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Effect of Hall Current on the Onset of MHD Convection in a Porous Medium Layer Saturated by a Nanofluid

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ABSTRACT

In this study, the effect of Hall current on the criterion for the onset of MHD convection in a porous medium layer saturated by a nanofluid is investigated. The model used for nanofluid combines the effect of Brownian motion and thermophoresis, while for a porous medium Brinkman model is used. A physically more realistic boundary condition than the previous ones on the nanoparticle volume fraction is considered i.e. the nanoparticle flux is assumed to be zero rather than prescribing the nanoparticle volume fraction on the boundaries. Using linear stability theory, the exact analytical expression for critical Rayleigh Darcy number is obtained in terms of various non-dimensional parameters. Results indicate that the magnetic field, Hall current, porous medium and nanoparticles significantly influence the stability characteristics of the system. The increase in the Hall current parameter, the Lewis number, the modified diffusivity ratio and the concentration Rayleigh Darcy number is to hasten the onset of convection while the magnetic Darcy number, the porosity parameter and the Darcy number has stabilized on the onset of convection.

Keywords: Hall current; Thermal instability; Porous medium; Nanofluid; Brownian motion; Thermophoresis.

1. INTRODUCTION

The study of the Magneto hydrodynamic (MHD) convection with or without Hall current effect has attracted considerable attention for the researchers due to its numerous applications in glass, crystal growing, MHD power generators, extrusion processes, cooling of nuclear reactors, as well as flow of laboratory plasma (Sutton and Sherman 1995). Hall currents is mainly effect where by a conductor carrying an electric current perpendicular to an applied magnetic field develops a voltage gradient which is transverse to the magnetic field. It was discovered by Hall in 1879 and known as Hall current effect. For strong magnetic field case, the effect of Hall current is important and the conventional MHD is not valid. The effect of Hall current on the thermal instability of a horizontal layer of conducting fluid was studied by Sherman (1966), Gupta (1967), Palese and Georgescu (2004), Rani and Tomar (2010). An extension to the porous medium case was made by Raptis and Ram (1984), Sharma and Gupta (1993), Sunil and Sigh (2000), Kumar and Mohan (2012) and Singh and Mehta (2013). They were found that the Hall current parameter has destabilizing effect on the system.

Heat transfer enhancement in MHD systems is an essential topic from an energy saving perspective. Convective heat transfer can be enhanced passively by enhancing thermal conductivity of the fluid. Various techniques have been proposed to enhance the heat transfer performance of fluids. Modern nanotechnology provides new opportunities to process and produce materials with average crystallite sizes (1-100) nm. Fluids with nanoparticles suspended in them are called nanofluids, a term proposed by Choi (1995). Nanofluids can be considered the next-generation heat transfer fluids as they offer exciting new possibilities to enhance heat transfer performance compared to pure fluids (Wang 2007). In order to get improved heat transfer performance of MHD devices, the use of nanofluid with higher thermal conductivity can be considered as a working medium (Sheikholeslami *et al*. 2013; Loganthan and Vimala 2015).

Buongiorno (2006) was the pioneer researcher who gave a comparatively satisfactory model including the effects of Brownian motion and thermophoresis of the nanoparticles suspended. He noted that the nanoparticles absolute velocity can be viewed as the sum of the base fluid velocity and a slip velocity.

With the help of these model, convective instability in nanofluid was conducted by many researchers including Tzou (2008a,b), Dhananjay (2011), Nield and Kuznetsov (2009, 2010, 2011), Kuznetsov and Nield (2010a,b), Yadav *et al*. (2011, 2012a,b, 2013a,b, 2014a,b,c, 2015a,b,c, 2016a,b,c,d,e), Umavathi *et al*. (2015) and Chand and Rana (2012, 2015 a,b). The effect of magnetic field effect on the thermal instability in nanofluids was studied by Yadav *et al*. (2013c, 2014d, 2015d,e,f, 2016f,g), Gupta *et al*. (2013) and Chand *et al*. (2014a,b). They found that the fluid under magnetic effects experiences a Lorentz force. This force, in turn, affects the buoyant flow field. Very recently, Sheikholeslami *et al*. (2015a,b) studied the two-phase simulation of nanofluid flow and heat transfer with the effect of magnetic field. They obtained that the temperature boundary layer thicknesses decreased with increasing aspect ratio and Hartmann number but increased with increasing Reynolds number, Schmidt number, Brownian parameter, thermophoresis parameter and Eckert number.

In all the investigations available in the literature, the thermal convection in a porous medium layer saturated by a nanofluid under the Hall current effect with zero flux boundaries for nanoparticles was not studied. Therefore, it would be of importance here to examine the effect of Hall current on the onset of MHD convection in a porous medium layer saturated by a nanofluid based on a new boundary condition for the nanoparticle fraction (zero flux boundaries with the combination of Brownian motion and thermophoresis for nanoparticles), which is physically more realistic than the other ones (Nield and Kuznetsov 2014). The model used for nanofluid combines the effects of Brownian motion and thermophoresis, while for porous medium Brinkman model with corrections to include effects due to the magnetic field, the Hall current and nanoparticles is considered.

2. Mathematical Formulation

Assuming that the nanoparticles being suspended in the base fluid, using either surfactant or surface charge technology, prevent the agglomeration and deposition of these on the porous matrix (Nield and Kuznetsov 2009), we consider an infinite horizontal layer of incompressible electrically conducting nanofluid heated from below (as shown in Fig. 1). A Cartesian co-ordinate system (x, y, z) is chosen in which z axis is taken at right angle to the boundaries. The nanofluid is confined between two parallel plates $z^* = 0$ and $z^* = L$, where the temperatures at the lower and upper boundaries are taken as T_0^* and T_1^*, respectively, T_0^* being greater than T_1^*. A uniform strong vertical magnetic field $\mathbf{H}^* = (0, 0, H_0^*)$ acts on the system. Asterisks are used to distinguish the dimensional variables from the non-dimensional variables (without asterisks).

