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Evaluation Study of Pressure-Strain Correlation Models in Compressible Flow

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ABSTRACT

This paper is devoted to the second-order closure for compressible turbulent flows with special attention paid to modeling the pressure-strain correlation appearing in the Reynolds stress equation. This term appears as the main one responsible for the changes of the turbulence structures that arise from structural compressibility effects. The structure of the gradient Mach number is similar to that of turbulence, therefore this parameter may be appropriate to study the changes in turbulence structures that arise from structural compressibility effects. Thus, the incompressible model (LRR) of the pressure-strain correlation and its corrected form by using the turbulent Mach number, fail to correctly evaluate the compressibility effects at high shear flow. An extension of the widely used incompressible model (LRR) on compressible homogeneous shear flow is the major aim of the present work. From this extension the standard coefficients C_i became a function of the compressibility parameters (the turbulent Mach number and the gradient Mach number). Application of the model on compressible homogeneous shear flow by considering various initial conditions shows reasonable agreement with the DNS results of Sarkar. The ability of the models to predict the equilibrium states for the flow in cases A_1 and A_4 from DNS results of Sarkar is examined, the results appear to be very encouraging. Thus, both parameters M_t and M_g should be used to model significant structural compressibility effects at high-speed shear flow.

Keywords: Compressible; Turbulence; Pressure-strain; Model of turbulence; Shear flow.

NOMENCLATURE

b_{ij}	Reynolds stress anisotropy	U_i	velocity in the direction, x_i
c	speed of sound		
M_t	turbulent Mach number	ε_s	solenoidal part of the dissipation
M_g	gradient Mach number	δ_{ij}	Kronecker delta
P	pressure	$( )'$	Reynolds fluctuation
R	specific gas constant	$( )_{,i}$	spatial gradient
T	time	Φ_{ij}	pressure-strain correlation
T	temperature	γ	specific heat ratio

1. INTRODUCTION

Compressible turbulence modeling is an essential element for calculations of many problems of practical engineering interest, such us combustion, environment and aerodynamics. The compressibility phenomena have been extensively studied and numerical simulation of compressible turbulent flows using compressible turbulence models have been performed by many authors. Previous studies carried out in the last 30 years have conjectured that compressibility effect was linked with dilatational dissipation and pressure–dilatation correlation, as it is represented by the models of Zeman (1990), Sarkar (1991) and others. During the last thirty years, many theoretical and experimental works have been developed primarily to understand and predict the behavior of turbulent flows. In fact, the models which developed turbulence gave full satisfaction in simple configurations for the homogeneous flows.

In this context, many studies, Sarkar and Lakshmanan (1991), Bradshaw (1977), H.

Marzougui *et al.* (2005) Carlos A. Gomez (2013), Hechmi *et al.* (2008) of the compressible shear flow which show the changes of the turbulence structures are principally due to the structural compressibility effects which significantly affect the pressure-strain correlation. Thus, the pressure-strain correlation appears as the main factor for the changes in the magnitude of the Reynolds stress anisotropies. The extension of the standard models to compressible flows represents a research topic of great scientific and industrial interest.

A major challenge related to this extension is to take into account the compressibility effects in the classical scheme closure of turbulence.

It is well concluded that the Favre Reynolds stress closure using the standard models of the pressure-strain correlation with the addition of the compressible dissipation and pressure-dilatation correlation models failed to predict high compressible flows (Speziale *et al.* (1995), Sarkar (1995) and Hamba (1999) also performed DNS results of compressible homogeneous shear flow and reached similar conclusions concerning the roles of dilatational terms.

The consequent effects on the pressure-strain correlation may cause significant changes on turbulence structures. According to the DNS results of Sarkar (1995), there is a reduction in the magnitude of the Reynolds shear stress anisotropy and an increase in the magnitude of the normal stress anisotropy. As a consequence, the pressure strain modeling seems to be an important issue in the second order closures for the compressible turbulent flows.

A method of including compressibility effects in the pressure strain correlation is the subject of the present study. The LRR model developed by Launder-Reece and Rodi (1975) has shown a great success in simulating a variety of incompressible complex turbulent flows. Thus, a compressible correction for the LRR model is the major aim of this study. In the present work, the correction concerns essentially the Ci coefficients which became in a compressible situation a function of the turbulent Mach number and the gradient Mach number.

In the present study, we concentrate on evaluating the models of the pressure-strain correlation in the homogeneous shear flow.

