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# Capillary Rise of Magnetohydrodynamics Liquid into Deformable Porous Material

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## ABSTRACT

We have developed a mathematical model for capillary rise of magnetohydrodynamic fluids. The liquid starts to imbibe because of capillary suction in an undeformed and initially dry sponge-like porous material. The driving force in our model is a pressure gradient across the evolving porous material that induces a stress gradient which in turn causes deformation that is characterized by a variable solid fraction. The problem is formulated as a non–linear moving boundary problem which we solve using the method of lines approach after transforming to a fixed computational domain. The summary of our finding includes a notable reduction in capillary rise and a decrease in solid deformation due to magnetic effects.

**Keywords:** Capillary Rise; Magnetohydrodynamics fluid; Deformable porous media; Mixture theory.

## 1. INTRODUCTION

The history of studying the phenomena of capillary rise goes back to the pioneering work of Washburn (1921) where he concluded that the volume of the liquid that penetrates into the porous material in a time t is proportional to $\sqrt{t}$. Later on, other authors including Delker *et al*. (1996), Lago and Araujo (2001), Zhmud *et al*. (2000), Cai *et al*. (2014) and Cai *et al*. (2010) contributed in this area to further understand the capillary rise phenomena via different mathematical models along with experiments. All of these efforts well describe the mechanism of capillary rise into rigid porous material. However in many industrial as well as biological processes, the porous material is not rigid anymore, instead they deform when liquid passes through them. These materials exist in biomechanics, magma mechanics, soil science, infiltration, paper printing and textile engineering.

The history of studying the deformation of porous materials coupled with fluid flow goes back to Terzaghi (1925). It was further extended by Biot (1955) Biot (1956) to study soil consolidation. To understand fluid flow through deformable porous material as well as the deformation in porous media, mixture theory approach was introduced by Atkin and Crain (1976) and Bowen Bowen (1980). Later on, mixture theory was also used in studying various biological applications such as articular cartilage Lai and Mow (1980), Holmes (1983), Holmes (1984), Holmes (1985), Holmes (1986), Holmes and Mow (1990), Hou *et al*. (1989), arterial tissue Kenyon (1976), Kenyon (1987), Barry *et al*.(1991), Barry and Aldis (1992), and skin Oomens *et al*. (1987). Similarly, many industrial applications such as paper and inkjet printing, textile engineering, and dyeing of colored fabrics that deforms when the fluid motions through them has been studied using mixture theory. In this study our aim is to extend the mixture theory modeling to include magnetic property of electrically conducting fluids.

Our particular interest here is to develop a mathematical model to examine one-dimensional fluid flow through a deformable sponge like porous material. Recently Sommer and Mortensen (1996) developed a mathematical model using mixture theory to examine a forced unidirectional infiltration in an initially dry deformable porous material where a constant pressure drives the fluid flow in the porous material. On similar lines, Preziosi *et al*. (1996) presented a mathematical model for unidirectional infiltration of an incompressible liquid into a deformable porous material where the porous material is allowed to deform and relax when fluid flows through it. In view of ink–jet printing application, Anderson (2005) developed a mathematical model for the imbibition of liquid droplet on a deformable porous substrate. In the absence of gravitational effects, swelling, swelling relaxation, and shrinking of a porous material was reported in this work for particular choices of permeability function and stress function.

The preceding short literature review summarizes two important ingredients (i.e mixture theory and magnetohydrodynamics (MHD) fluid) that serve as important building blocks for current study. Our work is natural extension of Siddique *et al*. (2009), where they studied the capillary rise of liquid into deformable porous material by using mixture theory to account for material deformation. Their predictions for initial times are in good agreement with Washburn model of capillary rise but deviate from this trend for longer times. Later on Siddique and Anderson (2011) developed a mathematical model for capillary rise of non–Newtonian liquid into deformable porous material. They reported that capillary rise of liquid and deformation in porous material depends on power law index n and power–law consistency index $\mu^*$.

