Journal of Applied Fluid Mechanics, Vol. 9, No. 6, pp. 2845-2853, 2016.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.

Influence of Magnetic Field on Fingering Instability in a Porous Medium with Cross-Diffusion Effect: a Thermal Non-equilibrium Approach

A. K. Srivastava[1†] and B. S. Bhadauria[2]

[1] *Department of Applied Sciences, KIET Group of Institutions, Ghaziabad, 201206, India,*
[2] *Department of Mathematics, Faculty of Science, Banaras Hindu University, Varanasi−221005, India.*

†Corresponding Author Email: atulshaswat@gmail.com

(Received December 19, 2015; accepted January 1, 2016)

ABSTRACT

The purpose of this paper is to investigate the thermal instability problem under the influence of three diffusing components on fluid saturated horizontal porous media under local thermal non-equilibrium effect. These three components are vertical magnetic field, solute and heat. We considered an electrically conducting two components fluid and anisotropic porous medium. The physical system is heated and salted from above. Flow in the porous medium is characterized by Darcy model, whereas the fluid and solid phases are not in local thermodynamic equilibrium (LTNE). Linear stability analysis is used to calculate critical Darcy-Rayleigh number and corresponding wave number for onset of stationary convection. The study is based on mainly four controlling parameters, Darcy-Chandrasekhar number (Q), dimensionless inter-phase heat transfer coefficient (H), Soret (Sr) and Dufour (Du) cross-diffusion parameters, among several. We illustrated the effects of these parameters on thermal instability and found parameters may have stabilizing or destabilizing effects, thus may advance or delay the onset of convection. Various comparative studies are also presented between different cases and conditions, such as for anisotropic and isotropic cases, cross-diffusion and without cross-diffusion cases and for different values of Sr and Du.

Keywords: Thermal non-equilibrium; Magneto-convection; Darcy-Rayleigh Number; Fingering instability; Cross-diffusion.

NOMENCLATURE

a	Horizontal wave number
c	specific heat capacity
C	concetration
d	length of the porous layer
D_M	solute diffusivity
D_F	DuFour coefficient
D_T	soret coefficient
D_u	duFour parameter
$\vec{g}$	acceleration due to gravity
H	non-dimensional inter-phase heat transfer coefficient
h_s	inter-phase heat transfer coefficient
h_z	perturbed magnetic field along z-axis
K	permeability of porous layer
K_x, K_x, K_y	characteristic permeabilities in the x, y and z directions
Le	Lewis number
l,m	x-component and y-component of wavenumber
p	reduced pressure
Pm	magnetic Prandtl number
)-(	magnetic field ( )-(1)-(2)-(3)
Q	Darcy-Chandersekhar number
$\vec{q}$	velocity
R	Darcy-Rayleigh number
R_C	Critical Rayleigh number
R_S	Solutal Rayleigh number
Sr	soret parameter
T_f	temperature of fluid,
T_s	temperature of solid,
ΔT	temperature difference across the porous layer
T_l	temperature of lower surface
T_U	temperature of upper surface
t	time
x,y,z	space coordinates
κ	thermal diffusivity
κ_f	thermal diffusivity for fluid
κ_s	thermal diffusivity for solid
β_C	Coefficient of concentration expansion

β_T	coefficient of thermal expansion
Λ	magnetic viscosity
η_f	fluid thermal conductivity ratio
η_s	solid thermal conductivity ratio
ε	porosity
μ	dynamic viscosity of the fluid
μ_m	magnetic permeability
ν	kinematic viscosity
ρ	density of fluid
ρ_0	reference density
γ	porosity modified conductivity ratio
χ	diffusivity ratio
ξ	anisotropy ratio
Θ	amplitude of fluid temperature perturbation
Φ	amplitude of solid temperature perturbation
Ψ	amplitude of fluid concentration perturbation

Other symbols

D	d/dz
∇_1^2	$\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}$

Subscripts

b	basic state
c	Critical
f	Fluid
S	Solid

superscripts

$'$	perturbed quantity
$*$	dimensionless quantity

1. INTRODUCTION

Double-diffusive convection in fluid-saturated porous media with thermo-diffusive effect is of practical interest in many engineering applications such as petrology, hydrology, solidification of binary alloys as well as many other applications, and well documented by Nield and Bejan (2013). When an imposed magnetic field is associated with double-diffusive convection in a fluid-saturated porous medium, then it has various applications in science, engineering and technology (Wallace *et al*. (1969)). One of the key importance of such type of convection is seen in petroleum reservoirs where the system comprises of both electrically inert solid and electrically conducting fluid phases. Due to its applications, as mentioned above, the problem has attracted many researchers to work in this field. Some of the recent literature in this area are: Sarvanan and Yamaguchi (2005), Bhadauria *et al*.(2008a,b,2010). Bhadauria *et al*. (2008 a, b, 2010) studied the effect of magnetic field on thermal modulated convection in porous medium. Specially in the paper of (2010), the present authors have studied the effect of thermal modulation in double-diffusive magneto-convection.

