Entropy Generation Analysis for the Peristaltic Flow of Cu-water Nanofluid with Magnetic Field in a Lopsided Channel

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(Received January 7, 2015; accepted February 24, 2015)

ABSTRACT

This article is intended for investigating the entropy generation analysis for the peristaltic flow of Cu-water nanofluid with magnetic field in a lopsided channel. The mathematical formulation is presented. The resulting equations are solved exactly. The obtained expressions for pressure gradient, pressure rise, temperature and velocity phenomenon are described through graphs for various pertinent parameters. The streamlines are drawn for some physical quantities to discuss the trapping phenomenon.

Keywords: Magnetic field; Peristaltic flow; Cu-Water; Lopsided channel.

1. INTRODUCTION

In the antiquity of fluid subtleties, the extent of peristaltic transference have gained momentous desirability due to its substantial influence in the arenas of manufacturing and biomechanics as this procedure remnants energetic in numerous organic devices and biomedical engineering. Unambiguously, it is extremely functional in the design of swallowing food through the esophagus, chyme motion in the gastrointestinal tract, vasomotion of small blood vessels such as venules, capillaries and arterioles, urine transport flow from kidney to bladder, sanitary fluid transportation, transportation of corrosive fluids and a toxic liquid flow in the nuclear industry etc. In the opinion of such huge influence of peristaltic movements in engineering and biomedical many researchers have absorbed on the study of peristaltic mechanism. Logically, the behavior of regularly used liquids in such type of phenomenons is typically non-Newtonian to concentrated degree. Custody in attention the difficulty of non-Newtonian fluids, numerous of the investigators have functioned on the peristaltic flows of dissimilar non-Newtonian models in the intelligence of constitutive relations. In the studies\cite{1-6} , the scholars have gained the numerous results concerning peristaltic flows in different flow geometries.

In present-days nanofluid is a topic of innumerable courtesy among the researchers. Choi (1995) was the chief who have habituated the word "nanofluid" which elucidate a fluid rescheduling including ultrafine units with span less than 50 nm. The increase in thermal conductivity of nanofluids has been accredited to a volume of varied geniuses with Brownian motion, bunching of nanoparticles and fluid layering at the liquid/solid border. In peristaltic rhyme few studies have been completed for nano fluid. Akbar and Nadeem (2014) present endoscopic properties on the peristaltic flow of a nanofluid. In a further article Akbar \textit{et al.} (2012) considered the peristaltic flow of a nanofluid in a non-uniform tube. Very recently Hamad and Ferdows (2012) explore the similarity solution of boundary layer stagnation-point flow towards a heated porous and nonlinear stretching sheet saturated with nano fluid with Cu-water nano fluid with water as base fluid.

In thermodynamics, entropy is a measure of the number of specific ways in which a thermodynamic system may be arranged, often taken to be a measure of disorder, or a measure of progressing towards thermodynamic equilibrium. Bejan (1979) studied the entropy generation in fundamental convective heat transfer. Non-Newtonian fluid flow in a pipe system with entropy generation is considered by Pakdemirli and Yilbas (2006). According to them entropy number increases with increasing Brinkman number. Entropy generation due to heat and fluid flow in backward facing step flow with various expansion ratios is studied by
Entropy generation for peristaltic flow is not explored so far, to fill this gap we have investigated the entropy generation analysis for the peristaltic flow of Cu-water nanofluid with magnetic field in a lopsided channel. The coupled differential equations are simplified under long wave length and low Reynolds number assumptions. Exact solutions are obtained for reduced coupled differential equations. The entropy generation is computed by evaluation of thermal and fluid viscosities contribution. The physical features of pertinent parameters have been discussed by plotting the graphs of velocity, temperature, entropy number and stream functions.

