Journal of Applied Fluid Mechanics, Vol. 8, No. 4, pp. 693-701, 2015.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.18869/acadpub.jafm.73.238.21562

MHD Mixed Convection Heat Transfer in a Vertical Channel with Temperature-Dependent Transport Properties

K. V. Prasad[1]†, H. Vaidya[2] and K. Vajravelu[3]

[1]*Department of Mathematics, Vijayanagra Sri Krishnadevaraya University, Vinayaka nagar, Bellary - 583 104, Karnataka, India*
[2] *Research Scholar, Department of Mathematics, Vijayanagra Sri Krishnadevaraya University, Vinayaka Nagar, Bellary - 583 104, Karnataka, India*
[3]*Department of Mathematics, University of Central Florida, Orlando, Florida 32816, USA*

†*Corresponding Author Email:prasadkv2000@yahoo.co.in*

(Received September 28, 2013; accepted November 17, 2014)

ABSTRACT

An analysis is carried out to study the effects of temperature-dependent transport properties on the fully developed free and forced MHD convection flow in a vertical channel. In this model, viscous and Ohmic dissipation terms are also included. The governing nonlinear equations (in non-dimensional form) are solved numerically by a second order finite difference scheme. A parametric study is performed in order to illustrate the interactive influences of the model parameters; namely, the magnetic parameter, the variable viscosity parameter, the mixed convection parameter, the variable thermal conductivity parameter, the Brinkmann number and the Eckert number. The velocity field, the temperature field, the skin friction and the Nusselt number are evaluated for several sets of values of these parameters. For some special cases, the obtained numerical results are compared with the available results in the literature: Good agreement is found. Of all the parameters, the variable thermo-physical transport property has the strongest effect on the drag, heat transfer characteristics, the stream-wise velocity, and the temperature field.

Keywords: Variable fluid properties; Mixed convection; Viscous dissipation; Keller-box method.

NOMENCLATURE

A	constant (Pa m^{-1})
a	constant defined in Equation (5)
B_0	uniform magnetic field (tesla)
Br	Brinkman number defined in (13)
C_p	specific heat at constant pressure $\left(J\,kg^{-1}K^{-1}\right)$
g	acceleration due to gravity $\left(ms^{-2}\right)$
Gr	Grashof number defined in (13)
k	thermal conductivity $\left(Wm^{-1}K^{-1}\right)$
L	channel width (m)
Mn	magnetic parameter defined in Eq.(13)
M_1	mass flux conversion parameter
Nu	Nusselt number
P	pressure (Pa)
Pr	Prandtl number defined in Eq.(13)
R_T	wall temperature ratio parameter defined in Eq.(13)
Re	Reynolds number defined in Eq.(13)
T_1, T_2	prescribed boundary temperatures (K)
T_0	reference temperature (K)
T	fluid temperature (K)
T_r	constant defined in Eq. (5)
U	velocity component in the X- direction (m s^{-1})
V	velocity component in Y-direction (m s^{-1})
X, Y	space co-ordinates (m)
U_0	reference velocity defined in Eq.(9) (m s^{-1})
u	dimensionless velocity component in the X- direction (m s^{-1})
α	thermal diffusivity $\left(m^2s^{-1}\right)$
β	volumetric coefficient of thermal expansion $\left(K^{-1}\right)$
ε	small parameter
θ	dimensionless fluid temperature

γ kinematic viscosity $\left(m^2 s^{-1}\right)$

σ electric conductivity (mho m^{-1})

ρ_0 is the ambient fluid density (kg m^{-3})

θ_r transformed dimensionless reference temperature

λ mixed convection parameter defined in Eq. (13)

τ_w wall shear stress (kg m^{-1} s^{-2})

μ dynamic viscosity (Pa s)

1. INTRODUCTION

The study of mixed convection flow and heat transfer in a vertical/ horizontal channel has gained momentum in the recent past because of its number of applications in several engineering processes, such as geothermal reservoirs; cooling of nuclear reactors; thermal insulation; energy storage and conservation; fire control; chemical, food and metallurgical industries; and petroleum reservoirs. This type of situation also occurs in electronic packages, microelectronic devices during their operations. Tou (1960) was the first among the others to initiate the study of laminar fully developed mixed convection flow in a vertical channel with uniform wall temperature. This was then extended by Habch and Acharya (1986) for asymmetric heating, where one plate is heated and the other is adiabatic. Thereafter numerous investigations have been carried out on mixed convection in a vertical/ horizontal channel with symmetric and/or asymmetric heating under different physical situations (Vajravelu and Sastri (1977), Aung and Worku (1986), Barletta (1988), Cheng et al. (1990), Bejan (1995), Barletta (1999), Chamkha (2000)). The mathematical models described in these papers deal with three kinds of thermal boundary conditions: (i) prescribed uniform temperature at both walls, which can be either equal or different; (ii) prescribed uniform temperature at one of the walls and prescribed uniform heat flux at the other wall; and (iii) prescribed uniform heat fluxes at both walls. Vajravelu and Sastri (1977) explored the effects of frictional heating on the fully developed free convective flow and heat transfer between parallel vertical walls kept at constant temperature. Aung and Worku (1986) analyzed the developing flow for asymmetric wall temperature or symmetric wall heat flux by a finite difference method. Cheng et al. (1990) studied the flow reversal with the boundary conditions (i), (ii) and (iii) mentioned above. Barletta (1999) analyzed the effects of viscous dissipation on the fully developed combined forced and free convection in a vertical channel for the case of prescribed wall heat flux.