According to fact that the fluid density depends on solute concentration, it leads to a competition between thermal and compositional gradients. So it is obvious to ask what will happen when cross-diffusion takes place in double-diffusive magneto-convection? In a system where three diffusing properties (magnetic field, solute, heat) are present, instabilities can occur only if at least one of the component is destabilizing. In the present paper, we are attempting to answer the above question. Lots of work is available on the onset of double-diffusive convection in a porous medium with cross-diffusion effects (Nield and Bejan (2013)). Thermal convection in a binary fluid driven by the Soret and DuFour effects, has been investigated by Knobloch (1980). He has shown that the equations are identical to the thermo-solutal problem except for a relation between the thermal and solute Rayleigh numbers. The double diffusive convection in a porous medium in the presence of Soret and DuFour coefficients has been analyzed by Rudraiah and Malashetty (1986). It is important to mention that most of the work related to double-diffusive convection with and without cross-diffusion is considered when system is heated and salted from below. What will happen if we consider the case of heating and salting from above? From the available literature, it can be stated that due to its occurrence in many situations such as in oceanography, it is matter of attraction for researchers. In this case double-diffusive convection differ from classical Rayleigh Be´nard convection. There is two fundamental new flow structures can appear: layers and fingers. Fingers are vertically oriented long and narrow regions of up or down welling fluid which can occur if the fluid property with large diffusion coefficient (for example temperature) imposes a stability gradient on the fluid, whereas the property with the small diffusion coefficient (the salt concentration in oceanography) is unstably stratified (Huge and Tilgner (2010)). A study of convective instability in a fluid mixture heated from above with negative separation ratio (Soret coefficient) was performed experimentally by La Porta and Surko (1998). This topic is well documented in the Huge and Tilgner (2010) and reference of it.

Thermal equilibrium is assumed between solid porous matrix and saturated fluid in most of the available literatures related to thermal convection in porous media. But for sufficiently large Rayleigh number or rapid heat transfer for high speed flow, it can be expected that the equilibrium will break down, so that the respective local mean values of the temperature of solid and fluid phases are not same. Local thermal non-equilibrium (LTNE) state will be more suitable than local thermal equilibrium (LTE) state in this situation. A two-phase model for energy equations was well documented in the book of Nield and Bejan (2013). Rees and Pop (2005), Banu and Rees (2002), Rees *et al*. (2008) have investigated thermal non-equilibrium effect on natural convection in horizontal porous layer. Recently Kuznetsov *et. al*. (2015) is investigated analytically, the effect of local thermal non-equilibrium on the onset of double-diffusive convection in a porous medium consisting of two horizontal layers, each internally heated.

It is also noticeable that most of works related to convective flow with LTNE effect have dealt with mechanical and thermal isotropic case. Mechanical term is used in the sense when porous matrix allow fluid flow with same permeability in all possible directions. Whereas thermal term indicates that heat is equally distributed in all possible directions in fluid as well as solid. However, in many practical situations the porous materials are anisotropic in their mechanical and thermal properties. Onset of thermal convection in horizontal porous layer with anisotropic permeability was first investigated by Castinel *et al*. (1974). Later on Epherre (1975) extended the stability analysis to media with anisotropic thermal diffusivity. Malashetty *et al.* (2005) investigated onset of convection in an anisotropic porous medium under LTNE effect, when porous medium is heated from below. The effect of mechanical and thermal anisotropic on the stability of gravity driven convection in rotating porous media in the presence of thermal non-equilibrium is investigated by Govender and Vadasz (2007). They considered the physical system heated from below.

The idea of this paper is influence by the recent work of Saravanan and Jegajothi (2010) and Srivastava *et al*. (2011). In the first paper, they have investigated the stationary fingering instability in a non-equilibrium porous medium with coupled molecular diffusion. They focused on the effects of cross-diffusion parameters and inter-phase heat transfer coefficient. Where as in the second paper, authors presented impact of magnetic field and anisotropic effect on onset of convection in horizontal porous media. A question arise, what will happen if we consider the magnetic field effect (i.e. three diffusing components) with mechanical and thermal anisotropic properties Motivated by the importance and impact of three diffusing components with more realistic anisotropic condition on the onset of convection, we would like to investigate the problem theoretically in this article. An outline of the paper is as follows. In section 2, the mathematical formulation and solution of the physical problem are given. Linear stability analysis presented in section 3. Results and discussions are reported in section 4. Finally, some important features of the analysis are concluded in section 5.

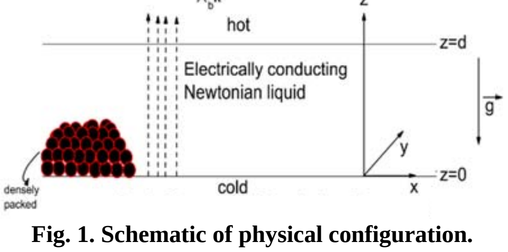

Fig. 1. Schematic of physical configuration.