2. MATHEMATICAL FORMULATION

Let us discussed an incompressible Cu-water nanofluid in an asymmetric channel of width \(d_1 + d_2\). The channel has a sinusoidal wave propagating with constant speed \(c\) on the channel walls induces the flow. The asymmetric of the channel is due to different amplitudes. Temperature \(T_0\), \(T_1\) and nanoparticle concentrations \(C_0\), \(C_1\) are given to the upper and lower wall of the channel. The wall surfaces are selected to satisfy the following expressions

\[
Y = H_1 = d_1 + a_1 \cos \left[ \frac{2\pi}{\lambda} (X - ct) \right],
\]

\[
Y = H_2 = -d_2 - b_1 \cos \left[ \frac{2\pi}{\lambda} (X - ct) + \phi \right].
\]

In the above equations \(a_1\) and \(b_1\) are the waves amplitudes, \(\lambda\) is the wave length, \(d_1 + d_2\) is the channel width, \(c_1\) is the wave speed, \(t\) is the time, \(X\) is the direction of wave propagation and \(Y\) is perpendicular to \(X\) . The phase difference \(\phi\) varies in the range \(0 \leq \phi \leq \pi\). When \(\phi = 0\) then symmetric channel with waves out of phase can be described and for \(\phi = \pi\), the waves are in phase. Moreover, \(a_1, b_1, d_1, d_2\) and \(\phi\) satisfies the following relation

\[
a_1^2 + b_1^2 + 2a_1b_1 \cos \phi \leq (d_1 + d_2)^2.
\]

The coordinates, velocity components and pressure between fixed and wave frames are related by the following transformations,

\[
\tilde{x} = X - ct, \quad \tilde{y} = Y, \quad \tilde{u} = U - c, \quad \tilde{v} = V, \quad \tilde{p}(y) = \tilde{P}(\tilde{x}, \tilde{t}),
\]

in which \((\tilde{x}, \tilde{y})\), \((\tilde{u}, \tilde{v})\) and \(\tilde{p}\) are the coordinates, velocity components and pressure in the wave frame.

With the transformation given Eq. (2) equations governing the flow and temperature in the presence of heat source or heat sink with viscous dissipation are

\[
\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0,
\]

\[
\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = \frac{1}{\rho_{nf}} \frac{\partial P}{\partial \tilde{x}} + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 \tilde{v}}{\partial \tilde{x}^2} - \frac{\sigma B^2}{\rho_{nf}} (\tilde{u} + c_1),
\]

\[
\frac{\partial T}{\partial \tilde{x}} + \frac{\partial \tilde{T}}{\partial \tilde{y}} = \frac{1}{\rho_{nf}} \frac{\partial P}{\partial \tilde{y}} + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2} + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2},
\]

\[
\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = \frac{\alpha_{nf}}{\sigma_{p, nf}} \left( \frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2} + \frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2} \right) + \frac{\mu_{nf}}{\sigma_{p, nf}} \left( \frac{\partial \tilde{u}}{\partial \tilde{y}} \frac{\partial \tilde{v}}{\partial \tilde{x}} \right)^2,
\]

where \(\tilde{x}\) and \(\tilde{y}\) are the coordinates along and perpendicular to the channel, \(\tilde{u}\) and \(\tilde{v}\) are the velocity components in the \(\tilde{x}\)– and \(\tilde{y}\)– directions, respectively, \(\tilde{T}\) is the local temperature of the fluid. Further, \(\rho_{nf}\) is the effective density, \(\mu_{nf}\) is the effective dynamic viscosity, \((\sigma_{p, nf})\) is the heat capacitance, \(\alpha_{nf}\) is the effective thermal diffusivity, and \(k_{nf}\) is the effective thermal conductivity of the nanofluid, which are defined as

\[
\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_f, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{\frac{5}{2}}},
\]

\[
(\sigma_{p, nf}) = (1 - \phi)(\sigma_{p, f}) + \phi(\sigma_{p, f}), \quad \alpha_{nf} = \frac{k_{nf}}{\sigma_{p, nf}},
\]

\[
k_{nf} = k_f \left[ \frac{k_f + 2k_f - 2\phi(k_f - k_x)}{k_x + 2k_f + 2\phi(k_f - k_x)} \right]
\]

where \(\phi\) is the solid volume fraction of the nanoparticles.