All the above investigators restricted their analyses to flow and heat transfer in the absence of a magnetic field. But in recent years, we find several applications of magnetic field, namely in MHD generators, cross field operators, shock tubes, pumps, and flow meters. The flow in an MHD generator channel is seldom fully developed over its entire length, and large heat fluxes occur at the entrance regions of these devices. In many cases the flow in these devices will be accompanied by heat either dissipated internally through viscous or Joule heating. In view of these practical applications, Umavathi and Malashetty (2005) investigated the laminar hydromagnetic flow and heat transfer in a vertical channel with symmetric/asymmetric wall temperatures in the presence of viscous dissipation. Also, the effect of magnetic field and viscous dissipation on fully developed mixed convection flow in a vertical channel was studied numerically by Saleh and Hashim (2010). Prathap Kumar et al. (2011) studied mixed convection magnetohydrodynamic and viscous fluid in a vertical channel. Liu and Lo (2012) studied numerically the entropy generation within a combined forced and free convective magnetohydrodynamic (MHD) flow in a parallel-plate vertical channel.

In all the above mentioned papers, the thermophysical transport properties of the ambient fluid were assumed to be constant. However, it is well known (Lai and Kulacki (1990), Setayesh and Sahai (1990), Hassanien (1997), Attia (1999), Abel et al. (2002), Andersson and Aarseth (2007), Prasad et al. (2010), Prasad et al. (2013), Vajravelu et al. (2014)) that these physical properties may change with temperature, especially the fluid viscosity and the thermal conductivity. For lubricating fluids, heat generated by internal friction and the corresponding rise in the temperature affects the physical properties of the fluid, and the properties of the fluid are no longer assumed to be constant. The increase in temperature leads to increase in the transport phenomena by reducing the physical properties across the thermal boundary layer. Thus the heat transfer at the wall is also affected. Therefore to predict the flow and heat transfer characteristics of the generator channel, it is necessary to take into account the temperature dependent fluid properties. Hence, the study on the variable fluid properties will help in preventing wear and tear of the equipments we use in day-to-day life.

In view of these applications, the problem studied here extends the work of Umavathi and Malashetty (2005) for the temperature dependent variable fluid properties. Thus in the present paper, we study the effects of variable transport properties on the magnetohydrodynamic mixed convection flow and heat transfer in a vertical channel. Here the thermo-physical transport properties namely, the density, the viscosity, and the thermal conductivity are assumed to be functions of temperature. In addition to this, we have also included the contribution of viscous dissipation and ohmic dissipation in the energy equation. Because of the intricacy of the physical model (due to the inclusion of the variable transport properties, the magnetic parameter, the viscous and ohmic dissipations), the momentum and energy equations are coupled and non-linear. The governing non-dimensional, non-linear differential equations are solved numerically by a second order finite difference scheme known as the Keller-Box method. Numerical computations were been carried out for temperature and velocity profiles, Nusselt number and slip velocity coefficient.

The effects of variable viscosity, variable thermal conductivity, mixed convection parameter, magnetic parameter, and the Brinkmann number on the flow behavior and heat transfer process are presented trough graphs and tables, and their salient features are discussed.

2. MATHEMATICAL FORMULATION

Consider the problem of laminar, hydromagnetic mixed convection flow in a vertical channel. Let the distance between the walls be L: A co-ordinate system is chosen such that the X-axis is parallel to the gravitational acceleration but with opposite direction. The Y-axis is perpendicular to the channel walls and the channel walls are represented by Y =0 and Y=L. The flow is assumed to be steady and the fluid properties are assumed to be constant except viscosity, density and thermal conductivity: They are assumed to vary with temperature.

A constant magnetic field of strength B_0 is applied across the channel. The external uniform magnetic filed is directed along Y-axis, and the induced electric and magnetic fields are neglected.

The flow is being fully developed, we get $V = 0,\ \frac{\partial V}{\partial Y} = 0,\ \frac{\partial P}{\partial Y} = 0$, and $\frac{\partial P}{\partial X} = \frac{dP}{dX} = A,$ where V is the velocity in the transverse direction P is the pressure and A is a constant. Thus from the continuity equation we have, $\frac{\partial U}{\partial X} = 0$, so that the velocity component along $X-$ axis depends only on Y, *i.e.* $U = U(Y)$. Based on the fact that the flow is being fully developed we can also assume $T = T(Y)$. Under these assumptions the equations governing the flow and heat transfer (by including the variation of thermo-physical transport properties, and viscous & Ohmic dissipations) can be written in usual notation as:

$$g\beta(T - T_0) - \frac{1}{\rho_0}\frac{dP}{dX} + \frac{1}{\rho_0}\frac{d}{dY}\left(\mu(T)\frac{dU}{dY}\right) - \frac{\sigma B_0^2}{\rho_0}U = 0, \quad (1)$$