2. GOVERNING EQUATIONS

We consider an electrically conducting two component fluid-saturating anisotropic porous layer of depth d, which is heated and salted from above confined between two parallel horizontal planes at $z = 0$ and $z = d$. Cartesian coordinates have been taken with the origin at the bottom of the porous medium, and the z-axis vertically upwards. The surfaces are extended infinitely in x and y directions and maintained at a constant gradient ΔT and salinity ΔS across the porous layer. A constant magnetic field $\vec{\mathcal{H}_b} = \mathcal{H}_b\hat{k}$ is maintained externally in the vertical upward direction. We use a two-phase model for heat equation with separate thermal conductivities for the solid and fluid phases respectively, because the solid and liquid phases of the medium are considered not to be in local thermal equilibrium. A schematic digram of physical configuration of the problem has been shown in Fig. 1. Under the Boussinesq approximation, the dimensional governing equations for the study of magneto-convection in an electrically conducting-fluid-saturated anisotropic porous layer are:

$$\nabla.\vec{q} = 0, \tag{1}$$

$$\nabla.\vec{\mathcal{H}} = 0, \tag{2}$$

$$\frac{1}{\rho_0}\nabla P + \frac{\rho_f}{\rho_0}\vec{g} - \nu K.\vec{q} + \frac{\mu_m}{\rho_0}(\vec{\mathcal{H}}.\nabla)\vec{\mathcal{H}} = 0, \tag{3}$$

$$\begin{aligned}\varepsilon(\rho_f c_f)\frac{\partial T_f}{\partial t} + \rho_f c_f(\vec{q}.\nabla)T_f = \nabla(\varepsilon\kappa_f\nabla T_f)\\ + D_f\nabla^2 C + h_s(T_s - T_f),\end{aligned} \tag{4}$$

$$(1-\varepsilon)(\rho_s c_s)\frac{\partial T_s}{\partial t} = \nabla((1-\varepsilon)\kappa_s\nabla T_s) - h_s(T_s - T_f), \tag{5}$$

$$\varepsilon\frac{\partial C}{\partial t} + (\vec{q}.\nabla)C = \varepsilon D_m\nabla^2 C + \varepsilon D_T\nabla^2 T_f, \tag{6}$$

$$\frac{\partial\vec{\mathcal{H}}}{\partial t} + \vec{q}.\nabla\vec{\mathcal{H}} - \vec{\mathcal{H}}\nabla\vec{q} = \eta\nabla^2\vec{\mathcal{H}}. \tag{7}$$

The constants and variables in the above equations have their usual meanings and are given in the nomenclature. The relation between the reference density and temperature is given by

$$\rho_f = \rho = \rho_0[1 - \beta_T(T_u - T_f) + \beta_C(C_u - C_f)]. \tag{8}$$

where $\beta_T > 0$ and $\beta_C > 0$ are the coefficient of thermal and concentration expansions, respectively. The boundary conditions are

$$\begin{aligned}T_f = T_S = T_l, C = C_l \text{ at } z = 0 \text{ and } T_f = T_S = T_u,\\ C = C_u \text{ at } z = d.\end{aligned} \tag{9}$$

Eqs. (1) − (7) may be non-dimensionalised using the transformation

$$(x,y,z) = d(x^*,y^*,z^*),(u,v,w) = \left(\frac{\varepsilon\kappa_{fz}}{(\rho_f c_f)d}\right)(u^*,v^*,w^*)$$

$$p = \left(\frac{\kappa_{fz}\mu}{(\rho_f c_f)K_z}\right)p^*, T_f = \Delta T T_f^*, T_s = \Delta T T_s^*$$

$$C = \Delta CC*,) - (=) - (_b) - (^*, t = \left(\frac{d^2(\rho_f c_f)}{\kappa_{fz}}\right) t* \quad (10)$$

where $'*'$ refers to the non-dimensional quantity. Taking the curl twice of momentum equation and considering its z-component, after dropping the asterisk, we get

$$\left(\nabla_1^2 + \frac{1}{\xi} D^2\right) w = -\mathrm{R}\nabla_1^2 T_f + R_S \nabla_1^2 C + \mathrm{QPm}\nabla^2 D h_z, \quad (11)$$

$$\frac{\partial T_f}{\partial t} + w\frac{\partial T_f}{\partial z} = \eta_f \nabla_1^2 T_f + \frac{\partial^2 T_f}{\partial z^2} + \mathrm{Du}\frac{\mathrm{R}_S}{\mathrm{R}}(\nabla_1^2 + D^2)C + H(T_s - T_f), \quad (12)$$

$$\chi\frac{\partial T_s}{\partial t} = \eta_s \nabla_1^2 T_s + \frac{\partial^2 T_s}{\partial z^2} - \gamma H(T_s - T_f), \quad (13)$$

$$\frac{\partial C}{\partial t} + w\frac{\partial C}{\partial z} = \frac{1}{\mathrm{Le}}(\nabla_1^2 + D^2)C + \frac{\mathrm{SrR}}{R_s \mathrm{Le}}(\nabla_1^2 + D^2)T_f, \quad (14)$$

$$\frac{\partial h_z}{\partial t} - \frac{\partial w}{\partial z} = \mathrm{Pm}\nabla^2 h_z. \quad (15)$$

together with the boundary conditions

$$T_f = T_S = C = 0 \text{ at } z = 0 \text{ and } T_f = T_S = C = 1, \text{ at } z = 1. \quad (16)$$

