \tau} = \frac{-1}{\gamma_T}(\Psi - T) \qquad (13)$$

where k is the thermal conductivity of the fluid, c_p is the specific heat capacity of the fluid, c_s is the specific heat capacity of the particles, ρ_p is the mass of dust particles per unit volume of fluid, γ_T is the temperature relaxation time and T and Ψ are the temperatures of the fluid and dust particles, respectively.

The initial and boundary conditions for the temperature of the fluid and dust particles are given by

$$T = \Psi = T_1; \quad \tau \le 0$$
$$T = T_1; \quad at\ Y = -h,\ \tau > 0 \qquad (14)$$
$$T = T_2; \quad at\ Y = +h,\ \tau > 0$$

A linear temperature dependent function is assumed $g(T) = 1 + b(T_2 - T_1)T$ for the electrical conductivity, and an exponentially temperature dependent function is assumed for the viscosity of the fluid $f(T) = e^{-a(T-T_1)}$, where a and b are constant.

$$\frac{\partial u}{\partial t} = -S\frac{\partial u}{\partial y} + \alpha + \frac{1}{\mathrm{Re}}f(\theta)\frac{\partial^2 u}{\partial y^2} + \frac{1}{\mathrm{Re}}\frac{\partial f(\theta)}{\partial y}\frac{\partial u}{\partial y} - \frac{1}{\mathrm{Re}}\frac{Ha^2}{1+m^2}(u+mv) - \frac{R}{\mathrm{Re}}(u-w) \qquad (15)$$

$$\frac{\partial v}{\partial t} = -S\frac{\partial v}{\partial y} + \frac{1}{\mathrm{Re}}f(\theta)\frac{\partial^2 v}{\partial y^2} + \frac{1}{\mathrm{Re}}\frac{\partial f(\theta)}{\partial y}\frac{\partial v}{\partial y} - \frac{1}{\mathrm{Re}}\frac{Ha^2}{1+m^2}(v-mu) - \frac{R}{\mathrm{Re}}(v-q)$$

(16)

$$\frac{\partial w}{\partial t} = \frac{1}{G_0}(u-w) \qquad (17)$$

$$\frac{\partial q}{\partial t} = \frac{1}{G_0}(v-q) \qquad (18)$$

$$u = w = v = q = 0; \quad t \le 0 \qquad (19)$$

$$u = v = 0; \quad at\ y = -1$$
$$u = 1, v = 0; \quad at\ y = +1 \qquad (20)$$

$$\frac{\partial \theta}{\partial t} = -S\frac{\partial \theta}{\partial y} + \frac{1}{\mathrm{Re\,Pr}}g(\theta)\frac{\partial^2 \theta}{\partial y^2} + \frac{1}{\mathrm{Re\,Pr}}\frac{\partial g(\theta)}{\partial y}\frac{\partial \theta}{\partial y} + \frac{Ec}{\mathrm{Re}}f(\theta)\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \frac{EcHa^2}{\mathrm{Re}\left(1+m^2\right)}\left(u^2+v^2\right) + \frac{2R}{3\mathrm{Re\,Pr}}(\psi - \theta) \qquad (21)$$

$$\frac{\partial \psi}{\partial t} = -L_0(\psi - \theta) \qquad (22)$$

$$\theta = \psi = 0; \quad t \le 0 \qquad (23)$$

$$\theta = 0; \quad at\ y = -1$$
$$\theta = 1; \quad at\ y = +1 \tag{24}$$

where

$$y = \frac{Y}{h}, \quad x = \frac{X}{h}, \quad t = \frac{\tau U_0}{h}, \quad u = \frac{U}{U_0},$$
$$w = \frac{W}{U_0}, \quad v = \frac{V}{U_0}, q = \frac{Q}{U_0}, \quad p = \frac{P}{\rho U_0^2}, \tag{25}$$
$$\theta = \frac{T - T_1}{T_2 - T_1}, \quad \psi = \frac{\Psi - T_1}{T_2 - T_1}$$