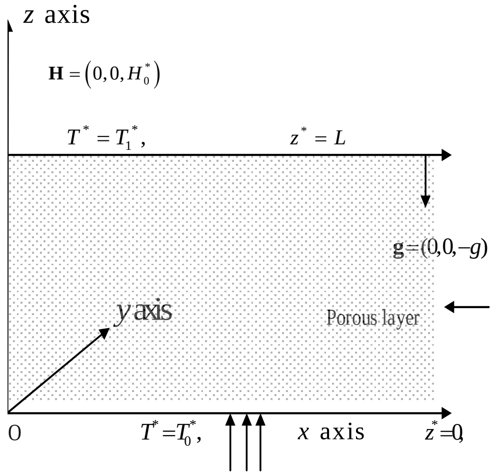

Fig. 1. Physical model and co-ordinate system.

2.1. Assumptions

The mathematical equations describing the physical model are based upon the following assumptions:

(i) The thermophysical properties except for density in the buoyancy force (Boussinesq Hypothesis) are constant;

(ii)The fluid phase and nanoparticles are in thermal equilibrium state and thus, the heat flow has been described using one equation model.

(iii) Nanofluid is incompressible, electrically conducting, Newtonian and laminar.

(iv) Nanoparticles are spherical and non-magnetic.

(v) Each boundary wall is assumed to be impermeable and perfectly thermal conducting.

(vi) Radiative heat transfer between the sides of wall is negligible when compared with other modes of the heat transfer.

2.2. Governing Equations

The continuity equation for the nanofluid is

$$\nabla^* \cdot \mathbf{v}_D^* = 0, \tag{1}$$

where $\mathbf{v}_D^*$ is the nanofluid Darcy velocity. If one introduces a buoyancy force, Lorentz force and adopts the Boussinesq approximation, then the momentum equation can be written as

$$\begin{aligned}\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{v}_\mathbf{D}^*}{\partial t^*} &= -\nabla^* p^* + \bar{\mu}\,\nabla^{*2}\mathbf{v}_\mathbf{D}^* - \frac{\mu}{K}\mathbf{v}_\mathbf{D}^* \\ &+ \left[\phi^*\rho_p + \left(1-\phi^*\right)\rho_0\left\{1-\beta\left(T^* - T_1^*\right)\right\}\right]\mathbf{g} \\ &+ \frac{\mu_e}{4\pi}\left(\nabla^* \times \mathbf{H}^*\right)\times \mathbf{H}^*,\end{aligned} \tag{2}$$

where ρ_0 is the density of the nanofluid at the reference temperature T_1^*, ε is the porosity of the porous medium, K is the permeability of the porous medium, β is the thermal expansion coefficient, t^* is the time, p^* is the pressure, ϕ^* is volumetric fraction of nanoparticle, ρ_p is the density of nanoparticle, $\mathbf{H}^*$ is the magnetic field, $\bar{\mu}$ is the effective viscosity, μ, ρ and μ_e are the viscosity, density and magnetic permeability of nanofluid, respectively.

The energy equation is:

$$(\rho c)_m \frac{\partial T^*}{\partial t^*} + (\rho c)_f \, \mathbf{v}_\mathbf{D}^* \cdot \nabla^* T^* = k_m \nabla^{*2} T^* + \varepsilon (\rho c)_p \left[D_B \nabla^* \phi^* \cdot \nabla^* T^* + \left(\frac{D_T}{T_1^*} \right) \nabla^* T^* \cdot \nabla^* T^* \right]. \tag{3}$$

where $(\rho c)_m$ is the effective heat capacity, k_m is the effective thermal conductivity, D_B is the Brownian diffusion coefficient and D_T is the thermophoretic diffusion coefficient of the nanoparticles, $(\rho c)_f$ and $(\rho c)_p$ are the heat capacity of the nanofluid and nanoparticles, respectively. The second term of the right hand side is the additional flow work due to Brownian motion and thermophoresis of nanoparticles relative to the flow velocities.

The conservation equation for the nanoparticles is

$$\left[\frac{\partial}{\partial t^*} + \frac{1}{\varepsilon} \left(\mathbf{v}_\mathbf{D}^* . \nabla^* \right) \right] \phi^* = D_B \nabla^{*2} \phi^* + \frac{D_T}{T_1^*} \nabla^{*2} T^* \tag{4}$$

When the strength of the magnetic field is strong, one cannot neglect the effect of Hall current. Therefore, the modified Maxwell's equations relevant to the problem are:

$$\varepsilon \left[\frac{\partial}{\partial t^*} + \frac{1}{\varepsilon} \left(\mathbf{v}_\mathbf{D}^* \cdot \nabla^* \right) \right] \mathbf{H}^* = \left(\mathbf{H}^* \cdot \nabla^* \right) \mathbf{v}_\mathbf{D}^* + \varepsilon \eta \nabla^{*2} \mathbf{H}^* - \frac{\varepsilon \vartheta}{4 \pi e N} \nabla^* \times \left[\left(\nabla^* \times \mathbf{H}^* \right) \times \mathbf{H}^* \right], \tag{5}$$

$$\nabla^* \cdot \mathbf{H}^* = 0, \tag{6}$$

where η, σ, ϑ, e, and N are the electrical resistivity, the electrical conductivity, the velocity of light, the charge of electron and the electron number density of the nanofluid, respectively.

In this paper, a physically more realistic boundary condition on the nanoparticle volume fraction is considered i.e. the nanoparticle flux is assumed to be zero rather than prescribing the nanoparticle volume fraction on the rigid impermeable boundaries. Thus the boundary conditions are:

$$w^* = 0, \; T^* = T_0^*, \quad D_B \frac{d\phi^*}{dz^*} + \frac{D_T}{T_1^*} \frac{dT^*}{dz^*} = 0 \quad \text{at} \quad z^* = 0, \tag{7a}$$

$$w^* = 0, \; T^* = T_1^*, \quad D_B \frac{d\phi^*}{dz^*} + \frac{D_T}{T_1^*} \frac{dT^*}{dz^*} = 0 \quad \text{at} \quad z^* = L. \tag{7b}$$

Introducing the following non dimensional parameters:

$$(x, y, z) = \left(x^*, y^*, z^* \right) / L, \; t = t^* \alpha_m / \sigma L^2,$$

$$(u, v, w) = \left(u^*, v^*, w^* \right) L / \alpha_m, \; p = p^* K / (\mu \alpha_m),$$

$$\left(H_x, H_y, H_z \right) = \left(H_x^*, H_y^*, H_z^* \right) / H_0^*,$$

$$\phi = \left(\phi^* - \phi_0^* \right) / \phi_0^*, \; T = \left(T^* - T_1^* \right) / \left(T_0^* - T_1^* \right), \tag{8}$$

where $\alpha_m = \dfrac{k_m}{(\rho c)_f}$, $\sigma = \dfrac{(\rho c)_m}{(\rho c)_f}$ and ϕ_0^* is a reference scale for the volumetric fraction of nanoparticle .