where

$$\mathrm{R} = \frac{\rho_f g\beta_T \Delta T K_z d}{\varepsilon\mu\kappa_{fz}}, \mathrm{R}_S = \frac{\rho_f g\beta_C \Delta C K_z d}{\varepsilon\mu\kappa_{fz}},$$
$$\mathrm{Q} = \frac{\mu_m ??K_z}{\rho_0 \nu \Lambda},$$
$$\mathrm{Pm} = \frac{\Lambda}{\kappa_{fz}}, \xi = \frac{K_x}{K_z}, H = \frac{h_s d^2}{\varepsilon\kappa_{fz}}, \mathrm{Du} = \frac{\mathrm{D}_f \beta_T}{\varepsilon\kappa_{fz}\beta_C},$$
$$\mathrm{Le} = \frac{\kappa_{fz}}{D_m}, \mathrm{Sr} = \frac{D_s \beta_C}{D_m \beta_T} \gamma = \frac{\varepsilon\kappa_{fz}}{(1-\varepsilon)\kappa_{fz}}, \chi = \frac{(\rho_s c_s)\kappa_{fz}}{(\rho_f c_f)\kappa_{sz}},$$
$$\eta_f = \frac{\kappa_{fx}}{\kappa_{fz}} = \frac{\kappa_{fy}}{\kappa_{fz}}, \eta_s = \frac{\kappa_{sx}}{\kappa_{sz}} = \frac{\kappa_{sy}}{\kappa_{sz}}. \quad (17)$$

are the Darcy-Rayleigh number, solutal Rayleigh number, Darcy-Chandrasekhar number, magnetic Prandtl number, non-dimensional inter-phase heat transfer coefficient, Dufour parameter, Lewis number, Soret parameter, porosity modified conductivity ratio and diffusivity ratio. Eqs. (11) − (16) allow a basic conduction state given by

$$\overline{q}_b = (0,0,0), T_{fb} = T_{sd} = C_b = z \quad (18)$$

Combining Eqs. (11) and (15), we get

$$\left[\left(\nabla_1^2 + \frac{1}{\xi}D^2\right)\left(\frac{\partial}{\partial t} - \mathrm{Pm}\nabla^2\right) - \mathrm{QPm}\nabla^2 D^2\right] w + \mathrm{R}\left(\frac{\partial}{\partial t} - \mathrm{Pm}\nabla^2\right)\nabla_1^2 T_f - \mathrm{R}\left(\frac{\partial}{\partial t} - \mathrm{Pm}\nabla^2\right)\nabla_1^2 C = 0. \quad (19)$$

3. LINEAR STABILITY ANALYSIS

We are concerned only with finding thresholds for the onset of convection, hence, it is sufficient to intigate the stability of the basic state by linear analysis. We perturbed the basic state by considering the infinitesimal perturbations. The perturbation variables are defined as:

$$[w, T_f, T_s, C] = [0, T_{fB}, T_{sB}, C_B] + [W, \Theta, \Phi, \Psi]. \quad (20)$$

Substituting expression (20) into the Eqs. (12) − (14) and (19), the linearised equations are

$$\left[\left(\nabla_1^2 + \frac{1}{\xi}D^2\right)\left(\frac{\partial}{\partial t} - \mathrm{Pm}\nabla^2\right) - \mathrm{QPm}\nabla^2 D^2\right] W + \mathrm{R}\left(\frac{\partial}{\partial t} - \mathrm{Pm}\nabla^2\right)\nabla_1^2\Theta - \mathrm{R}_S\left(\frac{\partial}{\partial t} - \mathrm{Pm}\nabla^2\right)\nabla_1^2\Psi = 0, \quad (21)$$

$$W + \left(\frac{\partial}{\partial t} + \eta_f \nabla_1^2 - D^2 + H\right)\Theta - H\Phi \quad \mathrm{Du}\frac{\mathrm{R}_S}{\mathrm{R}}\left(\nabla_1^2 + D^2\right)\Psi = 0, \quad (22)$$

$$-\gamma H\Theta + \left(\chi\frac{\partial}{\partial t} - \eta_s \nabla_1^2 - D^2 + \gamma H\right)\Phi = 0, \quad (23)$$

and

$$W - \frac{\mathrm{SrR}}{\mathrm{R}_S \mathrm{Le}}\left(\nabla_1^2 + D^2\right)\Theta + \left(\frac{\partial}{\partial t} - \frac{1}{\mathrm{Le}}(\nabla_1^2 + D^2)\right)\Psi = 0 \quad (24)$$

The corresponding boundary conditions reduce to

$$W = 0 \text{ at } z = 0,1 \text{ and } \Theta = \Psi = 0 \text{ at } z = 0,1. \quad (25)$$

In this study we restrict our self to investigate only stationary stability, as discussed by Saravanan and Jegajothi (2010). To solve our system of Eqs. (21) − (25), we apply normal mode technique, thus express the perturbed variables as given below 5)

$$[W, \Theta, \Phi, \Psi] = [A_1, A_2, A_3, A_4] e^{i(lx+my)} \sin \pi z \quad (26)$$

where A_1, A_2, A_3, A_4 are constants and l, m are wave numbers in x, y directions respectively. We Substitute Eq. (26) into Eqs. (21) − (24) and get the matrix equation $XA = 0$ where

$$X=\begin{pmatrix} \left(a^2+\dfrac{\pi^2}{\xi}\right) & a^2\mathrm{R} & 0 & -a^2\mathrm{R}_S \\ 1 & (\eta_f a^2+\pi^2+H) & -H & \mathrm{Du}\dfrac{\mathrm{R}_S(a^2+\pi^2)}{\mathrm{R}} \\ 0 & -\gamma H & (\eta_s a^2+\pi^2+\gamma H) & 0 \\ 1 & \dfrac{\mathrm{SrR}(a^2+\pi^2)}{\mathrm{R}_S\mathrm{Le}} & 0 & \dfrac{(a^2+\pi^2)}{\mathrm{Le}} \end{pmatrix} \tag{27}$$

