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Effects of Variable Viscosity on Power-Law Fluids over a Permeable Moving Surface with Slip Velocity in the Presence of Heat Generation and Suction

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ABSTRACT

In this paper, a numerical investigation on the effects of variable viscosity, slip velocity and heat generation or absorption on power-law fluids with heat and mass transfer over a moving permeable surface is carried out. The transformation of the governing boundary layer equations into ordinary differential equations has been performed by applying similarity transformations. The transformed governing equations are numerically solved by using MATLAB BVP solver bvp4c. The obtained results are presented graphically and discussed for various values of the viscosity parameter, the slip parameter, the heat generation or absorption parameter, the Eckert number and Lewis number. The result shows that, the variable viscosity parameter $\theta_c \to -\infty$, it is confirmed that the local skin-friction coefficient decreases while heat and mass transfer rates increases. The heat and mass transfer rates increases rapidly on increasing the Prandtl number. The rate of mass transfer is rapidly increased when the Lewis number increased.

Keywords: Power-law fluids; Variable viscosity; Slip velocity; Heat generation or absorption; Suction or injection; Heat and mass transfer.

NOMENCLATURE

C	concentration
C_∞	concentration at the free stream
Cf	skin-friction coefficient
c_p	specific heat at constant pressure
C_w	concentration at the wall
D_m	mass diffusivity
Ec	Eckert number
f	dimensionless streamfunction
f_w	suction or injection parameter
Le	Lewis number
n	flow behavior index
Nu	Nusselt number
p	pressure
Pr	Prandtl number
Q_0	volumetric heat generation
Re	Reynolds number
Sh	Sherwood number
T	temperature
T_∞	temperature of the uniform flow
v_w	constant velocity normal to the wall
x, y	co-ordinate system
α	thermal diffusivity
γ	heat generation or absorption parameter
η	dimensionless similarity variable
θ	dimensionless temperature
θ_c	variable viscosity parameter
λ	slip parameter
λ_1	slip coefficient
μ	viscosity
μ_∞	free stream dynamic viscosity
υ	kinematic viscosity
ρ	fluid density
ϕ	dimensionless concentration
ψ	streamfunction

Subscripts

w	condition at the wall

T_w	wall temperature
U	constant velocity
u, v	velocity components
∞	condition at the ambient medium

Superscript

'	differentiation with respect to η

1. INTRODUCTION

The study of flow, heat and mass transfer of power-law fluids over a continuously moving surface in a quiescent fluid has been received a considerable attention in literature due to its numerous industrial applications such as continuous casting, performance of lubricants, hot rolling, cooling of an infinite metallic plate in a cooling bath, food process, paper and petroleum production. Materials which are manufactured by extrusion processes and heat-treated materials travelling between a feed roll and a wind-up roll or on conveyor belts possess the characteristics of moving continuous surface. The mass transfer problems have great importance in extending the theory of separation processes and chemical kinetics. This phenomenon has attained a great significance in chemical industries, reservoir engineering and in various applications. Many of the power-law fluids encountered in engineering processes are known to follow the empirical Ostwald–de Waele power-law model. Since the physical properties of the ambient fluid effectively influence the boundary layer characteristics, the analysis of non-Newtonian power-law fluid flow over a moving sheet also plays a vital role in engineering applications. Similarity solutions for non-Newtonian fluids were obtained by Kapur and Srivastava (1963) and Lee and Ames (1966).

Boundary layer flow generated by a continuous solid surface in quiescent fluid has been studied by many authors. Sakiadis (1961) initiated the study of boundary layer flow over a continuous solid surface moving with a constant speed. Because of entrainment of ambient fluid, this boundary layer flow is quite different from that of a semi-infinite flat plate. Fox *et al.* (1969) investigated the laminar boundary layers on a continuous moving flat plate in the non-Newtonian fluids characterized by power-law model. Hassanien (1996) concluded that the solutions of the problems of flow past a moving continuous flat surface depend not only on the velocity difference but also on the velocity ratio. Howell *et al.* (1997) applied a general power series to describe the fluid velocity and temperature for the problem of combined momentum and heat transfer in the boundary layer of the moving sheet. Grosan *et al.* (2000) obtained the similarity solutions for boundary layer flows on a moving surface in non-Newtonian power law fluids. Gireesha *et al.* (2014) examined boundary layer flow and heat transfer over a stretching sheet.

Tsou *et al.* (1967) confirmed that the mathematically described boundary-layer problem on a continuous moving surface is physically reasonable. Tsai and Hsu (1995) proved that the temperature level is found to decay gradually with distance along the moving surface. An increase in the value of the Prandtl number results in an increase of the Nusselt number for each fluid. Sahu *et al.* (2000) obtained that the displacement thickness is much thicker for pseudo plastic fluids than for Newtonian and dilatant fluids. Zheng and Zhang (2002) investigated the applicability of boundary layer theory for the flow of power-law fluids on a moving plate. They concluded that thermal boundary layer thickness decreases with power-law index. Chen (2003) observed that the free stream velocity, choice of the fluid, and velocity difference between the ambient fluid and the surface affect the thermal behavior of the continuous sheet in materials processing and is useful to determine the quality of the final products. Fang (2003) found that the viscous dissipation heating becomes dominant when the Eckert number is increased. Zhang *et al.* (2007) reported that an increase in viscous dissipation enhances greatly the local heat transfer leading to temperature overshoots adjacent to the wall for low values of the velocity ratio. Chandra and Kumar (2013) studied the combined effect of chemical reaction, radiation on heat and mass transfer along a continuously moving surface in presence of thermophoresis. Gangadhar (2015) carried out an investigation on the boundary layer flow with a convective surface boundary condition. It was concluded that increasing the Eckert number tend to reduce the thermal boundary layer thickness.