2. GOVERNING EQUATIONS

The General equations governing the motion of a compressible fluid are the Navier-Stokes equations. They can be written as follows for mass, momentum and energy conservation:

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u_i)}{\partial x_i}=0 \tag{1}$$

$$\frac{\partial(\rho u_i)}{\partial t}+\frac{\partial(\rho u_i u_j)}{\partial x_j}=\frac{\partial \sigma_{ij}}{\partial x_j} \tag{2}$$

$$\frac{\partial(\rho e)}{\partial t}+\frac{\partial(\rho e u_j)}{\partial x_j}=\frac{\partial}{\partial x_j}\sigma_{ij}u_i-\frac{\partial}{\partial x_j}(kT_{,j}) \tag{3}$$

Here ρ is the density, u is the velocity, p is the pressure, e is the internal energy, T is the temperature, μ is the viscosity, k is the thermal conductivity and C_v is specific heat at constant volume. Where:

$$\sigma_{ij}=-p\delta_{ij}+\tau_{ij} \tag{4}$$

$$e=C_v T \tag{5}$$

$$\tau_{ij}=2\mu S_{ij} \tag{6}$$

$$S_{ij}=\left(u_{i,j}+u_{j,i}\right)/2 \tag{7}$$

For an ideal gas, the relation between pressure, density and temperature can be written as follows:

$$p=\rho RT \tag{8}$$

3. BASIC EQUATIONS OF THE FAVRE SECOND-ORDER CLOSURE IN COMPRESSIBLE FLOWS

For compressible homogeneous shear flow, the mean

gradient velocity is given by:

$$\widetilde{U}_{i,j}=S\delta_{i1}\delta_{j2}=\begin{cases}S\,, & \text{if } (i,j)=(1,2)\\ 0\,, & i\neq 1, j\neq 2\end{cases} \tag{9}$$

Where S is the constant mean shear rate. These considerations lead to:

$$\widetilde{U}_{k,k}=0, \bar{\rho}=\text{constante} \tag{10}$$

The Favre equations for conservation of mass, momentum and energy are:

$$\frac{\partial}{\partial t}\bar{\rho}+\frac{\partial}{\partial x_i}(\bar{\rho}\widetilde{U}_i)=0 \tag{11}$$

$$\frac{\partial}{\partial t}(\bar{\rho}\widetilde{U}_i)+\frac{\partial}{\partial x_j}(\bar{\rho}\widetilde{U}_i\widetilde{U}_j)+\frac{\partial}{\partial x_j}\overline{\rho u_i'' u_j''}=\frac{\partial}{\partial x_j}\left[\tilde{\tau}_{ij}+\overline{\tau_{ij}''}-\bar{p}\delta_{ij}\right] \tag{12}$$

$$\frac{\partial}{\partial t}(\bar{\rho}\,\bar{C}_v\widetilde{T})+\frac{\partial}{\partial x_i}(\bar{\rho}\,\bar{C}_v\widetilde{T}\,\widetilde{U}_i)=-\bar{p}\frac{\partial}{\partial x_i}\widetilde{U}_i-\bar{p}\,\frac{\partial}{\partial x_i}\overline{u_i''}-\overline{p'\frac{\partial}{\partial x_i}u_i''}+\overline{\Phi}+\frac{\partial}{\partial x_i}\overline{k\frac{\partial}{\partial x_i}T}-\frac{\partial}{\partial x_i}\left(\bar{\rho}C_v\widetilde{u_i''T''}\right) \tag{13}$$

Where :

$$\tilde{\tau}_{ij}=2\bar{\mu}\tilde{S}_{ij}-\frac{2}{3}\bar{\mu}\widetilde{U}_{k,k}\delta_{ij} \tag{14}$$

$$\tilde{S}_{ij}=\left(\widetilde{U}_{i,j}+\widetilde{U}_{j,i}\right)/2 \tag{15}$$

$$\tau_{ij}''=2\bar{\mu}s_{i,j}''-\frac{2}{3}\bar{\mu}u_{k,k}''\delta_{ij} \tag{16}$$

$$\overline{\Phi}=\overline{\tau_{ij}u_{i,j}} \tag{17}$$

Classically, the second-order closure requires a transport equation of the turbulent dissipation rate. The new concept of dissipation in compressible turbulence was proposed by Sarkar *et al.* (1995), Zeman (1991) and Ristorcelli (1997) and can be written as follows

$$\varepsilon_{ij}=\frac{2}{3}\varepsilon\delta_{ij} \tag{18}$$

Where for homogeneous turbulence $\bar{\rho}\varepsilon_s = \overline{\mu\omega_i\omega_i'}$, $\omega_i'$is the fluctuating vorticity and $\bar{\rho}\varepsilon_c = \frac{4}{3}\mu\left(u_{i,i}'\right)^2$ represent the solenoidal and compressible parts of ε respectively. Sarkar *et al.* (1990) have mentioned that for moderate Mach numbers, ε_s is insensitive to the compressibility changes. For ε_s, model transport equation, similar to what it was obtained in the incompressible case. Such a model equation is written in (1985), namely:

$$\bar{\rho}\frac{d\varepsilon_s}{dt} = -C_{\varepsilon 1}\bar{\rho}\frac{\varepsilon_s}{k}\widetilde{u_i''u_j''}\tilde{U}_{i,j} + \overline{p'u_{i,i}'} - C_{\varepsilon 2}\bar{\rho}\frac{\varepsilon_s^2}{k} \qquad (19)$$

Where $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ are respectively the model constants $C_{\varepsilon 1}$=1.44 and $C_{\varepsilon 2}$=1.83

