Evaluation Study of Pressure-Strain Correlation Models in Compressible Flow

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ABSTRACT

This paper is devoted to the second-order closure for compressible turbulent flows with special attention paid to modeling the pressure-strain correlation appearing in the Reynolds stress equation. This term appears as the main one responsible for the changes of the turbulence structures that arise from structural compressibility effects. The structure of the gradient Mach number is similar to that of turbulence, therefore this parameter may be appropriate to study the changes in turbulence structures that arise from structural compressibility effects. Thus, the incompressible model (LRR) of the pressure-strain correlation and its corrected form by using the turbulent Mach number, fail to correctly evaluate the compressibility effects at high shear flow. An extension of the widely used incompressible model (LRR) on compressible homogeneous shear flow is the major aim of the present work. From this extension the standard coefficients $C_i$ became a function of the compressibility parameters (the turbulent Mach number and the gradient Mach number). Application of the model on compressible homogeneous shear flow by considering various initial conditions shows reasonable agreement with the DNS results of Sarkar. The ability of the models to predict the equilibrium states for the flow in cases $A_1$ and $A_4$ from DNS results of Sarkar is examined, the results appear to be very encouraging. Thus, both parameters $M_t$ and $M_g$ should be used to model significant structural compressibility effects at high-speed shear flow.

Keywords: Compressible; Turbulence; Pressure-strain; Model of turbulence; Shear flow.

NOMENCLATURE

- $b_{ij}$: Reynolds stress anisotropy
- $c$: speed of sound
- $M_t$: turbulent Mach number
- $M_g$: gradient Mach number
- $P$: pressure
- $R$: specific gas constant
- $T$: time
- $T$: temperature
- $U_i$: velocity in the direction, $x_i$
- $\epsilon_s$: solenoidal part of the dissipation
- $\delta_{ij}$: Kronecker delta
- $(\cdot)'$: Reynolds fluctuation
- $(\cdot)_i$: spatial gradient
- $\Phi_{ij}$: pressure-strain correlation
- $\gamma$: specific heat ratio

1. INTRODUCTION

Compressible turbulence modeling is an essential element for calculations of many problems of practical engineering interest, such as combustion, environment and aerodynamics. The compressibility phenomena have been extensively studied and numerical simulation of compressible turbulent flows using compressible turbulence models have been performed by many authors. Previous studies carried out in the last 30 years have conjectured that compressibility effect was linked with dilatational dissipation and pressure-dilatation correlation, as it is represented by the models of Zeman (1990), Sarkar (1991) and others. During the last thirty years, many theoretical and experimental works have been developed primarily to understand and predict the behavior of turbulent flows. In fact, the models which developed turbulence gave full satisfaction in simple configurations for the homogeneous flows.

In this context, many studies, Sarkar and Lakshmanan (1991), Bradshaw (1977), H.
Marzougui et al. (2005) Carlos A. Gomez (2013),
Hechmi et al. (2008) of the compressible shear flow
which show the changes of the turbulence structures
are principally due to the structural compressibility
effects which significantly affect the pressure-strain
correlation. Thus, the pressure-strain correlation
appears as the main factor for the changes in
the magnitude of the Reynolds stress anisotropies.
The extension of the standard models to compressible
flows represents a research topic of great scientific
and industrial interest.

A major challenge related to this extension is to
take into account the compressibility effects in the
classical scheme closure of turbulence.

It is well concluded that the Favre Reynolds stress
closure using the standard models of the pressure-
strain correlation with the addition of the compressible
dissipation and pressure-dilatation correlation models
failed to predict high compressible flows (Speziale et al.
1995), Sarkar (1995) and Hamba (1999) also performed DNS
results of compressible homogeneous shear flow
and reached similar conclusions concerning the
roles of dilatational terms.

The consequent effects on the pressure-strain
correlation may cause significant changes on
turbulence structures. According to the DNS results
of Sarkar (1995), there is a reduction in the
magnitude of the Reynolds shear stress anisotropy
and an increase in the magnitude of the normal
stress anisotropy. As a consequence, the pressure
strain modeling seems to be an important issue in
the second order closures for the compressible
turbulent flows.