Erickson *et al.* (1966) discussed the problem involving the boundary layer flow over a continuous solid moving surface when the sheet is maintained at a constant temperature with suction or blowing. Chen and Char (1988) proved that the thermal boundary-layer thickness decreases with the increase in Prandtl number and suction or blowing parameter. Chen (1999) concluded that the heat transfer is enhanced due to increasing the values of the free stream velocity, the injection parameter, and Prandtl number. Rao *et al.* (1999) carried out a work on the momentum and heat transfer in the laminar boundary layer of a non-Newtonian power-law fluid flowing over a flat plate with injection/suction. Ishak *et al.* (2009) found that suction increases the skin friction coefficient compared to injection and the heat transfer rate at the surface delays the boundary layer separation. Mahmoud and Megahed (2009) observed that the skin-friction coefficient and the Nusselt number are increased as the suction parameter increases. Deswita *et al.* (2010) obtained that the effect of suction was found to increase the skin friction while injection decreases it. Gangadhar and Reddy (2013) found that an increase in wall suction enhances the boundary layer thickness and reduce the skin friction together with the heat and mass transfer rate at the moving plate surface.

The flow of a viscous fluid with temperature dependent properties has been discussed broadly in the literature since it has most powerful practical applications such as polymer processing industries, food processing, bio-chemical industries and enhanced recovery of petroleum resources. The viscosity of the fluids used in industries like polymer fluids, fossil fuels etc. varies rapidly with temperature. Consequently, consideration of variable viscosity of the fluid is of physical interest instead of constant viscosity. Lai and Kulacki (1990) examined the effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium. They concluded that a significant error in the heat transfer coefficient occurs if the variable viscosity is neglected. Pantokratoras (2004) studied the further results on the variable viscosity on the flow and heat transfer to a continuous moving flat plate. Soundalgekar *et al.* (2004) discussed the effect of variable viscosity on boundary layer flow along a continuously moving plate with variable surface temperature. Mureithi *et al.* (2013) observed that the viscosity variation parameter has notable effects on the temperature dependent viscosity and the velocity and temperature distribution within the boundary layer.

The inclusion of internal heat generation in a problem reveals that it affects the temperature distribution strongly. Since internal heat generation is related in the fields of disposal of nuclear waste, storage of radioactive materials, nuclear reactor safety analyses, fire and combustion studies and in many industrial processes, considering internal heat generation becomes a key factor in the engineering applications. Heat generation can be assumed to be constant, space dependent or temperature dependent. Crepeau and Clarksean (1997) applied a space dependent heat generation in their study on flow and heat transfer from vertical plate. They reported that the exponentially decaying heat generation model can be used in mixtures where a radioactive material is surrounded by inert alloys. Elbashbeshya and Bazidb (2004) studied the effect of temperature-dependent viscosity on heat transfer over a continuous moving surface with variable internal heat generation. Mahmoud (2011) observed that the local skin-friction coefficient is decreased as the slip parameter increased. Moreover, it is found that the local Nusselt number is decreased as the slip parameter increased.

All the previous studies cited above explain the free convection flow and the heat transfer characteristics in moving surface with or without variable viscosity in detail. Many works in the literature deals the significance of the existence or absence of injection/suction and heat generation/absorption. It is clear that the variable viscosity and mass transfer in the presence of heat generation and suction in a moving permeable surface is a special case of interest. This article aims to provide the knowledge about the effects of variable viscosity, slip velocity on a moving surface with non-Newtonian power-law fluids with heat generation or absorption with suction or injection for various values of the flow behavior index as a first effort of its kind.

2. MATHEMATICAL MODEL

Consider the coordinate system and flow model as shown in Fig. 1. The boundary layer flow under investigation is two-dimensional, steady state, laminar, viscous and incompressible. Further, the heat and mass transfer over a continuously moving permeable surface with constant velocity U through power-law fluids at rest are taken into account for the analysis.

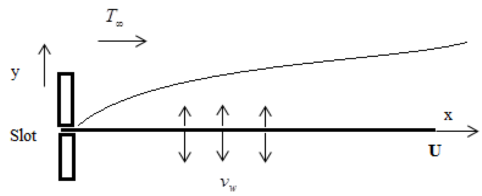


Fig. 1. Physical model and co-ordinate system.

With the usual Boussinesq and the boundary layer approximations, the governing equations of continuity, momentum, energy and concentration are written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\left(\mu\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left|\frac{\partial u}{\partial y}\right|^{n+1}+\frac{Q_0(T-T_\infty)}{\rho c_p} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2} \tag{4}$$

where u and v are the velocity components in the x and y directions, respectively. ρ and α denote, respectively, the fluid density and the thermal diffusivity. n is the power-law viscosity index, T is the temperature of the fluid, c_p is the specific heat at constant pressure, C is the concentration of the fluid and D_m is the mass diffusivity.

The viscosity of the fluid is assumed to be an inverse linear function of temperature as $\mu=\frac{\mu_\infty}{(1+\delta(T-T_\infty))}$, where δ is the thermal property of the fluid, and this is reasonable for liquids such as water and crude oil (Ling and Dybbs 1987).

