Radial and Axial Magnetic Fields Effects on Natural Convection in a Nanofluid-filled Vertical Cylinder

M. Battira and R. Bessaïh

1Department of Physics, Moulay Tahar University, Saida, 20000, Algeria
2Department of Mechanical Engineering, LEAP Laboratory, Constantine, 250000, Algeria

†Corresponding Author Email: mounamaache@yahoo.fr

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ABSTRACT

This work aims to study numerically the steady natural convection in a vertical cylinder filled with an Al₂O₃ nanofluid under two different external magnetic fields (Bₘ, B₀) either in the radial or axial directions. The cylinder having an aspect ratio H/R₀=1 is bounded by the top and the bottom disks at temperatures T₀ and Tₘ, and by an adiabatic side wall. The equations of continuity, Navier-Stokes and energy are non-dimensionalized and then discretized by the finite volume method. A computer program based on the SIMPLER algorithm is developed and compared with the numerical results found in the literature. The effects of nano-size solid volume fraction ranging from 0 to 0.1 and application of the magnetic field in either directions axial and radial for various values of Hartmann numbers on flow and thermal fields, and on local and average Nusselt numbers are presented and discussed for two values of Rayleigh numbers (Ra=10⁵ and 10⁶). The behaviors of average Nusselt number, streamlines, temperature contours, and the both components of velocity are illustrated. The results indicate that for small values of the Hartmann number, where the flow remains due to the convection, the average Nusselt number decreases when increasing the solid volume fraction and this decrease is more important if the magnetic field is applied in the axial direction and by increasing the Hartmann numbers. The increasing in the solid volume fraction increases the performance of heat transfer in the nanofluid.

Keywords: Natural convection; Nanofluid; Magnetic field.

NOMENCLATURE

B magnetic field, Tesla
Cₚ specific heat, J kg⁻¹ K⁻¹
g gravitational acceleration, m s⁻²
H cylinder height, m
Ha Hartmann number (BₘR₀√σₙf/ρₙfνf)
k thermal conductivity, W m⁻¹ K⁻¹
Nₙₑ local Nusselt number on the bottom hot disk
Nₙₑ₀ Average Nusselt number
P fluid pressure, Pa
Pr Prandtl number (νf/αf)
r, z cylindrical coordinates, m
R, Z dimensionless coordinates (r/R₀, z/R₀)
Ra Rayleigh number (gβfR₀²(Tₘ-T₀)/νfαf)
R₀ cylinder radius, m
Sₚ source term in Eq. (15)
T temperature, K
u, v radial, axial velocities, m s⁻¹
U, V dimensionless radial and axial velocities
α thermal diffusivity, m² s⁻¹
β thermal expansion coefficient, K⁻¹
φ solid volume fraction
ψ non-dimensional parameter in Eq. (15)
Γₚ diffusion term in Eq. (15)
μ dynamic viscosity, kg m⁻¹ s⁻¹
ν kinematic viscosity, m² s⁻¹
θ dimensionless temperature (T-Tₘ)/(Tₗ-Tₘ)
ρ density, kg m⁻³
σ electrical conductivity, μS cm⁻¹
ψ dimensionless stream function

Subscripts
ccold, hot
h hot
f fluid (pure water)
nf nanofluid
p nanoparticle
r, z radial and axial directions
1. INTRODUCTION