A method of including compressibility effects in
the pressure strain correlation is the subject of the
present study. The LRR model developed by
Lauder-Reece and Rodi (1975) has shown a great
success in simulating a variety of incompressible
complex turbulent flows. Thus, a compressible
correction for the LRR model is the major aim of
this study. In the present work, the correction
concerns essentially the Cl coefficients which
became in a compressible situation a function of the
turbulent Mach number and the gradient Mach
number.

In the present study, we concentrate on evaluating
the models of the pressure-strain correlation in the
homogeneous shear flow.

2. GOVERNING EQUATIONS

The General equations governing the motion of a
compressible fluid are the Navier-Stokes equations.
They can be written as follows for mass, momentum
and energy conservation:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0
\]

(1)

\[
\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = \nabla \cdot \mathbf{S} + \rho \mathbf{g} - \mathbf{p} \frac{\partial \mathbf{u}}{\partial x_j}
\]

(2)

\[
\frac{\partial (\rho e)}{\partial t} + \nabla \cdot (\rho \mathbf{u} e) = \nabla \cdot \mathbf{q} - \rho \mathbf{u} \cdot \mathbf{g} - \rho \mathbf{u} \cdot \mathbf{u} \cdot \mathbf{g} - \frac{\partial (P)}{\partial x_j}
\]

(3)

Here \( \rho \) is the density, \( \mathbf{u} \) is the velocity, \( P \) is the
pressure, \( e \) is the internal energy, \( T \) is the
temperature, \( \mu \) is the viscosity, \( v \) is the thermal
conductivity and \( C_v \) is specific heat at constant
volume. Where:

\[
\sigma_{ij} = -p \delta_{ij} + \tau_{ij}
\]

(4)

\[
e = C_v T
\]

(5)

\[
\tau_{ij} = 2 \mu S_{ij}
\]

(6)

\[
S_{ij} = \left( u_{ij} + u_{ji} \right) / 2
\]

(7)

For an ideal gas, the relation between pressure,
density and temperature can be written as follows:

\[
p = \rho RT
\]

(8)

3. BASIC EQUATIONS OF THE
FAVRE SECOND-ORDER
CLOSURE IN COMPRESSIBLE
FLOWS

For compressible homogeneous shear flow, the
mean gradient velocity is given by:

\[
\bar{U}_{ij} = S_{ij} \delta_{i,j} = \begin{cases}
(\mathbf{S} \cdot \mathbf{i}, \text{if } (i,j) = (1,2)) \\
(0, \text{if } i \neq 1, j \neq 2)
\end{cases}
\]

(9)

Where \( S \) is the constant mean shear rate. These
considerations lead to:

\[
\bar{U}_{i,k} = 0, \bar{p} = \text{constant}
\]

(10)

The Favre equations for conservation of mass,
momentum and energy are:

\[
\frac{\partial}{\partial t} \left( \rho \bar{u}_i \right) + \frac{\partial}{\partial x_j} \left( \rho \bar{u}_i \bar{u}_j \right) = \frac{\partial}{\partial x_j} \left[ \tau_{ij} + \bar{p} \delta_{ij} \right]
\]

(11)

\[
\frac{\partial}{\partial t} \left( \rho \bar{C}_v \bar{T} \right) + \frac{\partial}{\partial x_j} \left( \rho \bar{C}_v \bar{T} \bar{u}_j \right) = -\bar{p} \frac{\partial}{\partial x_i} \bar{U}_i
\]

(12)

\[
\frac{\partial}{\partial t} \left( \bar{p} \bar{C}_v \bar{T} \right) - \frac{\partial}{\partial x_j} \left( \bar{p} \bar{C}_v \bar{T} \bar{u}_j \right) = -\bar{p} \frac{\partial}{\partial x_i} \bar{U}_i
\]