The volumetric rate of heat generation is defined as

$$Q=\begin{cases}Q_0(T-T_\infty), T\geq T_\infty\\ 0 \qquad\qquad , T<0\end{cases}$$

where Q_0 is the heat generation/absorption coefficient (Mahmoud 2011).

The suitable boundary conditions are

$$y=0: u=U+\lambda_1\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right), v=-v_w, \tag{5}$$
$$T=T_w, C=C_w$$

$$y\to\infty: u\to 0, T\to T_\infty, C\to C_\infty \tag{6}$$

where T_w is the temperature of the plate, T_∞ is the temperature of the fluid far away from the plate, v_w is the constant velocity normal to the wall, λ_1 is the slip coefficient having dimension of length and U is the constant velocity.

3. METHOD OF SOLUTION

The equation of continuity is satisfied for the choice of a streamfunction $\psi(x,y)$ such that $u=\psi_y$ and $v=-\psi_x$.

Now, we introduce the following similarity transformations:

$$\eta=\left(\frac{U^{2-n}}{\upsilon x}\right)^{\frac{1}{n+1}} y \tag{7}$$

$$\psi(\eta)=\left(\upsilon x U^{2n-1}\right)^{\frac{1}{n+1}} f(\eta) \tag{8}$$

where $\upsilon=\dfrac{\mu}{\rho}$ is the kinematic viscosity, $f(\eta)=\dfrac{\psi(\eta)}{\left(\upsilon x U^{2n-1}\right)^{\frac{1}{n+1}}}$ is the dimensionless streamfunction.

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{9}$$

$$\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \tag{10}$$

After the substitution of these transformations (7) to (10) along with the boundary conditions (5) to (6) in the equations (2) to (4), the resulting non-linear ordinary differential equations are written as follows:

$$(n+1)\frac{d}{d\eta}\left[\frac{|f''|^{n-1}f''}{\left(1-\theta/\theta_c\right)}\right]+ff''=0 \tag{11}$$

$$\theta''+\frac{\Pr}{n+1}f\theta'+\frac{\Pr Ec|f''|^{n+1}}{\left(1-\theta/\theta_c\right)}+\gamma\Pr\theta=0 \tag{12}$$

$$\phi''+\frac{Le\Pr}{(n+1)}f\phi'=0 \tag{13}$$

where $\Pr=\dfrac{U}{\alpha x}\left(\dfrac{U^{2-n}}{\upsilon x}\right)^{\frac{-2}{n+1}}$ is the local Prandtl number, $Ec=\dfrac{U^2}{c_p(T_w-T_\infty)}$ is the local Eckert number, $\gamma=\dfrac{xQ_0}{U\rho c_p}$ is the local heat generation $(\gamma>0)$ or absorption parameter $(\gamma<0)$, $Le=\dfrac{\alpha}{D_m}$ is the Lewis number and $f_w=\dfrac{(n+1)\upsilon_w x^{\frac{n}{n+1}}}{\upsilon^{\frac{1}{n+1}}U^{\frac{2n-1}{n+1}}}$ is the local suction parameter $(f_w>0)$ or the local injection parameter $(f_w<0)$.

The parameter characterizing the influence of viscosity is given by $\theta_c=\dfrac{-1}{\delta(T_w-T_\infty)}$. The concept of this parameter θ_c was first introduced by Ling and Dybbs (1987) in their study of forced convection flow in porous media. For a given temperature differential, large values of θ_c implies either δ or (T_w-T_∞) are small. In this case, the effect of variable viscosity can be ignored. The effect of variable viscosity is significant if θ_c is small. The viscosity of liquids decreases with the increase in temperature whereas it increases for gases. Hence it is to be noted that θ_c is negative for liquids and positive for gases.

The corresponding boundary conditions are

$$\eta=0: f=f_w,\quad f'=1+\lambda\left(f''|f''|^{n-1}\right), \theta=1, \phi=1 \tag{14}$$

$$\eta\to\infty: f'\to 0, \theta\to 0, \phi\to 0 \tag{15}$$

where $\lambda=\dfrac{\lambda_1}{U}\left(\dfrac{U^3}{\upsilon x}\right)^{\frac{n}{n+1}}$ is the local slip parameter,

$Cf_x=-2(\mathrm{Re}_x)^{\frac{-1}{n+1}}|f''(0)|^{n-1}f''(0)$ is the local skin-friction co-efficient and $\mathrm{Re}_x=\dfrac{\rho U^{2-n}x^n}{\mu}$ is the local Reynolds number.

The most important characteristics of the problem are the rates of heat and mass transfer which are described by the local Nusselt number and the local Sherwood number are defined as

$$Nu_x=-\mathrm{Re}_x^{\frac{1}{n+1}}\theta'(0) \tag{16}$$

$$Sh_x=-\mathrm{Re}_x^{\frac{1}{n+1}}\phi'(0) \tag{17}$$

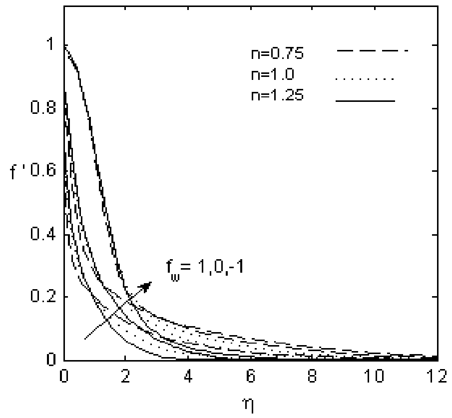
4. RESULTS AND DISCUSSION

The Equations (11) to (13) together with the boundary conditions (14) and (15) have been solved numerically by using the built in function bvp4c of the software MATLAB. This package is used to solve boundary value problems for ordinary differential equations of the form $y' = f(x, y, p)$, $a \le x \le b$, by applying a collocation method subject to general non-linear, two-point boundary conditions $f(y(a), y(b), p) = 0$. Here, p is a vector of unknown parameters. The first step in programming with bvp4c is to write the ordinary differential equation of any order into a system of first order ordinary differential equation. This software uses the higher order finite difference code that implements a collocation formula (Shampine *et al*., 2003).

