Flow and Heat Transfer Behavior of MHD Dusty Nanofluid past a Porous Stretching/Shrinking Cylinder at Different Temperatures

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ABSTRACT

In this study we analyzed the momentum and heat transfer behavior of CuO-water and Al₂O₃-water nanofluids embedded with micrometer sized conducting dust particles towards a porous stretching/shrinking cylinder at different temperatures in presence of suction/injection, uniform magnetic field, shape of nano particles, volume fraction of micro and nano particles. The governing boundary layer equations are transformed to nonlinear ordinary differential equations by using similarity transformation. Numerical solutions of these equations can be obtained by using Runge-Kutta Fehlberg technique. The influence of non-dimensional governing parameters on the flow field and heat transfer characteristics are discussed and presented through graphs and tables. Results indicates that spherical shaped nano particles showed better thermal enhancement compared with cylindrical shaped nano particles, increase in volume fraction of nano particles helps to enhance the uniform thermal conductivity. But it does not happen by increase in volume fraction of dust particles. Enhancement in fluid particle interaction reduces the friction factor and improves the heat transfer rate.

Keywords: MHD; Nanofluid, Dusty fluid; Stretching/shrinking; Suction/injection; Convection.

1. INTRODUCTION

The fluid flow and heat transfer over a stretching/shrinking cylinder have wide range of applications in engineering and its allied areas. Now a days, low thermal conductivity in convectional fluids like water, ethylene glycol, oil etc. encountered variety of problems in engineering electronic devices. To overcome this drawback and enhance the thermal conductivity in the convectional fluids, past few decades many researchers concentrated on mixing of nano or micrometer sized particles in the base fluids. Mixing of nano meter sized particles in to base fluid is called nanofluid, which helps to enhance the thermal conductivity of the mixture fluid compared with base fluid. Mixing of milli or micrometer sized particles (dust particles) in the base fluids are also helps to improve the thermal conductivity of the base fluid and it is called dusty fluid. Till now, researchers concentrated on investigating the momentum and heat transfer characteristics of either dusty or nano fluids. In this study we are taking initiation to analyze the momentum and heat transfer characteristics of dusty nanofluids. That is the mixture of milli or micro meter sized conducting dust particles in to the nanofluid. This also may help to enhance the thermal conductivity of the base fluid for some combination of dusty and nano mixtures.

Saffman (1962) was the first person who discussed about the laminar flow of the dusty gases. Choi (1995) introduced the concept of nanofluid by immersing the nano meter sized particles in to base fluids and he found the enhanced thermal conductivity in the base fluid due to the mixture of nano particles. Pulsatile flow and heat transfer behavior of dusty fluid over a long annular pipe was studied by Datta and Dalal (1995). Chamka (1998) discussed MHD flow and heat transfer over a non-isothermal stretching sheet immersed in a porous medium. Free convection of MHD flow over a cone with mixed thermal boundary conditions was discussed by Ece (2005). Buongiomo (2006) has given clear description on convective heat transfer in nanofluids. Das et al. (2007) written a book entitled “nanofluids science and technology” in this book they discussed the applications of nanofluids along with importance of convectional heat transfer in nanofluids. Palani and Ganeshan (2007) analyzed heat transfer behavior of dusty gases over infinite inclined plate.