(13)

Where:

\[
\bar{u}_i = 2 \bar{u}_i \delta_{i,j} - \frac{2}{3} \bar{U}_{i,k} \delta_{i,j}
\]

(14)

\[
\bar{S}_{ij} = \left( \bar{U}_{ij} + \bar{U}_{ji} \right) / 2
\]

(15)

\[
\bar{\tau}_{ij} = 2 \bar{S}_{ij} - \frac{2}{3} \bar{U}_{i,k} \delta_{i,j}
\]

(16)

\[
\bar{\Phi} = \bar{\tau}_{ij} \bar{u}_{ij}
\]

(17)

Classically, the second-order closure requires a
transport equation of the turbulent dissipation rate.
The new concept of dissipation in compressible
turbulence was proposed by Sarkar et al. (1995),
Zeman (1991) and Ristorcelli (1997) and can be
written as follows:

\[
\varepsilon_{ij} = \frac{2}{3} \varepsilon \delta_{ij}
\]

(18)
Where for homogeneous turbulence $\tilde{\rho}e_k = \frac{\mu_0}{\alpha} \omega_k - \rho e_k$ is the fluctuating vorticity and $\tilde{\rho}e_k = \frac{4}{3} \mu \left( \frac{\epsilon}{\alpha} \right)^2$ represent the solenoidal and compressible parts of $\epsilon$ respectively. Sarkar et al. (1990) have mentioned that for moderate Mach numbers, $\epsilon_k$ is insensitive to the compressibility changes. For $\epsilon_k$, model transport equation, similar to what it was obtained in the incompressible case. Such a model equation is written in (1985), namely:

$$\tilde{\rho} \frac{d\epsilon_k}{dt} = -C_{el} \tilde{\rho} \omega_k u_i u_j \tilde{u}_{ij} + \tilde{\rho} u_k \epsilon_{ij} - C_{el} \tilde{\rho} \omega_k^2$$  \hspace{1cm} (19)

Where $C_{el}$ and $C_{el}$ are respectively the model constants $C_{el}=1.44$ and $C_{el}=8.13$.

The turbulent stress is solutions of the transport equation:

$$\tilde{\rho} \frac{d \tilde{u}_i}{dt} = P_i + \phi_{ij} + \frac{2}{3} \tilde{\rho} \tilde{d} \delta_{ij} - \frac{8}{3} \tilde{\rho} \delta_{ij}$$  \hspace{1cm} (21)

Where $P$ the turbulent kinetic energy production:

$$P_{ij} = -\tilde{\rho} u_i \tilde{u}_j \tilde{U}_{ij}$$  \hspace{1cm} (22)

$$\phi_{ij} = \tilde{\rho} \left( \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right) - \frac{2}{3} \tilde{\rho} \tilde{d} \delta_{ij}$$  \hspace{1cm} (23)

The turbulent kinetic energy equation is

$$\tilde{\rho} \frac{d \tilde{\epsilon}}{dt} = -\tilde{\rho} u_i \tilde{u}_j \tilde{U}_{ij} + \tilde{\rho} \tilde{d} \tilde{\epsilon}$$  \hspace{1cm} (24)

The turbulent Mach number is described in (1985) by the transport equation as follows

$$\frac{d\tilde{M}_{ij}}{dt} = M_{ij} \tilde{P} s + M_{ij} \left[ 1 + \frac{1}{2} \gamma (\gamma - 1) M_{ij}^2 \right] \left( \tilde{\rho} \tilde{d} \tilde{\epsilon} - \tilde{\rho} \tilde{e} \right)$$  \hspace{1cm} (25)

4. MODEL OF TURBULENCE

It is generally accepted that the term of the pressure-dilatation plays an important role in the modeling of turbulent compressible flows and behavior analysis of the turbulence. Indeed, this term which appears in the equation of turbulent kinetic energy seems to have important effects on the evolution of turbulence. Such effect is manifested by a reduction in the turbulent kinetic energy when the turbulent Mach number increases. In this context, the results of the numerical simulation developed by Sarkar et al. (1991,1992,1995) and Blaisdell et al. (1991) have already shown that the pressure-dilatation correlation is an important indicator for the compressibility.