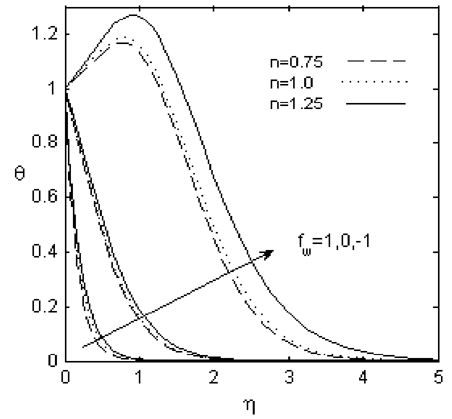
In the present study, numerical solutions have been obtained for various values of the power-law index in the range $0.75 \le n \le 1.25$. The parameter flow behavior index, n determines the nature of fluid. That is, $n < 1$ corresponds to pseudo-plastic, a number of non-Newtonian materials are in this category, including grease, molasses, paint, blood, starch, and many dilute polymer solutions, $n = 1$ to Newtonian and $n > 1$ to dilatant fluids such as beach sand mixed with water, titanium dioxide, and peanut butter. In order to gain physical insight, the velocity, temperature and concentration profiles have been discussed by varying the numerical values of the parameters encountered in the problem. The numerical results are tabulated and displayed with the graphical illustrations.

As shown in Fig. 2(a), the velocity is decreased by the suction parameter $(f_w > 0)$ whereas it is increased by injection $(f_w < 0)$. As far as the fluids are concerned, for all the fluids under investigation, namely, pseudo-plastic fluids $(n < 1)$, Newtonian fluid $(n = 1)$ and dilatant fluids $(n > 1)$, the velocity increases near the plate and decreases away from the plate for the imposition of both suction and injection. It is obvious that imposed suction decreases the boundary layer thickness. In the meantime, imposed injection increases the velocity in the boundary layer region signifying that injection helps the flow to penetrate more into the fluid.

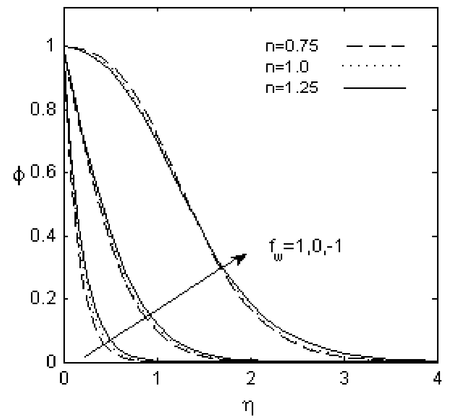
Fig. 2(b) demonstrates that suction $(f_w > 0)$ decreases the temperature and injection $(f_w < 0)$ increases the temperature for all types of power-law fluids. It can be noted that suction will lead to fast cooling of the surface. This is remarkably important in engineering and industrial applications. Further, it is seen that heat is transferred to the moving surface for the injection parameter and from the surface for suction. At the time of suction pseudo-plastic fluids $(n < 1)$ makes the plate cooler than that of Newtonian $(n = 1)$ and dilatant fluids $(n > 1)$, which is essential in many industrial processes. Moreover, it is observed that the temperature increases near the plate due to injection $(f_w < 0)$ and decreases until the value become zero at the outside of the boundary layer for all the fluids. On comparing the temperature increase among the fluids, it is strictly increased for dilatant fluids $(n > 1)$ than that for Newtonian $(n = 1)$ and pseudo plastic fluids $(n < 1)$. Fig. 2(c) exemplifies that suction parameter $(f_w > 0)$ decreases the concentration distribution and injection parameter $(f_w < 0)$ increases. It is observed that pseudo-plastic fluid reduces concentration distribution due to suction than the other fluids.