For a non-trivial solution we have $Det(X) = 0$. Thus we get

$$R=\frac{1}{(1+Sr)}[\mathrm{Du}\{R'-\alpha_5\}\frac{\alpha Sr}{a^2} + R'\frac{\alpha_2}{\alpha}+\frac{\alpha_2\alpha_5}{a^2} + \frac{R'\alpha_3 H}{\alpha\alpha_4}+\frac{\alpha_3\alpha_5 H}{a^2\alpha_4}]. \tag{28}$$

where

$\mathrm{R}'=\mathrm{R}_S\mathrm{Le}, \alpha=(a^2+\pi^2), \alpha_1=a^2+\dfrac{\pi^2}{\xi}, \alpha_2=\eta_f a^2+\pi^2, \alpha_3=\eta_s a^2+\pi^2, \alpha_4=\eta_s a^2+\pi^2+\gamma H$

The minimum value of R in Eq. (28) can be obtained by $\dfrac{\partial R}{\partial a}=0$. However it is difficult to obtain it in closed form, therefore, we have used the multidimensional Newton-Raphson iteration scheme as used by Banu and Rees (2002).

When we take Q = 0 and isotropic case, we get

$$R=\frac{1}{(1+Sr)}[\mathrm{Du}\{\mathrm{R}'-\frac{(\pi^2+a^2)^2}{a^2}Sr\} + \{R'+\frac{(\pi^2+a^2)^2}{a^2}\}\{1+\frac{H}{(\pi^2+a^2+\gamma H)}\}] \tag{29}$$

same as result of Saravanan and Jegajothi (2010).

4. RESULTS AND DISCUSSIONS

In this section, we have attempted to built up a picture of how LTNE affects the onset of double-diffusive thermal convection in electrically conducting fluid saturated anisotropic porous medium under cross-diffusion effects. The porous medium is considered as mechanically and thermally anisotropic. We will emphasis only the effects of Darcy-Chandrasekhar number (Q) (magnetic effect), non-dimensional inter-phase heat transfer coefficient (H) (LTNE effect), DuFour (Du) and Soret (Sr) effects (cross-diffusion effects) out of several. A comparative study has also been performed on the basis of isotopic and anisotropic effects, cross-diffusion and without crossdiffusion (pure double diffusive) and for different values of Sr and Du.

Since H is a quantity that is not easily measurable, therefore, we take nearly the same set of values of the parameters as taken by Govender and Vadasz (2007). The values of the Q are taken to be very small due to porous medium. Also due to the non-availability of accurate experimental values of Sr and Du, it is impossible to proceed to a practical quantitative study of cross-diffusion effects. The results obtained from Eq. (28) have been presented graphically in Figs. 2 − 15.

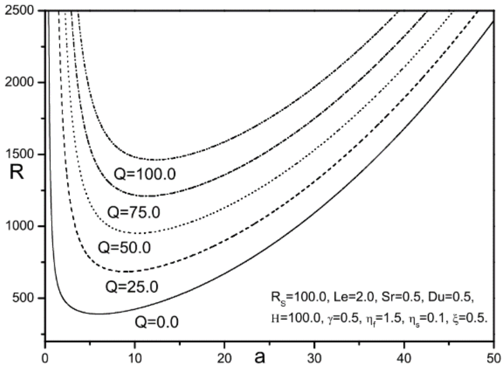

Fig. 2. Variation of R with a.

Initially we consider the neutral stability curves in Figs. 2 − 5 and depict the variation of the Rayleigh number R with respect to the wave number a for fixed values of the parameters Q = 25.0, $\eta_f = 1.5$, s = 0.1, ξ = 0.5, H = 100.0 Du = 0.5, Sr = 0.5, $\mathrm{R}_S = 100.0$, Le = 2.0 and γ = 0.5, while varying one of the parameters. From the figures we see that initially the value of R is large and decreases on increasing the wave number. It touches the minimum value at some value of a and then increases as the values of a increases. The minimum value of R is known as the critical value of Rayleigh number, denoted by R_c. The corresponding value of the wave number a is known as the critical value of the wave number and is denoted by a_c.

In Fig. 2, we find the effect of Q on thermal instability. From the figure it is observed that as the value of Q increases, the minimum value of R also increases, thus showing the stabilizing effect on the system. This can be explained for observing the behaviour of magnetic field lines. Increasing the value of Q implies that magnetic field strength permeating the medium is considerably strong. It induces viscosity into fluid, and the magnetic lines are distorted by convection. Then these magnetic lines hinder the growth of disturbances leading to the delay in onset of instability.