In this part, we will choose a number of models from literature that are well-known and that express the pressure-dilatation.

Model of Sarkar(1992)

The algebraic form of the model developed by Sarkar et al. (1992) and expressing the pressure-dilatation correlation is as follows:

$$\tilde{p} \frac{d\tilde{T}}{dt} = -\alpha_2 P_{\tilde{M}_s} + \alpha_3 \tilde{\rho} e_s M_{\tilde{f}}^2$$  \hspace{1cm} (26)

$$\alpha_2 = 0.15$$

$$\alpha_3 = 0.2$$

Model of Zeman(1990)

$$\tilde{p} \frac{d\tilde{T}}{dt} = -\frac{1}{2} \tilde{\rho} \left( \frac{C_P}{\gamma} \right) (\tilde{\rho} \tilde{T})^{-\frac{1}{2}} \left[ \frac{\tilde{D}_{\tilde{T}}^3 - \tilde{D}_{\tilde{T}}^3}{\tau} + \frac{(\tilde{\gamma} - 2\tilde{\gamma})}{(12)} \tilde{\rho}^2 \tilde{T} \tilde{D}_{\tilde{T}} \right]$$  \hspace{1cm} (27)

Where $\tau$ is the acoustic time scale,

$$\tau = \frac{k}{\tilde{\rho} M_{\tilde{f}}}$$  \hspace{1cm} (28)

$C_{p_{\tilde{f}}} = \frac{2\tilde{\rho}^2 k_\gamma R T \left( \frac{M_{\tilde{f}}^2 + M_{\tilde{f}}^2}{1 + M_{\tilde{f}}^2} \right)}{(29)}$

Model of F. Hamba(1999)

$$\tilde{p} \frac{d\tilde{T}}{dt} = \left[ 1 - (C_{pd} \tilde{\rho}) \right] \left[ C_{pd} \tilde{M}_{\tilde{f}}^2 \frac{D_{\tilde{f}} \tilde{U}_{ij}}{M_{\tilde{f}}} (\tilde{\rho} \tilde{T}) \right]$$  \hspace{1cm} (30)

Where

$$\tilde{\rho} \tilde{T} = \left( \frac{C_{pd}^{\tilde{f}}}{2>Cpd<\tilde{f}} \right)$$  \hspace{1cm} (31)

When $C_{pd}(i=1,3)$ are numerical constants

Model of El Baz and Launder (1996)

$$\tilde{p} \frac{d\tilde{T}}{dt} = -F \left( \tilde{\rho} - \frac{2}{3} \tilde{\rho} \tilde{U}_{ij} \tilde{U}_{ij} \right), F = 1.5 M_{\tilde{f}}^2$$  \hspace{1cm} (32)

Model of Ristocelli (1997)

$$\tilde{p} \frac{d\tilde{T}}{dt} = \frac{C_{rd} M_{\tilde{f}}^2}{C_{rd} + C_{rd} M_{\tilde{f}}^2} \left( R_{ij} \tilde{U}_{ij} - \tilde{e}_s \right)$$  \hspace{1cm} (33)

We note that there are other models using different compressibility parameters as the density variance, for example those developed by Taulbee et al. (1991), Hamba (1999), Simo et al. (1997), Hanafi (2013), and Bogdanoff et al. (1983).

For compressible homogeneous shear flow, the DNS results of Sarkar (1995) show the change in the magnitude of the components of Reynolds stress anisotropy tensor and the growth rate of turbulent kinetic energy. Thus, the compressibility has a significant effect on the pressure fields and in consequence on its correlations such as the pressure-strain, the pressure-dilatation. The work of Sarkar (1991) recommends to use the gradient Mach number with the turbulent Mach number in order to predict the correct behavior of compressible turbulence. This is the point on which the present work is centered. At first, we have examined some basic assumptions which were used in constructing turbulence models. For compressible homogeneous shear flow, the growth rate of the turbulent kinetic energy is given by the following equation.