Magnetohydrodynamics (MHD) fluids are electrically conducting fluids that occur in nature as well as in laboratory settings. The examples of these fluids are salt water, plasma, liquid metals, and electrolytes. An important fact in studying MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn creates forces on the fluid and also changes the magnetic field itself. In MHD mathematical modeling, a combination of the Navier-Stokes equations and Maxwell's equations of electromagnetism are coupled together to analyze the influence of force and changes in magnetic field. In this regard first effort to the best of authors knowledge where magnetic fields is combined with mixture is by Eldabe *et al*. (2011). In this study, they extended Barry *et al*. (1991) work by presenting a mathematical model for unsteady flow of a Newtonian fluid in a deformable channel walls.

Our aim in current study is to develop a mathematical model for capillary rise of MHD liquid into deformable porous material. Our model is based on governing equations used by many authors in their studies Sommer and Mortensen (1996), Preziosi *et al*. (1996), Barry *et al*. (1991), Barry and Aldis (1992), Anderson (2005) and Siddique *et al*.(2009). We are interested in analyzing effects of magnetic fields by using mixture theory to examine capillary rise phenomena in deformable porous material. Generally speaking our model is analog of the models of Washburn (1921) and Anderson (2005). The model adds significant new results about the capillary rise of MHD fluids into deformable porous materials. We believe that these results will impact the understanding of capillary rise dynamics in the deformable porous materials that are critically important in many industrial settings. The organization of our article is as follows: section 2 describes our mathematical model, section 3 reports the results of our experiments while section 4 draws the conclusions from these results.

## 2. MATHEMATICAL MODELING

Figure (1) shows the basic geometrical description of capillary rise of an incompressible magnetohydrodynamics (MHD) liquid into a spongelike material. At time $t = 0$, the interface of liquid and deformable porous material is represented by $z = 0$. We assume that the imbibition of liquid begins from an infinite bath of liquid whose upper surface is open to the atmospheric pressure which means $p = P_A$ at $z = 0$ for all times. When $t > 0$, the imbibition of MHD liquid into an initially dry deformable porous material starts due to ability of suction in the pore space of porous spongelike material with the assumption that $P_c < 0$, which results in deformation of the porous material. This process of imbibition deforms the porous material that eventually forms two interfaces. We identify these interfaces as solid interface denoted by $z = h_s(t)$ and liquid interface denoted by $z = h_l(t)$ as shown in Fig. (1).

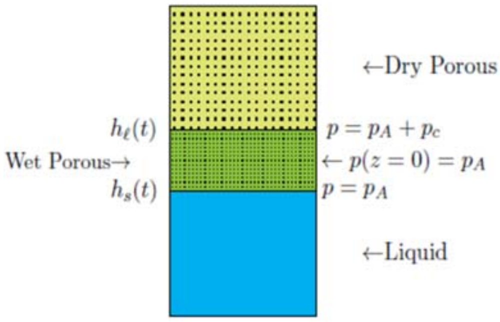


**Fig. 1. This figure shows the one dimensional capillary rise configuration.**

Incorporating the above assumptions leads us to the unknowns that are in the wet material and the boundary positions $h_s$ and $h_l$. The resulting variables of interest in the wet region are those of solid fraction $\phi$, the vertical velocity component of the liquid phase $u_l$, the vertical velocity component of solid phase $u_s$, the liquid pressure $p$ and the solid stress σ where σ = σ(I). Below we write down system of equations for the one dimensional deformation of spongelike material (see Appendix for details)

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial z}(\phi u_s) = 0, \tag{1}$$

$$\frac{\partial \phi}{\partial t} - \frac{\partial}{\partial z}\left[(1-\phi)u_l\right] = 0, \tag{2}$$

$$u_l - u_s = -\frac{K(\phi)}{(1-\phi)\mu}\frac{\partial p}{\partial z} + \frac{K(\phi)\sigma_0 B_0^2}{(1-\phi)^2 \mu}u_s \tag{3}$$

$$0 = -\frac{\partial p}{\partial z} + \frac{\partial \sigma}{\partial z}, \tag{4}$$