Effect of thermal radiation on
magnetohydrodynamics nanofluid flow and heat transfer by means of two phase model was discussed by Sheikholeslami et al. (2015). Nadeem et al. (2010) presented a homotopy analysis method to solve a boundary layer stagnation point flow over a stretching surface. Timofeeva and Dileep Singh (2009) studied effect of nano particle shape by considering Alumina based nanofluid. MHD effects on unsteady dusty viscous flow by considering volume fraction of dust particles was analyzed by Ibrahim Saidu et al. (2010). Makinde and Aziz (2011) analyzed boundary layer flow of a nanofluid by considering convective boundary conditions. Flow and heat transfer behavior over a nonlinearly stretching surface was studied by Cortell (2011). Akbar et al. (2014, 2014a) studied the stagnation point flow of a CNT suspended nanofluid and Prandtl fluid over a stretching and shrinking sheets. Partial slip effect on non-aligned stagnation point flow of a nanofluid over a stretching convective surface was discussed by Nadeem et al. (2015). Gireesha et al., (2012) discussed flow and heat transfer behavior of dusty fluid over stretching. Remeli et al. (2012) discussed boundary layer flow of a nanofluid by considering suction/injection effects. Pavithra and Gireesha (2013) discussed unsteady flow and heat transfer behavior of the dust particles suspended flow over a stretching sheet. Ramana Reddy et al. (2014) studied aligned magnetic field, radiation and chemical reaction effects on unsteady dusty viscous flow with heat generation/absorption. Swati Mukhopadhyay (2013) studied MHD boundary layer flow and heat transfer over an exponentially stretching sheet. Unsteady natural convection flow of a nanofluid over a vertical plate in presence of radiation was analyzed by Sandeep et al. (2013). Stagnation point flow of a Cu-water nanofluid over a stretching/shrinking sheet was discussed by Bachok (2013). Mohan Krishna et al. (2014) discussed influence of radiation on unsteady MHD nanofluid flow over infinitely vertical flat plate. Than et al. (2014) discussed mixed convection flow of a nanofluid over cylinder embedded in porous medium. Stagnation-point flow and mass transfer over stretching/shrinking cylinder in presence of chemical reaction was presented by Najib et al.(2014). Mishra and Singh (2014) illustrated dual solutions for mixed convection flow over a shrinking cylinder. Boundary layer analysis and convective heat transfer in nanofluids was discussed by MacDevette et al. (2014). Very recently Sulochana and Sandeep (2015) discussed the stagnation-point flow and heat transfer of Cu-water nanofluid towards horizontal and exponentially stretching or shrinking cylinders and concluded that heat transfer in a Cu-water nanofluid through horizontal and exponential cylinders is non-unique.

To the authors’ knowledge no studies have been reported on the flow and heat transfer behavior of CuO-water and Al2O3-water nanofluids embedded with micrometer sized conducting dust particles towards a porous stretching/shrinking cylinder at different temperatures, in presence of suction/injection, uniform magnetic field, shape of nano particles, volume fraction of dust and nano particles. Numerical results are presented in this study. The influence of non-dimensional governing parameters on the flow field and heat transfer characteristics are discussed and presented through graphs and tables.

2. MATHEMATICAL FORMULATION

Consider a steady two dimensional boundary layer flow of anano fluid embedded with conducting dust particles over a horizontal porous stretching/shrinking cylinder with radius $R$ along the axis of the cylinder. The $X$-axis is measured along the axis of the cylinder and $F$-axis is measured in the radial direction as displayed in Fig.1. It is assumed that the stretching/shrinking velocity of the cylinder is $u_w = cx/L$, where $c$ is a constant and $L$ is the characteristic length. A uniform magnetic field of strength $B_0$ is applied in radial direction. Here magnetic Reynolds number is assumed to be small so that the induced magnetic field to be neglected. The dust particles are assumed as uniform in size, spherical and cylindrical shaped nano particles are considered and number density of dust particles along with volume fraction of dust and nano particles are taken into account.

The boundary layer equations as per above assumptions are given by (Gireesha et al. (2012), Than (2014))

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0, \quad (1)$$

$$\rho_f(1-\phi_f)\left(\frac{\partial u}{\partial x} + \frac{v}{r}\frac{\partial u}{\partial r}\right) = \left(1-\phi_f\right)\mu_f\left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right)$$

$$+ KN(u_p - u) - \sigma R_d\frac{e^2}{k}u, \quad (2)$$

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial r} = \frac{K}{m}(u - u_p), \quad (3)$$

$$\frac{\partial}{\partial x}(ru_p) + \frac{\partial}{\partial r}(rv_p) = 0, \quad (4)$$
\[
\rho_f \frac{\partial v}{\partial t} + (\rho v) \frac{\partial v}{\partial x} = -\frac{1}{
\frac{1}{\rho_f} \left[ k_f \left( \frac{\partial T}{\partial x} \right) - \phi (\rho \nu) \frac{\partial T}{\partial x} \right] } + \frac{1}{\tau_i} (u_p - u)^2, \tag{5}
\]