$$\frac{d}{dY}\left(\alpha(T)\frac{dT}{dY}\right) + \frac{1}{\rho_0 C_P}\mu(T)\left(\frac{dU}{dY}\right)^2 + \frac{\sigma B_0^2}{\rho_0 C_P}U^2 = 0 \quad (2)$$

subject to the boundary conditions

$$U = 0,\ T = T_1 \ \ at\ Y = 0,\qquad U = 0,\ T = T_2\ \ at\ Y = L. \quad (3)$$

Here, ρ_0 is the ambient fluid density, g is the acceleration due to gravity, β is the volumetric co-efficient of thermal expansion, σ is the electric conductivity, B_0 is the uniform magnetic field, T is the temperature, C_p is the specific heat at constant pressure and T_0 is the reference temperature and it is assumed that $T_0 = (T_1 + T_2)/2$. In order to determine the pressure gradient from the equation (1) the mass flux conservation M_1 is required and can be written as: $\int_0^L U dY = M_1$.

Here the fluid properties, viscosity and thermal conductivity are assumed to vary with temperature and they are defined successively as follows:

2.1 Variation of Viscosity

The coefficient of viscosity is considered to vary as an inverse function of temperature (Lai and Kulacki (1990)) as follows:

$$\frac{1}{\mu} = \frac{1}{\mu_0}\left[1 + \gamma(T - T_0)\right]. \quad (4)$$

This is reasonably a good approximation for liquids such as water and crude oil (Ling and Dybbs (1987)). The equation (4) can be rewritten as

$$\frac{1}{\mu} = a(T - T_r), \quad (5)$$

where $a = \frac{\gamma}{\mu_0}$ and $T_r = T_0 - \frac{1}{\gamma}$. Here a and T_r are constants, and their values depend on the reference state and the thermal property of the fluid: In general $a > 0$ for liquids and $a < 0$ for gases. This is due the fact that the viscosity of a liquid usually decreases with increasing temperature while it increases for gases. To demonstrate further the appropriateness of equation (4), correlations between viscosity and temperature for air and water are given below since these two fluids are most commonly used in engineering applications:

For air,

$$\frac{1}{\mu} = -123.2(T - 742.6)\ based\ on\ T_0 = 293\ K\ (20^0C); \frac{1}{2} \quad (6)$$

and for water,

$$\frac{1}{\mu} = -29.83(T - 258.6)\ based\ on T_0 = 288\ K\ (15^0C). \quad (7)$$

The data used for these correlations are taken from reference (CRC hand book (1986)). While equation (6) is good up to an error within 1.2% to the temperature difference from 278 K (5^0 C) to 373 K (100^0 C), equation (7) is good to an error within 5.8% to the temperature difference from 283 K (10^0 C) to 373 K (100^0 C). Hence, the reference temperatures selected here for the correlations are very useful for applications.

2.2 Variation of Thermal Conductivity

Finally the thermal conductivity α varies with temperature in a linear fashion and useful for the range of 0^0 F to 400^0 F. As in Chiam (1996), we assume that the thermal conductivity α is of the form

$$\alpha = \alpha_0\left(1+\varepsilon\frac{(T-T_0)}{(T_2-T_0)}\right),\ \alpha_0 = \frac{k_0}{(\rho_0 C_p)}, \tag{8}$$

where ε is a small parameter known as variable thermal conductivity parameter. Here α_0 is the thermal diffusivity and k_0 is the thermal conductivity of the fluid. The governing equations can be non-dimensionalized using the following set of dimensionless variables and parameters based on the characteristic width L of the channel:

$$u = \frac{U}{U_0},\ \theta = \frac{T-T_0}{(T_2-T_0)},\ y = \frac{Y}{L},\ \mathrm{Re} = \frac{U_0 L}{\nu_0} \tag{9}$$

The reference velocity U_0 is given by

$$U_0 = -\frac{AL^2}{\mu_0}.$$

By using quantities in (5), (8) & (9) in to the equations (1)-(2), and in to conditions (3); we get

$$\lambda\theta + 1 + \frac{1}{\left(1-\frac{\theta}{\theta_r}\right)^2}\frac{d\theta}{dy}\frac{du}{dy}\frac{1}{\theta_r} + \frac{1}{\left(1-\frac{\theta}{\theta_r}\right)}\frac{d^2u}{dy^2} - M_n u = 0, \tag{10}$$

$$(1+\varepsilon\theta)\frac{d^2\theta}{dy^2} + \varepsilon\left(\frac{d\theta}{dy}\right)^2 + \frac{Br}{\left(1-\frac{\theta}{\theta_r}\right)}\left(\frac{du}{dy}\right)^2 + M_n Br u^2 = 0, \tag{11}$$

$$u = 0,\ \theta = R_T \quad at \quad y = 0,$$
$$u = 0,\ \theta = 1 \quad at \quad y = 1, \tag{12}$$

where λ, M_n, θ_r Pr, Br and R_T are respectively the mixed convection parameter, the magnetic parameter, the viscosity parameter , the Prandtl number, the Brinkmann number and the wall temperature ratio parameter. They are defined as

$$\lambda = \frac{Gr}{\mathrm{Re}} = \frac{g\beta(T_2-T_0)L^3}{\nu_0^2\frac{U_0 L}{\nu_0}},\ Mn = \frac{\sigma B_o^2 L^2}{\mu_0},$$

$$\theta_r = \frac{1}{\delta(T_2-T_0)},$$

$$Br = \frac{\mu_0 U_0^2}{k_0(T_2-T_0)} \text{ and } R_T = \frac{T_1-T_0}{T_2-T_0}. \tag{13}$$

The fluid viscosity parameter is determined by the viscosity of the fluid in consideration and the operating temperature difference. A large value of θ_r implies either γ or (T_0-T_2) is small and the effect of variable viscosity can thus be neglected. On the other hand, for smaller value of θ_r either the fluid viscosity changes markedly with temperature or the operating temperature difference is high. In either case the variable fluid viscosity is expected to become more important.