where equation (1) and (2) are the mass balance equations for solid and liquid phases respectively and equations (3) and (4) are derived from solid and liquid momentum balances [see Preziosi *et al*.(1996), Anderson (2005) for details]. In this set of equations $\mu$ is the dynamics viscosity, $K(\phi)$ is the permeability and σs is the solid stress. It is important to note that stress is a function of solid volume fraction $\sigma_s = \sigma_s(\phi)$ i.e. equation (4) becomes $\partial p / \partial z = \sigma_s'(\phi)\partial\phi / \partial z$. The other quantities $u_s$ fluid velocity, us solid velocity, $\phi$ solid volume fraction, and $p$ is the pressure. The above set of equations are consistent with one dimensional models examined by Siddique *et al*. (2009), Preziosi *et al*. (1996) and Barry *et al*. (1991), Barry and Aldis (1992). Note that our new contribution is inclusion of uniform magnetic flux $B_0$ relative to fluid imbibition in the deformable porous sponge and $\sigma_0$ represents the electric conductivity. Below we will examine in detail solutions of these equations and corresponding boundary conditions in order to assess the effect of uniform magnetic flux on the flow and deformation. In order to obtain a single partial differential equation (PDE) for solid volume fraction $\phi$, we follow the procedure used in Siddique *et al*. (2009), Anderson (2005) to obtain

$$\frac{\partial\phi}{\partial t} + C(t)\frac{\partial}{\partial z}\left[\frac{\phi(1-\phi)}{1-\phi+\alpha}\right] = -\frac{\partial}{\partial z}\left[\frac{\phi(1-\phi)}{1-\phi}\frac{K(\phi)\sigma'(\phi)}{\mu}\frac{\partial\phi}{\partial z}\right] \tag{5}$$

where $C(t)$ is constant of integration and $\alpha = (K(\phi)\sigma_0 B_0^2)/\mu$ The constant of integration $C(t)$ can be determined by subtracting equation (2) from equation (1) and integrating with respect to $z$ to get

$$\phi u_s + (1-\phi)u_l = C(t). \tag{6}$$

Combining equation (6) with equation (3) gives

$$u_s = C(t)\left(\frac{1-\phi}{1-\phi+\alpha}\right) + \frac{1-\phi}{1-\phi+\alpha}\frac{K(\phi)\sigma'(\phi)}{\mu}\frac{\partial\phi}{\partial z} \tag{7}$$

$$u_l = \frac{C(t)}{(1-\phi)}\left(1-\frac{\phi(1-\phi)}{1-\phi+\alpha}\right) + \left(\frac{\phi}{1-\phi+\alpha}\right)\frac{K(\phi)\sigma'(\phi)}{\mu}\frac{\partial\phi}{\partial z} \tag{8}$$

In all of the above equations if we set $\alpha = 0$, we obtain the system of equation used in Anderson (2005). Before we derive the boundary conditions we will determine $C(t)$ by using the same argument as in (see Preziosi *et al*. (1996) and Anderson (2005) for details) in combination with equation (7) gives the following formula for $C(t)$

$$C(t) = \frac{(1-\phi_0)(1-\phi)}{\alpha(\phi-\phi_0) - \phi_0(1-\phi)^2}\frac{K(\phi)\sigma'(\phi)}{\mu}\frac{\partial\phi}{\partial z} \tag{9}$$

where φ0 represents the uniform solid volume fraction of dry rigid porous material. The process of imbibition forms liquid–wet material interface $z = h_s(t)$ and wet material–dry material interface $z = h_l(t)$ .The boundary conditions at $z = h_s(t)$ are

$$u_s(h_s^+, t) = \frac{\partial h}{\partial t}, \quad p(h_s^+, t) = p_A, \quad \sigma(h_s^+, t) = 0(10) \tag{10}$$

and boundary conditions at $z = h_l(t)$ are

$$u_s(h_s^-, t) = \frac{\partial h}{\partial t}, \quad p(h_l^-, t) = p_A + p_c, \tag{11}$$