Then, the non-dimensional form of Eqs. (1)-(7) are:

$$\nabla \cdot \mathbf{v} = 0, \tag{9}$$

$$\frac{D_a}{P_r} \frac{\partial \mathbf{v}}{\partial t} = -\nabla p + D_a \nabla^2 \mathbf{v} - \mathbf{v} - R_m \hat{\mathbf{e}}_Z + R_D T \hat{\mathbf{e}}_Z - R_n \phi \hat{\mathbf{e}}_Z + Q \frac{P_r}{P_{rM}} \left[(\nabla \times \mathbf{H}) \times \mathbf{H} \right], \tag{10}$$

$$\left[\frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) \right] T = \nabla^2 T + \frac{N_B}{L_e} (\nabla T \cdot \nabla \phi) + \frac{N_A N_B}{L_e} (\nabla T \cdot \nabla T), \tag{11}$$

$$\left[\frac{1}{\sigma} \frac{\partial}{\partial t} + \frac{1}{\varepsilon} (\mathbf{v} \cdot \nabla) \right] \phi = \frac{N_A}{L_e} \nabla^2 T + \frac{1}{L_e} \nabla^2 \phi, \tag{12}$$

$$\varepsilon \left[\frac{1}{\sigma} \frac{\partial}{\partial t} + \frac{1}{\varepsilon} (\mathbf{v} \cdot \nabla) \right] \mathbf{H} = (\mathbf{H} \cdot \nabla) \mathbf{v} + \varepsilon \frac{P_r}{P_{rM}} \nabla^2 \mathbf{H} - M \frac{P_r}{P_{rM}} \nabla \times \left[(\nabla \times \mathbf{H}) \times \mathbf{H} \right], \tag{13}$$

$$\nabla \cdot \mathbf{H} = 0, \tag{14}$$

$$w = 0, \; T = 1, \; \frac{d\phi}{dz} + N_A \frac{dT}{dz} = 0 \quad \text{at} \quad z = 0 \tag{15a}$$

$$w = 0, \; T = 0, \; \frac{d\phi}{dz} + N_A \frac{dT}{dz} = 0 \quad \text{at} \quad z = 1. \tag{15b}$$

Here $R_D = \rho_0 g \beta L K \left(T_0^* - T_1^* \right) / (\mu \alpha_m)$ is the Rayleigh Darcy number, $D_a = \bar{\mu} K / \left(\mu L^2 \right)$ is the Darcy number, $L_e = \alpha_m / D_B$ is the Lewis number, $P_r = \mu / (\rho_0 \alpha_m)$ is the Prantdl number,

$P_{rM} = \mu/(\rho_0\eta)$ is the magnetic Prantdl number, $N_B = (\rho c)_p \phi_0^* / (\rho c)$ is the modified particle density increment, $N_A = D_T \left(T_0^* - T_1^*\right) / \left(D_B T_1^* \phi_0^*\right)$ is the modified diffusivity ratio, $R_m = \left[\rho_p\phi_0^* + \rho_{fo}\left(1-\phi_0^*\right)\right] gLK / (\mu\alpha_m)$ is the basic density Rayleigh Darcy number, $R_n = \left(\rho_p - \rho_{fo}\right)\phi_0^* gLK / (\mu\alpha_m)$ is the concentration Rayleigh Darcy number, $Q = \mu_e H_0^{*2} K / (4\pi\rho_0 \nu\eta)$ is the magnetic Darcy number and $M = \varepsilon\vartheta H_0^* / (4\pi e N \eta)$ is the Hall current parameter.

2.3. Basic Flow

The basic state of the nanofluid is assumed to be time independent and is described by

$$\mathbf{v} = 0,\ T = T_b(z),\ p = p_b(z),\ \phi = \phi_b(z),$$
$$\mathbf{H} = \hat{\mathbf{e}}_z.$$

Here subscripts b represents basic state i.e. T_b is the basic temperature, p_b is the basic pressure, ϕ_b is the basic volumetric fraction of nanoparticle and $\hat{\mathbf{e}}_\mathbf{z}$ is the unit vector in z- direction.

In basic state, Eqs. (11) and (12) can be written as:

$$\frac{d^2T_b}{dz^2} + \frac{N_B}{L_e}\frac{d\phi_b}{dz}\frac{dT_b}{dz} + \frac{N_A N_B}{L_e}\left(\frac{dT_b}{dz}\right)^2 = 0, \quad (16)$$

$$\frac{d^2\phi_b}{dz^2} + N_A \frac{d^2T_b}{dz^2} = 0, \quad (17)$$

under the following boundary conditions:

$$T_b = 1,\ \frac{d\phi_b}{dz} + N_A\frac{dT_b}{dz} = 0 \text{ at } z = 0, \quad (18a)$$

$$T_b = 0,\ \frac{d\phi_b}{dz} + N_A\frac{dT_b}{dz} = 0 \text{ at } z = 1. \quad (18b)$$

On solving Eqs. (16) and (17) subject to the boundary conditions (18), we found that

$$\frac{dT_b}{dz} = -1 \text{ and } \frac{d\phi_b}{dz} = N_A, \quad (19a,b)$$

The pressure and magnetic field are of no consequence here as it will be eliminated subsequently.