where \( r \) is the coordinate measured in the radial direction, \((u, v)\) and \((u_p, v_p)\) are the velocity components of the fluid and particle phase respectively in \( x \) and \( r \) directions, \( \phi_d \) is the volume fraction of the dust particles, \( K = 6 \pi \eta_d \) is the stokes resistance with \( \mu_f \) is the dynamic viscosity of the base fluid and \( d \) is the radius of the dust particle, \( N \) is the number density of the dust particles, \( m \) is the mass concentration of dust particles, \( \sigma_f \) is the electrical conductivity, \( \rho_{nf} \) and \( \mu_{nf} \) are the density and the dynamic viscosity of the nano-fluid respectively, \( k \) is the permeability of the porous medium, \( T \) is the temperature in the boundary layer, \((\rho c_p)_{nf} \) is the heat capacitance of nanofluid, \( k_{nf} \) is the effective thermal conductivity of nanofluid, \( \rho_p = Nm \) is the density of the particle phase and \( \tau_i \) is the relaxation time of the dust particle.

Boundary conditions for the proposed problem is

\[
u = u_{u}, v = v_{w}, T = T_{w} = T_{e} + A(x/L)^{2} \text{ at } r = R, \tag{6}
\]

\[
u \to 0, u_p \to 0, v_p \to v_{c}, T \to T_{e} \text{ as } r \to \infty,
\]

where \( v_{w} \) is the suction \((v_{w} < 0)\) or injection \((v_{w} > 0)\) velocity. The nanofluid constants are given by

\[
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho, \quad (\rho c_{p})_{nf} = (1 - \phi) (\rho c_{p})_f + \phi (\rho c_{p})_p, \quad k_{nf} = \frac{k_f}{(1 - \phi)^{-2}}, \quad \frac{k_{nf}}{k_f} = \frac{k_f}{(1 - \phi)^{-2}}, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{-2}}.
\tag{7}
\]

To measure the thermal conductivity of the nanofluid, \( k_{nf} \) for different shapes of nano particles, we adopted the formula, which is proposed by Hamilton and Crosser (1962). Where \( \phi \) is the volume fraction of the nano particles, \( n \) is the nano particle shape, \( n = 3/2 \) for cylindrical shaped nano particles and \( n = 3 \) for spherical shaped nano particles. The subscripts \( f \) and \( s \) refer to fluid and solid properties respectively.

For similarity solutions of equations (1)-(5) with respect to the boundary conditions (6), we are introducing the following similarity transformation

\[
 u = \frac{\alpha x}{L} f'(y), v = \frac{1}{r^2} \left[ -\frac{1}{R^2} + \frac{2}{R} \right] f'(y), \quad \frac{\mu}{L} \left[ \frac{\rho_f}{\eta_d} + \frac{\rho_p}{\eta_p} \right] \frac{\partial^2 T}{\partial r^2} = \frac{1}{r^2} \frac{\partial^2 T}{\partial r^2} \tag{8}
\]

where \( T = T_{e} = A(x/L)^{2} \) \( \theta(y) \), \( \eta \) is the similarity variable, equation (8) identically satisfies equations (1) and (4), by defining \( \eta \) in this form, the boundary conditions at \( r = R \) reducing to the boundary conditions at \( \eta = 0 \) which is more convenient for numerical computations. Substituting (8) into equations (2), (3) and (5), we get the nonlinear ordinary differential equations of the form

\[
(1 - \phi_f) (1 + 2\eta \delta) f'' + 2\delta f' + \left[ 1 - \phi_f \right] \left[ 1 + \phi \left( \frac{\rho_p}{\rho_f} \right) \right] \left[ (f')^{2} - f''^{2} \right] + \alpha \beta (G - f') - (M + \gamma) f' = 0,
\tag{9}
\]

\[
G F - G^{2} + \beta (f')^{2} - G = 0,
\tag{10}
\]

\[
\left[ 1 - \phi_f \right] \left[ 1 + 2\eta \delta \right] \left[ \theta^{2} + 2\delta \theta \right] + Pr ce Ec (G - f')^{2} + \left[ 1 - \phi \right] \left[ (\rho c_{p})_{nf}/(\rho c_{p})_f \right] \left[ (f')^{2} - 2f' \right] \theta = 0.
\tag{11}
\]