ε_c is generally taken to be proportional to $\varepsilon_s$through the following equation algebraic equation:

$$\varepsilon_c = f(M_t)\varepsilon_s \qquad (20)$$

As it is suggested in model (1991), one can write $f(M_t)=0.5M_t^2$

f (Mt) is a function of the turbulent Mach number The Reynolds stress is solutions of the transport equation:

$$\bar{\rho}\frac{d}{dt}\left(\widetilde{u_i''u_j''}\right) = P_{ij} + \Phi_{ij}^* + \frac{2}{3}\overline{p'd'}\delta_{ij} - \frac{2}{3}\bar{\rho}\varepsilon\delta_{ij} \qquad (21)$$

Where P the turbulent kinetic energy production:

$$P_{ij} = -\bar{\rho}\widetilde{u_i''u_j''}\tilde{U}_{i,j} \qquad (22)$$

$$\Phi_{ij}^* = \overline{p'\left(\frac{\partial u_i''}{\partial x_j} + \frac{\partial u_j''}{\partial x_i}\right)} - \frac{2}{3}\overline{p'd'}\delta_{ij} \qquad (23)$$

The turbulent kinetic energy equation is

$$\bar{\rho}\frac{dk}{dt} = -\bar{\rho}\widetilde{u_i''u_j''}\tilde{U}_{i,j} + \overline{p'd'} - \bar{\rho}\varepsilon \qquad (24)$$

The turbulent Mach number is described in (1985) by the transport equation as follows

$$\frac{dM_t}{dt} = M_t\frac{P}{2k} + \frac{M_t}{2\bar{\rho}k}\left[1 + \frac{1}{2}\gamma(\gamma-1)M_t^2\right]\left(\overline{p'd'} - \bar{\rho}\varepsilon\right) \qquad (25)$$

4. MODEL OF TURBULENCE

It is generally accepted that the term of the pressure-dilatation plays an important role in the modeling of turbulent compressible flows and behavior analysis of the turbulence. Indeed, this term which appears in the equation of turbulent kinetic energy seems to have important effects on the evolution of turbulence. Such effect is manifested by a reduction in the turbulent kinetic energy when the turbulent Mach number increases. In this context, the results of the numerical simulation developed by Sarkar *et al.* (1991,1992,1995) and Blaisdell *et al.* (1991) have already shown that the pressure-dilatation correlation is an important indicator for the compressibility.

In this part, we will choose a number of models from literature that are well-known and that express the pressure-dilatation.

Model of Sarkar(1992)

The algebraic form of the model developed by Sarkar *et al.* (1992) and expressing the pressure-dilatation correlation is as follows:

$$\overline{p'd'} = -\alpha_2 PM_t + \alpha_3\bar{\rho}\varepsilon_s M_t^2 \qquad (26)$$

$$\alpha_2 = 0.15$$

$$\alpha_3 = 0.2$$

Model of Zeman(1990)

$$\overline{p'd'} = -\frac{1}{2}\frac{D}{Dt}\left(\frac{\overline{p'^2}}{\gamma\bar{p}}\right) = (\gamma\bar{p})^{-1}\left[\frac{\overline{p'^2} - p_e^2}{\tau} + \left(\frac{5-3\gamma}{12}\right)\overline{p'^2}\tilde{U}_{k,k}\right] \qquad (27)$$

Where τ is the acoustic time scale,

$$\tau = \frac{k}{\varepsilon}M_t \qquad (28)$$

$$p_e^2 = 2\bar{\rho}^2 k\gamma R\tilde{T}\left(\frac{M_t^2 + M_t^4}{1 + M_t^2 + M_t^4}\right) \qquad (29)$$

Model of F. Hamba(1999)

$$\overline{p'd'} = -\left(1 - C_{pd_3}\chi_p\right)\left[C_{pd1}M_t^2\frac{D}{Dt}(\bar{\rho}k) + C_{pd_2}\gamma\bar{\rho}M_t^2 k\tilde{U}_{i,i}\right] \qquad (30)$$

Where

$$\chi_p = \left(\frac{\overline{p'^2}}{2\bar{\rho}^2C^2k}\right) \qquad (31)$$

When C_{pdi}, $(i = 1,3)$ are numerical constants

Model of El Baz and Launder (1996)

$$\overline{p'd'} = -F\left(P - \frac{2}{3}kU_{k,k}\right), F = 1.5M_t^2 \qquad (32)$$

Model of Ristorcelli (1997)

$$\overline{p'd'} = \frac{C_1M_t^2}{C_2 + C_1M_t^2}\left(R_{ij}\tilde{U}_{i,j} - \varepsilon_s\right) \qquad (33)$$

We note that there are other models using different compressibility parameters as the density variance, for example those developed by Taulbee *et al.* (1991), Hamba (1999), Simone *et al.* (1997), Hanafi (2013), and Bogdanoff *et al.* (1983) .