$f(\theta) = e^{-a\theta}$, the exponential temperature dependent function of viscosity, $g(\theta) = 1 + b\theta$, the linear temperature dependent function of electrical conductivity, S=v0/U0, the suction parameter, $\alpha = -dp/dx$ the constant pressure gradient, $Ha^2 = \sigma\beta_0^2 h^2/\mu_0$, Ha is the Hartman number, $\mathrm{Re} = \rho U_0 h/\mu_0$ is the Reynolds number, $R = KNh^2/\mu_0$ is the particle concentration parameter, $G_0 = \mu_0/\rho h^2 K$ is the particle mass parameter, $\Pr = \mu_0 c/k$ is the Prandtl number, $Ec = U_0^2/c_p(T_2 - T_1)$ is the Eckert number, $L_0 = \rho h^2/\mu_0\gamma_T$ is the temperature relaxation time parameter.

To solve the coupled nonlinear partial equations (Eq. (15) to Eq. (24)) in the domain $t \in [0,T]$ and $y \in [-1,1]$ using hybrid DTM and FDM, we applied finite difference approximation on y direction and take DTM on t. The following finite difference scheme is used based on a uniform mesh. The length in direction of y is divided into N_y equal intervals. The y coordinates of the grid points can be obtained by $y_j = j(\Delta y)$, $j = 0 : N_y$, where Δy is the mesh size.

After taking second order accurate central finite difference approximation with respect to y and applying DTM on Eq. (15) to Eq. (18) and Eqs. (21) and (22) for time domain, the following recurrence relations can be obtained:

$for \quad 1 \le j \le N_y$

$$U(j,k+1) = \frac{H}{k+1}\left\{-S\frac{U(j+1,k)-U(j-1,k)}{2\Delta y} + \alpha\delta(k)\right.$$
$$+\frac{1}{\mathrm{Re}\Delta y^2}\sum_{r=0}^{k}F(j,k-r)\big(U(j+1,r)-2U(j,r)+U(j-1,r)\big)$$
$$+\frac{1}{4\mathrm{Re}\Delta y^2}\sum_{r=0}^{k}\big(F(j+1,k-r)-F(j-1,k-r)\big)\big(U(j+1,r)-U(j-1,r)\big)$$
$$\left.-\frac{Ha^2}{\mathrm{Re}(1+m^2)}\big(U(j,k)+mW(j,k)\big)-\frac{R}{\mathrm{Re}}\big(U(j,k)-W(j,k)\big)\right\} \tag{26}$$

$$V(j,k+1) = \frac{H}{k+1}\left\{-S\frac{V(j+1,k)-V(j-1,k)}{2\Delta y}\right.$$
$$+\frac{1}{\mathrm{Re}\Delta y^2}\sum_{r=0}^{k}F(j,k-r)\big(V(j+1,r)-2V(j,r)+V(j-1,r)\big)$$
$$+\frac{1}{4\mathrm{Re}\Delta y^2}\sum_{r=0}^{k}\big(F(j+1,k-r)-F(j-1,k-r)\big)\big(V(j+1,r)-V(j-1,r)\big)$$
$$\left.-\frac{Ha^2}{\mathrm{Re}(1+m^2)}\big(V(j,k)-mU(j,k)\big)-\frac{R}{\mathrm{Re}}\big(V(j,k)-Q(j,k)\big)\right\} \tag{27}$$

