MHD Mixed Convection Oscillatory Flow over a Vertical Surface in a Porous Medium with Chemical Reaction and Thermal Radiation

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ABSTRACT

The present paper concerns with the study of thermal radiation and magnetohydrodynamic effects on mixed convection flow of a viscous incompressible electrically-conducting fluid through a porous medium with variable permeability in the presence of oscillatory suction. The influence of a first-order homogeneous chemical reaction, heat source and Soret effects are analyzed. The resultant governing boundary layer equations are highly nonlinear and coupled form of partial differential equations which are solved analytically using two-term harmonic and non-harmonic functions. The effects of different physical parameters on the velocity, temperature and concentration fields are discussed in detail. The results are presented graphically and discussed qualitatively.

Keywords: Heat and mass transfer; MHD; Chemical reaction; Radiation; Mixed convection; Skin-friction.

NOMENCLATURE

- $b_0$: magnetic induction
- $C'$: species concentration in the boundary layer
- $C_p$: specific heat at constant pressure
- $D$: diffusion coefficient
- $g$: gravitational acceleration
- $G_c$: Solutal Grashof number
- $Gr$: Grashof number
- $K'$: Darcy permeability
- $K$: magnetic parameter
- $Pr$: Prandtl number
- $Q_i$: coefficient of proportionality of the radiation
- $Q_0$: heat absorption coefficient
- $q_r$: heat flux
- $R$: radiation parameter
- $Sc$: Schmidt number
- $So$: Soret number
- $T'$: thermal temperature in the boundary layer
- $u', v'$: velocity components in $x', y'$ directions
- $\delta$: chemical reaction parameter
- $\eta$: heat source parameter
- $\beta$: coefficient of thermal expansion
- $\beta^*$: coefficient of concentration expansion
- $\rho$: density of the fluid
- $\sigma$: electric conductivity

1. INTRODUCTION

In recent years, the problems of free convective and heat transfer flows through a porous medium under the influence of a magnetic field have been attracted the attention of a number of researchers because of their possible applications in many branches of science and technology, such as its applications in transportation cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers. On the other hand, flow through a porous medium have numerous engineering and geophysical applications, for example, in chemical engineering for filtration and purification process; in agriculture engineering to study the underground water resources; in
petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs. In view of these applications, many researchers have studied MHD free convective heat and mass transfer flow in a porous medium; some of them are Raptis and Kafousias (1982), Sattar (1993) and Kim (2004). Due to the importance of Soret (thermal-diffusion) and Dufour (diffusion-thermo) effects for the fluids with very light molecular weight as well as medium molecular weight many investigators have studied and reported results for these flows of whom the names are Eckert and Drake (1972), Dursunkaya and Worek (1992), Anghel et al. (2000), Postelnicu (2004) are worth mentioning. Alam and Rahman (2005) studied the Dufour and Soret effects on steady MHD free convective heat and mass transfer flow past a semi-infinite vertical porous plate embedded in a porous medium.

Many practical diffusive operations, the molecular diffusion of a species are involved in the presence of chemical reaction within or at the boundary. There are two types of reactions i.e. homogeneous and heterogeneous reactions. A homogeneous reaction occurs uniformly throughout a given phase. In such type of reaction the species generation is analogous to internal source of heat generation. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. It is so treated as a boundary condition similar to the constant heat flux condition in heat transfer. All industrial chemical processes are so designed that the cheaper raw materials can be transformed to high value products by chemical reaction. For a specific chemistry, the reactor performance is a complex function of the underlying transport processes. An analysis of the transport processes and their interaction with chemical reactions are quite difficult and is directly connected to the underlying fluid dynamics.

Moreover, the chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification have been presented by Kandasamy et al. (2005). Al-Odat and Al-Azab (2007) studied the influence of magnetic field on unsteady free convective heat and mass transfer flow along an impulsively started semi-infinite vertical plate taking into account a homogeneous chemical reaction of first order. The effect of radiation on the heat and fluid flow over an unsteady stretching surface has been analyzed by El-Aziz (2010). Ramana Reddy et al. (2009) has studied the mixed convective MHD flow and mass transfer past an accelerated infinite vertical porous plate. Singh et al. (2010) investigated MHD oblique stagnation point flow towards a stretching sheet with heat transfer for steady and unsteady cases. Elbashbeshy et al. (2009) investigated the effects of thermal radiation and magnetic field on unsteady boundary layer mixed convection flow and heat transfer problem from a vertical porous stretching surface. The opposing buoyancy effects on simultaneous heat and mass transfer by natural convection in a fluid saturated porous medium investigated by Angirasa et al. (1997). Ahmed (2007) investigates the effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate. Chamkha and Khaled (2000) considered hydromagnetic combined heat and mass transfer by natural convection from a permeable surface embedded in a fluid-saturated porous medium. Chamkha et al. (2010) reported similarity solution for unsteady heat and mass transfer from a stretching surface embedded in a porous medium with suction/injection and chemical reaction effects.