In Fig. 3, we investigate the effect of variation of dimensionless inter phase heat transfer coefficient (H) on the onset of convection. We observed that as H increases the minimum value of R increases thus the system becomes more stable. To explain this: if we increases the value of $H=\dfrac{h_s d^2}{\varepsilon\kappa_{fz}}$, only the value of inter phase heat transfer coefficient h_s increases as d, ε and κ_f are used in fixed parameters. That indicates heat transfer between solid and fluid is increases (maximum heat involve in transferring not in convection), therefore delay the onset of convection.

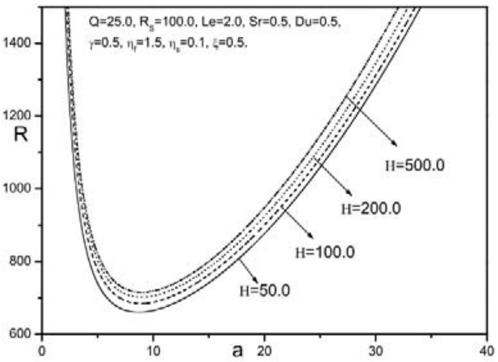

Fig. 3. Variation of R with a.

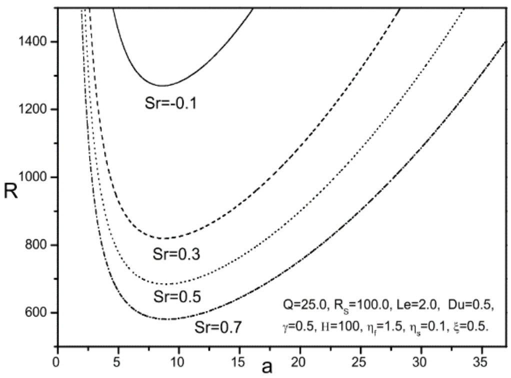

Fig. 4. Variation of R with a.

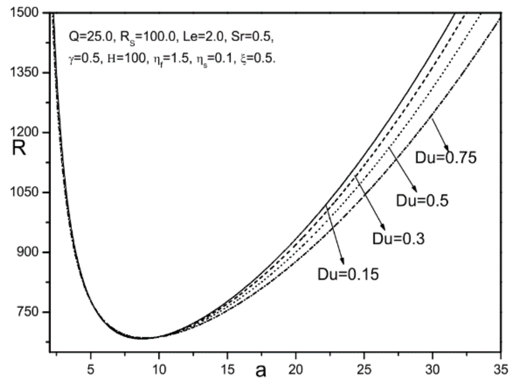

Fig. 5. Variatio n of R with a.

In Fig. 4, we show the effect of Soret parameter (Sr). It is observed that on increasing the value of Sr, decreasing the minimum (critical) value of R, thus destabilizes the system. It is also remarkable that maximum stability obtained for negative value of Sr. It can be explain as: positive Sr means solute diffuse towards cooler plates, whereas negative Sr represents that solute diffuse towards hotter plate. Therefore negative Sr opposes the convection and delay the onset of convection.

In Fig. 5, we considered the effect of DuFour parameter Du. We find that critical value of Rayleigh number (R_c) increases on increasing the values of Du, thus making the system stable.

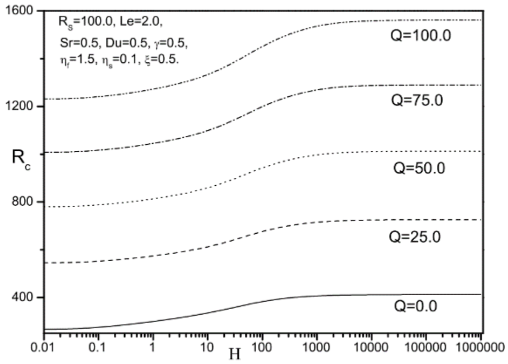

Fig. 6. Variation of R_c with H.

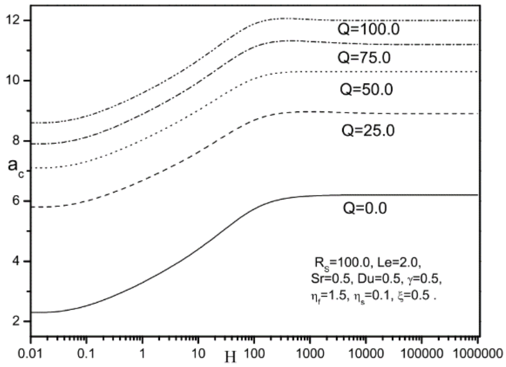

Fig. 7. Variation of a_c with H.