Model of Sarkar(1992)
\[ \frac{\Delta}{\Delta t} = 1 \ \frac{dK}{SK} = \frac{P - \varepsilon + \frac{p'd'}{S}}{SK} = -2b \varepsilon + \frac{\varepsilon}{\beta} - \varepsilon c \]  

(Vreman et al. 1996) performed DNS results for compressible mixing layers. From which, they deduced that the diagonal rapid part of the pressure-strain correlation are approximately proportional to the growth rate. Pantano and Sarkar (2003) used DNS results to study the compressibility effects in the high-speed turbulent shear layer. They showed that the components of the pressure-strain correlation and the pressure variance normalized by its incompressible counterparts decrease similarly.

\[ \frac{\Phi_{ij}}{(\varepsilon_{ij})} \sim \frac{\frac{\Delta^2}{p^2}}{\frac{\Delta^2}{p^2}} \]  

The hypothesis about an approximate proportionality between \( \frac{\varepsilon_{ij}}{(\varepsilon_{ij})} \) and \( \Delta^j \) can be supported as in H.Marzougui and al.(2005) and we can write.

\[ \Phi_{ij} \sim \frac{\Delta^j}{(\varepsilon_{ij})} \]  

Now, we consider the equation

\[ \frac{\Phi_{ij}}{(\varepsilon_{ij})} = \frac{\Delta^j}{(\varepsilon_{ij})} \]  

It comes

\[ \Phi_{ij} = \frac{\Delta^j}{(\varepsilon_{ij})} \]  

The turbulent kinetic energy \( K \) and \( K' \) satisfied the transport equations

\[ \frac{dK}{dt} = P - \varepsilon c - \varepsilon + \frac{p'd'}{S} \]  

\[ \frac{dK'}{dt} = P' - \varepsilon c \]  

From the last two equations, the time derivative ratio in RHS of Eq. (38) can be written as

\[ \frac{\Delta^j}{(\varepsilon_{ij})} = \left( 1 - \frac{\varepsilon}{\varepsilon c} \right) \left( 1 + \frac{p'd'}{p - \varepsilon} \right) \]  

The DNS results of Sarkar(1995) show that the relative dissipation \( \varepsilon c/Pc \) less affected by compressibility and it shows a similar equilibrium value as its incompressible counterpart \( \varepsilon c/Pc \). Thus, the term between the square brackets in (41) can be approximated by 1 and we can obtain

\[ \frac{dK}{dt} = \left( 1 - \frac{\varepsilon}{\varepsilon c} \right) \left( 1 + \frac{p'd'}{p - \varepsilon} \right) \]  

\[ \frac{dK'}{dt} = \left( 1 - \frac{\varepsilon}{\varepsilon c} \right) \left( 1 + \frac{p'd'}{p - \varepsilon} \right) \]  

\[ \beta = \left( \frac{1}{\varepsilon c} \right) \]  

Where, \( \beta \) is a numerical constant can be determined by the equilibrium value of \( \varepsilon c/Pc \). From eqs.(38,42), we have

\[ \frac{dK}{dt} = \left( 1 - \frac{\varepsilon}{\varepsilon c} \right) \left( 1 + \frac{p'd'}{p - \varepsilon} \right) \]  

Pantano and Sarkar (1991) use two scales to express \( \Phi_{ij}/\Phi_{ij}' \)by a function of the turbulent Mach number and gradient Mach number.

\[ \frac{\Phi_{ij}}{(\varepsilon_{ij})} = 1 - f(Mv, Mg) \]  

Where \( f(Mv, Mg) \) is a function model of turbulent Mach number and gradient Mach number.

\[ f(Mv, Mg) = M_V^2 \frac{b - \left( \frac{3}{a} \right) (\varepsilon c + b)}{b + \left( \frac{3}{a} \right) (\varepsilon c + b)} \]  

\[ p'd' = -f(Mv, Mg)P + aM_V^2 \varepsilon c \]  

\( a=0.9, \beta=0, b=4.2 \)

The present work deals with the problem of modeling of the pressure-dilatation, for this reason we choose a number of models from literature that are well-known.