Recently, Ramana Reddy et al. (2011) have investigated unsteady MHD free convective mass transfer flow past an infinite vertical porous plate with variable suction and Soret effect using perturbation analysis. Thus the paper, we make an attempt to study an oscillatory hydromagnetic mixed convection flow through a porous medium with periodic temperature variation. It is assumed that the surface absorbs fluid with a constant velocity and free stream velocity of the fluid vibrates about a mean constant value. We have also considered the effects of first-order chemical reaction and thermal radiation on mixed convective flow and mass transfer of a viscous incompressible fluid past an infinite vertical plate subject to a time-dependent suction velocity in the presence of a uniform transverse magnetic field, internal heat generation/absorption and Soret effects.

2. MATHEMATICAL ANALYSIS

We consider an unsteady two-dimensional flow of an incompressible, viscous, electrically conducting and heat-absorbing fluid past a semi-infinite vertical plate is subject to slip boundary condition at the interface of porous and fluid layers. A uniform transverse magnetic field of magnitude $B_0$ is applied in the presence of thermal radiation, temperature and concentration buoyancy effects in the direction of $y$-axis. The transversely applied magnetic field and magnetic Reynolds number are assumed to be very small so that the induced magnetic field and the Hall Effect are negligible. It is assumed that there is no applied voltage which implies the absence of an electric field. Since the motion is two-dimensional and length of the plate is large enough so all the physical variables are independent of $x$. Oscillatory wall temperature and oscillatory wall concentration are also considered which are higher than the ambient temperature $T_{in}$ and concentration $C_{in}$, respectively. Also, it is assumed that there exists a homogeneous first-order chemical reaction with rate constant $R'$ between the diffusing species and the fluid. It is assumed that the porous medium is homogeneous and present everywhere in local thermodynamic equilibrium. Rest of properties of the fluid and the porous medium are assumed to be constant. The flow variable are functions of $y'$ and $t'$ only. The governing equations for this investigation are based on the balances of mass, linear momentum, energy
and concentration species. Taking into consideration of these assumptions, the equations that describe the physical situation can be written in Cartesian frame of references, as follows:

**Flow configuration and coordinate system**

**Continuity Equation:**
\[
\frac{\partial v}{\partial y} = 0
\]  
(1)

**Momentum Equation:**
\[
\frac{\partial u}{\partial y} + v \frac{\partial v}{\partial y} = \frac{1}{\rho_{\infty}} \left[ \frac{\partial^2 T}{\partial y^2} + Q_0(T^*-T_\infty) \right] - \frac{1}{\rho_{\infty}} \frac{\partial p}{\partial y},
\]  
(2)

**Energy Equation:**
\[
\frac{\partial T}{\partial y} + \frac{\partial v}{\partial y} \frac{\partial T}{\partial y} = \frac{1}{\rho_{\infty} C_p} \left[ \frac{\partial^2 T}{\partial y^2} + Q_0(T^*-T_\infty) \right] - \frac{1}{\rho_{\infty} C_p} \frac{\partial q_r}{\partial y},
\]  
(3)

**Mass diffusion Equation:**
\[
\frac{\partial C}{\partial y} + v \frac{\partial C}{\partial y} = \frac{D_C}{\gamma^2} \frac{\partial^2 C}{\partial y^2} + D_T \frac{\partial^2 T}{\partial y^2} - K_T \left( C'-C_\infty \right)
\]  
(4)

The boundary conditions for the velocity, temperature and concentration fields are

\[
u' = u' = t = 0, \quad T^* = T_\infty + c(\bar{T}_w - T_\infty) [1 + \alpha T'], \quad C' = C_\infty + c(C_w - C_\infty) [1 + \alpha T']
\]  
(5)

From Equation (1), it is clear that the suction velocity at the plate is either a constant and or a function of time. Hence the suction velocity normal to the plate is assumed in the form
\[
u' = -v_0(1 + \epsilon A e^{\alpha t'})
\]  
(6)

where \(A\) is a real positive constant, \(\epsilon\) and \(\epsilon A\) is small values less than unity, and \(v_0\) is scale of suction velocity which is non zero positive constant.