Now we consider the Figs. 6 − 11, and depict the variation of R_c and the corresponding α_c with respect to the inter-phase heat transfer coefficient H for the fixed values of the parameters Q = 25.0, $\eta_f = 1.5$, $\eta_s = 0.1$, ξ = 0.5, Du = 0.5, Sr = 0.5, $\mathrm{R}_S = 100.0$, Le = 2.0 and γ = 0.5, while varying one of the parameters. From these figures we observe that initially when *H* is small the value of R_c is also small, increases for intermediate values of *H* and then becomes constant as the values of *H* increased further. Figs. 6 and 10, depict the effects of Q and Du respectively, on thermal instability. It is observed from the figures that on increasing Q and Du, the value of R_c increases, thus stabilizing the system. Further from Fig. 8 we find that on increasing Sr the value of R_c decreases. This shows that increase in the value of Sr makes the system unstable. From the Figs. 7, 9, and 11, we find that initially when *H* is small α_c is also small. At some particular value of *H* we see that a_c takes on a maximum value and then decreases and becomes constant at very large values of H. From Fig. 7, which depicts the effect of Q on magneto-convection, we find that value of α_c increases on increasing Q. However from Figs. 9 and 11 having same patterns, which show the effect of Sr and Du on thermal instability of the system, we observed that initially ac increases on decreasing Sr and Du, attains its maxi-mum values at some H and then decreases on increasing Sr and Du respectively. To understand the above phenomena, we can recall the physics of inter phase heat transfer coefficient. It states that for small values (i.e. $H \rightarrow 0$) and large values (i.e. $H \rightarrow \infty$) of H system behaves like closed to local thermal equilibrium (CLTE) not to LTNE. A simple physics involve behind this, when H is small that indicates that heat transfer between solid and fluid is so small that it cannot influence the convection. Similarly when H is very large, heat transfer between solid and fluid is going to negligible and cannot influence the convection. So that we can see the effect of LTNE only for intermediate value of H.

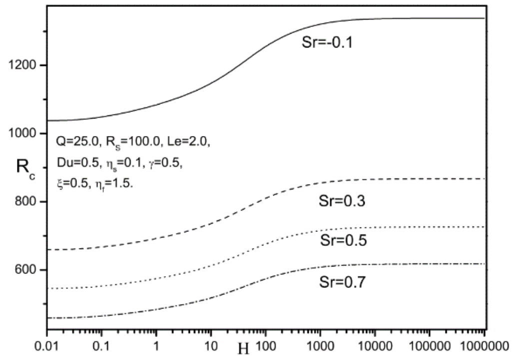

Fig. 8. Variation of R_c with H.

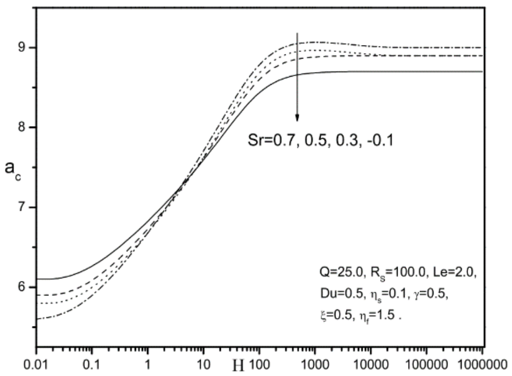

Fig. 9. Variation of a_c with H.

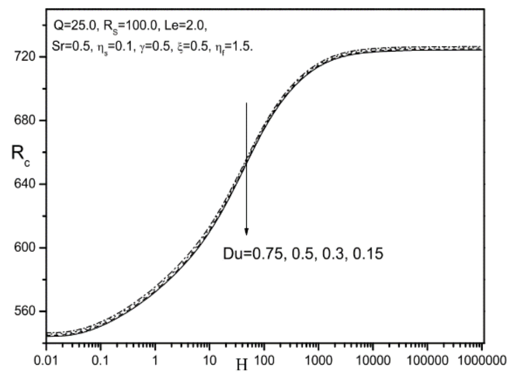

Fig. 10. Variation of R_c with a_c.

Now in upcoming figures, we are going to discuss a comparative study between different conditions as isotropic (mechanical as well as thermal) and anisotropic porous medium, cross diffusion and without cross diffusion and in last for different values of Du and Sr.

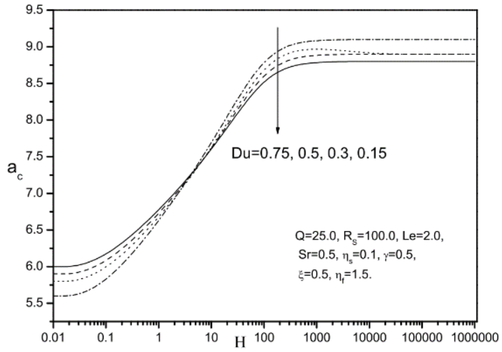

Fig. 11. Variation of a_c with H.

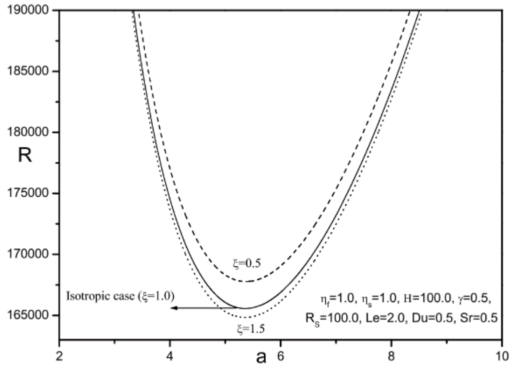

Fig. 12. Comparison between iso tropic and anisotropic case in terms of ξ.