Numerous studies were devoted to numerical and experimental modeling of the natural convection heat transfer problem, because of its wide presence in industrial and technological applications, such as nuclear reactor systems, heating and cooling buildings, solar technology, grain storage, food and metallurgical industries. Enhancement of heat transfer in such systems is very fundamental from the energy saving perspectives. A principal constraint against enhancing the heat transfer in such systems is low thermal conductivity of the usually used fluids like air, water and oil. An innovative technique, which insert within the fluid, metallic particles of nanometer size hope to increase the thermal conductivity of the mixture was first introduced by Choi (1995). The term nanofluid was then generally used to characterize this type of mixture. Many investigations have been carried out in rectangular and square enclosures which showed an immense increase in thermal conductivity with addition of small amounts of nanoparticles (Purta et al., 2003; Oztop and Abu-Nada, 2008; Abu-Nada, 2009). However, very few numerical models have been proposed to predict the natural convection in vertical cylinder. Haddad et al. (2012) review experimental, numerical and theoretical studies for natural convection of nanofluid in different types of enclosures. They find that most of numerical results showed that nanofluids significantly improve the heat transfer capability of conventional heat transfer fluid. Whereas, experimental results showed that the presence of nanoparticles deteriorates heat transfer systematically. In some fields of industries, such as crystal growth of semi-conductors, material manufacturing and the geothermal energy extraction, the natural convection is under the influence of a magnetic field. There exist several studies (Okada and Ozo, 1989 and 1992; Garandet et al., 1992; Kolsi et al. 2007; Kherief et al., 2012) dedicated to understanding the influence of magnetic field on the flow behavior and the heat transfer in the enclosures that are filled with electrically conducting fluids. The most significant results occur in this context is the existence of Lorentz force that affects heat transfer rate. The application of magnetic field, decrease the heat transfer rate and the use of nanofluid enhance the heat transfer in the enclosure. In some engineering systems like the magnetic fields sensors, the magnetic storage media and the cooling systems of electronic equipment, the application of magnetic field is necessary and the enhanced heat transfer rate is desirable. The idea of using nanofluids as working fluid in an enclosure to improve the heat transfer performance in such devices was first presented by Ghasemi et al. (2011). The authors investigated the effect of transverse magnetic field on the natural convection in a nanofluid filling a square enclosure and they studied the effects of appropriate parameters such as Rayleigh number, Hartmann number and solid volume fraction on the heat transfer performance of the enclosure. They find that the heat transfer rate increases with an increase of Rayleigh number but it decreases with an increase of the Hartmann number. Depending on the value of Hartmann number and Rayleigh number, an increase of the solid volume fraction may result in enhancement or deterioration of the heat transfer performance.

Ahammad et al. (2011) examined numerically the laminar forced convection of a water-Al\textsubscript{2}O\textsubscript{3} nanofluid flowing through a horizontal micro-channel. The middle section of the micro-channel is heated by a constant and uniform heat flux and influenced by a transverse uniform magnetic field. The results show that the micro-channel performs better heat transfers at higher values of the Reynolds and Hartmann numbers. For all values of the Reynolds and Hartmann numbers considered in this study, the average Nusselt number on the middle section surface of the micro-channel increases as the solid volume fraction increases. The rate of this increase is considerably more at higher values of the Reynolds number and at lower values of the Hartmann number. Abouali et al. (2012); made a critical review on the subject of numerical simulation of natural convection in enclosures filled with nanofluids. It was shown that based on the assumption of single phase flow idea for the nanofluids, the same correlations exist for prediction of pure fluids heat transfer rate could also be used to predict the overall heat performance of enclosures filled with nanofluids. It was also found that the results of numerical simulations and those existing correlations would coincide if the correlation proposed by Corcione (2011) is used for calculation of the effective thermal conductivity and viscosity. Syam et al. (2012) evaluated experimentally the convection heat transfer coefficient and friction factor characteristics of Fe\textsubscript{3}O\textsubscript{3} nanofluid for flow in a circular tube. They found that nanofluid heat transfer is higher compared to water and increases with volume concentration. Mahmoudi et al. (2013) investigated numerically the entropy generation and enhancement of heat transfer in natural convection flow and heat transfer using copper Cu-water nanofluid in the presence of a constant magnetic field. The analysis uses a two-dimensional trapezoidal enclosure with the left vertical wall and inclined walls kept in a low constant temperature and a heat source with constant heat flux placed on the bottom wall of the enclosure. Their results show that at Ra=10\textsuperscript{4} and 10\textsuperscript{5} the enhancement of the Nusselt number due to the presence of nanoparticles increases with the Hartmann number, but at higher Rayleigh number, a reduction has been observed. Reza Ashorynejad et al. (2013) investigated numerically the effect of static radial magnetic field on natural convection heat transfer in a horizontal cylindrical annulus enclosure filled with nanofluid. The inner and the outer cylinder surfaces are maintained at the different uniform temperatures. The results reveal that the flow oscillations can be suppressed effectively by imposing an external radial magnetic field. Also, it is found that the average Nusselt number is an increasing function of nanoparticle volume fraction and Rayleigh number, while it is a decreasing function of Hartmann number.
numerically the influence of nanofluid with double nanoparticles on the forced convection in a flat plate solar collector, they compared heat transfer phenomena among four nanofluids. They found that the average Nusselt number for water-Ag nanofluid is higher than others.