For compressible homogeneous shear flow, the DNS results of Sarkar (1995) show the change in the magnitude of the components of Reynolds stress anisotropy tensor and the growth rate of turbulent kinetic energy. Thus, the compressibility has a significant effect on the pressure fields and in consequence on its correlations such as the pressure-strain, the pressure-dilatation. The work of Sarkar (1991) recommends to use the gradient Mach number with the turbulent Mach number in order to predict the correct behavior of compressible turbulence. This is the point on which the present work is centered. At first, we have examined some basic assumptions which were used in constructing turbulence models. For compressible homogeneous shear flow, the growth rate of the turbulent kinetic energy is given by the following equation.

$$\wedge = \frac{1}{SK}\frac{dK}{dt} = \frac{P - \varepsilon + \overline{p'd'}}{SK} = -2b_{12} - \frac{\varepsilon_s}{Sk} + \frac{\frac{\overline{p'd'}}{\bar{\rho}} - \varepsilon_c}{Sk} \quad (34)$$

Vreman *et al.* (1996) performed DNS results for compressible mixing layers. From which, they deducted that the diagonal rapid part of the pressure-strain correlation are approximately proportional to the growth rate. Pantano and Sarkar (2003) used DNS results to study the compressibility effects in the high-speed turbulent shear layer. They showed that the components of the pressure-strain correlation and the pressure variance normalized by its incompressible counterparts decrease similarly.

$$\frac{\Phi_{ij}}{(\Phi_{ij})^I} \sim \frac{\sqrt{\overline{p'^2}}}{\sqrt{\overline{p'^2}^I}} \quad (35)$$

The hypothesis about an approximate proportionality between $\frac{\Phi_{ij}}{(\Phi_{ij})^I}$, $\frac{\sqrt{\overline{p'^2}}}{\bar{\rho}K} / \left(\frac{\sqrt{\overline{p'^2}}}{\bar{\rho}K}\right)^I$ and $^\wedge/_{\wedge I}$ can be supported as in H.Marzougui and al.(2005) and we can write.

$$\frac{\frac{\Phi_{ij}}{sk}}{\left(\frac{\Phi_{ij}}{SK}\right)^I} \sim \frac{\wedge}{\wedge I} \sim \frac{\frac{\sqrt{\overline{p'^2}}}{\bar{\rho}K}}{\left(\frac{\sqrt{\overline{p'^2}}}{\bar{\rho}K}\right)^I} \quad (36)$$

Now, we consider the equation

$$\frac{\frac{\Phi_{ij}}{sk}}{\left(\frac{\Phi_{ij}}{SK}\right)^I} = \frac{\wedge}{\wedge I} \quad (37)$$

It comes

$$\frac{\Phi_{ij}}{(\Phi_{ij})^I} = \frac{\frac{dK}{dt}}{\frac{dK^I}{dt}} \quad (38)$$

The turbulent kinetic energy K and K^I satisfied the transport equations

$$\frac{dK}{dt} = P - \varepsilon_s - \varepsilon_c + \overline{p'd'} \quad (39)$$

$$\frac{dK^I}{dt} = P^I - \varepsilon_s \quad (40)$$

From the two last equations, the time derivative ratio in RHS of Eq. (38) can be written as

$$\frac{\frac{dK}{dt}}{\frac{dK^I}{dt}} = \left[\frac{\frac{P}{\varepsilon_s} - 1}{\frac{P^I}{\varepsilon_s} - 1}\right]\left(1 - \frac{\frac{\varepsilon_c}{\varepsilon_s}}{\frac{P}{\varepsilon_s} - 1}\right)\left(1 + \frac{\overline{p'd'}}{P - \varepsilon}\right) \quad (41)$$

The DNS results of Sarkar(1995) show that the relative dissipation $\varepsilon_s/P$is less affected by compressibility and it shows a similar equilibrium value as its incompressible counterpart ε_s/P^I. Thus, the term between the square brackets in (41) can be approximated by 1 and we can obtain

$$\frac{\frac{dK}{dt}}{\frac{dK^I}{dt}} = \left(1 - \beta\frac{\varepsilon_c}{\varepsilon_s}\right)\left(1 + \frac{\overline{p'd'}}{P - \varepsilon}\right) \quad (42)$$

$$\beta = \left(\frac{1}{\frac{P}{\varepsilon_s} - 1}\right) \quad (43)$$

Where, β is a numerical constant can be determined by the equilibrium value of ε_s/P^I .From eqs.(38,42), we have

$$\overline{p'd'} = \left[\frac{\frac{\Phi_{ij}}{\Phi_{ij}^I} - 1 + \beta\frac{\varepsilon_c}{\varepsilon_s}}{1 - \beta\frac{\varepsilon_c}{\varepsilon_s}}\right](P - \varepsilon) \quad (44)$$

Pantano and Sarkar (1991) use two scales to express $\Phi_{ij}/\Phi_{ij}^I$by a function of the turbulent Mach number and gradient Mach number.

$$\frac{\Phi_{ij}}{\Phi_{ij}^I} = 1 - f(M_t, M_g) \quad (45)$$

Where $f(M_t, M_g)$is a function model of turbulent Mach number and gradient Mach number.

$$f(M_t, M_g) = M_t^2\left(\frac{b - e^{\left[-(\alpha M_g - \beta)^2\right]}}{1 + bM_t^2}\right) \quad (46)$$

$$\overline{p'd'} = -f(M_t, M_g)P + \alpha M_t^2 \varepsilon_s \quad (47)$$

α=0.9, β=0, b= 4.2

The present work deals with the problem of modeling of the pressure-dilatation, for this reason we choose a number of models from literature that are well-known.