The negative sign indicates that the suction is towards the plate.

Outside the boundary layer, Equation (2) gives
\[
\frac{\partial^2 T}{\partial y^2} = \frac{1}{\rho_{\infty} K} \left[ \frac{\epsilon^2 T}{\gamma^2} + Q_0(T^*-T_\infty) \right] - \frac{1}{\rho_{\infty} K} \frac{\partial q_r}{\partial y}
\]  
(7)

By using the Rosseland diffusion approximation and following among other researchers, the radiative heat flux, \(q_r\), is given by
\[
q_r = \frac{4\sigma T^4}{3K_s} \frac{\partial T^4}{\partial y}
\]  
(8)

where \(\sigma\) and \(K_s\) are the Stefan-Boltzmann constant and the Roseland mean absorption coefficient, respectively. We assume that the temperature differences within the flow are sufficiently small such that \(T^4\) may be expressed as a linear function of temperature.

\[
T^4 \approx 4T_\infty^4T - 3T_\infty^4
\]  
(9)

Using (8) and (9) in the last term of Equation (3) we obtain
\[
\frac{\partial q_r}{\partial y} = \frac{16\sigma T_\infty^3}{3K_s} \frac{\partial^2 T}{\partial y^2}
\]  
(10)

In order to write the governing Equations and the boundary conditions in dimensional following non-dimensional quantities are introduced.

\[
y = \frac{y}{\gamma_0}, \quad u' = \frac{u'}{v_0}, \quad T = \frac{T'}{T_\infty}, \quad \theta = \frac{T' - T_\infty}{T_w - T_\infty}, \quad \eta = \frac{\nu \theta_0}{\nu_0}
\]

\[
U_p = \frac{u_p'}{U_0}, \quad \theta = \frac{T' - T_\infty}{T_w - T_\infty}, \quad \eta = \frac{\nu \theta_0}{\nu_0}
\]

\[
Gr = \frac{g\beta v(\bar{T}_w - T_\infty)}{v_0}, \quad C' = C - C_\infty, \quad \omega = \frac{\nu \theta_0}{\nu_0},
\]

\[
Gm = \frac{g\beta v(C_w - C_\infty)}{v_0}, \quad \text{Sc} = \frac{v}{D}, \quad \text{Pr} = \frac{v\nu_0}{\nu}
\]

\[
M = \frac{\sigma \theta_0^2}{\rho_{\infty}v_0^2}, \quad \text{K}' = K_{\infty}, \quad \text{K}' = K_{\infty}, \quad \delta = \frac{K_{\infty}v_0^2}{\nu_0}, \quad R = \frac{4\sigma T_\infty^3}{K_s}
\]  
(11)

In view of Equations (6)-(11), Equations (2)-(4) reduce to the following dimensional form.

\[
\frac{\partial u}{\partial t} - (1 + \epsilon A e^{\alpha t'}) \frac{\partial u}{\partial y} + Gr\theta
\]

\[+GmC + \frac{\partial \theta}{\partial y} + N(U_w - u)
\]  
(12)

\[
\frac{\partial \theta}{\partial t} - \frac{1 + \epsilon A e^{\alpha t'}}{Pr} \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \left( 1 + \frac{AR}{3} \right) \frac{\partial^2 \theta}{\partial y^2} + \eta \theta
\]  
(13)

\[
\frac{\partial C}{\partial t} - \frac{1 + \epsilon A e^{\alpha t'}}{Sc} \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + \frac{S_C}{Sc} \frac{\partial \theta}{\partial y} + \delta C
\]  
(14)

The corresponding boundary conditions in dimensionless form are

\[
u = u', \quad \theta = 1 + \epsilon e^{\alpha t'}, \quad C = 1 + \epsilon e^{\alpha t'} \quad \text{at} \quad y = 0
\]

\[
u = U_w = 1 + \epsilon e^{\alpha t'}, \quad \theta = 0, \quad C = 0 \quad \text{as} \quad y \rightarrow \infty
\]  
(15)
3. SOLUTION OF THE PROBLEM