To the best of our knowledge, no attempts have been made as yet to study the effect of the direction of an external applied magnetic field on nanofluid natural convection in a vertical cylinder. The main objective of the present research is an investigation of the effect of various parameters such as Rayleigh number, Hartmann number and solid particle volume fraction on the steady natural convection in a vertical cylinder filled with an Al₂O₃ nanofluid under two different external magnetic fields either in the radial (Br) or axial (Bz) directions.

2. PROBLEM FORMULATION

Figure 1 shows a schematic diagram of the vertical cylindrical enclosure having an aspect ratio H/R=1 and filled with water-Al₂O₃ nanofluid. The bottom disk is maintained at a hot temperature T_h, while the top disk is maintained at a cold temperature T_c (T_c<T_h). The sidewall of the cylinder is assumed to be adiabatic. It is assumed that the Al₂O₃ nanoparticles and water are in thermal equilibrium and the nanofluid is Newtonian and incompressible.

![Fig. 1. Geometry of the problem.](image)

Two different external magnetic fields either in the radial (Br) or axial (Bz) directions are applied. The thermo-physical properties of the base fluid and the nanoparticles presented in Table 1 are given by Abu-Nada et al. (2008). The Boussinesq approximation for buoyancy flow is adopted. Therefore, the governing equations are written in dimensional form as follows:

<table>
<thead>
<tr>
<th>Table 1 Thermo-physical properties of water and nanoparticles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Pr</td>
</tr>
<tr>
<td>β (kg/m²)</td>
</tr>
<tr>
<td>C_p (J/kg·K)</td>
</tr>
<tr>
<td>k (W/m·K)</td>
</tr>
<tr>
<td>β (K⁻¹)</td>
</tr>
</tbody>
</table>

Continuity equation in both directions of magnetic field:

\[
\frac{\partial (\rho u)}{\partial r} + \frac{\partial (\rho v)}{\partial z} = 0
\]  

The magnetic field is radial:

r-Momentum equation:

\[
u \frac{\partial u}{\partial r} + \frac{1}{\rho_{nf}} \left( \frac{\partial p}{\partial r} + \mu_{nf} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{\partial^2 u}{\partial z^2} \right) \right)
\]

z-Momentum equation:

\[
u \frac{\partial v}{\partial z} + \frac{1}{\rho_{nf}} \left( \frac{\partial p}{\partial z} + \mu_{nf} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v}{\partial r} \right) + \frac{\partial^2 v}{\partial z^2} \right) \right) + \rho \beta_{nf} (T - T_c) \]

The magnetic field is axial:

r-Momentum equation:

\[
u \frac{\partial u}{\partial r} + \frac{1}{\rho_{nf}} \left( \frac{\partial p}{\partial r} + \mu_{nf} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{\partial^2 u}{\partial z^2} \right) \right)
\]

z-Momentum equation:

\[
u \frac{\partial v}{\partial z} + \frac{1}{\rho_{nf}} \left( \frac{\partial p}{\partial z} + \mu_{nf} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v}{\partial r} \right) + \frac{\partial^2 v}{\partial z^2} \right) + \rho \beta_{nf} (T - T_c) \]

Energy equation in both directions of magnetic field:

\[
u \frac{\partial T}{\partial r} + \frac{\partial (\rho \beta T)}{\partial z} = \alpha_{nf} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right)
\]

In the above equations, the density of nanofluid is assumed to be constant, except in the buoyancy term \((\partial p/\partial z - \rho_{nf}g)\) in the z-momentum equation. The buoyancy term can be written as \((\partial \tilde{p}/\partial z) + (\rho_c - \rho_{nf})g\), where \(\tilde{p}\) is the modified pressure \((\tilde{p} = p + \rho_c g z)\) and \(\rho_c\) is the density of nanofluid at the reference temperature, \(T_c\).