We introduce the following non-dimensional quantities

\[
x = \frac{2\tilde{x}}{\lambda}, \quad y = \frac{\tilde{y}}{d_1}, \quad u = \frac{\tilde{u}}{c}, \quad v = \frac{\tilde{v}}{c}, \quad t = \frac{2\tilde{t}}{\lambda}, \quad \delta = \frac{2n_1}{\lambda},
\]

\[
h_1 = \frac{h_1}{d_1}, \quad h_2 = \frac{h_2}{d_2}, \quad \text{Re} = \frac{\rho c d_1}{\mu_f}, \quad \text{Sc} = \frac{d_1}{d_1}, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \quad \epsilon = \frac{c^2}{(T_1 - T_0)(e_p)}, \quad \delta = \frac{d_2}{d_1}, \quad \epsilon = \frac{2n_1^2 P}{\mu_f c^2},
\]

\[
P_f = \frac{c_p \mu_f}{k_f}.
\]

in above equations \(P_f\) is the Prandtl number, \(M_f\) is Hartmann number and \(\epsilon_f\) is the Eckert number.

Stream function and velocity field are related by the expressions
In view of the Eqs. (7–9) under the long wavelength and low Reynolds number assumption we have the following equations

\[ \left( \frac{\mu_{nf}}{\mu_f} \right) \frac{\partial^4 \Psi}{\partial y^4} - M^2 \frac{\partial^3 \Psi}{\partial y^3} = 0, \tag{10} \]

\[ \frac{dP}{dx} = \frac{\partial}{\partial y} \left[ \left( \frac{\mu_{nf}}{\mu_f} \right) \frac{\partial^2 \Psi}{\partial y^2} - M^2 \left( \frac{\partial^2 \Psi}{\partial y^2} + 1 \right) \right]. \tag{11} \]

\[ \left( \frac{k_{nf}}{k_f} \right) \frac{\partial^2 \theta}{\partial y^2} + B_r \left( \frac{\mu_{nf}}{\mu_f} \right) \left( \frac{\partial^2 \Psi}{\partial y^2} \right)^2 = 0. \tag{12} \]

The non-dimensional boundary conditions are

\[ \Psi = \frac{F}{2} \frac{\partial \Psi}{\partial y} = -1, \theta = 0 \text{ at } y = b_1 = 1 + a \cos x, \]

\[ \Psi = -\frac{F}{2} \frac{\partial \Psi}{\partial y} = -1, \theta = 1 \text{ at } y = b_2 = -d - b \cos(x + \epsilon), \]

The flow rates in fixed and wave frame are related by

\[ Q = F + 1 + d. \tag{13} \]

### 3. VISCOUS DISSIPATION AND ENTROPY GENERATION

The dimensional viscous dissipation term \( \Phi \) can be obtained from equations of motion, i.e.,

\[ \Phi = \mu_{nf} \left\{ 2 \left( \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right)^{\frac{3}{2}} \left( \frac{\partial \theta}{\partial y} \right)^2 \right\}. \tag{14} \]

The dimensional volumetric entropy generation is defined as

\[ S''''_{gen} = \frac{k_{nf}}{\theta_0^2} \left( \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right) + \Phi. \tag{15} \]

Dimensionless form of the Entropy Generation in terms of stream function is given as:

\[ N_s = \frac{S''''_{gen}}{S''''_{G}} = \frac{k_{nf}}{k_f} \left( \frac{\partial \theta}{\partial y} \right)^2 + \Lambda B_r \left( \frac{\mu_{nf}}{\mu_f} \right) \left( \frac{\partial \Psi}{\partial y} \right)^2, \tag{16} \]

where

\[ S''''_{G} = \frac{k_f (T_1 - T_0)^2}{\overline{\theta}_0 \overline{a}_1^2}, \quad B_r = \frac{c^2 \mu_f}{k_f (T_1 - T_0)}, \quad \Lambda = \frac{\overline{\theta}_0}{T_1 - T_0}. \]

\( \overline{\theta}_0 \) the reference temperature.