Subject to the boundary conditions

\[
f(0) = S, f'(0) = \lambda, \theta(0) = 1, \quad f'(\eta) \to 0, \quad G(\eta) \to 0, \quad F(\eta) \to -f'(\eta), \theta(\eta) \to 0, \quad \text{as } \eta \to \infty\tag{12}
\]

where \( \delta \) is the curvature parameter, \( \alpha \) is the mass concentration of the dust particles, \( \beta \) is the fluid particle interaction parameter , \( \tau_{vi} \) is the relaxation time of the dust particles, \( M \) is the magnetic field parameter, \( \gamma \) is the porosity parameter, \( Pr \) is the Prandtl number, \( Ec \) is the Eckert number, \( S \) is the suction/injection parameter, here \( S > 0 \) for suction and \( S < 0 \) for injection and \( \lambda \) is the stretching/shrinking parameter, here \( \lambda > 0 \) for stretching and \( \lambda < 0 \) for shrinking, these are given by

\[
\delta = \left( \frac{v_{w} L}{a R^{2}} \right)^{1/2}, \quad \alpha = \frac{\rho_p}{\rho_f}, \quad \beta = 1/\alpha \tau_{vi},
\gamma = \nu / a k, \quad \tau_{vi} = m / K, \quad M = \sigma B_{0}^{2} / a \rho_f,
\Pr = \frac{(\mu c_{p})_f / k_f}{\nu_w} = -S \left( R^{2} v_{w} / a r^{2} L \right)^{1/2}, \quad \lambda = c / a, Ec = a^{2} L / A(c) f,
\tag{13}
\]

The main physical quantities are interest the skin friction coefficient and Nusselt number are given by
By using non-dimensional variables, we have
\[ C_f \operatorname{Re}_f^{1/2} = (1 - \phi)^{2.5} f(0), \]
\[ \operatorname{Nu}_f \operatorname{Re}_f^{1/2} = -k_f ((\phi/\rho) \theta(0)), \]
Where \( \operatorname{Re}_f = u_f x_f / v_f \) is the local Reynolds number.

3. NUMERICAL PROCEDURE

Equations (9) to (11), subject to the boundary conditions (12) are solved numerically by using Runge-Kutta Fehlberg scheme. We considered
\[ f = x_1, f' = x_2, f'' = x_3, F = x_4, G = x_5, G'' = x_6, \]
\[ \theta = x_7, \phi = x_8. \]
Equations (9) to (11) are transformed into systems of first order differential equations. Subject to the following initial conditions
\[ x_1(0) = S, x_2(0) = \lambda, x_3(0) = s_1, \]
\[ x_4(0) = -S, x_5(0) = s_2, x_7(0) = s_3 \]

(16)

We assumed the unspecified initial conditions in equation (16), transformed first order differential equations are integrated numerically as an initial valued problem to a given terminal point. We can check the accuracy of the assumed missing initial condition, by comparing the calculated value of the different variable at the terminal point with the given value by the existence of the difference in improved values so that the missing initial conditions must be obtained. The calculations are carried out by the program using MATLAB.

4. RESULTS AND DISCUSSION

Equations (9) to (11), subject to the boundary conditions (12) are solved numerically. For numerical results we considered \( \eta = 10, \delta = \alpha = \beta = \gamma = 0.5, \lambda = M = 1, \phi = \phi_d = 0.1, \]
\( n = 3, S = 0.5 \) and \( Ec = 0.1 \). These values are kept as common in entire study except the varied values as displayed in respective figures and tables. Results shows the influence of non-dimensional governing parameters like curvature parameter \( \delta \), volume fraction of nano particles \( \phi \) (in nanometers), volume fraction of dust particles \( \phi_d \) (in micrometers), suction/injection parameter \( S \), mass concentration of dust particles \( \alpha \), nano particle shape \( n \), fluid particle interaction parameter \( \beta \) and stretching/shrinking parameter \( \lambda \) on velocity and temperature profiles along with skin friction coefficient and Nusselt number. In this study we analyzed the flow and heat transfer behavior of CuO-water and Al₂O₃-water nanofluids embedded with micro meter sized dust particles at different temperatures. Table 1 shows the thermophysical properties of nano particles and base fluid at different temperatures.