$$W(j,k+1) = \frac{H}{(k+1)G_0}\big(U(j,k)-W(j,k)\big) \tag{28}$$

$$Q(j,k+1) = \frac{H}{(k+1)G_0}\big(V(j,k)-Q(j,k)\big) \tag{29}$$

$$\Theta(j,k+1) = \frac{H}{k+1}\left\{-S\frac{\Theta(j+1,k)-\Theta(j-1,k)}{2\Delta y}\right.$$
$$+\frac{1}{\mathrm{Re}\Pr\Delta y^2}\sum_{r=0}^{k}G(j,k-r)\begin{pmatrix}\Theta(j+1,r)-2\Theta(j,r)\\+\Theta(j-1,r)\end{pmatrix}$$
$$+\frac{1}{4\mathrm{Re}\Pr\Delta y^2}\sum_{r=0}^{k}\begin{pmatrix}G(j+1,k-r)\\-G(j-1,k-r)\end{pmatrix}\big(\Theta(j+1,r)-\Theta(j-1,r)\big)$$
$$+\frac{Ec}{4\mathrm{Re}\Delta y^2}\sum_{r=0}^{k}\sum_{s=0}^{r}F(j,s)\left[\begin{pmatrix}U(j+1,r-s)\\-U(j-1,r-s)\end{pmatrix}\big(U(j+1,k-r)-U(j-1,k-r)\big)\right.$$
$$\left.\begin{pmatrix}W(j+1,r-s)\\-W(j-1,r-s)\end{pmatrix}\big(W(j+1,k-r)-W(j-1,k-r)\big)\right]$$
$$+\frac{EcHa^2}{\mathrm{Re}(1+m^2)}\sum_{r=0}^{k}\big[U(j,r)U(j,k-r)+W(j,r)W(j,k-r)\big]+\frac{2R}{3\mathrm{Re}\Pr}\big(\Psi(j,k)-\Theta(j,k)\big) \tag{30}$$

$$\Psi(j,k+1) = \frac{-HL_0}{(k+1)}\big(\Psi(j,k)-\Theta(j,k)\big) \tag{31}$$

where $F(j,k)$ and $G(j,k)$ are the differential transform form of the $f(\theta)$ and $g(\theta)$ functions respectively. By applying DTM on initial conditions in Eqs. (19) and (23), we have:

$for \ 0 \le j \le N_y$

$$U(j,0) = 0, \quad V(j,0) = 0, \quad W(j,0) = 0, \tag{32}$$
$$Q(j,0) = 0, \quad \Theta(j,0) = 0, \quad \Psi(j,0) = 0$$

The boundary conditions in Eqs. (20) and (24) can be transformed as follow:

$$\begin{cases}U(0,k) = 0, & k \ge 0\\ U(N_y,0) = 1,\ U(N_y,k) = 0, & k \ge 1\end{cases} \tag{33}$$

$$\begin{cases}W(0,k) = 0, & k \ge 0\\ W(N_y,k) = 0, & k \ge 0\end{cases} \tag{34}$$

$$\begin{cases}\Theta(0,k) = 0, & k \ge 0\\ \Theta(N_y,0) = 1,\ \Theta(N_y,k) = 0, & k \ge 1\end{cases} \tag{35}$$

4. RESULTS AND DISCUSSION

In this paper, some parameters are considered to be constant as:

$$G_0 = 0.8, \quad L_0 = 0.7, \quad \Pr = 1, \quad R = 0.5,$$
$$\alpha = 5, \quad a = 0.5, \quad b = 0.5 \tag{36}$$

Step sizes are chosen: $H = 0.001$ and $\Delta y = 0.05$ for time and position respectively. There is no significant change for the smaller step sizes. In this section, a parameter study is performed for determining the effect of each parameter on the velocity and temperature field of the clean fluid and dust particles.

Firstly, the validation of the present solution is investigated comparing to FDM. The results of the present problem are validated for a special case that dust particles and suction velocity don't exist. Therefore, the governing equations reduce to a system of partial equations with three equations. It means that we can use the following parameters to confirm our results with the Attia (2008)

$$G_0 = 0, \quad L_0 = 0, \quad S = 0, \quad R = 0 \tag{37}$$

Fig. 2 Compares the results of Hybrid-DTM and Attia (2008) when $Ec = 0.2$, $a = 0.5$, $b = 0.5$, $\mathrm{Re} = 1$, $m = 3$ and $Ha = 1$. As seen in Fig. 2, which presents the velocity and temperature distributions as functions of y for various values of time (0.1, 0.5 and 2), Hybrid-DTM is completely accurate and efficient. This figure reveals that temperature in Couette flow can exceeds to hot plate temperature in large times (Fig. 2-c, t=2s). As seen, Hybrid-DTM has an good agreement with Attia (2008), furthermore its solution is obtained using a simple iterative procedure.