2.4. Perturbation Equations

For small disturbances onto the primary flow, we assume that:

$$p = p_b(z) + p',\ \mathbf{H} = \hat{\mathbf{e}}_z + \mathbf{H}',$$
$$\rho = \rho_b(z) + \rho',\ \mathbf{v} = \mathbf{v}',\ T = T_b(z) + T',$$
$$\phi = \phi_b(z) + \phi', \quad (20)$$

where prime indicates perturbation quantities over their equilibrium counterparts and assumed to be small. On substituting Eq. (20) into Eqs. (9)-(15) and neglecting the product of prime quantities, we have:

$$\nabla\cdot\mathbf{v}' = 0, \quad (21)$$

$$\frac{D_a}{P_r}\frac{\partial\mathbf{v}'}{\partial t} = -\nabla p' + \frac{D_a}{P_r}\nabla^2\mathbf{v}' - \mathbf{v}' + R_D T'\hat{\mathbf{e}}_Z - R_n\phi'\hat{\mathbf{e}}_Z + Q\frac{P_r}{P_{rM}}\left[\left(\nabla\times\mathbf{H}'\right)\times\hat{\mathbf{e}}_Z\right], \quad (22)$$

$$\frac{\partial T'}{\partial t} - w' = \nabla^2 T' - \frac{N_B}{L_e}\left(N_A\frac{\partial T'}{\partial z} + \frac{\partial\phi'}{\partial z}\right), \quad (23)$$

$$\frac{1}{\sigma}\frac{\partial\phi'}{\partial t} + \frac{1}{\varepsilon}N_A w' = \frac{1}{L_e}\nabla^2\phi' + \frac{N_A}{L_e}\nabla^2 T', \quad (24)$$

$$\frac{\varepsilon}{\sigma}\frac{\partial\mathbf{H}'}{\partial t} = \frac{\partial w'}{\partial z}\hat{e}_z + \varepsilon\frac{P_r}{P_{rM}}\nabla^2\mathbf{H}' - M\frac{P_r}{P_{rM}}\nabla\times\left[\left(\nabla\times\mathbf{H}'\right)\times\hat{e}_z\right], \quad (25)$$

$$\nabla\cdot\mathbf{H}' = 0, \quad (26)$$

Operating on Eq. (22) with $\hat{\mathbf{e}}_\mathbf{z}\cdot\nabla\times\nabla\times$ and using the identity $\nabla\times\nabla\times = \nabla\ \nabla\cdot - \nabla^2$ together with Eqs. (21) and (26), we obtain z - component of the momentum equation as:

$$\left[D_a\nabla^4 - \frac{D_a}{\mathrm{P_r}}\frac{\partial}{\partial t}\nabla^2 - \nabla^2\right]w' + R_D\nabla_P^2 T' - R_n\nabla_P^2\phi' + Q\frac{P_r}{P_{rM}}\nabla^2\left(\frac{\partial H_z'}{\partial z}\right) = 0, \quad (27)$$

where ∇_P^2 is the Laplacian operator in the horizontal plane.

Equation (25) can be written in three directions:

$$\frac{\varepsilon}{\sigma}\frac{\partial H_x'}{\partial t} = \varepsilon\frac{P_r}{P_{rM}}\nabla^2 H_x' - M\frac{P_r}{P_{rM}}\frac{\partial\zeta_x'}{\partial z}, \quad (28)$$

$$\frac{\varepsilon}{\sigma}\frac{\partial H_y'}{\partial t} = \varepsilon\frac{P_r}{P_{rM}}\nabla^2 H_y' - M\frac{P_r}{P_{rM}}\frac{\partial\zeta_y'}{\partial z}, \quad (29)$$

$$\frac{\varepsilon}{\sigma}\frac{\partial H_z'}{\partial t} = \frac{\partial w'}{\partial z} + \varepsilon\frac{P_r}{P_{rM}}\nabla^2 H_z' - M\frac{P_r}{P_{rM}}\frac{\partial\zeta_z'}{\partial z}, \quad (30)$$

By differentiate Eq. (28) with respect to y and differentiate Eq. (29) with respect to x, and then subtract first one by second, we have

$$\left(\varepsilon\frac{P_r}{P_{rM}}\nabla^2 - \frac{\varepsilon}{\sigma}\frac{\partial}{\partial t}\right)\zeta_z' + M\frac{P_r}{P_{rM}}\nabla^2\frac{\partial H_z'}{\partial z} = 0. \quad (31)$$

On eliminating H_z' from Eq. (27) with the help of Eqs. (30) and (31), we have

$$\left[D_a\nabla^4-\frac{D_a}{\mathrm{P_r}}\frac{\partial}{\partial t}\nabla^2-\nabla^2\right]\Gamma_2 w'+R_D\nabla_p{}^2\Gamma_2 T'$$
$$-R_n\nabla_P{}^2\Gamma_2\phi'-Q\,\frac{P_r}{P_{rM}}\nabla^2\Gamma_1\frac{\partial^2 w'}{\partial z^2}=0, \tag{32}$$

where $\Gamma_1\equiv\left(\varepsilon\frac{P_r}{P_{rM}}\nabla^2-\frac{\varepsilon}{\sigma}\frac{\partial}{\partial t}\right)$ and

$$\Gamma_2\equiv\left[(\Gamma_1)^2+\left(M\,\frac{P_r}{P_{rM}}\right)^2\nabla^2\frac{\partial^2}{\partial z^2}\right].$$

Assuming the perturbation quantities are of the form as:

$$(w',\ T',\ \phi')=\left[W\ (z\ ),\ \Theta(z\ ),\ \Phi(z\ )\right]$$
$$\times\exp\left[ik_x x+ik_y y+\omega t\right], \tag{33}$$

where k_x, k_y are the wave number along the x and y directions respectively and $a=\sqrt{k_x{}^2+k_y{}^2}$ is the resultant wave number.

On substituting Eq. (33) into the differential Eqs. (23), (24) and (33), the linearized equations in dimensionless form are as follows:

$$W+\left(D^2-\frac{N_A N_B}{L_e}D-a^2-\omega\right)\Theta-\frac{N_B}{L_e}D\Phi=0, \tag{34}$$

$$\left[\frac{1}{L_e}\left(D^2-a^2\right)-\frac{\omega}{\sigma}\right]\Phi+\frac{N_A}{L_e}\left(D^2-a^2\right)\Theta-\frac{N_A W}{\varepsilon}=0, \tag{35}$$

$$\left(D^2-a^2\right)\left[D_a\left(D^2-a^2\right)-\left(1+\frac{\omega D_a}{\mathrm{P_r}}\right)\right]\Gamma_4 W$$
$$-R_D a^2\Gamma_4\Theta+R_n a^2\Gamma_4\Phi-Q\frac{P_r}{P_{rM}}\left(D^2-a^2\right)\Gamma_3 D^2 W=0, \tag{36}$$

where $\Gamma_3\equiv\left[\varepsilon\frac{P_r}{P_{rM}}\left(D^2-a^2\right)-\frac{\varepsilon}{\sigma}\omega\right]$,

$\Gamma_4\equiv\left[(\Gamma_3)^2+\left(M\,\frac{P_r}{P_{rM}}\right)^2\left(D^2-a^2\right)D^2\right]$ and

$D\equiv\dfrac{d}{dZ}$.