By using the definition of nanofluid thermal expansion coefficient \(\beta_{nf} = -\left(1/\rho_{nf}\right)(\partial p/\partial T)\rho\), the buoyancy term in the z direction can be rewritten as: \((\partial \tilde{p}/\partial z) + (\rho \beta_{nf} g(T - T_c))\). The classical models reported in the literature by Brinkman (1952) are used to determine the properties of the nanofluid.

The thermal diffusivity in equation (6) is given as:

\[\alpha_{nf} = k_{nf}/(\rho C_p)_{nf}\]  

The electrical conductivity of the nanofluid is given as:

\[\sigma_{nf} = (1 - \varnothing)\sigma_f + \varnothing \sigma_p\]  

The effective density of the nanofluid is expressed as:

\[\rho_{nf} = (1 - \varnothing)\rho_f + \varnothing \rho_p\]  

The heat capacitance of the nanofluid is expressed as:

\[C_{nf} = (1 - \varnothing)C_p + \varnothing C_p\]
as:
\[
(\rho c_p)_n = (1 - \emptyset)(\rho c_p)_f + \emptyset(\rho c_p)_p
\]
(10)
The thermal expansion of the nanofluid is given as:
\[
(\rho \beta)_{nf} = (1 - \emptyset)(\rho \beta)_{f} + \emptyset(\rho \beta)_{p}
\]
(11)
The effective dynamic viscosity and thermal conductivity of the nanofluid are modeled by
Brinkman (1952) and Maxwell (1873)
\[
\mu_{nf} = \mu_f(1 - \emptyset)^{-2.5}
\]
(12)
\[
k_{nf} = k_f \left[ \frac{b_{nf} + 2k_f}{b_{nf} + 2k_f} \right]^{-2(\emptyset - \emptyset_f)} \left( \frac{b_{nf} + 2k_f}{b_{nf} + 2k_f} \right) + \emptyset(\emptyset - \emptyset_f)
\]
(13)
By introducing the dimensionless parameters defined as follows:
\[
R = \frac{r}{R_0}, \quad Z = \frac{Z}{H_0}, \quad U = \frac{u}{(\alpha_f/\rho_f)}, \quad V = \frac{v}{(\alpha_f/\rho_f)},
\]
\[
P = \frac{\rho R_0}{\rho_f \alpha_f^2} \theta = \frac{r^2 - r_h^2}{r_h^2 - r_c^2} \quad \text{Ra} = \frac{\alpha_f \beta_f \left( T_h - T_c \right)}{\nu \alpha_f}
\]
(14)
The dimensionless form of the governing equations (1)-(6) are presented as:
\[
\frac{\partial(U \phi)}{\partial R} + \frac{\partial(V \phi)}{\partial Z} = \frac{\partial}{\partial R} \left( \Gamma \frac{\partial \phi}{\partial R} \right) + \frac{\partial}{\partial Z} \left( \Gamma \frac{\partial \phi}{\partial Z} \right) + S_{\phi}
\]
(15)
Where \( \phi \) stands for the dependent dimensionless parameters \( U, V \) and \( \Gamma_{\phi} \) and \( S_{\phi} \) are the corresponding diffusion and source terms, respectively, as summarized in Table 2.

The governing equations are subject to the following boundary conditions:

**Symmetry axe:**
At \( R=0 \)
\[
U = \frac{\partial V}{\partial R} = 0, \quad \frac{\partial \theta}{\partial R} = 0
\]
(16a)

**Adiabatic lateral wall:**
At \( R=1 \), \( U=V=0 \)
\[
\frac{\partial \theta}{\partial R} = 0
\]
(16b)

**Hot bottom disk:**
At \( Z=0 \)
\[
U=V=0, \quad \theta = 1
\]
(16c)

**Cold top disk:**
At \( Z = \frac{H}{R_0} \)
\[
U=V=0, \quad \theta = 0
\]
(16d)

The local Nusselt number on the top cold disk is defined by:
\[
\text{Nu}_R(R) = -\frac{k_{nf}}{k_f} \frac{\partial \theta}{\partial Z} \bigg|_{Z=H/R_0}
\]
(17)
The average Nusselt number \( (\text{Nu}_m) \) is determined by integrating the local Nusselt along the hot disk:
\[
\text{Nu}_m = \int_0^1 \text{Nu}_R(R) dR
\]
(18)

### 3. NUMERICAL APPROACH AND VALIDATION

The non-dimensional governing equations (15) with the associated boundary conditions (16) were discretized using a finite-volume formulation given by Patankar (1980). The vectorial quantities \( (u, v) \) are stored on the faces of volumes and the scalar quantities \( (P, \theta) \) are stored in the centers. For the convection and diffusion fluxes were approximated by a second-order accurate central differencing scheme and the SIMPLER algorithm was utilized to handle the pressure-velocity coupling. Finally, the discretized algebraic equations are solved by the line-by-line tri-diagonal matrix algorithm (TDMA).