Also it is to be noted that liquid viscosity varies differently with temperature than that of gas, therefore it is important to note that θ_r is negative for liquids and positive for gases. It can seen that in the absence of thermo-physical transport properties, equations (10) and (11) reduce to those of Umavathi and Malasheety (2005), while in the absence of Brinkamann number and the magnetic parameter equations reduce to those of Hamadah and Wirtz (1991), and in the presence of therm-ophysical transport properties with the buoyancy force and the magnetic parameter are absent equations reduce to those of Satayesh and Sahi (1990).

Furthermore, when the Brinkmann number and the magnetic parameter are absent, the analytical solution can be obtained for the constant thermo-physical transport properties. It is found that our results agree well with the existing results of Aung and Worku (1986). For all practical purpose, the physical quantities of interest in this problem are the skin friction and the Nusselt number, and are given by respectively

$$C_f = \frac{\mu_0}{\rho_0 U_0^2}\left(\frac{\partial u}{\partial y}\right)_{y=0,L},$$

$$Nu = \frac{L}{(T_2-T_0)}\left(\frac{\partial T}{\partial y}\right)_{y=0,L}. \tag{14}$$

3. EXACT SOLUTIONS FOR SOME SPECIAL CASES

Here we present exact solutions in certain special cases. Such solutions are useful and serve as a base-line for comparison with the solutions obtained via numerical schemes.

3.1. No Variable Fluid Properties, No Magnetic Field and No Viscous Dissipation $(\theta_r \to \infty,\ \varepsilon = 0, Mn = 0 \ and\ Br = 0)$

In the limiting case of $(\theta_r \to \infty,\ \varepsilon = 0, Mn = 0 \ and\ Br = 0)$ the flow and

Table 1 Nusselt numbers as functions of λ and R_T for isothermal-isothermal boundary for $\theta_r \to \infty,\ \varepsilon = 0.0,\ Mn = 0.0\ and\ Br = 0.0$

λ	Cheng et al (1990)				Present Method			
	R_T =0.0		R_T =0.5		R_T =0.0		R_T =0.5	
	Nu_1	Nu_2	Nu_1	Nu_2	Nu_1	Nu_2	Nu_1	Nu_2
100.0	2.769	1.565	2.323	1.756	2.768	1.567	2.320	1.759
10.0	2.057	1.946	2.028	1.973	2.056	1.948	2.025	1.975
1.0	2.000	1.994	2.003	1.997	2.000	1.991	2.001	1.994

heat transfer problem reduces to

$$\lambda\theta + 1 + \frac{d^2u}{dy^2} = 0, \tag{15}$$

$$\frac{d^2\theta}{dy^2} = 0 \tag{16}$$

(16)

and using boundary conditions (12) for θ can be integrated to give the following fully developed temperature profile

$\theta = (1 - R_T)\, y + R_T.$

Integration of equation (15), using the velocity boundary conditions results in

$$u = -\lambda\left[(1-R_T)\frac{y^3}{6} + R_T\frac{y^2}{2}\right] - \frac{y^2}{2} + \left\{\lambda\left[(1-R_T)\frac{1}{6} + \frac{R_T}{2}\right] + \frac{1}{2}\right\} y$$

3.2 No Variable Fluid Properties But The Presence of Magnetic Field

$(\theta_r \to \infty,\ \varepsilon = 0,\ and\ Br = 0)$

With and $Mn \neq 0$, the convective flow is coupled with heat transfer phenomena. The exact analytical solution to the equation (10) satisfying the required conditions is given by

$$u = C_1 e^{\sqrt{Mn}\, y} - \left(C_1 + \frac{(1+\lambda R_T)}{Mn}\right) e^{-\sqrt{Mn}\, y} + \frac{(1 + \lambda y - \lambda R_T y + \lambda R_T)}{Mn}$$

$$where\ C_1 = \frac{1 - e^{\sqrt{Mn}} - e^{\sqrt{Mn}}\lambda + R_T\lambda}{\left(e^{2\sqrt{Mn}} - 1\right) Mn}.$$

4. NUMERICAL PROCEDURE

The coupled nonlinear ordinary differential equations (10) and (11) subject to the conditions (12) are solved by a second order finite difference scheme known as the Keller-Box method (Keller (1992), Prasad et al.(2009,2010)). The numerical solutions are obtained in four steps as follows:

- reduce equations (10) and (11) to a system of first-order equations;
- write the difference equations using central differences;
- linearize the algebraic equations by Newton's method, and write them in matrix-vector form; and
- solve the linear system by the block tri-diagonal elimination technique.