Equation (16) consists of two parts. The first part is the entropy generation due to finite temperature difference (Nscond) and the second part is the entropy generation due to viscous effects (Nsvisc). The Bejan number is defined as

\[ B_r = \frac{N_s_{cond}}{N_s_{cond} + N_s_{visc}}. \tag{17} \]

### 4. SOLUTION PROFILES

Exact solutions for stream function, temperature profile using Mathematica 9 can be written as

\[ \Psi(x, y) = c_3 \sinh \left( \frac{My}{\sqrt{L}} \right) - c_4 \sinh \left( \frac{My}{\sqrt{L}} \right) \]

\[ + c_3 \cosh \left( \frac{My}{\sqrt{L}} \right) + c_4 \cosh \left( \frac{My}{\sqrt{L}} \right) \]

\[ = \frac{c_1 y}{M^2} \frac{c_2}{M^2} \]

\[ B_r \left( \frac{k_f}{k_{nf}} \right) M^2 \left( \frac{c_3^2 + c_4^2}{M^2} \right) \sinh \left( \frac{2M y}{\sqrt{L}} \right) \]

\[ + \left( \frac{c_3^2 + c_4^2}{M^2} \right) \cos \left( \frac{2M y}{\sqrt{L}} \right) \]

\[ \theta(x, y) = - \frac{2L}{c_6 y + c_5} \left( \frac{c_3^2 + c_4^2}{M^2} \right) \sinh \left( \frac{2M y}{\sqrt{L}} \right) \]

\[ + \frac{c_3 y c_4 M^2}{2L} \left( \frac{c_3^2 + c_4^2}{M^2} \right) \cos \left( \frac{2M y}{\sqrt{L}} \right) \tag{19} \]

where \( c_1 \cdots c_6 \) are constants evaluated using Mathematica 9.

The pressure rise \( \Delta p \) in non-dimensional form is defined as

\[ \Delta p = \int_0^1 \frac{dp}{dx} \, dx. \tag{20} \]

### 5. RESULTS AND DISCUSSION

In this section, we present a brief graphical analysis of the exact analytical solutions of the governing problem. Fig. 1(a) and 1(b) represent the magnitude of the horizontal velocity of the fluid inside the channel. We see that with the increase in the Hartmann number \( M \), i.e. ratio of electromagnetic force to the viscous force, the
Fig. 1: Velocity profile a) different M and b) different Q.

Fig. 2: Pressure rise $\Delta P$ for a) different M, b) different a and c) different b.

Fig 3: Pressure rise $\Delta P$ for a) different d and b) different $\varepsilon$

Fig. 4: Pressure gradient $\frac{dP}{dx}$ for a) different M and for different a.
velocity decreases in the center of the tube and increases near the walls of the tube, while as we increase the flow rate \( Q \), the magnitude of velocity takes a positive shift all around the tube. In both cases, it is observed that Cu-water has more variation as that of pure water. Also we note that the velocity attains its highest values in the center of the channel at \( y = 0 \), while it sufficiently decreases at the walls of the channel. Figs. 2(a) to 3(b) depict that with the addition of copper to the base fluid, the pressure rise gradually increases with the increase in Hartmann number \( M \) and amplitude \( \varepsilon \) while decreases with the increase in amplitudes ratio \( a, b \) and \( d \) in the peristaltic pumping region \( \Delta P > 0 \), while in the augmented pumping region \( \Delta P < 0 \) results are opposite pressure rise gradually decreases with the increase in Hartmann number \( M \) and amplitude \( \varepsilon \) while increases with the increase
in amplitudes ratio a and b. Free pumping hold for \( \Delta P = 0 \). It is observed that pressure rise for Cu-water has more variation as compare to pure water. We note that the pressure gradient certainly increases with an increase in the Hartmann number, amplitudes a and b but pressure gradient decreases with the rise in amplitude d for both pure and Cu-water see Figs.5 (a) to 6 (b).