The numerical method was implemented in a FORTRAN program.

#### 3.1 Grid Independence Study

The effect of grid resolution was examined for \( \phi = 0.1, \text{Ra} = 10^4, \text{and} \text{Ha} = 10 \) (Bc), in order to select the appropriate grid density. Results given in Fig. 2 show that a grid size of 112 \times 112 nodes satisfies the grid independence. This grid is therefore adopted for all numerical simulations, in order to optimize the CPU time and the cost of computations.

![Fig. 2. Result of grid independence at \( \phi = 0.1 \), \( \text{Ra} = 10^4, \text{Ha} = 10 \) (Bc).](image)

#### 3.2 Validation of the Computer Code

The results of our numerical simulations have been
It can be seen, for $5\times 4$ are given in $3\times 5$ his number $5\times 40$. b)

due to the magnetic field, the flow field is affected and subsequently, the dominant mechanism of heat transfer changes. The magnetic field has a negative effect on buoyancy force and decreases the flow motion. This reduction in velocity causes a decrease in the convection heat transfer. At $\text{Ha}=10$, it is clear that the isotherms are affected by variation in the solid volume fraction and by the magnetic field direction.

The domination of the conduction regime which appear in horizontal isotherms is more important if the magnetic field is oriented axially and with the increasing of solid volume fraction ($\text{Ha}=10$, $\phi=0.1$). The results also show a conduction-dominated regime with horizontal isotherms at high Hartmann numbers ($\text{Ha}=80$). Figure 7 presents the dimensionless axial velocity profiles along the midsection (middle plane) of the cylindrical enclosure. It can be seen from this figure the effects of the solid volume fraction on the axial velocity and the effect of the application of the magnetic field ($\text{Ha}=40$ and $\text{Ha}=80$) in either directions axial and radial for two values of Rayleigh number $Ra=10^3$ and $10^4$. It is clear that as the solid volume fraction of the Al$_2$O$_3$ nanoparticles increases the absolute magnitude of vertical velocity decreases. This decrease is due to the suppression of buoyant flows of the nanofluid at a higher solid volume fraction of the nanoparticles in the nanofluid. In addition, the increase in Hartmann number dramatically decreases the amount of vertical velocity. The absolute magnitude of velocity increases with Rayleigh number, because increasing this number means increasing buoyancy force. In fig.8, it is observed that the Hartmann number has an insignificant influence on the temperature profiles, because the heat transfer of these important values of Hartmann numbers ($\text{Ha}=40$, $\text{Ha}=80$) at these two values of the Rayleigh number ($Ra=10^3$ and $Ra=10^4$) is mainly due to the conduction. In addition, this figure shows that the
Fig. 5. Streamlines at $Ra=10^4$ for different Hartmann numbers $Ha$ (radial $B_r$ and axial $B_z$ magnetic fields) and solid volume fractions $\phi$.

Increasing of the solid volume fraction of nanoparticles in nanofluid increases the temperature at the mean part of the cylindrical enclosure and decreases it near the adiabatic sidewall.

Figures 9a-b illustrate the variation of dimensionless radial velocity $U$ along the height of the cylinder at $Ha=10$ (axial $B_z$ and radial $B_r$ magnetic fields) and for two values of $\phi$. It shows...
that the enhancement in the buoyancy-force due to the increase of Rayleigh number increment substantially the radial velocity, and the enhancement of the solid volume fraction reduce the radial velocity and this reduction is more considerable if the magnetic field direction is axial and this has a good corresponding with results found previously. Figure 10 shows the effect of Hartmann number $Ha$ (radial $B_r$ and axial $B_z$ magnetic fields) on the variation of the average
Fig. 7. Variation of dimensionless axial velocity \( V \) along the mid-span at \( Ra=10^3 \) and \( Ra=10^4 \) and different values of \( \phi \).