The step size Δy, and the position of the edge of the boundary layer y_L are to be adjusted for different values of parameters to maintain accuracy. For brevity, the details of the solution procedure are not presented here. To demonstrate the accuracy of the present numerical method, results of the Nusselt numbers for the isothermal boundary are compared with the available results in the literature, for a special case (That is, in the absence of thermo-physical properties, Cheng et al. (1990).). It can be seen from Table 1 that the present results agree very well with those of Cheng et al. (1990).

5. RESULT AND DISCUSSION

Employing the above numerical method, the governing equations of the problem are solved for several sets of values of the physical parameters. L The numerical results thus obtained are presented for velocity and temperature in Figures 1-7. Also the numerical results for the Nusselt number and the skin friction are presented in Table 2. Since it is not possible to present the results here for all possible permutations and combinations of all the physical parameters, we focus our attention on the effects of the new parameters (related to the thermo-physicatransport properties) on the flow and heat transfer fields. Velocity profiles are depicted in Figures 1-3, whereas the temperature profiles are shown graphically in ures 4-7.

In Figures 1-3 the profiles for the stream-wise velocity u are presented for several sets of values of the governing parameters. The velocity u increases from its value zero at the left wall, reaching to its maximum around the midway of the channel, and attains zero as the other wall of the channel.

Figure 1 is the graphical representation of the stream-wise velocity u for different values of the magnetic parameter Mn . The velocity profiles show that the rate of transport is considerably reduced with an increase in Mn . It clearly indicates that the transverse magnetic field opposes the transport phenomena. This is because of the fact that the variation of Mn leads to the variation of Lorentz force, due to magnetic field, and the Lorentz force

Table 2 Values of the skin friction and the Nusselt number for different values of the physical parameters.

Br	ε	*Mn*	λ	θ_r	$R_T = 0.0$			$R_T = 0.2$		
					$f'(0)$	$\theta'(0)$	$\theta'(1)$	$f'(0)$	$\theta'(0)$	$\theta'(1)$
0.01	0.01	0.0	50.0	∞	8.144124	0.820637	1.328645	11.218529	0.522618	1.215133
				-10.0	8.300903	0.809321	1.345021	11.576994	0.505657	1.237367
				-5.0	8.440881	0.798429	1.360770	11.921870	0.489235	1.258907
				-1.0	9.338680	0.697143	1.511198	14.367629	0.373366	1.412506
		1.0		∞	7.481717	0.868239	1.266422	10.444075	0.590283	1.134041
				-10.0	7.606374	0.861136	1.276954	10.749396	0.579847	1.148013
				-5.0	7.715908	0.854413	1.286881	11.041740	0.570001	1.161193
				-1.0	8.242496	0.811615	1.350740	13.079421	0.507171	1.246144
				-0.5	8.470123	0.775896	1.407380	13.830455	0.487229	1.273697
		0.0	0.0	-10.0	0.502858	1.014726	1.005519	0.511252	0.810878	0.805304
			5.0		1.348300	1.010704	1.001140	1.708050	0.805218	0.812699
			10.0		2.188508	1.002414	1.024357	2.897641	0.793034	0.829235
			50.0		8.300903	0.809321	1.345021	11.576994	0.505657	1.237367
			75.0		10.727517	0.662151	1.605673	14.260253	0.345573	1.475133
		1.0	0.0		0.463902	1.014837	1.005409	0.471269	0.810991	0.805191
			5.0		1.218606	1.011950	1.009930	1.554021	0.806908	0.810766
			10.0		1.970231	1.005999	1.019870	2.632067	0.798114	0.823307
			50.0		7.606374	0.861136	1.276954	10.749396	0.579847	1.148013
			75.0		10.548641	0.709833	1.560979	15.051808	0.353762	1.499368
		0.0	50.0	-10.0	11.5776994	0.505657	1.237367	8.300903	0.809321	1.345021
		1.0			10.749396	0.579847	1.148013	7.606374	0.861136	1.276954
		5.0			8.248505	0.749338	0.935606	5.563474	0.976472	1.116201
		10.0			6.330556	0.826616	0.828675	4.050596	1.027476	1.034128
		20.0			4.345513	0.861388	0.767235	2.540242	1.049781	0.983884
		30.0			3.361934	0.862323	0.755478	1.823170	1.050058	0.972204
	0.0	0.0	50.0	-1.5	8.908285	0.748586	1.435482	13.382675	0.415921	1.359124
	0.3				9.492467	0.866014	1.254215	13.872817	0.498877	1.187705
	0.5				9.823563	0.947170	1.170586	14.142502	0.552316	1.108307
	0.0	1.0			8.062163	0.823208	1.333971	12.265627	0.527360	1.220844
	0.3				8.577590	0.951629	1.165541	12.668258	0.615403	1.070100
	0.5				8.866882	1.039385	1.099342	12.886137	0.670995	1.000824
	0.0	1.0	50.0	-10.0	7.587638	0.856773	1.283750	10.735861	0.576773	1.153877
	0.1				7.769167	0.900572	1.221051	10.866006	0.607216	1.099699
	0.3				8.097063	0.989351	1.122972	11.095063	0.665945	1.014802
	0.5				8.384879	1.079515	1.049505	11.290030	0.721789	0.951115
0.0	0.01	1.0	50.0	-10.0	8.018701	1.015152	1.005100	11.308831	0.811307	0.804880
0.01					7.606374	0.861136	1.276954	10.749396	0.579847	1.148013
0.03					7.250573	0.731134	1.516387	10.277404	0.387163	1.443653

produces resistance to the transport phenomena. This is consistent with the physics.