where pc is the constant capillary pressure and pA is atmospheric pressure. Equation (5) along with equation (6) and boundary conditions in (10)–(11) forms a closed system on a moving domain $h_s \le z \le h_l$ . There are many choices that authors have employed to predict physical of their models for example Sommer and Mortensen (1996), Preziosi *et al*. (1996) but we follow Anderson (2005)–Siddique *et al*. (2009) and Siddique and Anderson (2011) for the choices of $K(\phi)$ and $\sigma(\phi)$ . These choices serve dual purpose; it simplifies the set of equation in a great deal and also physically consistent with realistic trends of capillary rise phenomena under study

$$K(\phi) = \frac{K_0}{\phi}, \quad \sigma(\phi) = m(\phi_r - \phi). \tag{12}$$

The permeability function in (12) is inversely proportional to the solid fraction where K0,m > 0 and $\sigma_s(\phi) = -m < 0$ and $M = (K_0 B_0^2 \sigma_0)/\mu$ .

## 2.1 Nondimensionalization:

The following set of dimensionless quantities are used to nondimensionalize the set of equations (5), (6) ,(10) and (11) along with transformation that helps to transform the problem from moving domain $h_s \le z \le h_l$ to a fixed domain $0 \le z \le 1$. The resulting PDE along with appropriate boundary conditions after dropping the prime notation is given as

$$z' = \frac{z - h_s(t)}{h_l(t) - h_s(t)}, \quad t' = \frac{t}{T}, \quad h_s'(t) = \frac{h_s}{H}, \quad h_s'(t) = \frac{h_l}{H}, \quad p' = \frac{p}{m} \tag{13}$$

$$\frac{\partial\phi}{\partial t} + \left[\frac{z-1}{h_l - h_s}\frac{dh_s}{dt} - \frac{z}{h_l - h_s}\frac{dh_l}{dt}\right]\frac{\partial\phi}{\partial z} + \frac{C(t)}{h_l - h_s}\left[\frac{\phi(2M + \phi - 3M\phi - 2\phi^2 + \phi^3)}{(\varphi - \varphi^2 + M)^2}\right]\frac{\partial\phi}{\partial z} = \frac{1}{(h_l - h_s)^2}\left[\frac{\phi(1-\phi)}{\phi - \phi^2 + M}\frac{\partial^2\phi}{\partial z^2}\phi - \frac{\alpha(1-2\phi)}{(\phi - \phi^2 + M)^2}\left(\frac{\partial\phi}{\partial z}\right)^2\right] \tag{14}$$

along with boundary conditions

$$\frac{dh_s}{dt} = C(t)\left[\frac{\phi(1-\phi)}{\phi(1-\phi) + M}\right] - \left[\frac{\phi(1-\phi)}{\phi(1-\phi) + M}\right]\frac{1}{(h_l - h_s)\phi}\left.\frac{\partial\phi}{\partial z}\right|_{z=0} \tag{15}$$

$$\frac{dh_l}{dt} = \frac{C(t)}{(1-\phi)}\left[\frac{\phi(1-\phi)^2 + M}{\phi(1-\phi)+M}\right] - \left[\frac{\phi}{\phi(1-\phi)+M}\right]\frac{1}{(h_l - h_s)\phi}\frac{\partial \phi}{\partial z}\bigg|_{z=1} \tag{16}$$

Note that introducing non-dimensional parameters yield ordinary differential equations for interface positions. These boundary conditions can be obtained by equating equations (7) with (10) and (8) with (11) and

$$C(t) = -\frac{(1-\phi_0)(1-\phi)}{M(\phi-\phi_0) - \phi_0(1-\phi)^2}\frac{1}{h_l - h_s}\frac{\partial \phi}{\partial z}\bigg|_{z=1} \tag{17}$$

The boundary conditions for solid volume fraction are derived from zero stress conditions and stress equilibrium conditions

$$\phi = \phi_r \quad \text{at} \quad \text{and} \quad z = 0, \quad \text{and} \quad \phi = \phi_l^* \quad \text{at} \quad z=1 \tag{18}$$

where $\phi_l^* = p_c / m$ and the initial conditions for the interface positions are given as follows

$$h_l(t=0) = 0, \quad \text{and} \quad h_s(t=0) = 0. \tag{19}$$

After setting magnetic parameter $M = 0$ in (14)–(17), we obtain system of equations in the absence of gravity given in Siddique *et al*. (2009) and for Newtonian case of Siddique and Anderson (2011) when we set power law index $n = 1$ in the absence of gravity effect. Note that for the magneto hydrodynamics (MHD), the similarity solution do not exist and we have to explore the other possible ways to get the solution. In the following section we present the solution procedure.