Equations (12) – (14) are coupled, non-linear partial differential Equations and these cannot be solved in closed form. However, these Equations can be reduced to a set of ordinary differential Equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighborhood of the fluid in the neighborhood of the plate as

\[ u(y,t) = u_0(y) + \varepsilon \varphi + o(\varepsilon^2) + \ldots \]
\[ \theta(y,t) = \theta_0(y) + \varepsilon \varphi^2 + o(\varepsilon^2) + \ldots \]
\[ C(y,t) = C_0(y) + \varepsilon \varphi C + o(\varepsilon^2) + \ldots \]  \hspace{5pt} (16)

Substituting (16) in Equations (12) – (14) and neglecting the higher order terms of \( \varepsilon^2 \), we obtain

\[ u_0 + u_0' = \left[ M + \frac{1}{K} \right] u_0 \]
\[ \left[ M + \frac{1}{K} + i \omega \right] u_0' = -\left[ M + \frac{1}{K} + i \omega \right] \left[ \varphi \theta_0 - GmC_0 \right] - Au_0' \]
\[ \left[ 1 + \frac{4R}{3} \right] \theta_0' + Pr \theta_0' - Pr \eta \theta_0 = 0 \] \hspace{5pt} (17)
\[ \left[ 1 + \frac{4R}{3} \right] \theta_0' + Pr \theta_0' = -A Pr \theta_0' \] \hspace{5pt} (18)

where prime denotes ordinary differentiation with respect to \( y \).

The corresponding boundary conditions can be written as

\[ u_0 = u_p, u_1 = 0, \theta_0 = 0, \theta_1 = 1, \]
\[ C_0 = 1, C_1 = 1 \] at \( y = 0 \)
\[ u_0' = 0, u_1' = 0, \theta_0' = 0, C_0 = 0, C_1 = 0 \] as \( y \to \infty \) \hspace{5pt} (23)

Solving Equations (17) – (22) under the boundary condition (23) we obtain the velocity, temperature and concentration distribution in the boundary layer as

\[ u(y,t) = \sum_{m=0}^{\infty} A_m e^{(-\lambda_m y)} + e^{(x-wz)} \left[ e^{-z(\lambda_m y)} + \sum_{n=0}^{\infty} A_n e^{(-\lambda_n y)} \right] \]
\[ \theta(y,t) = e^{(-\lambda_m y)} + e^{(x-wz)} \left[ e^{-z(\lambda_m y)} \right] \]
\[ C(y,t) = e^{(-\lambda_m y)} + e^{(x-wz)} \left[ e^{-z(\lambda_m y)} \right] \]

The skin-friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow.

3.1 Skin Friction

Knowing the velocity field, the skin friction at the plate can be obtained, which in non-dimensional form is given by

\[ C_n = \left[ \frac{\partial u}{\partial y} \right]_{y=0} = \left[ \frac{e^{(x-wz)}}{e^{(x-wz)}} \left[ e^{-z(\lambda_m y)} \right] \right] \]

3.2 Nusselt Number

Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in non-dimensional form is given, in terms of the Nusselt number, is given by

\[ N_u = \left[ \frac{\partial \theta}{\partial y} \right]_{y=0} = \left[ \frac{e^{(x-wz)}}{e^{(x-wz)}} \left[ e^{-z(\lambda_m y)} \right] \right] \]

3.3 Sherwood Number

Knowing the concentration field, the rate of mass transfer coefficient can be obtained, which in non-dimensional form, in terms of the Sherwood number, is given by

\[ S_h = \left[ \frac{\partial C}{\partial y} \right]_{y=0} = \left[ \frac{e^{(x-wz)}}{e^{(x-wz)}} \left[ e^{-z(\lambda_m y)} \right] \right] \]

4. RESULTS AND DISCUSSION

The present paper deals with the problem of free convective unsteady flow with heat and mass transfer of a viscous incompressible electrically conducting fluid past a vertical plate under oscillation suction and Soret effects in the presence of a transverse magnetic field and internal heat source. The solutions for velocity field, temperature and concentration fields are obtained using perturbation technique. The effects of various physical parameters such as magnetic parameter \( M \), thermal radiation parameter \( R \), Soret number \( S_o \), chemical reaction \( \delta \), Schmidt number \( Sc \), heat source parameter \( \eta \), Prandtl number \( Pr \), porous...
parameter \( K \), Grashof number for heat and mass transfer \( Gr \) and \( Gc \) on the velocity, temperature and concentration fields have been studied analytically and computed results of the analytical solutions are presented in Fig. 1-15.