Figure 2 shows the effect of mixed convection parameter λ on the stream-wise velocity. It is observed that the increase in λ is to increase the velocity profile in the mid region of the channel. Physically, $\lambda > 0$ means heating of the fluid or cooling of the surface, $\lambda < 0$ means cooling of the fluid or heating of the surface, and $\lambda = 0$ corresponds to the absence of mixed convection currents.

Increase of λ means an increase of the temperature difference $(T_2 - T_0)$. This leads to enhancement in the stream-wise velocity due to the enhanced convection. In the absence of λ, the velocity profile is linear. The stream-wise velocity profiles for different values of variable viscosity parameter θ_r are shown graphically in Figure 3. The effect of increasing (in absolute sense) θ_r is to decrease the parabolic nature in the mid region of the channel. From the results presented here, it clear that θ_r has strong effect on the stream-wise velocity and hence on the skin friction. The numerical results for temperature profiles are presented in Figures 4-7 for different values of the governing parameters.

Figure 4 depicts the temperature $\theta(y)$ for different values of Mn. The effect of increasing values of Mn is to increase the temperature. Physically it means that when a transverse magnetic field is applied to an electrically conducting fluid, the fluid experiences a resistive force known as the Lorentz force, increasing the friction between its layers. Due to this, there is an increase in the temperature in the channel.

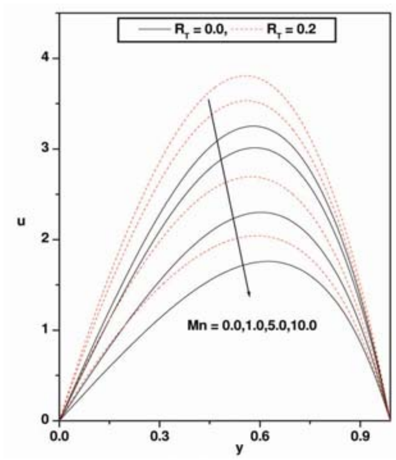

Fig. 1. Stream-wise velocity profiles for different valuesof magnetic parameter when e **= 0.01, Br = 0.01,**l **= 50.0,** q**= -10.0.**

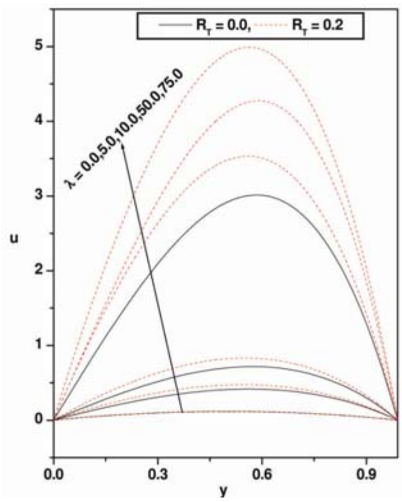

Fig. 2. Stream-wise velocity profiles for different values of mixed convection parameter when Mn= 1.0, e **= 0.01, Br = 0.01,** q**= -10.0.**

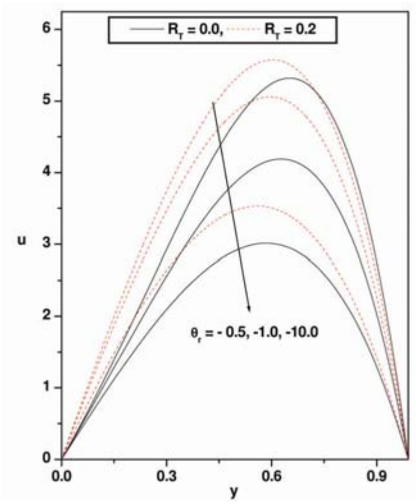

Fig. 3. Stream-wise velocity profiles for different values of variable viscosity parameter when e **= 0.01,** l **= 50.0, Br = 0.01, Mn = 1.0.**

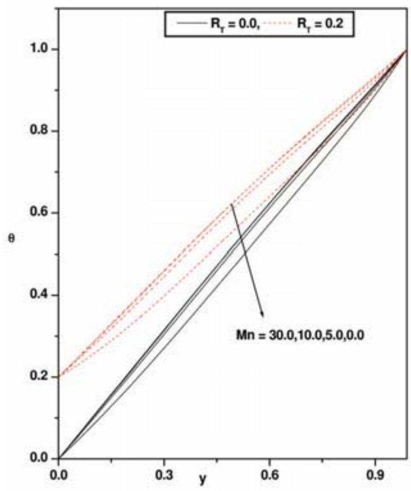

Fig. 4. Temperature profiles for different values of magnetic parameter when e **= 0.01, Br = 0.01,** l **= 50.0,** q**= -10.0.**

Figure 5 depicts the temperature profiles for different values of λ. An increase in λ results in a decrease in a thermal boundary layer and hence in the Nusselt number. This phenomenon holds even for non-zero values of the temperature ratio parameter.