a

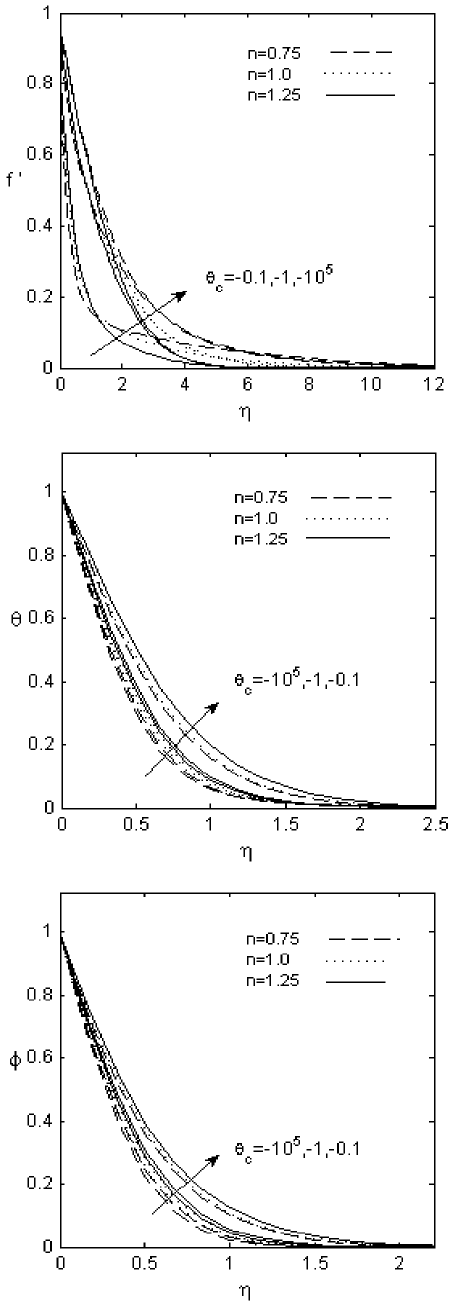


b



c

Fig. 2. Effects of f_w on (a) velocity (b) temperature and (c) concentration distributions for different values of n with $\theta_c = -0.2,\ \lambda = 0.1,\ \gamma = 0.1,\ \Pr = 10,\ Ec = 0.1$ **and** $Le = 1.0$.



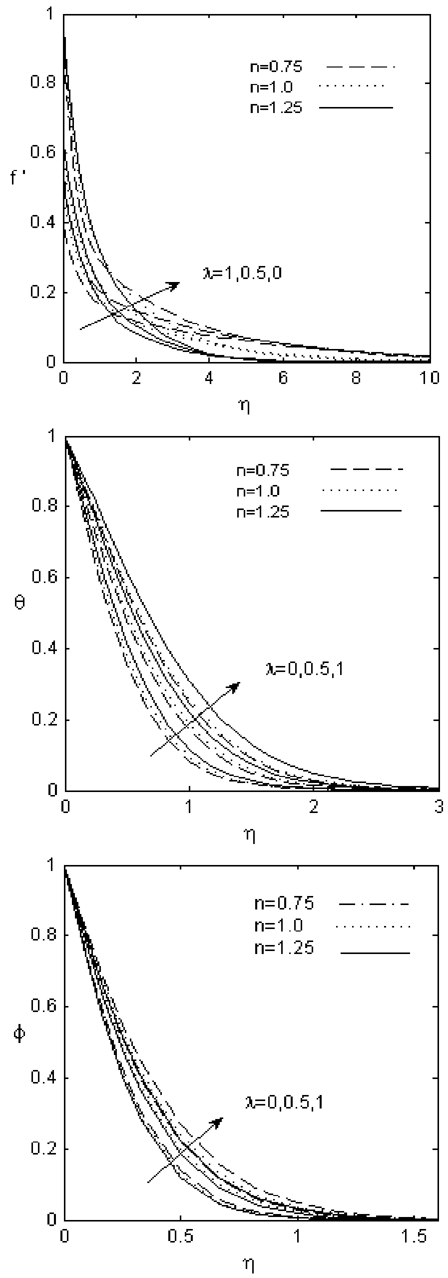
a

b

c

Fig. 3. Effects of θ_c on (a) velocity (b) temperature and (c) concentration distributions for different values of n with $f_w = 0.25,\ \lambda = 0.1,\ \gamma = 0.1,\ \Pr = 10,\ Ec = 0.1$ and $Le = 1.0$.

The effects of variable viscosity parameter θ_c on the dimensionless velocity $f'(0)$, temperature $\theta(0)$ and concentration $\phi(0)$ for all the fluids are depicted in Fig. 3. The other parameters are set constants as $\lambda = 0.1$, $\gamma = 0.1$, $Ec = 0.1$, $\Pr = 10$, $f_w = 0.25$ and $Le = 1.0$. When the variable viscosity is varied from higher value $(\theta_c = -0.1)$ to moderate viscosity $(\theta_c = -1)$ and then to the ambient viscosity level $(\theta_c = -10^5)$, it is understood that the velocity increases for all the types of fluid considered. But, in the cases of temperature and concentration distribution, the result is reversed. In other words, heat and concentration distributions increases as $\theta_c \to 0$. Clearly, the results obtained in this work reveals that θ_c, which is an indicator of the variation of viscosity with temperature, has a substantial effect on the velocity, temperature and concentration distributions within the boundary layer over a moving heat surface as well as the mass and heat transfer characteristics for all the fluids under consideration.



a

b

c

Fig. 4. Effects of λ on (a) velocity (b) temperature and (c) concentration distributions for different values of n with $f_w = 0.25,\ \theta_c = -0.2,\ \gamma = 0.1,\ \Pr = 10,\ Ec = 0.1$ and $Le = 1.0$

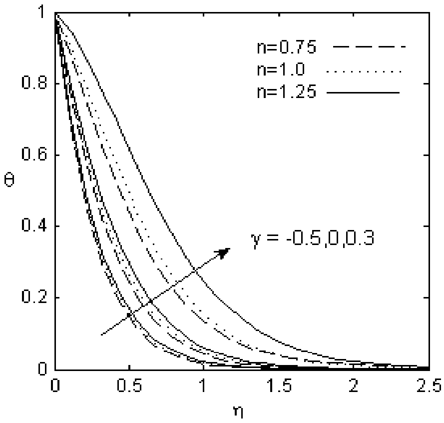


Fig. 5. Effetcs of γ on temperature distribution for different values of n with $f_w = 0.25,\ \theta_c = -0.2,\ \lambda = 0.1,\ \Pr = 10,\ Ec = 0.1$ **and** $Le = 1.0$.