Lauder Reece and Rodi(1975)

The models established in compressible turbulent flows are deduced by a simple extension of their incompressible counterparts. This extension concerned only one incompressible model, it is due to Lauder, Reece and Rodi (1975), this model is written in the following form:

\[ \Phi_{ij} = -C_1 \varepsilon c b_{ij} + C_3 \varepsilon c \delta_{ij} \]  

\[ C_1 b_{ij} S_{ij} + C_3 b_{ij} S_{ij} - \frac{2}{3} b_{ij} S_{ij} \delta_{ij} \]  

\[ C_1 b_{ij} S_{ij} + b_{ij} \delta_{ij} \]  

Where, \( C_1, C_2, C_3 \) and \( C_4 \) are constants that take on the values of: \( C_1=3, C_2=0.8 \) et \( C_3=1.75 \) et \( C_4=1.31 \)
of this term is written as:

$$\Phi_{ij}^t = -C_1 \bar{p} s_i b_{ij} + \bar{p} k \left( \frac{1}{2} + \frac{1}{2} \delta_{ij} \right) \left( 1 - \frac{1}{2} \delta_{ij} \delta_{ij} \right) +$$

$$2 \bar{p} k \left( 1 - C_3 + 2 d_2 \right) \left[ b_{ik} \bar{s}_{jk} + b_{jk} \bar{s}_{ik} -$$

$$\frac{1}{2} b_{im} \bar{s}_{mj} \delta_{ij} \right] - (1 - C_4 - 2 d_2) \bar{p} k \left( b_{ik} \bar{d}_{jk} + b_{jk} \bar{d}_{ik} -$$

$$\frac{1}{2} b_{im} \bar{d}_{mj} \delta_{ij} \right)$$

(52)

Where: $C_3 = C_3^l + 0.3 M_t$

$C_4 = C_4^l - 0.3 M_t$

**Present Model**

$$\bar{p} \bar{d}^t = -f(M_t, M_g) p + \alpha M_t^2 \delta_s$$

(53)

Where: $C_3 = C_3^l + M_t^2 \left( \frac{b \bar{e} - \left( \alpha M_g - \beta \right)^2}{1 + \alpha M_t^2} \right)$

$C_4 = C_4^l - M_t^2 \left( \frac{b \bar{e} - \left( \alpha M_g - \beta \right)^2}{1 + \alpha M_t^2} \right)$

In Eqs. (52) and (53), the coefficients $C_3^l$ and $C_4^l$ are those of the incompressible LRR model.

$C_3^l = 1.75$

$C_4^l = 1.31$

5. **RESULTS AND DISCUSSION**

The transport Eqs. (19), (21), (24) and (25) on which the second order closure for compressible homogeneous shear flow is based, are solved using the fourth-order accurate Runge-Kutta numerical scheme.

The ability of the proposed model to predict the anisotropy of compressible homogeneous turbulent shear flow will be now considered. The model predictions will be compared with DNS results conducted by Sarkar (1995) for cases $A_1$ and $A_4$. These cases correspond to different initial conditions for which the initial values of the gradient Mach number $M_g$ change by changing the initial values of $\frac{Sk}{\varepsilon}$ and taking $M_t$ constant as it is listed in Table 1.