And, the boundary conditions become:

$$W=\Theta=0,\ \mathrm{D}\Phi+N_A D\Theta=0\ \text{ at }\ z=0\ \text{and}\ 1. \tag{37}$$

Here the growth rate ω is in general a complex quantity such that $\omega=\omega_r+i\,\omega_i$, the system with $\omega_r<0$ is always stable, while for $\omega_r>0$, it will become unstable. For neutral stability, the real part of ω is zero. Hence, we now write $\omega=i\,\omega_i$, where ω_i is real and is a dimensionless frequency.

Equations (34)–(36) together with the boundary conditions (37) constitute a linear eigenvalue problem of the system. The resulting eigenvalue problem is solved analytically using the Galerkin weighted residuals method. In this method, the test (weighted) functions are the same as the base (trial) functions. Accordingly W, Θ and Φ are taken in the following way:

$$W=\sum_{s=1}^{N}A_s W_s,\ \Theta=\sum_{s=1}^{N}B_s\Theta_s,\ \Phi=\sum_{s=1}^{N}C_s\Phi_s, \tag{38}$$

where A_s, B_s and C_s are constants. The base functions W_s, Θ_s and Φ_s represented by power series as trivial functions satisfying the respective boundary conditions and are assumed in the following form:

$$W_s=\Theta_s=\sin s\pi z,\ \Phi_s=-N_A\sin s\pi z,\ \ s=1,2,3,... \tag{39}$$

Using Eq. (38) into Eqs. (34)-(36) and multiplying Eq. (34) by Θ_s, Eq. (35) by Φ_s and Eq. (36) by W_s; performing the integration by parts with respect to z between $z=0$ and 1, we obtain a system of $3s$ linear algebraic equations in the $3s$ unknowns A_s, B_s and C_s. For the existence of non trivial solution, the determinant of the coefficient matrix must vanish, which gives the characteristic equation for the system, with Rayleigh Darcy number R_D as the eigenvalue of the characteristic equation. For a first approximation, we take $s=1$, and this gives the following characteristic equation for the system

$$\begin{aligned}&\frac{N_A{}^2}{8\,\mathrm{L_e P_{rM}}{}^2\sigma^2}\left[\frac{-a^2}{\varepsilon}\left\{J\left(\varepsilon+\mathrm{L_e}\right)+i\mathrm{L_e}\omega_i\right\}N_A R_n\right.\\&\times\left\{JM^2\pi^2\mathrm{P_r}{}^2\sigma^2-\varepsilon^2\left(\omega_i\mathrm{P_{rM}}-iJ\mathrm{P_r}\sigma\right)^2\right\}\\&+\frac{1}{\sigma}\left(-i\mathrm{L_e}\omega_i-J\sigma\right)\left[\frac{1}{\mathrm{P_r}}J\left(-J-i\,\omega_i\right)\right.\\&\times\left\{\varepsilon^2\omega_i{}^2\left(-iD_a\omega_i-\left(1+D_aJ\right)\mathrm{P_r}\right)\mathrm{P_{rM}}{}^2\right.\\&+i\,\varepsilon\omega_i\mathrm{P_r P_{rM}}\left(2\varepsilon J\left(iD_a\omega_i+\mathrm{P_r}+D_aJ\,\mathrm{P_r}\right)+\pi^2\mathrm{P_r}Q\right)\sigma\\&+J\mathrm{P_r}{}^2\left\{\left(a^2\varepsilon^2+\left(\varepsilon^2+M^2\right)\pi^2\right)\left(iD_a\omega_i+\mathrm{P_r}+D_aJ\,\mathrm{P_r}\right)\right.\\&\left.\left.+\varepsilon\pi^2\mathrm{P_r}Q\right\}\sigma^2\right\}+a^2R_a\left(JM^2\pi^2\mathrm{P_r}{}^2\sigma^2\right.\\&\left.\left.\left.-\left(\varepsilon\omega_i\mathrm{P_{rM}}-i\,\varepsilon J\mathrm{P_r}\sigma\right)^2\right)\right]\right],\end{aligned} \tag{40}$$

where $J=a^2+\pi^2$.

3. Results And Discussion

3.1 Stationary Convection

First, consider the case of stationary convection i.e. $\omega_i=0$. Then, Eq. (40) gives the following expression for the Rayleigh Darcy number

$$R_D = \frac{\left(a^2+\pi^2\right)^2}{a^4\varepsilon^2 + a^2\left(\varepsilon^2+M^2\right)\pi^2}\left[\left\{a^2\varepsilon^2+\left(\varepsilon^2+M^2\right)\pi^2\right\} \times\left\{1+D_a\left(a^2+\pi^2\right)\right\}+\varepsilon\pi^2 Q\right]-\left(\frac{1}{\varepsilon}+\frac{1}{L_e}\right)R_n N_A L_e \quad (41)$$

It is clear from Eq. (41) that the Rayleigh Darcy number R_D is a function of the wave number a, porosity parameter ε, Darcy number D_a, Lewis number L_e, modified diffusivity ratio N_A, nanoparticle concentration Rayleigh number R_n, magnetic Darcy number Q and Hall current parameter M.

The critical wave number is at the onset of instability, which is obtained from the condition $\left(\frac{dR_D}{d\tilde{x}}\right)_{\tilde{x}=\tilde{x}_c} = 0$, where $\tilde{x} = a^2/\pi^2$ and satisfies the following relation

$$\begin{aligned}&2D_a\varepsilon^4\pi^2\tilde{x}^4+\left\{\varepsilon^4+D_a\varepsilon^2\pi^2\left(5\varepsilon^2+4M^2\right)\right\}\tilde{x}^3\\&+\left\{\varepsilon^4+2M^2\varepsilon^2+D_a\pi^2\left(3\varepsilon^4+6\varepsilon^2M^2+2M^4\right)\right\}\tilde{x}^2\\&+\left\{\left(M^4-\varepsilon^4\right)\left(1+D_a\pi^2\right)+\varepsilon\left(M^2-\varepsilon^2\right)Q\right\}\tilde{x}\\&-\left(\varepsilon^2+M^2\right)\left\{\left(\varepsilon^2+M^2\right)\left(1+D_a\pi^2\right)+\varepsilon Q\right\}=0.\end{aligned} \quad (42)$$

It is clear from Eq. (42) that the critical wave number does not depend on nanofluids parameters. It depends on magnetic Darcy number Q, Hall current parameter M, porosity parameter ε and Darcy number D_a.