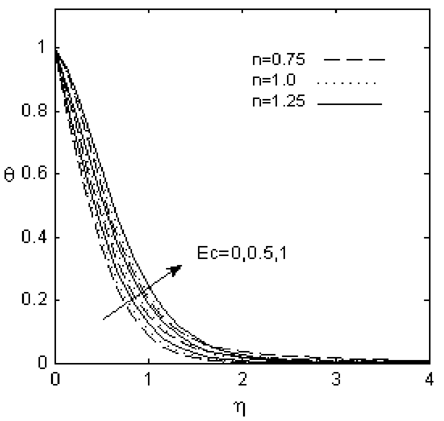


Fig. 6. Effects of Ec on temperature distribution for different values of n with $f_w = 0.25,\ \theta_c = -0.2,\ \lambda = 0.1,\ \gamma = 0.1,\ \Pr = 10$ **and** $Le = 1.0$ **.**

In order to discuss the effect of slip parameter (λ = 0, 0.5 and 1), the other controlling parameters are taken as $\theta_c = -0.2$, γ = 0.1, Ec = 0.1, Pr = 10, f_w = 0.25 and Le =1.0. Fig. 4 describes the effects of the slip parameter on the dimensionless velocity, temperature and concentration profiles. It is observed that the dimensionless velocity decreases near the surface but increases away from the surface when slip parameter λ is increased whereas the dimensionless temperature and concentration in the boundary layer region increases with the increase of the slip parameter. When slip parameter increases, the temperature distribution is better in the case of dilatant fluids than the other fluids. Pseudo-plastic fluids exhibit more concentration distribution than other fluids under consideration.

The effect of heat generation $(\gamma > 0)$ or absorption $(\gamma < 0)$ parameter on the dimensionless temperature is exhibited in Fig. 5. It can be viewed that the effect of heat absorption results in a fall of temperature since heat resulting from the wall is absorbed. Obviously, the heat generation $(\gamma > 0)$ leads to an increase in temperature throughout the entire boundary layer. Furthermore, it should be noted that for the case of heat generation the fluid temperature becomes maximum in the fluid layer adjacent to the wall rather at the wall. In fact, the heat generation effect not only has the tendency to increase the fluid temperature but also increases the thermal boundary layer thickness. In the presence of heat generation, the temperature distribution is better in the case of dilatant fluid $(n > 1)$ than the other fluids. Due to heat absorption $(\gamma > 0)$, it is seen that the fluid temperature as well as the thermal boundary layer thickness are decreased. No significance in heat distribution is observed among the fluids in the presence of heat absorption.

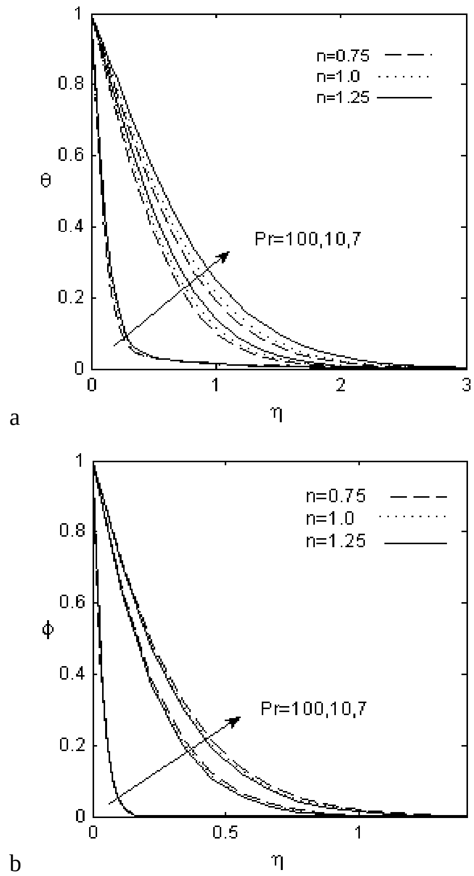


Fig. 7. Effetcs of Pr **on** (a) **temperature and** (b) **concentration distributions for different values of n with** $f_w = 0.25,\ \theta_c = -0.2,\ \lambda = 0.1,$ $\gamma = 0.1,\ Ec = 0.1$ **and** $Le = 1.0$.

The choice of the other parameters is fixed as in the previous discussion. The influence of different Eckert number (Ec = 0, 0.5 and 1) on the dimensionless temperature distribution within the boundary layer region is shown in Fig. 6. As compared to the case no viscous dissipation, it is understood that the dimensionless temperature is increased on increasing the Eckert number for all types of fluids. Due to viscous heating, the increase in the fluid temperature is enhanced and appreciable for higher value of Eckert number. In other words, increasing the Eckert number leads to a cooling of the wall. Dilatant fluids make the surface cooler