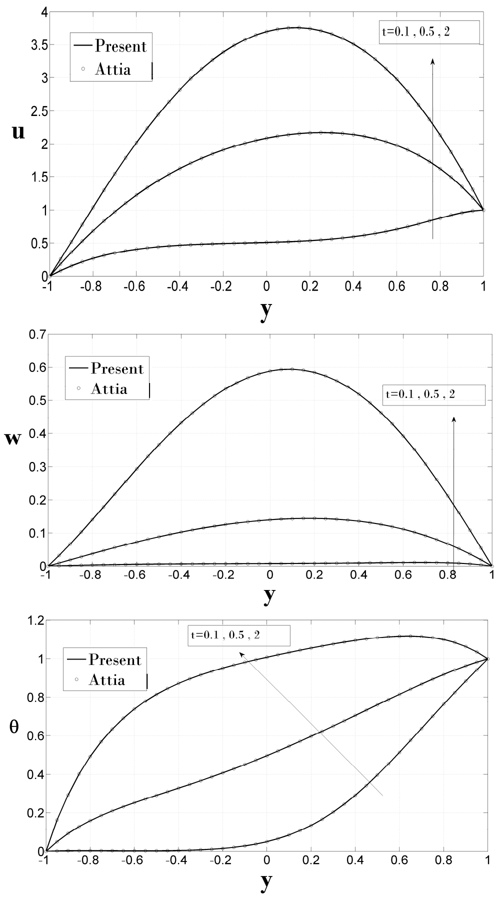

Fig. 2. Comparison of the present results and numerical solution in Attia (2008).

Fig. 3 demonstrates the profiles of the velocity and temperature of the fluid and dust particles at center of the channel (y=0) for different Reynolds numbers. Reynolds number of the flow has a direct relation with time required to reach the steady state. In other word, by increasing the Reynolds number, the time of unsteady state increases (see Fig. 3). Regarding Fig. 3, the time of the unsteady state for temperature of the fluid is larger than the velocity of the fluid. Both u and v increase with time until a maximum value and then decrease up to the steady state. In other words, the primary and secondary flows in directions x and z overshoot and have maximum values.

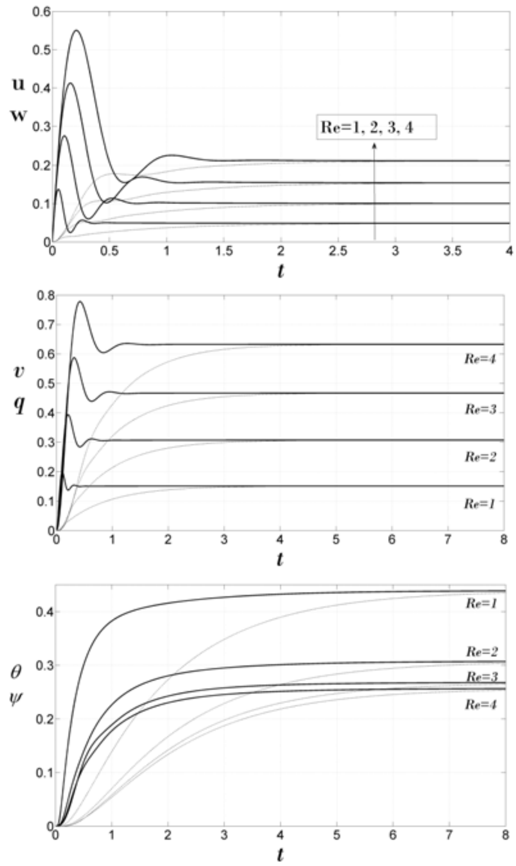

Fig. 3. Effects of Re on velocity and temperature profiles of the fluid (solid line) and dust particles (dotted) when $Ec = 0.2, m = 3, Ha = 10, S = 1$.