Launder Reece and Rodi(1975)

The models established in compressible turbulent flows are deduced by a simple extension of their incompressible counterparts. This extension concerned only one incompressible model, it is due to Launder, Reece and Rodi (1975), this model is written in the following form:

$$\Phi_{ij}^* = -C_1\bar{\rho}\varepsilon_s b_{ij} + C_2\bar{\rho}k\left(\tilde{S}_{ij} - \frac{1}{3}\tilde{S}_{kk}\delta_{ij}\right) + C_3\bar{\rho}k\left(b_{ip}\tilde{S}_{jp} + b_{jp}\tilde{S}_{ip} - \frac{2}{3}b_{pq}\tilde{S}_{pq}\delta_{ij}\right) + C_4\bar{\rho}k\left(b_{ip}\tilde{\Omega}_{jp} + b_{jp}\tilde{\Omega}_{ip}\right) \quad (48)$$

Where C_1, C_2, C_3 and C_4 are constants that take on the values of: C_1=3, C_2 =0.8 et C_3=1.75 et C_4=1.31

$$\tilde{S}_{ij} = \frac{1}{2}(\tilde{u}_{i,j} + \tilde{u}_{j,i}) \quad (49)$$

$$\tilde{\Omega}_{ij} = \frac{1}{2}(\tilde{u}_{i,j} - \tilde{u}_{j,i}) \quad (50)$$

$$b_{ij} = \frac{1}{2k}\widetilde{u_i'' u_j''} - \frac{1}{3}\delta_{ij} \quad (51)$$

Model of Adumitroaie et al. (1999)

Adumitroaie *et al*. (1999) assumed that incompressible modeling approach of the pressure-strain correlation can be used to develop turbulent models taking into account compressibility effects. Considering a none zero divergence for the velocity fluctuation called the compressibility continuity constraint and using different models for the pressure dilatation which is proportional to the trace of the pressure strain, their model for the linear part of this term is written as:

$$\Phi_{ij}^{*} = -C_1\bar{\rho}\varepsilon_s b_{ij} + \bar{\rho}k\left(\frac{4}{5}+\frac{2}{5}d_1\right)\left(\tilde{S}_{ij}-\frac{1}{3}\tilde{S}_{ll}\delta_{ij}\right) + 2\bar{\rho}k(1-C_3+2d_2)\left[b_{ik}\tilde{S}_{jk}+b_{jk}\tilde{S}_{ik}-\frac{2}{3}b_{ml}\tilde{S}_{ml}\delta_{ij}\right] - (1-C_4-2d_2)\bar{\rho}k\left(b_{ik}\tilde{\Omega}_{jk}+b_{jk}\tilde{\Omega}_{ik}-\frac{4}{3}d_2\tilde{S}_{kk}b_{ij}\right) \quad (52)$$

Where: $C_3 = C_3^I + 0.3M_t$;

$C_4 = C_4^I - 0.3M_t$

Present Model

$$\overline{p'd'} = -f(M_t, M_g)P + \alpha M_t^2\varepsilon_s \quad (53)$$

Where ; $C_3 = C_3^I + M_t^2\left(\frac{b-e^{\left[-(\alpha M_g-\beta)^2\right]}}{1+bM_t^2}\right)$

$$C_4 = C_4^I - M_t^2\left(\frac{b-e^{\left[-(\alpha M_g-\beta)^2\right]}}{1+bM_t^2}\right)$$

In Eqs. (52) and (53), the coefficients C_3^I and C_4^I are those of the incompressible LRR model

$C_3^I = 1.75$

$C_4^I = 1.31$

5. RESULTS AND DISCUSSION

The transport Eqs. (19), (21), (24) and (25) on which the second order closure for compressible homogeneous shear flow is based, are solved using the fourth-order accurate Runge-Kutta numerical scheme.

The ability of the proposed model to predict the anisotropy of compressible homogeneous turbulent shear flow will be now considered. The model predictions will be compared with DNS results conducted by Sarkar (1995) for cases A_1 and A_4. These cases correspond to different initial conditions for which the initial values of the gradient Mach number Mg change by changing the initial values of Sk/ε and taking Mt constant as it is listed in Table1.

Figures (1, 2), (3, 4) and (5, 6) show the non-dimensional time (St) variation of the Reynolds stress anisotropies b_{11}, b_{22}, and b_{12}. From these figures, it is clear that at low-Mg values, Figs. (1, 3 and 5), the Adumitroaie *et al.* (1999) model is in agreement with DNS data but at high-Mg values Figs. (2, 4, 6), the proposed model appears to be able to predict correctly the significant decrease in the normalized turbulent production term $-2b_{12}$ and the increase in the streamwise term b_{11}, as well as the transverse b_{22} Reynolds stresses anisotropies with increasing the gradient Mach number. Results obtained with Adumitroiae *et al.*'s model (1999) disagree with DNS data, especially at high-Mg values in case A_4, this model is still unable to predict the changes in the magnitude of the Reynolds stress anisotropy when the compressibility is higher.