Table 1 Values of $(Cf_x \mathrm{Re}_x^{\frac{1}{n+1}})/2$ and $Nu_x / \mathrm{Re}_x^{\frac{1}{n+1}}$

f_w	θ_c	λ	γ	Pr	Ec	$(Cf_x \mathrm{Re}_x^{\frac{1}{n+1}})/2$			$Nu_x / \mathrm{Re}_x^{\frac{1}{n+1}}$		
						n = 0.75	n = 1.0	n = 1.25	n = 0.75	n = 1.0	n = 1.25
-1	-0.2	0.1	0.1	10	0.1	0.1588	0.0868	0.0379	0.2187	0.2536	0.3046
0	-0.2	0.1	0.1	10	0.1	1.2695	1.1249	1.0028	0.9856	0.9304	0.8468
1	-0.2	0.1	0.1	10	0.1	2.9380	2.6714	2.4579	5.7859	5.1050	4.5584
0.1	-0.1	0.1	0.1	10	0.1	1.7819	1.6385	1.5116	1.1752	1.1017	0.9845
0.1	-1.0	0.1	0.1	10	0.1	0.8557	0.7209	0.6206	1.5956	1.4545	1.3221
0.1	-10^5	0.1	0.1	10	0.1	0.5347	0.4433	0.3769	1.6912	1.5297	1.3873
0.1	-0.2	0	0.1	10	0.1	1.7438	1.5696	1.4088	1.4519	1.3552	1.2396
0.1	-0.2	0.5	0.1	10	0.1	0.8770	0.7738	0.6988	1.0942	1.0146	0.9030
0.1	-0.2	1.0	0.1	10	0.1	0.5936	0.5280	0.4815	0.8635	0.8050	0.6837
0.1	-0.2	0.1	-0.5	10	0.1	1.6086	1.4267	1.2771	2.8999	2.8461	2.7921
0.1	-0.2	0.1	0	10	0.1	1.4890	1.3230	1.1866	1.7195	1.6389	1.5506
0.1	-0.2	0.1	0.25	10	0.1	1.3471	1.0169	1.0520	0.5274	-1.8838	-0.0694
0.1	-0.2	0.1	0.1	7	0.1	1.3842	1.2332	1.1095	1.0441	0.9612	0.8564
0.1	-0.2	0.1	0.1	10	0.1	1.4488	1.2873	1.1539	1.3648	1.2674	1.1525
0.1	-0.2	0.1	0.1	100	0.1	1.9181	1.6648	1.4641	7.3837	6.7651	6.1895
0.1	-0.2	0.1	0.1	10	0	1.4634	1.2985	1.1623	1.4856	1.3724	1.2477
0.1	-0.2	0.1	0.1	10	0.5	1.3973	1.2481	1.1240	0.8979	0.8604	0.7816
0.1	-0.2	0.1	0.1	10	1.0	1.3450	1.2083	1.0933	0.3443	0.3748	0.3364

than the other fluids. Consequently, a transfer of heat to the fluid occurs, which causes a rise in the temperature of the fluid.

The effects for the choices of the Prandtl numbers as Pr = 7, 10 and 100 on the dimensionless temperature and concentration profiles are illustrated in Figs. 7 (a) and (b) respectively. It can be noticed that the dimensionless temperature and concentration are decreased on increasing Prandtl number. Physically, increasing Prandtl number becomes a key factor to reduce the thickness of the thermal and concentration boundary layers for all type of fluids.

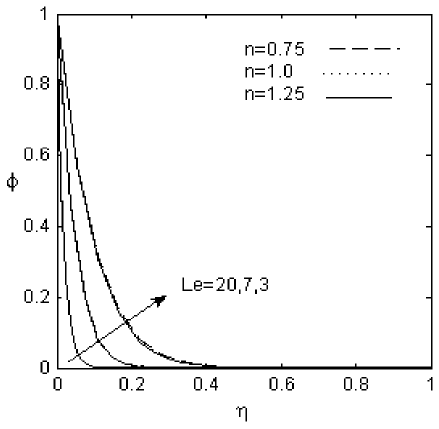


Fig. 8. Effetcs of Le on concentration distribution for different values of n with $f_w = 0.25,\ \theta_c = -0.2,\ \lambda = 0.1,\ \gamma = 0.1,\ \Pr = 10$ **and** $Ec = 0.1$**.**

Setting the other parameters to be constants as taken before, the discussion is now made only by varying the Lewis number as Le = 3, 7 and 20. The effect for the variation of the Lewis number Le on the dimensionless concentration profile is exemplified in Fig. 8. It is confirmed that the concentration distribution is significantly decreased while the value of Lewis number is increased.

The effects of the slip parameter λ, the suction or injection parameter f_w, heat generation or absorption parameter γ, variable viscosity parameter θ_c, Prandtl number Pr and Eckert number Ec, on the local skin-friction coefficient and Nusselt number for all types of fluids under consideration are illustrated in Table 1. From this table, it is observed that the local skin-friction coefficient and Nusselt number increases when suction is imposed whereas they are decreased for injection. Furthermore, as $\theta_c \to -\infty$, it is confirmed that the local skin-friction coefficient decreases while Nusselt number increases.