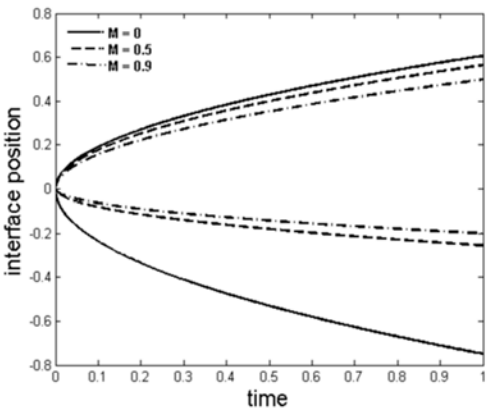


**Fig. 2. This figure shows the evolution of the interface positions $h_s$ and $h_l$ for different values of Magnetic parameter M. We have used $\phi_l = 0.2,\ \phi_r = 0.1,$ and $\phi_0 = 0.33$**

## 2.2 Numerical Solution

The system of equations (14)–(17) does not admit similarity solutions as was the case for non–MHD Newtonian case Anderson (2005). In order to obtain the numerical solution we come across a singularity at $t = 0$, which we deal by using the similarity solution obtained in Anderson (2005), for Newtonian case as an initial condition for $t > t_I$ where $t_I$ is initial time that we chose to be small enough so that our numerical solution is independent of any further reduction in $t_I$ . For details on how to obtain the similarity solution, (see Anderson (2005) for details). The similar procedure was followed by Siddique *et al*. (2009) where they used the similarity solution calculated by Anderson (2005) as an initial condition for non-zero gravity case for Newtonian fluids. For the case of power law fluid Siddique and Anderson (2011) obtain similarity solution numerically for zero gravity case. This similarity solution is then used as an initial condition for non–zero gravity case to avoid singularity. In our study we use methods of lines along with second order finite difference scheme in space to convert the PDEs (14) into a system of ODEs. These ODEs along with equations (15) and (16) are solved numerically using Matlab's ode15s solver.

## 3. RESULTS

Figure (2) shows the capillary rise and deformation dynamics for various values of *M*. In case of no magnetic effects $h_s(t)$ evolves downward and $h_l(t)$ evolves upward following a square root in time dynamics. This solution was first identified by Anderson (2005), which was later confirmed in Siddique *et al*. (2009) and Siddique and Anderson (2011). Interestingly, the $h_l(t)$ and $h_s(t)$ for magnetohydrodynamics (MHD) fluids evolve more slowly when compared to $M = 0$ case. The process of evolution for $h_s(t)$ is even slower than $h_l(t)$ . In other words, for magnetohydrodynamic fluids both $h_l(t)$ and $h_s(t)$ curves follow a dynamics that may still be proportional to $\sqrt{t}$ but with different constant. This trend of evolution is different than classical prediction of Washburn (1921), Lago and Araujo (2001) and Zhmud *et al*. (2000) for capillary rise in rigid porous material and for deformable porous material Siddique *et al*. (2009) and Siddique and Anderson (2011).

Figure (3) shows the plot of permeability as a function of space. Note that there are many choices that have been used by different researchers for permeability of porous material and the summary of the list of permeability function can be found in Anderson and Siddique (2012). This plot represents the choice of permeability function that have been used in this study. The permeability function represents more non–linear dynamics as we move from the absence of magnetohydrodynamics fluid $M = 0$ case to magnetohydrodynamic $M \neq 0$ case. Note that increasing the magnetic parameter values decreases the permeability $K(\phi)$ which is consistent with Fig. (2).

Figure (4) shows the solid volume fraction as a function of infiltration height. It is important to note that as M decreases the dynamics of capillary rise for local volume fraction tend to move toward the linearity. This is evidence that penetration of liquid through the deformable porous material is slower as magnetic parameter increases. This can also be seen in (2), where the curve $h_s(t)$ shows the slower dynamics for the liquid height when value of magnetic parameter is increased. In general, as magnetic parameter increases, the solid volume fraction increases throughout the deformable porous material.