Temperature of the fluid in the tube significantly increases with an increase in flow rate Q and Brinkman number \( B_r \), and a decrease in Hartmann number \( M \). However with less copper in the fluid, the temperature substantially decreases inside the tube. In comparison to the walls of the tube, higher temperature exists in the center - We note that with higher the values of the Brinkman number, i.e. the ratio of viscous heat generation to external heating, the lesser will be the conduction of heat produced by viscous dissipation and hence larger the temperature rise see Fig.7(a-c). Temperature for Cu-water is observed to be higher as compared to pure water.

Figs. 7(a)–8(c) are prepared to analyze the entropy generation with respect to change in different physical constraints involved. Figs. 8(a) to 8(c) depict that entropy generation is directly proportional to Hartmann number, flow rate Q and ratio of Brinkman number \( B_r \) with \( \Lambda \), and that entropy generation for pure water is higher than that of Cu-water. It has larger values near the walls of the channel as compared to the center of the channel. It is to be noticed that for significantly larger values of these two parameters, entropy generation can be larger in the center of the tube than to those generated at the walls.

Figs. 9(a)–9(c) are prepared to analyze the Bejan number with respect to change in different physical constraints involved. Fig. 9(a)–9(c) depict that with the increase in Hartmann number, and flow rate ratio heat transfer irreversibility is high as compare to the total irreversibility due to heat transfer, fluid friction and magnetic field while results are opposite for the case of ratio of Brinkman number \( B_r \) with \( \Lambda \).

Fig. 10. Shows streamlines for different values of Hartmann number \( M \) for Copper water. It is seen that the size and number of trapped bolus increases for increasing M in upper part of the channel, while size of bolus decreases but number of bolus.
increases with the increase in Hartmann number $M$ in lower part of the channel.

A line connecting points of equal temperature is called an isotherm. See Figs. 11, the small orange numbers are contour labels, which identify the value of an isotherm (75, 85 degrees Fahrenheit). So it analyzed that when we increases ratio of Brinkman number $Br$ with $\Lambda$ than temperature is 75, 85 degrees Fahrenheit.

**CONCLUSION**

This article is intended for investigating the entropy generation analysis for the peristaltic flow of Cu-water nanofluid with magnetic field. Key points of the present work are as follows.

1. We see that with the increase in the Hartmann number $M$, i.e. ratio of electromagnetic force to the viscous force, the velocity decreases in the center of the tube and increases near the walls of the tube, while as we increase the flow rate $Q$, the magnitude of velocity takes a positive shift all around the channel.

2. It is observed that Cu-water has more variation as that of pure water.

3. Temperature of the fluid in the channel significantly increases with an increase in flow rate $Q$ and Brinkman number $Br$, and a decrease in Hartmann number $M$.

4. Entropy generation is directly proportional to Hartmann number, flow rate $Q$ and ratio of Brinkman number $Br$ with $\Lambda$, and that entropy generation for pure water is higher than that of Cu-water.

5. It is seen that the size and number of trapped bolus increases for increasing $M$ in upper part of the channel, while size of bolus decreases but number of bolus increases with the increase in Hartmann number $M$ in lower part of the channel.

6. It analyzed that when we increases ratio of Brinkman number $Br$ with $\Lambda$ than temperature is 75, 85 degrees Fahrenheit.

**ACKNOWLEDGEMENTS**

The Author is thankful to the Higher Education Commission Pakistan for the financial support to complete this work.
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