### Table 1 Thermophysical properties of base fluid and nano particles at different temperatures

<table>
<thead>
<tr>
<th>Fluid</th>
<th>ρ (kg/m³)</th>
<th>c_p (J/kgK)</th>
<th>k (W/mK)</th>
<th>Pr</th>
<th>Temp in °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂O</td>
<td>999.6</td>
<td>4090</td>
<td>0.5884</td>
<td>8.80</td>
<td>10</td>
</tr>
<tr>
<td>H₂O</td>
<td>987.7</td>
<td>4066</td>
<td>0.6440</td>
<td>3.35</td>
<td>50</td>
</tr>
<tr>
<td>CuO</td>
<td>6320</td>
<td>531.8</td>
<td>76.5</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Al₂O₃</td>
<td>3970</td>
<td>765</td>
<td>40</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

Figs. 2-4 illustrates the influence of magnetic field parameter on velocity profiles of fluid phase, velocity profiles of dust phase and temperature profiles of both CuO-water and Al₂O₃-water dusty nanofluids at different temperatures. It is evident from figures that an increase in magnetic field parameter depreciates the velocity profiles of the fluid and dust phases and enhances the temperature profiles. It is due the fact that a rise in magnetic field parameter develops the opposite force to the flow, called Lorentz force this force declines the velocity profiles of the fluid phase. Since the dust particles are immersed in nanofluid, the velocity profiles of dust phase also follow the fluid phase. An increase in magnetic field parameter enhances the thermal boundary layer thickness and hence improves the temperature profiles of the flow. We observed an interesting result that there is no significant difference in the velocity profiles of the fluid and dust phases due to increase in temperature from 10°C to 50°C. But in temperature profiles we noticed hike in temperature profiles of the fluid due to the temperature difference and also observed that CuO-water dusty nanofluid showed better thermal enhancement compared with Al₂O₃-water dusty nanofluid.

![Fig. 2. Velocity profiles of fluid phase for different values of magnetic field parameter M.](image-url)
Figs. 3-7 depict the effect of volume fraction of nano particles on velocity profiles of fluid phase, velocity profiles of dust phase and temperature profiles of both CuO-water and Al$_2$O$_3$-water dusty nanofluids at different temperatures. It is clear from figures that enhancement in volume fraction of nano particles increases the velocity profiles of fluid and dust phases along with temperature profiles of the flow. Generally increase in volume faction of nano particles enhances the thermal conductivity of the flow. Due to this reason we observed hike in velocity and temperature profiles. Also, we noticed that the velocity profiles of Al$_2$O$_3$-water dusty nanofluid at 10°C temperature showed better enhancement in velocity boundary layer thickness compared with CuO-water dusty nanofluid at the same temperature. Here we can conclude that at lower temperatures CuO-water dusty nanofluid improves the velocity boundary layer compared with CuO-water dusty nanofluid.

Figs. 8-10 display the variations in velocity profiles of fluid phase, velocity profiles of dust phase and temperature profiles of both CuO-water and Al$_2$O$_3$-water dusty nanofluids at different temperatures for different values of nano particle volume fraction $\phi$. The enhancement in volume fraction of dust particles depreciates the velocity profiles of fluid and dust phases and improves the temperature profiles of the flow. This may happen due to the fact that an increase in volume fraction of the dust particles improves the conduction in particles. This causes to declines the velocity profiles of both fluid and dust phases. From fig.10 it is very clear that there exists a gradual enhancement in temperature profiles due to the temperature difference from 10°C to 50°C. Which
shows that after critical temperature level dust particles effectively enhances the temperature profiles of the flow. If we compare this result with fig.7 we have seen a uniform temperature distribution in fig.7 due to increase in volume fraction of nano particles. But in fig. 10 we noticed abnormal hike in temperature profiles for small temperature difference. From this we can say that dusty nanofluids help to gradual enhancement in temperature profiles after reaching critical temperature level. It is also noticed that CuO-water dusty nanofluid showed better heat transfer performance compared with Al2O3-water dusty nanofluid.

Figs. 11-13 illustrate the influence of curvature parameter on velocity and temperature profiles for both Al2O3-water and CuO-water dusty nanofluids at different temperatures. It is clear from figures that increases in curvature parameter enhance the velocity profiles of fluid and dust phases along with temperature profiles. This is due to the fact that increase in \( \delta \) enhances the radius of the cylinder, this helps to reduce the contact area of the cylinder with the fluid and hence improves the velocity boundary layer thickness along with boundary layer thickness of temperature. Also, we observed that due to increase in curvature parameter we don’t have significant differences in enhancement in temperature profiles of CuO-water and Al2O3-water dusty nanofluids.

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Figs. 14 present the effect of mass concentration of dust particles on velocity profiles of the flow. It is evident from figure that an increase in mass concentration of dust particles depreciates the velocity profiles of the fluid phase. We observed similar type of results as we explained in volume fraction of dust particles case. This agrees the general physical behavior of mass concentration of dust particles on the flow.