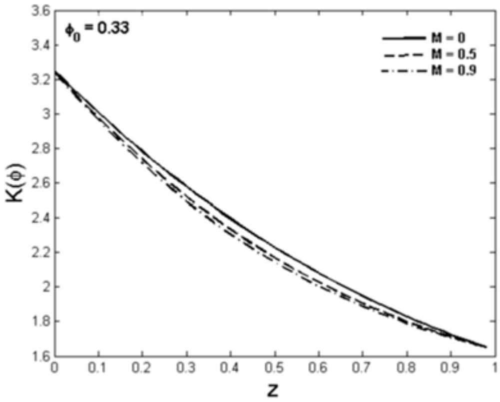


**Fig. 3. Permeability as a function of *z* for different values of Magnetic parameter *M*.**

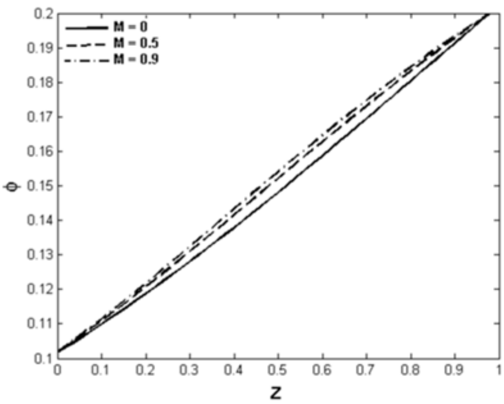


**Fig. 4. Solid volume-fraction distribution as a function of *z* for different values of Magnetic parameter *M*.**

Note that the choice of solid stress $\sigma(\phi)$ and permeability $K(\phi)$ plays an important role in capillary rise dynamics.

Figure (5) shows the liquid volume fraction as a function of infiltration length. This figure shows the fraction of liquid in the deformable porous material. The capillary rise dynamics is slower as we decrease the magnetic parameter *M*. The fluid possessing MHD properties follow slower dynamics which in turn results in slower deformation in the deformable porous material. This behavior can also be seen in Fig. (2) shows by the curve represented by $h_l(t)$. This was the main objective to investigate the effects of magnetohydrodynamic flow during the capillary rise into deformable porous material.

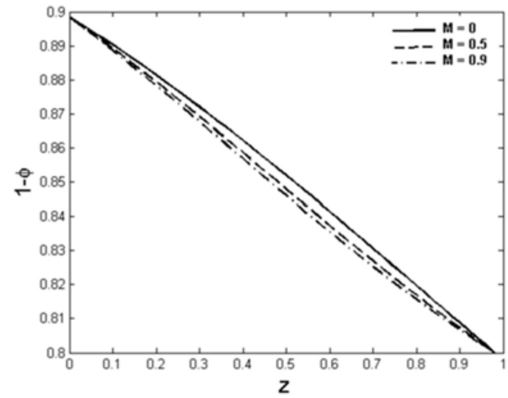


**Fig. 5. Liquid volume-fraction distribution as a function of *z* for different values of Magnetic parameter *M*.**

## 4. CONCLUSION

In this paper we have investigated the capillary rise of MHD fluid into deformable porous material. This work is motivated from experimental as well as theoretical studies Sommer and Mortensen (1996), Preziosi *et al*. (1996), Barry *et al*. (1991), Anderson (2005), Siddique *et al*. (2009), Siddique and Anderson (2011), Eldabe *et al*. (2011) that have highlighted the physical significance of these phenomenon. Our finding in this work carries numerical as well as physical significance. Summary of our work is based on magnetic parameter *M* and compared with that of Siddique *et al*. (2009) in the absence of gravitational effects. Our model is an analog of classical Washburn model Washburn (1921) for rigid porous material and Sommer and Mortensen (1996), Anderson (2005), Siddique *et al*. (2009) for deformable porous materials.