Table 1 Initial condition for the DNS results of Sakar (1995) in compressible homogeneous shear flow

Case	Mt	Mg	Sk/ε
A_1	0.4	0.22	1.6
A_2	0.4	0.44	3.6
A_3	0.4	0.66	5.4
A_4	0.4	1.32	10.8

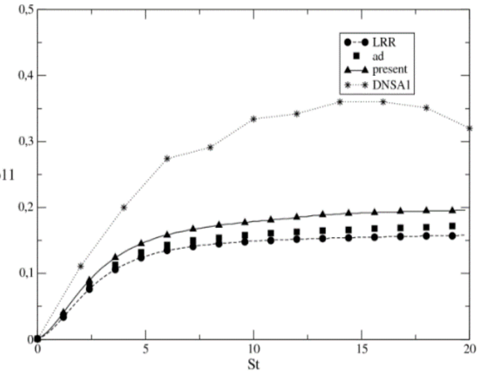

Fig. 1. Time evolution of the Reynolds-stress anisotropy b_{11} in case A_1.

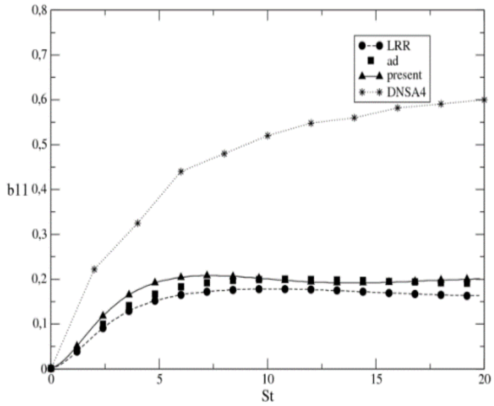

Fig. 2. Time evolution of the Reynolds-stress anisotropy b_{11} in case A_4.

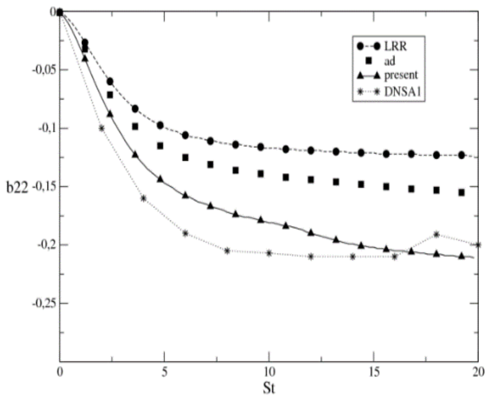

Fig. 3. Time evolution of the Reynolds-stress b_{22} in case A_1.

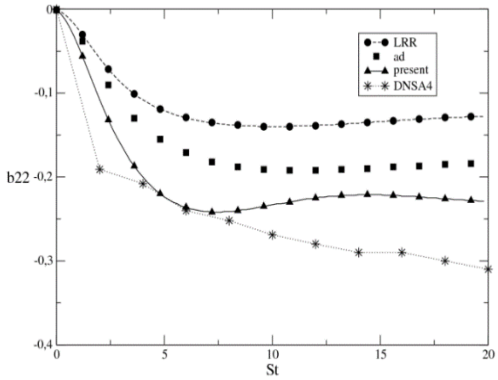

Fig. 4. Time evolution of the Reynolds-stress b_{22} in case A_4.

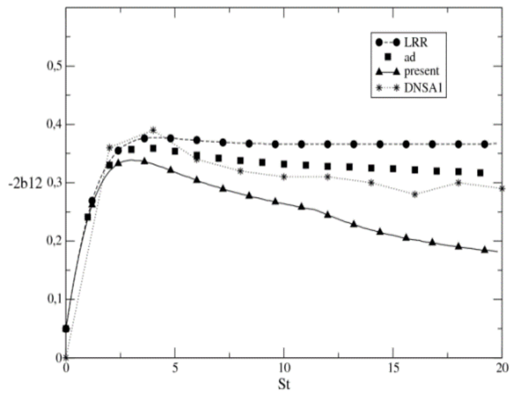

Fig. 5. Time evolution of the Reynolds-stress anisotropy $-2b_{12}$ in case A_1.

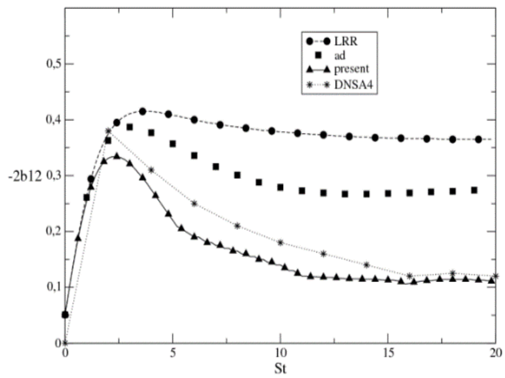

Fig. 6. Time evolution of the Reynolds-stress anisotropy $-2b_{12}$ in case A_4.