The effect of the Hall parameter (*m*) on velocity and temperature profiles is shown in Fig. 4. Hall parameter has a very important role in MHD Couette flows, because this parameter causes the start of the secondary flow in z direction.

Magnetic field plays the role of body force for momentum equations and increases the values of velocity in the center of the channel. This effect is explained by Eq. (6) where the effective conductivity takes the form of $\sigma B_0^2 / \left(1 + m^2\right)$ and the damping force of the magnetic field decreases when m takes higher values. Increasing the Hall value, the distributions of primary and secondary velocities will have greater changes.

Both primary and secondary velocities of the fluid exceed their steady state values and then go down towards the steady state. The overshooting time occurs within increases with increasing the Hall parameter (*m*). As we can see in Fig. 4, the velocities components and temperature of the dust particles go down towards their values for the clean fluid. Primary velocity of dust particles (w) also overshoots and has a peak value (see Fig. 4a) while the secondary flow of dust particles (q) hasn't peak (see Fig. 4b). As we can see, the Hall parameter has great effect on the velocity distribution. This parameter has little effect on the temperature of the fluid at the channel center.

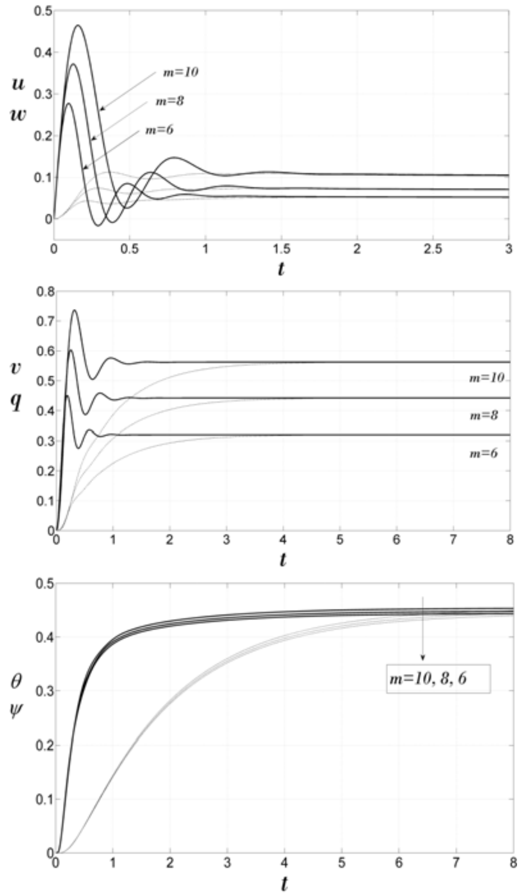

Fig. 4. Effects of Hall parameter (m) on velocity and temperature profiles of the fluid (solid line) and dust particles (dotted) when $Ec = 0.2, \mathrm{Re} = 1, Ha = 10, S = 1$.

The time dependent velocity and temperature of the flow at center of the channel (y=0) plotted in Fig. 5 for different Hartman numbers. This figure indicates that increasing Ha decreases the primary velocity of the flow while increasing Ha increases the secondary velocity of the flow. Therefore the effect of rising Ha can be predicted as a negative body force for x- momentum equation and positive body force for z-momentum equation of the fluid.

Figs. 6, 7 and 8 elucidate the variation of dimensionless local skin friction (surface shear stress) and Nusselt number of the upper and lower plates for different parameters such as Hartman number (Ha), suction velocity (S), Reynolds number (Re), Eckert number (Ec) and Hall parameter (m). In these figures, subscripts "1" and "2" show the properties of the lower and upper plates, respectively.