In the absence of Hall current, magnetic field and nanoparticles i.e. M, Q and R_n are all equal to zero, then Eqs. (41) and (42) give, respectively:

$$R_D = \frac{\left(a^2+\pi^2\right)^2\left[1+D_a\left(a^2+\pi^2\right)\right]}{a^2}, \quad (43)$$

$$\tilde{x}_c = \frac{-1-D_a\pi^2+\sqrt{1+10D_a\pi^2+9{D_a}^2\pi^4}}{4D_a}. \quad (44)$$

These are the same results as given in (Nield and Bejan 2006).

To study the effect of M, Q, D_a, ε, L_e, N_A and R_n, we examine the behaviour of $\frac{dR_D}{dM}$, $\frac{dR_D}{dQ}$, $\frac{dR_D}{dD_a}$, $\frac{dR_D}{d\varepsilon}$, $\frac{dR_D}{dL_e}$, $\frac{dR_D}{dN_A}$ and $\frac{dR_D}{dR_n}$ analytically. Eq. (41) yields:

$$\frac{dR_D}{dM} = -\frac{2\varepsilon M\pi^4\left(a^2+\pi^2\right)^2 Q}{a^2\left\{a^2\varepsilon^2+\left(\varepsilon^2+M^2\right)\pi^2\right\}^2},$$

$$\frac{dR_D}{dQ} = \frac{\varepsilon\pi^2\left(a^2+\pi^2\right)^2}{a^2\left\{a^2\varepsilon^2+\left(\varepsilon^2+M^2\right)\pi^2\right\}^2},$$

$$\frac{dR_D}{dD_a} = \frac{\left(a^2+\pi^2\right)^3}{a^2},$$

$$\frac{dR_D}{d\varepsilon} = \frac{R_n N_A L_e}{\varepsilon^2} - \frac{\pi^2\left(a^2+\pi^2\right)^2\left\{a^2\varepsilon^2+\left(\varepsilon^2-M^2\right)\pi^2\right\}Q}{\left\{a^3\varepsilon^2+a\left(\varepsilon^2+M^2\right)\pi^2\right\}^2},$$

$$\frac{dR_D}{dL_e} = -\frac{R_n N_A}{\varepsilon},\quad \frac{dR_D}{dN_A} = -\frac{\left(\varepsilon+L_e\right)R_n}{\varepsilon}$$

and $$\frac{dR_D}{dR_n} = -\frac{\left(\varepsilon+L_e\right)N_A}{\varepsilon}.$$

These show that, for the stationary convection, the Hall current parameter M, Lewis number L_e, modified diffusivity ratio N_A and nanoparticles concentration Rayleigh number R_n have always a destabilized effect, while the magnetic Darcy number Q and Darcy number D_a have always stabilized the system. The porosity parameter ε have a stabilizing effect if

$$\frac{R_n N_A L_e}{\varepsilon^2} > \frac{\pi^2\left(a^2+\pi^2\right)^2\left\{a^2\varepsilon^2+\left(\varepsilon^2-M^2\right)\pi^2\right\}Q}{\left\{a^3\varepsilon^2+a\left(\varepsilon^2+M^2\right)\pi^2\right\}^2}.$$

3.2 Oscillatory Convection

The present section is devoted to find the possibility as to whether the instability due to the presence of Hall current and magnetic field may occur as oscillatory convection. Since we wish to determine the Rayleigh Darcy number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which Eq. (40) will admit of solutions with ω_i real. Equating real and imaginary parts of (40) and eliminating R_D between them, we have

$$F_1\left({\omega_i}^2\right)^3+F_2\left({\omega_i}^2\right)^2+F_3\left({\omega_i}^2\right)+F_4=0, \quad (45)$$

where $F_1 = -\varepsilon^5{L_e}^2\left\{\mathrm{P_r}+D_aJ\left(1+\mathrm{P_r}\right)\right\}{\mathrm{P}_{rM}}^4$,

$$\begin{aligned}F_2 = {} & \varepsilon^3{\mathrm{P}_{rM}}^2\sigma\left[-\varepsilon L_e\mathrm{P_r}\mathrm{P}_{rM}\left\{-JL_e\pi^2\mathrm{P_r}Q\right.\right.\\&\left.+a^2\left(\varepsilon+L_e\right)N_A\mathrm{P_r}\mathrm{P}_{rM}R_n\right\}\\&-\left\{J\left(\mathrm{P_r}+D_aJ\left(1+\mathrm{P_r}\right)\right)\left(2{L_e}^2\left(\varepsilon^2J-M^2\pi^2\right),\right.\right.\\&\left.\times{\mathrm{P_r}}^2+\varepsilon^2J{\mathrm{P}_{rM}}^2\right)+\varepsilon J{L_e}^2\pi^2{\mathrm{P_r}}^3Q\\&\left.\left.-a^2\varepsilon L_eN_A\mathrm{P_r}{\mathrm{P}_{rM}}^2R_n\right\}\sigma\right]\end{aligned}$$

And $J = a^2+\pi^2$. The coefficients F_3 and F_4 are very lengthy and not needed in the discussion of overstability, and have not been written here. Since ω_i is real for overstability, the three values of $(\omega_i)^2$ should be positive. The sum of roots of the Eq. (45) is $-\frac{F_2}{F_1}$. For oscillatory convection to occur, $F_2 > 0$ (because $F_1 < 0$), and on using the parametric values of nanofluid, the values of F_2 which cannot be positive, ruling out the possibility of oscillatory convection in this case.