For different values of the magnetic field parameter \( M \), the velocity profiles are plotted in Figure. 1. It is obvious that the effect of increasing values of the magnetic field parameter results in a decreasing velocity distribution across the boundary layer. Figure 2 presents typical velocity profiles in the boundary layer for various values of the thermal Grashof number, while all other parameters are kept at some fixed values. The thermal Grashof number defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force.

![Fig. 1. Effects of magnetic parameter on velocity profiles.](image1)

![Fig. 2. Effects of Grashof number on velocity profiles.](image2)

The influence of the solutal Grashof number \( Gc \) on the velocity is presented in Figure 3. The Solutal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Here, the positive values of \( Gc \) correspond to cooling of the plate. Also, as \( Gc \) increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity.

![Fig. 3. Effects of modified Grashof number on velocity profiles.](image3)

![Fig. 4. Effects permeability parameter on velocity profiles.](image4)

Figure 4 illustrates the variation of velocity distribution across the boundary layer for various values of the permeability parameter \( K \). The velocity increases with an increase in permeability parameter \( K \). Figures 5 and 6 illustrate the velocity and temperature profiles for different values of the Prandtl number \( Pr \). The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity (Fig. 5). From Figure 6, it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of \( Pr \) are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly.
than for higher values of Pr. Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

Fig. 5. Effects of Prandtl number on velocity profiles.

Fig. 6. Effects of Prandtl number on temperature profiles.

For different values of the radiation parameter $R$ the velocity and temperature profiles are plotted in Figures 7 and 8. It is obvious that an increase in the radiation parameter $R$ results an increasing in the velocity and temperature profiles within the boundary layer, as well as an increasing in the momentum and thermal thickness. This is because the large $R$ values correspond to an increased dominance of conduction over radiation thereby increasing buoyancy force (thus, vertical velocity) and thickness of the thermal and momentum boundary layers.

Figure 9 and 10 display the effects of the Schmidt number ($Sc$) on the velocity and concentration profiles, respectively. As the Schmidt number increases, the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reduction in the velocity and concentration profiles are accompanied by simultaneous reductions in the momentum and concentration boundary layers thickness. These behaviors are clearly shown in Figures 9 and 10.

Fig. 7. Effects of radiation parameter on velocity profiles.

Fig. 8. Effects of radiation parameter on temperature profiles.

Fig. 9. Effects of Schmidt number on velocity profiles.

Figures 11 and 12 display the effects of the Soret number ($So$) on the velocity and concentration profiles respectively. As the Soret number increases, the velocity and concentration profiles both decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The influence of the heat source parameter on the temperature profiles is presented...
in Figure 13. It is observed that the temperature is decreases with an increasing heat source parameter. velocity distribution moves further away from the surface.

Figures 14 and 15, display the effects of the chemical reaction parameter (\(\delta\)) on the velocity and concentration distributions respectively. It is seen, that the velocity and concentration increases with decreasing the chemical reaction parameter. Also, we observe that the magnitude of the stream wise velocity decreases and the inflection point for the

5. CONCLUSIONS

The governing equations for unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat absorption was
formulated. The plate velocity was maintained at a constant value and the flow was subject to a transverse magnetic field. The computed values obtained from analytical solutions for the velocity, temperature, concentration fields as well as skin-friction coefficient, Nusselt number and the Sherwood number with their amplitude and phase are presented graphically and in tabular form. After a suitable transformation, the governing partial differential equations were transformed to ordinary differential ones. These equations were solved analytically by using two-term harmonic and non-harmonic functions.

We conclude the following after analyzing the graphs:

The velocity decreases with increasing the Prandtl number, and magnetic field parameter whereas reverse trend is seen with increasing the heat generation parameter, radiation parameter, porous parameter, Soret number, thermal and solutal Grashof numbers.

The temperature decreases as the values of Prandtl number increase and reverse trend is seen by increasing the values of the thermal radiation parameter, heat source parameter.

The concentration decreases as the values of the chemical reaction parameter and Schmidt number whereas concentration increases with increase the value of Soret number.

REFERENCES


Postelnicu, A. (2004). Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous