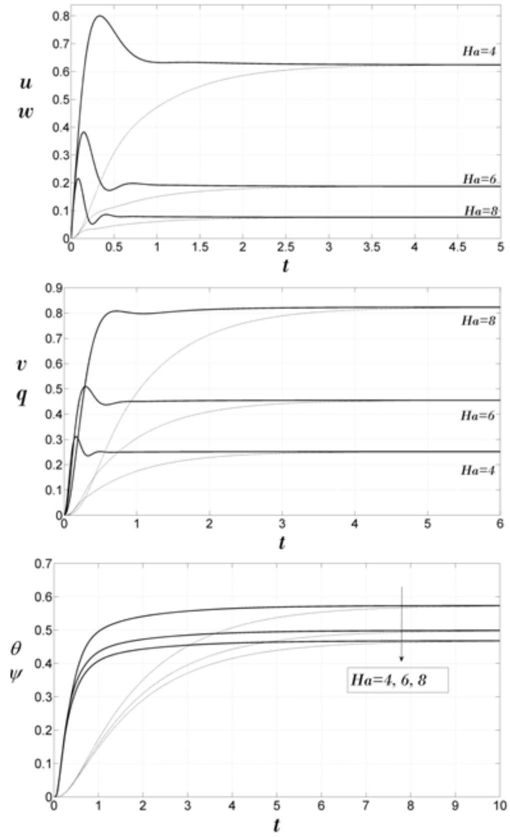

Fig. 5. Effects of Hartman number (Ha) on velocity and temperature profiles of the fluid (solid line) and dust particles (dotted) when $Ec = 0.2, \mathrm{Re} = 1, m = 3, S = 1$.

The effects of Hartman value and suction velocity on the skin friction and the Nusselt values on both upper and lower plates are shown in Fig. 6. Increasing the suction parameter decreases the skin friction coefficient of the lower plate. Suction in this problem

decreases horizontal velocity near the lower plate which causes lower velocity gradient (skin friction coefficient). Skin friction coefficient is so low and leans toward zero for Hartman high values (Fig. 6a).

The effect of suction parameter is not very sensible on the skin friction coefficient of the upper plate. The changes trend of C_{f2} with Hartman value is elevating. This means that when Ha increases, skin friction of the upper plate increases too. Surprisingly, in regard to the Fig. 6b, C_{f2} has negative values

for $Ha \le 4$. This means that fluid velocity value near the upper plate is more than 1 (upper plate velocity).

Increasing the Hartman number makes a decrease and increases in the Nusselt values in lower and upper plates respectively. These changes are nonlinear and linear with low gradient for low and high Ha values respectively. The velocity gradient is high near of the hot plate and so the temperature of the fluid increases as the Joule and viscous dissipations increase. Nusselt value in the upper plate has negative value for $S = 0, Ha \le 5$ and $S = 1, Ha \le 2$. This means that fluid temperature near the upper plate is more than the plate temperature.