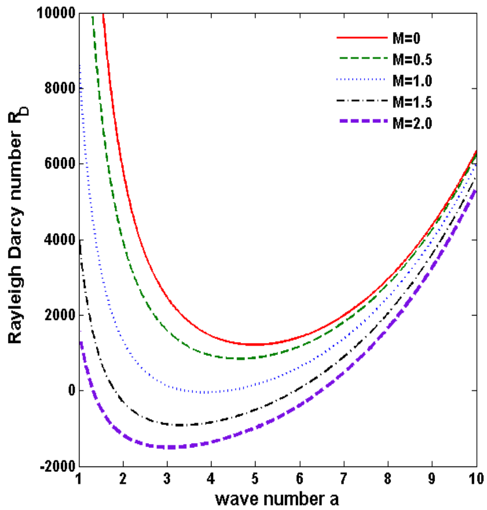

Fig. 2. Variation of Rayleigh Darcy number R_D with wave number a, for various values of Hall current parameter M with $R_n = 0.1$, $N_A = 5$, $\varepsilon = 0.8$, $L_e = 5\times10^3$, $D_a = 0.5$, $Q = 200$.

The Eq. (41) is also solved numerically to obtain the critical Rayleigh Darcy number $R_{D,c}$ and the corresponding critical wave number a_c for alumina/water nanofluid as shown in Figs. 2-8 for various parameter values. According to Buongiorno (2006) and Yadav *et al*. (2013c, 2014d), the following parameter values for alumina/water nanofluid are taken: $\phi_0^* = 0.001$, $\rho_0 = 1000$ kg/m^3, $\rho_p = 4\times10^3$ kg/m^3, $\alpha = 2.0\times10^{-7}$ m^2/s, $D_B = 4\times10^{-11}$ m^2/s, $D_T = 6\times10^{-11}$ m^2/s, $(\rho c)_p = 3.1\times10^6$ J/m^3, $\mu = 10^{-3}$ Pas, $\beta = 3.4\times10^{-3}$ 1/K, $(\rho c)_f = 4\times10^6$ J/m^3, $T_0^* - T_1^* = 1$K, $T_1^* = 300$K.

The parameter values given above give the following representative values of dimensionless parameters: $L_e = 5\times10^3$, $P_r = 5$, $N_A = 5$. The values of R_n can be controlled by changing the distance between the boundaries and changing the reference scale for the nanoparticle fraction. Finally, we fix the values for the parameters as $L_e = 5\times10^3$, $R_n = 0.1$, $N_A = 5$, $P_r = 5$, Q=200, $M = 0.5$, $D_a = 0.5$ and $\varepsilon = 0.8$ except the varying parameters.

In Fig. 2, Rayleigh Darcy number R_D is plotted against wave number a, for different values of Hall current parameter M. Here we observed that the critical Rayleigh Darcy number decreases as M increases and hence the Hall current is having a destabilizing factor to make the system more less stable. This is due to the fact that the Hall current effect produces a cross-flow i.e. right angle to the primary flow in the presence of a transverse magnetic field. This breakdown of the primary flow may be presumably attributed to the inherent instability.

The significant characteristics of the magnetic number Q on the stability of the system are exhibited graphically in Fig. 3. From Fig. 3, it is found that the when the magnetic number Q increases, in terms of the larger value of the critical Rayleigh number R_c, the system becomes more stable. This is due to the fact that the variation of Q leads to the variation of the Lorentz force and the Lorentz force produces more resistance to transport phenomenon. Hence, magnetic field has a stabilizing effect on the stability of the system.

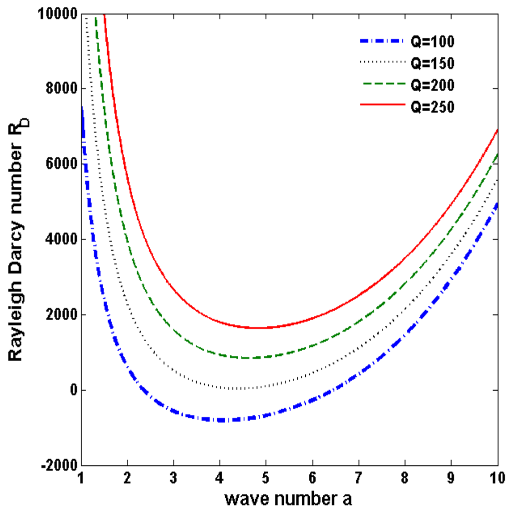

Fig. 3. Variation of Rayleigh Darcy number R_D with wave number a, for various values of magnetic Darcy number Q with $R_n = 0.1$, $N_A = 5$, $\varepsilon = 0.8$, $L_e = 5\times10^3$, $D_a = 0.5$, $M = 0.5$.

Figs. 4, 5, and 6 show, respectively, the effect of the concentration Rayleigh Darcy number R_n, the Lewis number L_e and the modified diffusivity ratio N_A on the stability of the system. It can be easily be said that an increases in the values of the concentration Rayleigh Darcy number R_n, Lewis number L_e and modified diffusivity ratio N_A lead to the decrease in the value on Rayleigh Darcy number R_D, thus indicating an increase in the onset of convection. Hence nanoparticles parameters (the concentration Rayleigh Darcy number R_n, the Lewis number L_e and the modified diffusivity ratio N_A) have a destabilizing effect on the stability of the system. This may be understand that as an increase in volumetric fraction of nanoparticles, increases the Brownian motion of the nanoparticles, which causes the destabilizing effect of nanoparticles parameters.

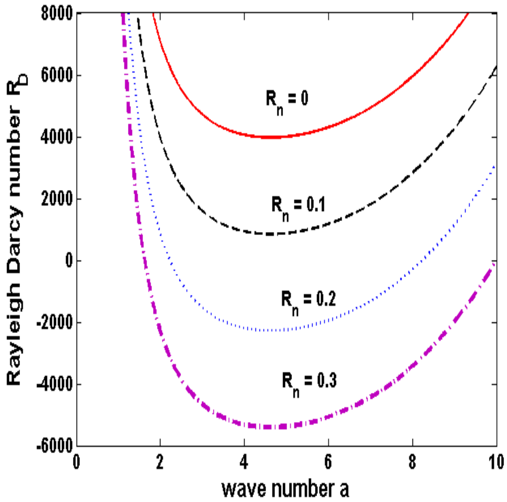

Fig. 4. Variation of Rayleigh Darcy number R_D with wave number a, for various values of concentration Rayleigh number R_n with $N_A = 5$, $\varepsilon = 0.8$, $D_a = 0.5$, $L_e = 5\times10^3$, $Q = 200$, $M = 0.5$.