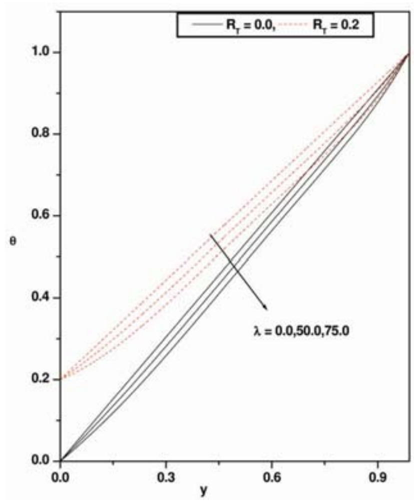

Fig. 5. Temperature profiles for different values ofmixed convection parameter when , e **= 0.01, Br = 0.01,** q **= -10.0, Mn =1.0**

Temperature profiles for different values of the viscosity parameter θ_r and the variable thermal conductivity parameter ε are shown graphically in Figures 7(a) and 7(b). From the graphical representation we see that the effect of increasing values of viscosity parameter is to decrease the temperature. This is because of the fact that the increase in θ_r results in an increase in the thermal boundary layer thickness within the channel. From these graphs we also notice that the temperature distribution is lower throughout the channel in the absence of ε and increases with by increasing values of ε.

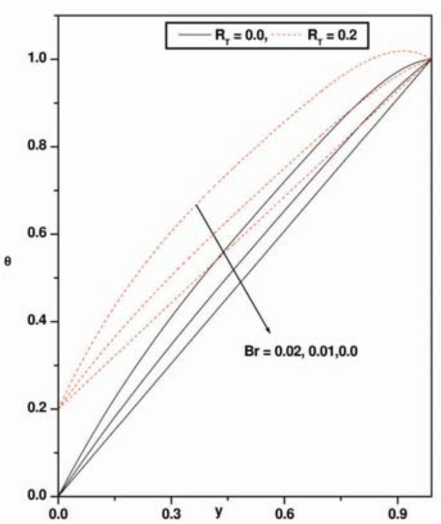

Fig. 6. Temperature profiles for different values of Brinkmann number when e = 0.01, Mn = 1.0,q =-10.0, l = 50.0.

The profiles of the temperature distribution for different values of the Brinkmann number Br are shown in Figure 6. It is to be noted that the effect of the increasing values of Br is to decrease the temperature distribution.

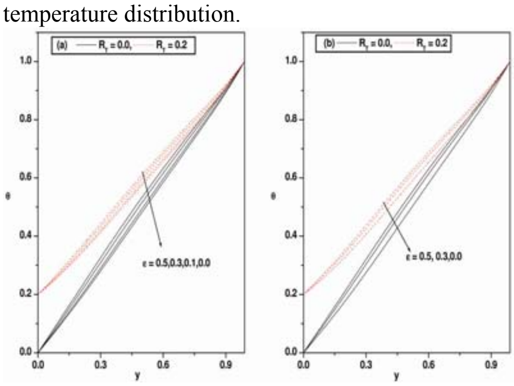

Fig. 7. Temperature profiles for different values of variable thermal conductivity parameter (a) q = - 10.0 , and (b) q = -1.5, when (b) Mn = 10.0,Br = 0.01, l = 50.0.

The impact of all physical parameters on the skin friction coefficient and Nusselt number may be analyzed from Table 2. It is of interest to note that the effects of the variable viscosity parameter and the mixed convection parameter are to increase the skin friction and to decrease the Nusselt number. Quite opposite is true with the magnetic parameter.

6. CONCLUSIONS

This study provides theoretical results for a class of MHD mixed convection flow of a viscous fluid between parallel plates maintained at constant temperatures. The temperature dependent transport properties of the fluid namely, the viscosity, the density and the thermal conductivity are used. The coupled, non-linear non-dimensional equations subjected to appropriate conditions are solved numerically by a second order finite difference scheme known as Keller-Box method, which is found to be accurate and stable for a wide range of the values of the parameters. Some of the important findings are:

- An increase in the magnetic parameter reduces the flow in the channel. This is true even in the presence of temperature dependent transport properties;
- The effect of increasing mixed convection parameter is to increase the velocity in the channel and hence produces a reduction in the skin friction;
- An increase in the variable thermal conductivity parameter is to enhance the temperature in the channel and this phenomenon is true even with the variable viscosity parameter;
- The effect of viscosity parameter is to reduce the Nusselt number. However, this effect is found to be minimum for the non-zero values of the magnetic parameter; and
- Of all the parameters, the variable thermo-physical transport property has the strongest effect on the drag, heat transfer characteristics, the stream-wise velocity and the temperature field.

ACKNOWLEDGEMENT

We thank the anonymous referees for their helpful comments, which have greatly improved the quality of the manuscript.

REFERENCES

Aung, W and G. Worku (1986). Theory of fully developed combined convection including flow reversal, *ASME J. Heat Trans*. 108, 485-488.

Aung, W and G. Worku (1986). Developing flow and flow reversal in a vertical channel with asymmetric wall temperature, *J. Heat Trans*. 108, 299-304.