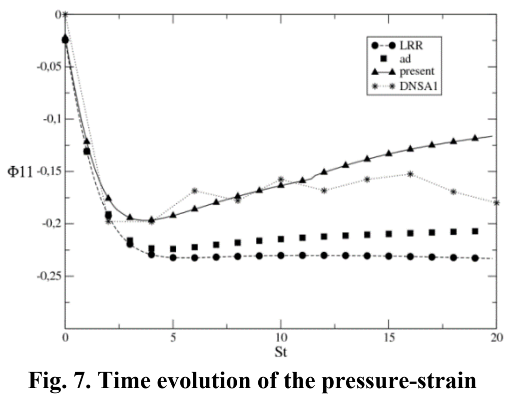

Fig. 7. Time evolution of the pressure-strain correlation Φ_{11} in case A_1.

From all the figures, the incompressible Launder, Reece and Rodi model (1975) is unable to predict the dramatic changes in the magnitude of the Reynolds stress anisotropy that arise from compressibility, while the present Model provides an acceptable performance in reproducing the DNS results in cases A_4. This model explains the importance of the evolving of M_g with the commonly used the parameter M_t in modeling the high compressible turbulent flow.

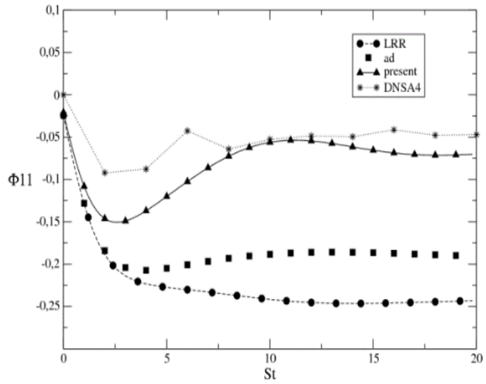

Fig. 8. Time evolution of the pressure-strain correlation Φ_{11} in case A_4.

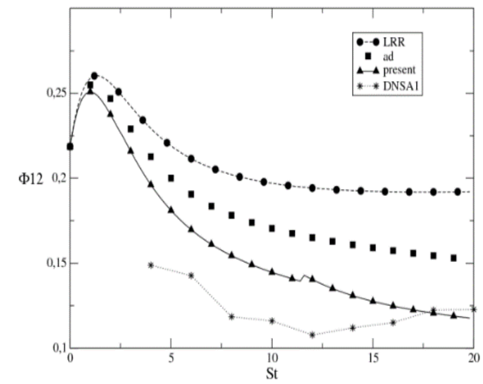

Fig. 9. Time evolution of the pressure-strain correlation Φ_{12} in case A_1.

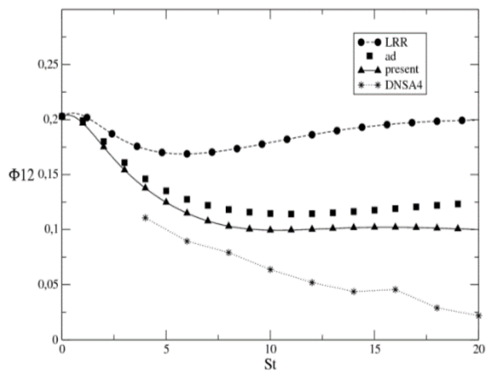

Fig. 10. Time evolution of the pressure-strain correlation Φ_{12} in case A_4.

Figs. ((7, 8); (9, 10) and (11, 12)) present the behavior of the pressure-strain correlation. As can be seen in these figures, the present model yields acceptable results that are in good qualitative agreement with the DNS data, especially at high gradient Mach number (case A_4) .

Figs. (13, 14) present the behavior of the normalized dissipation ε_s/Sk, ($\varepsilon_s/Sk=-2b_{12}\ \varepsilon_sP$) for cases A_1 and A_4 . It can be seen that there is a decrease in ε_s/Sk when M_g increases, since the compressibility effects cause the significant reduction in the Reynolds turbulent shear stress b_{12} from numerical simulation cases A_1 and A_4 of the previous DNS results. It is clear that the proposed model is in accordance with the DNS results.

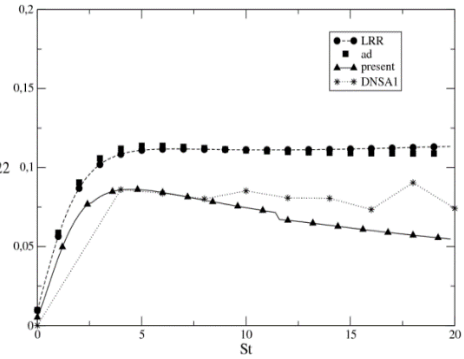

Fig. 11. Time evolution of the pressure-strain correlation Φ_{22} in case A_1.

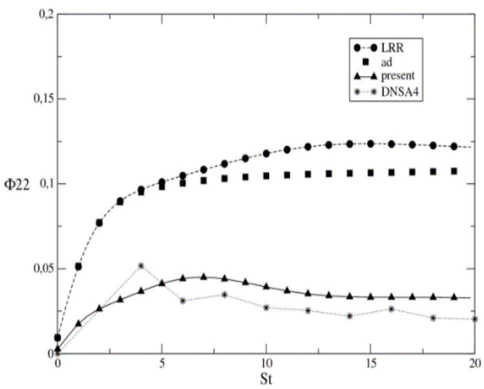

Fig. 12. Time evolution of the pressure-strain correlation Φ_{22} in case A_4.