Fig. 8. Variation of dimensionless temperature \( \theta \) along the mid-span at \( Ra=10^3 \) (left) and \( Ra=10^4 \) (right) and for different values of \( \phi \).
Nusselt number $Nu_m$ at $\phi=0.04$ and for two values of Rayleigh number when the cylindrical enclosure is filled with a water-Al$_2$O$_3$ nanofluid ($\phi=0.04$). For $Ra=10^3$, where the heat transfer is only due to conduction and the magnetic field does not have an important effect on the heat transfer performance, the average Nusselt number remains unchanged when the Hartmann number increases. The average Nusselt number does, however, decrease for $Ra=10^4$ if $Ha<20$ when the heat transfer is mainly due to convection and this decrease is more important if the magnetic field is applied in axial direction ($B_z$). For $Ha>20$, the magnetic field suppresses the convection flows, so that the average Nusselt number becomes practically constant.

Fig. 9. Variation of dimensionless radial velocity $U$ along the height of the cylinder at $Ha=10$ (axial $B_r$ and radial $B_z$ magnetic fields) and for two values of $\phi$.

For $Ra=10^3$, table 3 illustrates the variation of the average Nusselt number ($Nu_m$) and the maximum stream function ($|\Psi|_{max}$) with the solid volume fraction at different values of the Hartmann number either in the radial or axial directions of the magnetic field. The values show that the solid volume fraction has an important influence on the heat transfer performance of the cylindrical enclosure at all values of the Hartmann numbers. For $Ha<20$, it is clear that the Nusselt number decreases strongly with the increasing Hartmann number, especially when the magnetic field is applied in axial direction.

![Graph](image)

Fig. 10. The effect of Hartmann number (radial $B_r$ and axial $B_z$ magnetic fields) on the variation of the average Nusselt number at $\phi=0.04$ and for two values of Rayleigh number.

Table 3 Average Nusselt number and the maximum stream function at various $Ha$ and $\phi$ ($Ra=10^4$)

<table>
<thead>
<tr>
<th>$Ha$</th>
<th>$\phi=0$</th>
<th>$\phi=0.04$</th>
<th>$\phi=0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Nu_m$</td>
<td>$Nu_m$</td>
<td>$Nu_m$</td>
</tr>
<tr>
<td>0</td>
<td>2.2053</td>
<td>2.2616</td>
<td>2.2566</td>
</tr>
<tr>
<td>10</td>
<td>$B_z$ 1.6634</td>
<td>1.5754</td>
<td>1.3762</td>
</tr>
<tr>
<td></td>
<td>$B_r$ 1.8445</td>
<td>1.7806</td>
<td>1.5840</td>
</tr>
<tr>
<td>20</td>
<td>$B_z$ 0.9956</td>
<td>1.1140</td>
<td>1.3090</td>
</tr>
<tr>
<td></td>
<td>$B_r$ 1.0430</td>
<td>1.1141</td>
<td>1.3108</td>
</tr>
<tr>
<td>40</td>
<td>$B_z$ 0.9952</td>
<td>1.1138</td>
<td>1.3106</td>
</tr>
<tr>
<td></td>
<td>$B_r$ 0.9954</td>
<td>1.1125</td>
<td>1.3107</td>
</tr>
<tr>
<td>80</td>
<td>$B_z$ 0.9953</td>
<td>1.1138</td>
<td>1.3105</td>
</tr>
<tr>
<td></td>
<td>$B_r$ 0.9953</td>
<td>1.1138</td>
<td>1.3105</td>
</tr>
</tbody>
</table>

As the Nusselt number describes the intensity of the convective heat transfer, this observation indicates clearly the significant inhibition of the convective heat transfer by the magnetic field at $Ha=20$. For all values of Hartmann number, if the solid volume fraction increases $|\Psi|_{max}$ is reduced. The addition of solid nanoparticles to the base fluid involves an increase of $\mu_{nf}/\mu_{nf}$ in the diffusive term. This means weaker circulation flow and lower values of the maximum stream function. Also, for $Ha=10$, as the solid volume fraction increases the heat transfer rate decreases. For high values of the Hartmann number ($Ha>20$), an increase of the solid volume fraction results in the increase of the heat transfer performance.
Figure 11 illustrates the effect of Hartmann number Ha (axial magnetic field, Bz) on the variation of the average Nusselt number Nu$_m$ at Ra=10$^4$ and for different values of $\phi$. We can observe the increase in the heat transfer performance in the case of an axial direction of the magnetic field with the increasing in solid volume fraction $\phi$.