Fig. 15 depicts the influence of shape of the nano particles on the temperature profiles of both Al₂O₃-water and CuO-water dusty nanofluids at different temperatures cylinder. It is clear that enhances in the value of $n$ improves the temperature profiles of the flow. Here $n = 3/2$ indicates cylindrical shaped nano particles and $n = 3$ for spherical shaped nano particles. From this we conclude that spherical shaped nano particles have higher thermal conductivity compared with cylindrical shaped nano particles.

Tables 2 and 3 depict the influence of non dimensional governing parameters on friction factor and Nusselt number for Al₂O₃-water and CuO-water dusty nanofluids at different temperatures. It is evident from the tables that increase in curvature parameter and volume fraction of nano particles increases the skin friction coefficient and reduces the Nusselt number. Enhancement in volume fraction of dust particles and mass concentration of dust particles depreciates the friction factor and heat transfer rate. Increase in suction/injection parameter depreciates the skin friction coefficient and enhances the Nusselt number. A raise in the value of fluid particle interaction parameter depreciates the friction factor and enhances the rate of heat transfer for both dusty nanofluids. Shape of the
nano particles does not influence the friction factor. But spherical shaped nano particles reduce the heat transfer rate. And these two tables agree the general behavior of the heat transfer that at lower temperatures rate of heat transfer is more compared with higher temperatures.

5. CONCLUSIONS

In this study we are investigated the flow and heat transfer behavior of behavior of CuO-water and Al2O3-water nanofluids embedded with conducted dust particles towards a porous stretching/shrinking cylinder at different temperatures, in presence of suction/injection, uniform magnetic field, shape of nano particles, volume fraction of dust and nano particles. Numerical results are carried out. The conclusions are made as follows:

- Increase in volume fraction of dust particles and mass concentration of dust particles depreciates the heat transfer rate.
- Increase in fluid particle interaction parameter reduces the friction factor and improves the heat transfer rate.
- CuO-water dusty nanofluid shows better heat transfer performance compared with Al2O3-water dusty nanofluid.
- Increase in temperature does not shown significant difference in velocity profiles of CuO-water and Al2O3-water dusty nanofluids.
- A rise in the nano particle volume fraction improves the temperature profiles of the flow.
- Solutions exist only for certain range of stretching/shrinking parameter, for higher values of stretching/shrinking parameter velocity of the fluid is equal to free stream velocity.
- At lower temperatures Al2O3-water dusty nanofluid improves the velocity boundary layer thickness compared with CuO-water dusty nanofluid.
- Spherical shaped nano particles have better heat transfer performance compared with cylindrical shaped particles.
- A raise in the value of fluid particle interaction parameter enhances the heat transfer rate.
- Suction parameter have tendency to improve the heat transfer rate.

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### Table 2 Variation in $f'(0)$ and $-\theta'(0)$ for Al$_2$O$_3$-water dusty nanofluid at different values of $S, \phi, \phi_l, \delta, \alpha, \beta$ and $n$

<table>
<thead>
<tr>
<th>Fluid Temperature</th>
<th>$S$</th>
<th>$\phi$</th>
<th>$\phi_l$</th>
<th>$\delta$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$n$</th>
<th>$f'(0)$</th>
<th>$-\theta'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100°C</td>
<td>-0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>3</td>
<td>-0.490786</td>
<td>0.961668</td>
<td></td>
</tr>
<tr>
<td>100°C</td>
<td>0</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>3</td>
<td>-0.515944</td>
<td>1.109492</td>
<td></td>
</tr>
<tr>
<td>50°C</td>
<td>-0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>3</td>
<td>-0.491056</td>
<td>0.597824</td>
<td></td>
</tr>
<tr>
<td>50°C</td>
<td>0</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>3</td>
<td>-0.515551</td>
<td>0.625308</td>
<td></td>
</tr>
<tr>
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1121-1134.


### Table 3 Variation in $f'(0)$ and $-\theta'(0)$ for CuO-water dusty nanofluid at different values of $S, \phi, \phi_d, \delta, \alpha, \beta$ and $n$

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<th>$\phi_d$</th>
<th>$\delta$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$n$</th>
<th>$f'(0)$</th>
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