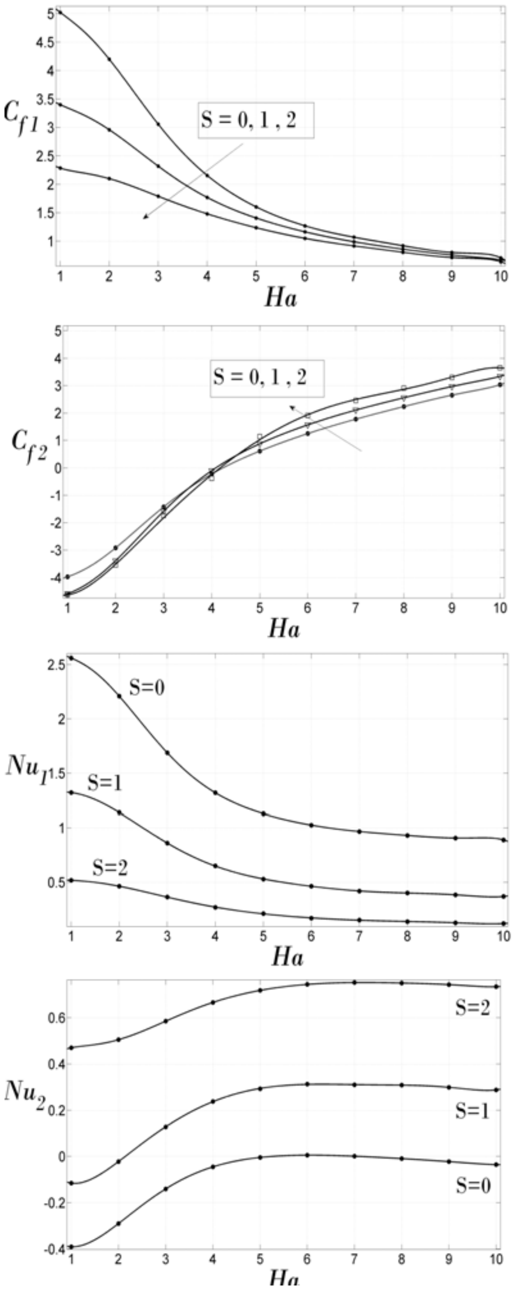

Fig. 6. Variation of the skin friction and Nusselt number with Ha and different values of S when $Ec = 0.2, \mathrm{Re} = 1, m = 3$.

In Fig. 7, effects of Eckert and Reynolds numbers on the Nusselt of upper and lower plates are demonstrated. Because Ec number has a little effect on the skin friction coefficient, it's not presented. In the below Reynolds numbers, the changes of Ec number has a little effect on the Nusselt value of the upper and lower plates. As the Reynolds number increases, the effects of Ec number on Nusselt value will increase.

The effect of Reynolds number on Nusselt values are shown in the Fig. 7 for Ec=0. As Reynolds increases, Nusselt number in lower and upper plates decreases and increases respectively. But on the other hand, Ec has a reverse effect on the Nusselt in comparison to Reynolds. Based on this reason, the variation of Nusselt plates has a relative maximum and minimum for $Ec \ne 0$. This means that from the relative minimum and maximum points on the effects of viscous dissipation on Reynolds ones are dominant and it changes the direction of graph curve. Based on this fact, Nusselt has continues variation for $Ec = 0$ in Fig. 7.

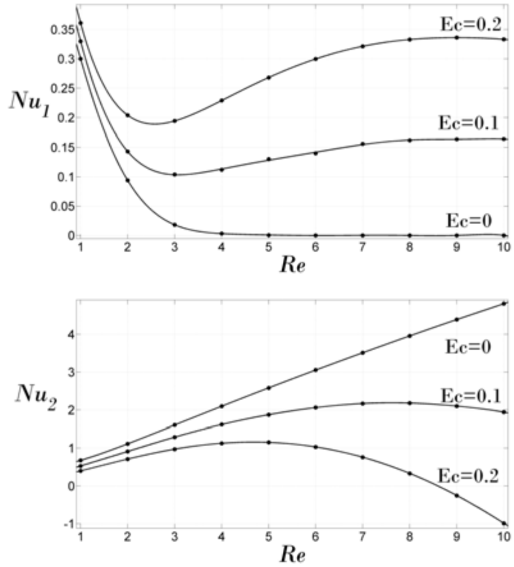

Fig. 7. Variation of the Nusselt number with Re and different values of Ec when $Ha = 10, S = 1, m = 3$.

From Fig. 7-b, it can be observed that Nusselt number of the upper plate has a negative value for $Ec = 0.2, 8.5 \le \mathrm{Re}$. As mentioned before, it indicates that the fluid temperature near the upper wall is higher than wall temperature. On the other word, viscous dissipation term produces heat and finally increases the fluid temperature near the wall.