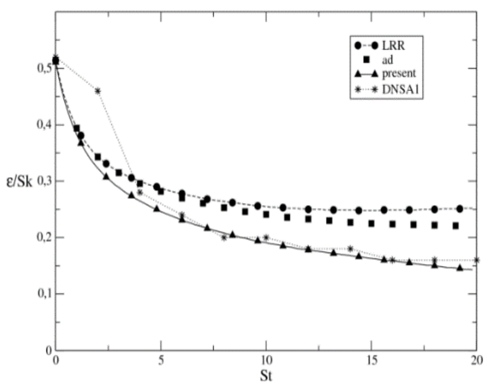

Fig. 13. Time evolution of the turbulent dissipation rate: ε_s/Sk in cases A_1.

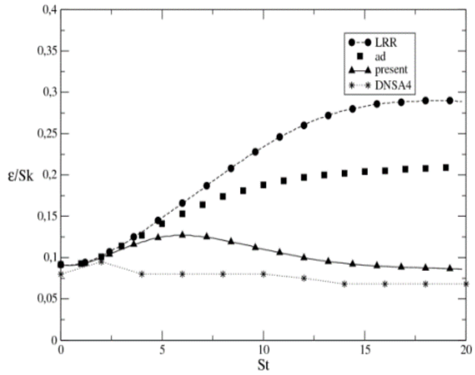

Fig. 14. Time evolution of the turbulent dissipation rate: ε_s/Sk in cases A_4.

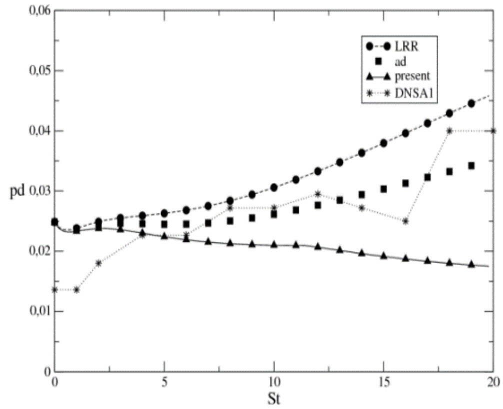

Fig. 15. Time evolution of the pressure-dilatation correlation $pd = \frac{(-\overline{p'd'}/\bar{\rho}+\varepsilon_c)}{Sk}$ in case A_1.

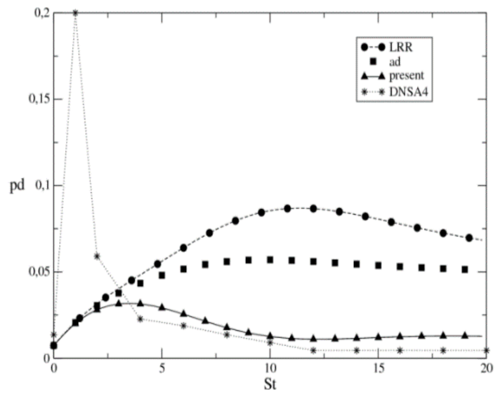

Fig. 16. Time evolution of the pressure-dilatation correlation $pd = \frac{(-\overline{p'd'}/\bar{\rho}+\varepsilon_c)}{Sk}$ in case A_4.

Figs. (15, 16) present the behavior of the dilatational terms in cases A_1 and A_4.It will be shown that these terms are much smaller to explain the compressibility effect on the turbulence. Using Eq. (34. One can notice that the compressibility effect of decreased growth rate of turbulent kinetic energy is due to a decrease of the normalized production term. It will be shown from cases A_1, A_2, A_3 and A_4that the asymptotic values of turbulent parameters are highly dependent on the initial conditions when Mg is changed. This shows that the gradient Mach number is an important parameter that describes the level of stabilizing effect of compressibility.

Our model is actually designed to study cases of high- compressibility and to the limit moderate compressibility. Whereas, the case A1 correspond to a low-compressibility, it is clear that even the incompressible models can describe the evolution of the pressure-dilatation term such as is shown in Fig. 15.

Generally the modeling of the pressure-strain correlation is calibrated for a large time relatively to equilibrium conditions, therefore, all the models cannot predict accurately these terms for enough small time.

6. CONCLUSION

In this study, the compressible models are used to describe the evolution of the turbulence and the performances of these models which are compared to the results of numerical simulation of Sarkar (1995). The standard model for the pressure-strain correlation of L. R. R. yields poor predictions for compressible homogeneous shear flow. It is clear that this model is unable to predict the effect of compressibility, while the predictions of the compressible models using the turbulent Mach number yield encouraging result. The present model is an extension of the LRR model involving the gradient Mach number Mg with the commonly used Mt appears to be able to predict accurately the structural compressibility effects. Therefore, the gradient Mach number Mg is concluded to be an important parameter in addition to Mt in the modeling of the pressure-strain correlation for high compressible homogeneous turbulence.

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