Figure 12 shows the variation of local Nusselt number Nu$_R$ along the top cold disk at Ra=10$^4$ and Ha=10 (radial B$_r$ and axial B$_z$ magnetic fields) for different values of $\phi$. The results show that if the solid volume fraction increases the local Nusselt number decreases in both directions of the magnetic field. Also, it is shown that the local Nusselt number when the magnetic field is applied axially is less than the one when the magnetic field is applied radially and this result is in good agreement with earlier finding. The effect of solid volume fraction $\phi$ on the variation of the average Nusselt number ratio (Nu$_{m}$/Nu$_{m,\phi=0}$) at Ra=10$^4$ and for different Hartmann numbers (radial B$_r$ and axial B$_z$ magnetic fields) is presented in fig. 13. The results presented for Rayleigh number Ra=10$^4$ show that as the Hartmann number increases from Ha=0 to Ha=10, the average Nusselt number ratio decreases when the solid volume fraction increases and the rate of this decrease is more important if the magnetic field direction is axial. For Ha $\geq$ 20, the average Nusselt number ratio increases as the solid volume fraction increases. This behavior is due to the great effect of the magnetic field and its stronger suppression of the buoyant flows in the nanofluid.

Table 4 presents the effect of the solid volume fraction ($\phi$) on the Nusselt number (Nu$_m$) and the maximum stream function ($|\Psi|_{max}$) at two values of the Rayleigh number (Ra=10$^3$ and Ra=10$^4$).

The value of Hartmann number considered is Ha=20. For these two values of the Rayleigh number, the buoyant forces are weak and diffusion governs the behavior of the fluid. The addition of nanoparticles results in weaker buoyancy-driven circulations and low values of maximum stream function. That being said, the heat transfer rate still expected to increase as the solid volume fraction increases. This finding is similar to that of the previous results.

5. CONCLUSION

Table 4 Average Nusselt number and the maximum stream function at various $\phi$ and at Ra=10$^3$ and Ra=10$^4$ (Ha=20)

| $\phi$ | Nu$_m$ | $|\Psi|_{max}$ | Nu$_m$ | $|\Psi|_{max}$ |
|-------|--------|---------------|--------|---------------|
|       | Ra=10$^3$ | Ra=10$^4$ | Ra=10$^3$ | Ra=10$^4$ |
| 0     | 0.9954 | 0.9956 | 0.9953 | 1.0432 |
| 0.02  | 1.0523 | 1.0524 | 1.0533 | 1.0540 |
| 0.06  | 1.1757 | 1.1756 | 1.1756 | 1.1769 |
| 0.1   | 1.3087 | 1.3090 | 1.3107 | 1.3108 |

Natural convection in a cylindrical enclosure filled with a water-Al$_2$O$_3$ nanofluid and under two different external magnetic fields (B$_r$, B$_z$) either in the radial or axial directions has been studied numerically. The effect of the magnetic field
direction and some parameters such as the solid volume fraction and the Hartmann number on the flow and temperature fields, and heat transfer rate have been examined.

The main conclusions are summarized as follows:

The magnetic field reduces the circulation in the cylindrical cavity, when the magnetic field becomes stronger.

If the conduction regime dominates, the strength and direction of the magnetic field have a weakness influence on the heat transfer rate, but the increasing of solid volume fraction increment considerably the heat transfer performance for all values of Hartmann number.

For low values of Hartmann numbers (Ha<20) and at Rayleigh number Ra=10^4 where the convection mode remains dominate, the increasing in solid volume fraction in the nanofluid decreases the heat transfer performance, especially if the magnetic field is applied axially.

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REFERENCES


Okada, K. and H. Ozoe (1992). Transient Responses of Natural Convection Heat Transfer with Liquid Gallium under an External Magnetic Field in Either the x, y or z Direction.

**Industrial & Engineering Chemical Research**
31, 700-706.