The effects of Hall parameter and Reynolds number on the skin friction and Nusselt number of the upper and lower plates are shown in Fig. 8. As Reynolds elevates, skin friction of the lower and upper plates increases and decreases respectively. This variation is linear for $m = 0$ and nonlinear for $m \ne 0$. As was observed in Fig. 4, the increase of Hall parameter m, increases primary and secondary flows velocity. This causes the increase of velocity from the upper plate one which this can come to a negative skin friction coefficient (Fig. 8-b for $m = 5$). Hall parameter increases the Nusselt of lower plate and decreases the upper plate Nusselt.

5. CONCLUSION

In this work, the unsteady MHD Couette flow and heat transfer of dust particles between two parallel plates are studied. Viscosity and thermal conductivity of the fluid considered as an

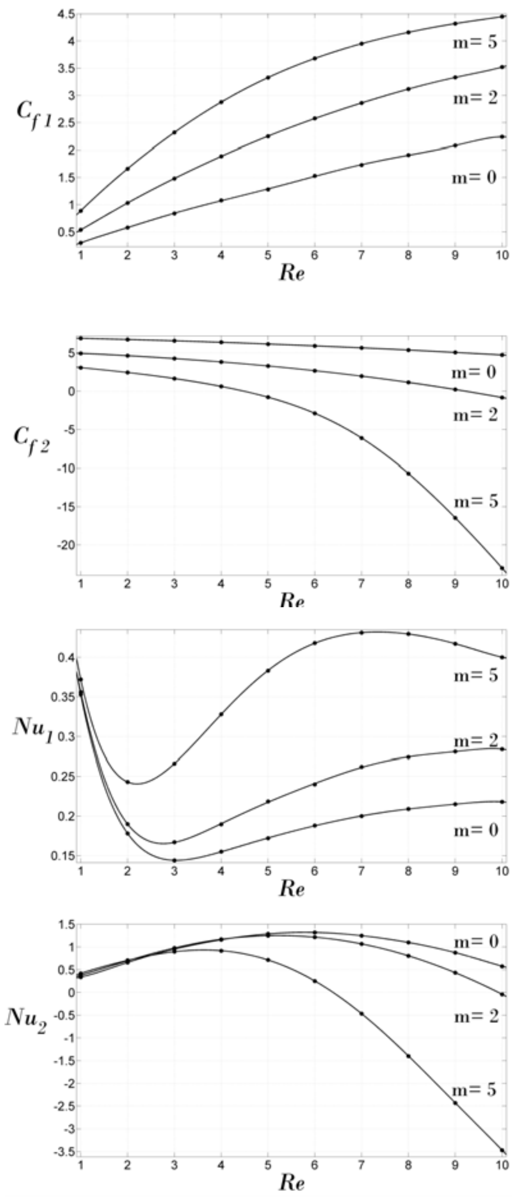

Fig. 8. Variation of the skin friction and Nusselt number with Re and different values of m when $Ha = 10, S = 1, Ec = 0.2$.

exponential and linear functions of the temperature. The problem is solved analytically using a hybrid technique based on the differential transform method and finite difference approximation. The results were presented for different values of Hartman number Ha , Reynolds number Re , Eckert number Ec , Hall parameter m and suction parameter S . The effect of these parameters is discussed by considering transient profiles of the velocity and temperature of the both clean fluid and dust particles.

Following conclusions are concluded from the results:

- Reynolds number of the flow has a direct relation with time required to reach the steady state. In other word, increasing the Reynolds number increases the time of unsteady state.
- Hall parameter (m) causes the start of the secondary flow in z direction.
- Hall parameter has great effect on the velocity distribution. This parameter has little effect on the temperature of the fluid at the center of channel.
- In some cases, skin friction coefficient and Nusselt number of the upper plate had negative values. This means that fluid velocity and temperature near the upper plate are more than unit.
- By increasing the suction parameter, the skin friction coefficient of the lower plate decreases.
- When Hartman number increases, skin friction of the upper plate decreases, but for lower plate increases.
- Increasing the Hartman value decreases the Nusselt number of lower plate and increases Nusselt of the upper plates.

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