Attia, H. A. (1999). Transient and heat transfer between two parallel olates with temperature dependent viscosity, *Mech. Res. Comm*. 26, 115-121.

Abel, M. S., S. K Khan and K. V. Prasad (2002). Study of visco-elastic fluid flow and heat transfer over a stretching sheet with variable fluid viscosity, *Int. J. Non-Linear Mech*. 37, 81-88.

Andersson, H. I and J. B. Aarseth (2007). Sakisdis flow with variable fluid properties, *Int. J. Engng. Sci*. 45(2), 554-561.

Barletta, A. (1988). Laminar mixed convection with viscous dissipation in a vertical channel, *Int. J. Heat Mass Trans*. 41 ,3501-3513.

Bejan, A. (1995). Convective heat transfer, 2nd

edition, Wiley, New York.

Barletta, A. (1999). Analysis of combined forced and free flow in a vertical channel with viscous dissipation and isothermal–isoflux boundary conditions, *Int. J. Heat Trans*. 121 ,349-356.

Barletta, A. (1999). Heat transfer by fully developed flow and viscous heating in a vertical channel with prescribed wall heat fluxes, *Int. J. Heat Mass Transfer* 42, 3873-3885.

Cheng, C. H., H. S. Kou and W. H. Haung (1990). Flow reversal and heat transfer of fully developed mixed convection in vertical channels, *J. Thermo Phy. Heat Trans*. 4, 375-382.

Chiam, T. C. (1996). Heat transfer with variable thermal conductivity in a stagnation point flow towards a stretching sheet, *Int. Comm. Heat Mass Trans*. 23, 239-248.

Chamkha, A. (2000). Flow of two-immiscible fluids in porous and non-porous channel, *ASME J. Fluid Eng*. 122, 117-124.

CRC Hand book of Chemistry and Physics, 67th edition, *CRC Press, Boca Raton Florida* (1986-1987).

Habchi, S. and S. Acharya (1986). Laminar mixed convection in a symmetrically or asymmetrically heated vertical channel, *Numer. Heat Trans*. 9 605-618.

Hassanien, I. A. (1997). The effect of variable viscosity on flow and heat transfer on a continuous stretching surface, *ZAMM*. 77, 876-880.

Keller, H. B. (1992). Numerical Methods for Two-point Boundary Value Problems, *Dover Publ, New York* .

Ling, J. X. and A. Dybbs (1987). Forced convection over a flat plate submersed in a porous medium: variable viscosity case, *ASME paper 87-WA/HT*-23, ASME, New York .

Lai, F. C. and F. A. Kulacki (1990). The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium, *Int. J. Heat Mass Trans.* 33, 1028-1031.

Liu, C. C. , C. Y. Lo (2012). Numerical analysis of entropy generation in mixed-convection MHD flow in vertical channel, *Int. Comm. in Heat and Mass Transfer* 39(9),1354–1359.

Prasad, K. V., D. Pal and P. S. Datti (2009). MHD flow and heat transfer in the flow of a power law fluid over a non-isothermal stretching sheet, *Comm. Nonlinear Science and Numerical Simulation* 14, 2178-2189.

Prasad, K. V., P. S. Datti and K. Vajravelu (2010). Hydromagnetic flow and heat transfer of a non-Newtonian Power law fluid over a vertical stretching sheet, *Int. J. Heat and Mass Trans*. 53, 879-888.

Prasad, K. V., K. Vajravelu and P. S. Datti (2010). The effects of variable fluid properties on the hydro-magnetic flow and heat transfer over a non-linearly stretching sheet, *Int. J. Thermal Sci*. 49(3), 603-610.

Prasad, K. V., K. Vajravelu, P. S. Datti and B. T. Raju (2013). MHD Flow and Heat Transfer in a Power-law Liquid Film at a Porous Surface in the Presence of Thermal Radiation, *Journal of Applied Fluid Mechanics* 6(3), 385-395.

Prathap Kumar, J., J. C. Umavathi and M. B. Biradar (2011). Mixed convection of magentohydrodynamic and viscous fluid in a vertical channel, *Int. J. Non-Linear Mech.* 46, 278–285.

Setayesh, A. and V. Sahai (1990). Heat transfer in developing magneto hydrodynamic Poiseuille flow and variable transport properties, *Int. J. Heat Mass Trans.* 33, 1711-1720.

Saleh, H. and I. Hashim (2010). Flow reversal of fully developed mixed MHD convection in vertical channels, *Chin Phys Lett*. 27,24401-24403.

Tao, L. N. (1960). On combined free and forced convection in vertical channels, *ASME J. Heat Trans.* 82, 233-238.

Umavathi, J. C. and M. S. Malashetty (2005). MHD mixed convection in a vertical channel, *Int. J. Non-linear Mech*. 40, 91-101.

Vajravelu, K and K. Sastri (1977). Fully developed laminar free convection flow between two parallel vertical walls I, *Int. J. Heat Mass Trans*. 20, 655-660.

Vajravelu, K., K. V. Prasad, S. R. Santhi and V. Umesh (2014). Fluid Flow and Heat Transfer over a Permeable Stretching Cylinder, *Journal of Applied Fluid Mechanics* 7(1), 111-120.