We found that the imbibition of magnetohydrodynamics (MHD) fluid into deformable porous material is slower as compared to in the absence of no magnetic effects. This information could be helpful in industrial processes such as inkjet printing and dyeing the fabric. Studies in the past i.e Lago and Araujo (2001), Zhmud *et al*. (2000), Siddique *et al*. (2009) have shown that the volume of the liquid that penetrates into the porous material in time *t* is proportional to $\sqrt{t}$ for early times but for longer times the dynamics deviates from this trend. Our finding confirms this as well and leads to the conclusion that capillary rise or imbibition process of liquid depends on the fluid properties as well. In case of deformable porous material these properties of fluids play an important role in material deformation process.

Our findings indicate that this model needs to be further investigated both theoretically as well as experimentally. In addition to these, other complications such as chemical interaction between the liquid and solid phases, using models of different permeability and solid stress available in the literature and gravitational effects could lead us in predicting dynamics of magnetohydrodynamic fluids. We hope that this simple model could lead to more sophisticated models as well as experimental investigations.

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### A GOVERNING SYSTEM OF EQUATIONS

We consider the porous deformable porous material as a continuous binary mixture of solid and fluid phase, where each point in the mixture is occupied by both fluid and solid. We also assume that both fluid and solid are intrinsically incompressible and porous material to be isotropic. The variable of interest in wet porous material are the solid volume fraction $\phi$ , the fluid and solid velocities ( $\mathbf{u}_l, \mathbf{u}_s$ ) respectively, fluid pressure $p$, and solid stress σ. We assume that the density of solid and fluid to be constant and the conservation of mass for both phases can be written as follows

$$\frac{\partial \phi}{\partial t} + \nabla.\, (\phi \mathbf{u}_s) = 0 \tag{20}$$

$$\frac{\partial \phi}{\partial t} + \nabla.\left[(1-\phi)\mathbf{u}_l\right] = 0. \tag{21}$$

Momentum balance for both phases can be written as

$$r^m \left( \frac{\partial \mathbf{v}^m}{\partial t} + \mathbf{v}^m \nabla \mathbf{v}^m \right) = \nabla \mathbf{T}^m + \pi^m + J \wedge B \tag{22}$$

where $m = l, s.$ The stress tensor is expressed as

$$\mathbf{T}^m = -\phi^m P \mathbf{I} + \sigma^m, \tag{23}$$

The friction force term is given as

$$\pi_l = -\pi_s = K(\mathbf{u}_s - \mathbf{u}_l) - P \nabla \phi^s. \tag{24}$$

Maxwell’s equations

$$\nabla \times \mathbf{B} \;=\; \mu_c \mathbf{J}, \tag{25}$$

$$\nabla \mathbf{B} \;=\; 0, \tag{26}$$

$$\nabla \times \mathbf{E} \;=\; -\frac{\partial \mathbf{B}}{\partial t}. \tag{27}$$

And Ohm’s law is written as

$$\mathbf{J} \sigma_o (\mathbf{E} + v^m \times \mathbf{B}). \tag{28}$$

## REFERENCES

Anderson, D. and J. I. Siddique (2012). *Modeling Wicking in Deformable Porous Media Using Mixture Theory (Chapter 11) Wicking in Porous Materials: Traditional and Modern Modeling Approaches*, CRC Press, USA.

Anderson, D. M. (2005). Imbibition of a liquid droplet on a deformable porous substrate. *Phys. Fluids* 17(8).

Atkin, R. J. and R. E. Crain (1976). Continuum theories of mixture: Basic theory and historical development. *Quart. J. Mech. Appl. Math*. 29(2), 229-244.

Barry, S. I. and G. K. Aldis (1992). Flow induced deformation from pressurized cavities in absorbing porous tissues. *Bull. Math. Biol*. 54(6), 977-997.

Barry, S. I., K. H. Parker and G. K. Aldis (1991). Fluid flow over a thin deformable porous layer. *Journal of Applied Mathematics and Pysics (ZAMP)* 42(5), 633-648.

Biot, M. A. (1955). Theory of elasticity and consolidation for a porous anisotropic solid. *J. Appl. Phys*. 26(2), 182-185.