Figures (1, 2), (3, 4) and (5, 6) show the non-dimensional time ($St$) variation of the Reynolds stress anisotropies $b_{11}$, $b_{22}$, and $b_{12}$. From these figures, it is clear that at low-$M_g$ values, Figs. (1, 3 and 5), the Adumitroae $et al.$ (1999) model is in agreement with DNS data but at high-$M_g$ values Figs. (2, 4, 6), the proposed model appears to be able to predict correctly the significant decrease in the normalized turbulent production term $-2b_{12}$ and the increase in the streamwise term $b_{11}$, as well as the transverse $b_{22}$ Reynolds stresses anisotropies with increasing the gradient Mach number. Results obtained with Adumitroae $et al.$’s model (1999) disagree with DNS data, especially at high-$M_g$ values in case $A_4$, this model is still unable to predict the changes in the magnitude of the Reynolds stress anisotropy when the compressibility is higher.

### Table 1: Initial condition for the DNS results of Sarkar (1995) in compressible homogeneous shear flow

<table>
<thead>
<tr>
<th>Case</th>
<th>$M_t$</th>
<th>$M_g$</th>
<th>$\frac{Sk}{\varepsilon}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.4</td>
<td>0.22</td>
<td>1.6</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.4</td>
<td>0.44</td>
<td>3.6</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.4</td>
<td>0.66</td>
<td>5.4</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.4</td>
<td>1.32</td>
<td>10.8</td>
</tr>
</tbody>
</table>
From all the figures, the incompressible Launder, Reece and Rodi model (1975) is unable to predict the dramatic changes in the magnitude of the Reynolds stress anisotropy that arise from compressibility, while the present Model provides an acceptable performance in reproducing the DNS results in cases A. This model explains the importance of the evolving of $M_g$ with the commonly used the parameter $M_t$ in modeling the high compressible turbulent flow.

Figs. ((7, 8); (9, 10) and (11, 12)) present the behavior of the pressure-strain correlation. As can
be seen in these figures, the present model yields acceptable results that are in good qualitative agreement with the DNS data, especially at high gradient Mach number (case A4).

Figs. (13, 14) present the behavior of the normalized dissipation $\varepsilon_s/\Sk$, $(\varepsilon_s/\Sk=-2b_{12} \varepsilon_P)$ for cases A1 and A4. It can be seen that there is a decrease in $\varepsilon_s/\Sk$ when $M_g$ increases, since the compressibility effects cause the significant reduction in the Reynolds turbulent shear stress $b_{12}$ from numerical simulation cases A1 and A4 of the previous DNS results. It is clear that the proposed model is in accordance with the DNS results.

Figs. (15, 16) present the behavior of the dilatational terms in cases A1 and A4. It will be shown that these terms are much smaller to explain the compressibility effect on the turbulence. Using Eq. (34. One can notice that the compressibility effect of decreased growth rate of turbulent kinetic energy is due to a decrease of the normalized production term. It will be shown from cases A1, A2, A3 and A4 that the asymptotic values of turbulent parameters are highly dependent on the initial conditions when M_g is changed. This shows that the
gradient Mach number is an important parameter that describes the level of stabilizing effect of compressibility.

Our model is actually designed to study cases of high-compressibility and to the limit moderate compressibility. Whereas, the case A1 correspond to a low-compressibility, it is clear that even the incompressible models can describe the evolution of the pressure-dilatation term such as is shown in Fig. 15.

Generally the modeling of the pressure-strain correlation is calibrated for a large time relatively to equilibrium conditions, therefore, all the models cannot predict accurately these terms for enough small time.

6. CONCLUSION

In this study, the compressible models are used to describe the evolution of the turbulence and the performances of these models which are compared to the results of numerical simulation of Sarkar (1995). The standard model for the pressure-strain correlation of L. R. R. yields poor predictions for compressible homogeneous shear flow. It is clear that this model is unable to predict the effect of compressibility, while the predictions of the compressible models using the turbulent Mach number yield encouraging result. The present model is an extension of the LRR model involving the gradient Mach number M_{g} with the commonly used M_{t} appears to be able to predict accurately the structural compressibility effects. Therefore, the gradient Mach number M_{g} is concluded to be an important parameter in addition to M_{t} in the modeling of the pressure-strain correlation for high compressible homogeneous turbulence.

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