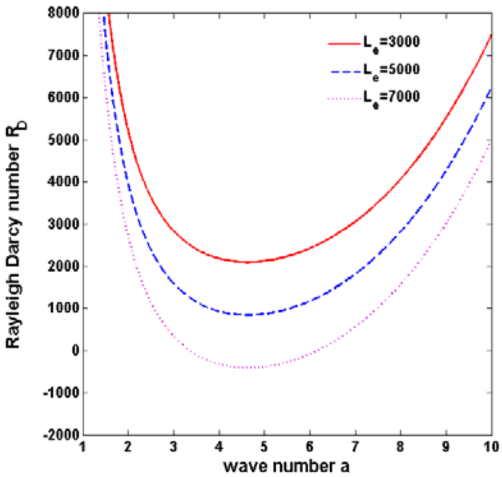

Fig. 5. Variation of Rayleigh Darcy number R_D with wave number a, for various values of Lewis number L_e with $R_n = 0.1$, $N_A = 5$, $\varepsilon = 0.8$, $D_a = 0.5$, $Q = 200$, $M = 0.5$.

To assess the effect of porous medium on the stability of the system, the variation of Rayleigh number R_D as a function of wave number a for different values of porosity parameter ε is shown in Fig. 7. We found that with an increase in the value of ε, the critical Rayleigh number R_D increases, indicating that it delays the onset of convection in nanofluid saturated in porous medium.

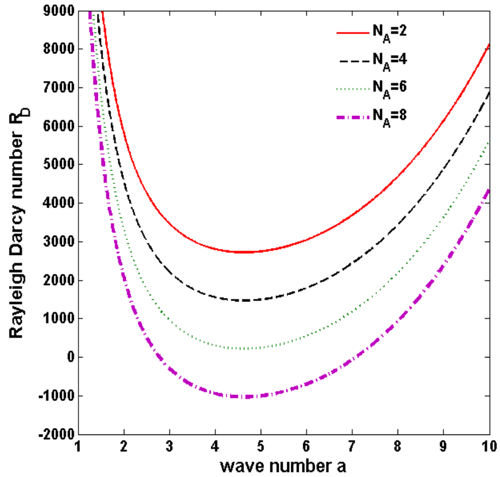

Fig. 6. Variation of Rayleigh Darcy number R_D with wave number a, for various values of modified diffusivity ratio N_A with $R_n = 0.1$, $\varepsilon = 0.8$, $D_a = 0.5$, $Q = 200$, $M = 0.5$.

The effect of Darcy number D_a, on the natural curve is depicted in Fig. 8. The critical Rayleigh number R_D increases with an increase in the Darcy number D_a which shows that the effect of Darcy number D_a delays the onset of convection in the nanofluid-saturated porous media. This is because; increase in the value of Darcy number D_a is related to increase in the effective viscosity which has the tendency to retard the fluid flow.

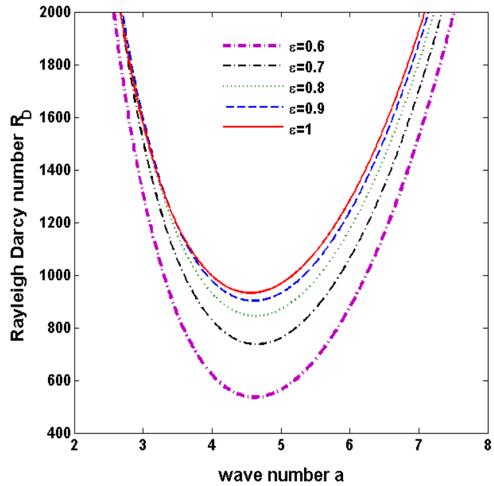

Fig. 7. Variation of Rayleigh Darcy number R_D with wave number a, for various values of porosity parameter ε with $R_n = 0.1$, $N_A = 5$, $L_e = 5\times10^3$, $D_a = 0.5$, $Q = 200$, $M = 0.5$.

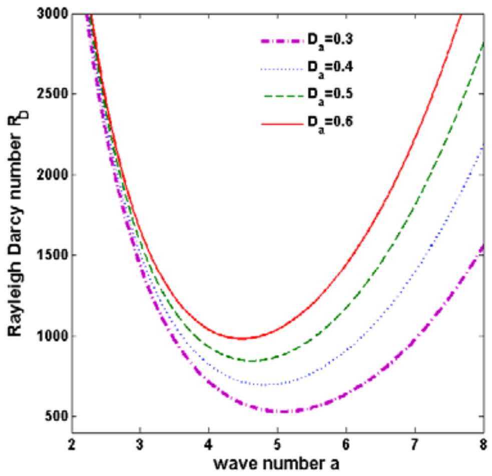

Fig. 8. Variation of Rayleigh Darcy number R_D with wave number a , for various values of Darcy number D_a with $R_n = 0.1$, $N_A = 5$, $L_e = 5\times10^3$, $\varepsilon = 0.8$, $Q = 200$, $M = 0.5$.

Also, from Figs. 2, 7 and 8, we observed that an increase in the values of the Hall current parameter M , the porosity parameter ε and the Darcy number D_a , the critical wave number a_c decreases and thus its effect is to increase the size of convection cell, while an increase in the magnetic Darcy number Q decreases the size of convection cell as observed from the Fig.3.

4. Conclusions

The onset of MHD nanofluid convection in a porous medium layer with the presence of Hall current effect was analyzed analytically using the linear stability theory. A physically more realistic boundary condition than the previous ones on the nanoparticle volume fraction was considered i.e. the nanoparticle flux was assumed to be zero rather than prescribing the nanoparticle volume fraction on the boundaries. The linear stability theory gives the condition for the onset of stationary convection and show that the oscillatory convection cannot occur with the new boundary conditions. The expression for the stationary convection show that the Hall current parameter and nanoparticles parameters (the Lewis number, the modified diffusivity ratio and the concentration Rayleigh Darcy number) accelerate the onset of convection, while the magnetic Darcy number, porosity parameter and Darcy number delay the onset of convection. The critical wave number a_c decreases with an increase in the values of the Hall current parameter, the porosity parameter and the Darcy number, while it increases with the magnetic Darcy number.

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