Biot, M. A. (1956). Mechanics of deformation and acoustic propogation in porous media. *J. Appl. Phys*. 27.

Bowen, M. R. (1980). Incompressible porous media models by use of the theory of mixtures. *Int. J. Engng. Sci.* 18(9), 1129-1148.

Cai, J., B. Yu, M. Zou and L. Luo (2010). Fractal characterization of spontaneous co–current imbibition in porous media. *Energy Fuels 24, 1860–1867.*

Cai, J., E. Perfect, C. L. Cheng and X. Hu (2014). Generalized modeling of spontaneous imbibition based on hagenpoiseuille flow in tortuous capillaries with variable shaped apertures. *Langmuir* 30, 5142–5151.

Delker, T., D. B. Pengra and P. Z. Wong (1996). Interface pinning and the dynamics of capillary rise in porous media. *Phys. Rev. Lett*. 76(16), 2902.

Eldabe, N. T. M., G. Saddeek and K. A. S. Elagamy (2011). Magnetohydrodynamic flow of a bi–viscosity fluid through porous medium in a layer of deformable material. *Journal of Porous Media* 14(3), 273–283.

Holmes, M. H. (1983). A nonlinear diffusion equation arising in the study of soft tissue. *Q. Appl. Math*. 41, 209-220.

Holmes, M. H. (1984). Comparison theorems and similarity solution approximations for a nonlinear diffusion equation arising in the study of soft tissue. *SIAM J. Appl. Math*. 44(3), 545-556.

Holmes, M. H. (1985). A theoretical analysis for determining the nonlinear hydraulic permeability of a soft tissue from a permeation experiment. *SIAM J. Appl. Math*. 47(5), 669-683..

Holmes, M. H. (1986). Finite deformation of soft tissue: analysis of a mixture model in uniaxial compression. *J. Biomech. Eng*. 108(4), 372-381.

Holmes, M. H. and V. C. Mow (1990). The nonlinear characteristic of soft gels and hydrated connective tissue in ultrafiltration. *J. Biomech*. 23(11), 1145-1156.

Hou, J. S., M. H. Holmes, W. M. Lai and V. C. Mow (1989). Boundary conditions at the cartilage-synovial fluid interface for joint lubrication and theoretical verifications. *J. Biomechanics Engng*. 111(1), 78-87.

Kenyon, D. E. (1976). The theory of an incompressible solid–fluid mixture. *Arch. Rat. Mech. Anal.* 111.

Kenyon, D. E. (1987). A mathemtical model of water flux through aortic tissue. *Bull. Math. Biology* 49.

Lago, M. and M. Araujo (2001). Capillary rise in porous media. *Journal of Colloid and Interface Science* 234.

Lai, W. M. and V. C. Mow (1980). Drag induced compression of articular cartilage during a permeation experiment. *Biorheology* 17(1-2), 111-123.

Oomens, C. W. J., D. H. V. Campen and H. J. Grootenboer (1987). A mixture approach to the mechanics of skin. *J. Biomechanics* 20(9), 877-885.

Preziosi, L., D. D. Joseph and G. S. Beavers (1996). Infiltration of initially dry, deforamable porous media. *Int. J. Multiphase Flow* 22(6), 1205-1222.

Siddique, J. I. and D. M. Anderson (2011). Capillary rise of non-newtonian liquid into deformable porous material. *Journal of Porous Media* 12(14), 1087-1102.

Siddique, J. I., D. M. Anderson, and A. Bondarev (2009). Capillary rise of a liquid droplet on a deformable porous substrate. *Phys. Fluids* 21(1).

Sommer, J. L. and A. Mortensen (1996). Forced unidirectional infiltration of de-formable porous media. *J. Fluid Mech*. 311.

Terzaghi, K. (1925). In *Erdbaumechanik auf Bodenphysikalischen Grundlagen,* Deuticke, Wien.

Washburn, E. W. (1921). The dynamics of capillary flow. *Phys. Rev*. *17*(3), 273.

Zhmud, B. V., F. Tiberg and K. Hallstensson (2000). Capillary rise in porous media. *Journal of Colloid and Interface Science* 228.