Increasing the slip parameter λ strictly decreases the values of both local skin-friction coefficient and Nusselt number. Heat absorption parameter increases both the local skin-friction coefficient and the Nusselt number while heat generation decreases those values. Moreover, the local Nusselt number and the local skin-friction coefficient are increased on increasing the Prandtl number Pr. This signifies that a fluid with larger Pr possesses larger heat

Table 2 Values of $Sh_x / \mathrm{Re}_x^{\frac{1}{n+1}}$

f_w	θ_c	λ	Pr	Le	$Sh_x / \mathrm{Re}_x^{\frac{1}{n+1}}$		
					n = 0.75	n = 1.0	n = 1.25
-1	-0.2	0.1	10	1	0.0035	0.0038	0.0040
0	-0.2	0.1	10	1	2.0300	2.1022	2.1439
1	-0.2	0.1	10	1	10.4760	10.5320	10.5667
0.1	-0.1	0.1	10	1	2.5144	2.6186	2.6775
0.1	-1.0	0.1	10	1	2.9265	2.9568	2.9745
0.1	-10^5	0.1	10	1	3.0481	3.0426	3.0481
0.1	-0.2	0	10	1	2.8323	2.9049	2.9411
0.1	-0.2	0.5	10	1	2.3261	2.4298	2.4952
0.1	-0.2	1.0	10	1	2.0989	2.2213	2.2973
0.1	-0.2	0.1	7	1	2.1096	2.1752	2.2119
0.1	-0.2	0.1	10	1	2.6808	2.7587	2.8035
0.1	-0.2	0.1	100	1	14.2092	14.3908	14.5057
0.1	-0.2	0.1	10	3	5.8106	5.9103	5.9708
0.1	-0.2	0.1	10	7	10.9311	11.0440	11.1168
0.1	-0.2	0.1	10	20	25.4607	25.5890	25.6783

Table 3 Comparison values of $Nu_x / \mathrm{Re}_x^{\frac{1}{n+1}}$ **when** $\lambda = 0.1$**,** $\mathrm{Pr} = 10$, $Ec = 0.1$, $Le = 0$ **and** $\theta_c \to -\infty$

f_w	γ	Mahmoud (2011)		Present case	
		n = 0.8	n = 1.2	n = 0.8	n = 1.2
0.5	0.1	0.0303	0.0122	0.0307	0.0122
0	0.1	1.2914	1.1105	1.2921	1.1105
0.5	0.1	3.3460	2.7990	3.3465	2.7989
0.1	-1	3.8046	3.6892	3.8046	3.6890
0.1	0.2	1.3223	1.0243	1.3228	1.0242

Table 4 Comparison values of $(Cf_x \mathrm{Re}_x^{\frac{1}{n+1}})/2$ **when** $\lambda = 0.1$**,** $\gamma = 0.1$**,** $\mathrm{Pr} = 10$, $Ec = 0.1$, $Le = 0$ **and** $\theta_c \to -\infty$

f_w	Mahmoud (2011)		Present case	
	n = 0.8	n = 1.2	n = 0.8	n = 1.2
0.5	0.3619	0.2439	0.36151	0.24399
0	0.4865	0.3625	0.48604	0.36252
0.5	0.6326	0.5004	0.63223	0.50040

capacity and hence improves the heat transfer. On the other hand, the local Nusselt number and the local skin-friction coefficient are decreased with increasing Eckert number.

Table 2 shows the values of Sherwood number obtained for the variation of the slip parameter λ, the suction or injection parameter f_w, variable viscosity parameter θ_c, Prandtl number Pr and Lewis number Le for all types of fluids. From the table, the mass transfer rate shows a rapid rise for imposed suction than for injection. Sherwood number increases when $\theta_c \to -\infty$. Further, on increasing Prandtl number and Lewis number, it is noted that mass transfer rate increases rapidly. In the case of local slip parameter, mass transfer rate decreases along with the increase of λ.

In order to verify the accuracy of our present results, comparisons have been made with the available results of Mahmoud (2011) in the literature, which are shown in Table 3 and Table 4. It is established that the results obtained in the present work shows a good agreement with the previous results.

5. CONCLUSION

In this work the effects of variable viscosity, slip velocity and heat generation or absorption on power-law fluids with heat and mass transfer over a moving permeable surface are numerically investigated. The governing boundary layer equations for the present problem are transformed into non-linear ordinary differential equations by using similarity transformations. MATLAB BVP solver bvp4c is used to compute the numerical results and to represent the obtained solutions graphically for the controlling parameters such as viscosity parameter, the slip parameter, the heat generation or absorption parameter, the Eckert number and Lewis number. The drawn conclusions for the present work are summarized as follows:

(i) At the time of suction pseudo-plastic fluids makes the plate cooler than that of Newtonian and dilatant fluids, which is essential in many industrial processes. On injection, comparing the temperature increases among the fluids, it is strictly increased for dilatant fluids than that of other fluids under consideration. Pseudo-plastic fluids reduce concentration distribution due to suction than the other fluids.

(ii) From this analysis, it is observed that the local skin-friction coefficient, Nusselt and Sherwood numbers increases rapidly when suction is imposed whereas they are decreased for injection.

(iii) In the presence of heat generation, the temperature distribution is better in the case of dilatant fluids than the other fluids. Heat absorption parameter increases both the local skin-friction coefficient and the Nusselt number while heat generation decreases those values.

(iv) Pseudo-plastic fluids exhibit more concentration distribution than the other fluids while wall slip parameter increases.

(v) Increasing the slip parameter strictly decreases the values of local skin-friction coefficient, Nusselt and Sherwood numbers.

(vi) As, it is confirmed that the local skin-friction coefficient decreases while heat and mass transfer rates increases.

(vii) On increasing the Eckert number leads to a cooling of the surface and dilatant fluids make the surface cooler than the other fluids.

(viii) The heat and mass transfer rates increases rapidly on increasing the Prandtl number for all types of fluids.

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