In Figs. 12 and 13, we compare the results of isotropic and anisotropic cases with the help of graphs. In Fig. 12, mechanical anisotropic and thermal isotropic cases are considered. From the figure, it is clear that as mechanical anisotropic parameter (ξ) increases from 0.5 to 1.5, the value of critical Rayleigh number decreases and thus destabilize the system. We represent our study as isotropic case (ξ= 1) and anisotropic case (ξ < 1) and (ξ > 1). To understand the phenomena, we recall definition of $\xi = \frac{k_x}{k_z}$. As k_z is used in other fixed parameters, so kz

increasing ξ from 1 means increasing the value of $k_x (k_x > k_z)$ that is flow permeability is more in horizontal direction than vertical and similarly if the value of ξ decreases from 1, decreasing the value of $k_x (k_x < k_z)$. From figure, system is most stable at = 0.5, then for ξ = 1.0 and least stable at ξ = 1.5. When ξ < 1, horizontal permeability is smaller and slow down horizontal motion thus conduction solution is stabilized. Whereas when ξ > 1, horizontal permeability is larger and enhance horizontal motion thus conduction solution is destabilized.

In Fig. 13, we compare the stability criteria for mechanical isotropic and thermal anisotropic porous medium. The study based on the results for three conditions (i) when thermal conductivity for solid is greater than fluid i.e. $(\eta_s > \eta_f)$, (ii) isotropic case $(\eta_s = \eta_f)$ and (iii) when $\eta_s < \eta_f$. It is clear from figure that system is most stable for condition (i) (maximum value of the R_c) and least stable for condition (iii) (minimum value of the R_c). This behaviour of system is explained as: if thermal conductivity of solid is more than fluid then most of the heat transfer in solid and delay onset of convection. Whereas when conductivity of fluid is more than solid, it enhance the onset of convection.

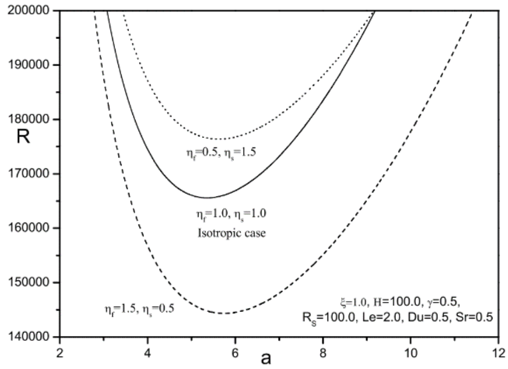

Fig. 13. Comparison between isotropic and anisotropic case in terms of η_f and η_s.

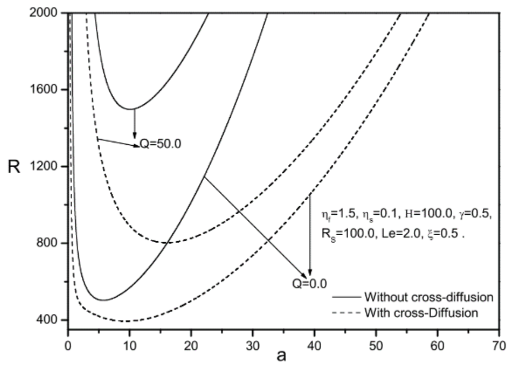

Fig. 14. Comparison between with and without cross-diffusion.

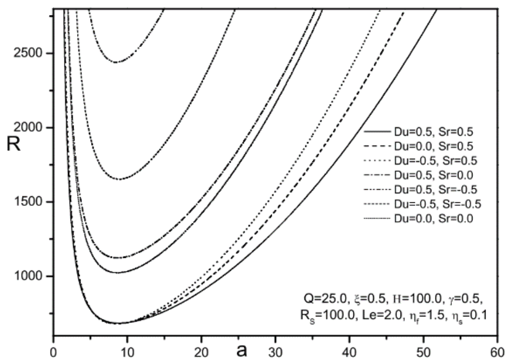

Fig. 15. Comparison of R_c for different cases related with value of Du and Sr.

In Fig. 14, we compare the results for two conditions (i) with cross-diffusion (Sr = 0.5,Du = 0.5) and (ii) without cross-diffusion (Sr = 0,Du = 0) that is purely double-diffusive convection. It is observed from figure that system is more stable for purely double-diffusion effect than with diffusion effect. The reason of this behaviour is explained as: we considered the positive Sr and we mentioned above that has destabilizing effect. It is also well known fact that the effect of Du on onset of convection in porous media is negligible in comparison to Sr.

In Fig. 15, we considered different conditions related to the values of Sr and Du. We conclude from figure that $R_{c(Soret+,Dufor+)} < R_{c(Soret+)} < R_{c(Soret-,Dufor+)} < R_{c(nocross-diffusion)} < R_{c(Dufour+)} < R_{c(Soret-,Dufour+)} < R_{c(Soret-,Dufour-)}$, thus for fixed value of other parameters, positive Sr and positive Du highly destabilize the system where as negative Sr and negative Du stabilize the system. These patterns of our results are similar to those obtained by Saravanan and Jegajothi (2010).

5. CONCLUSIONS

In the present article, linear stability analysis is performed to investigate the LTNE effect on finger type double-diffusive, magneto-convection in an anisotropic porous medium, heated and salted from above. The Darcy model is used for the momentum equation. We divided our study into two parts. In first part, we studied the effect of main controlling parameters in $R - a$ and $R_c / a_c - \log H$ planes. Whereas in second part, we illustrated a comparative study between different cases and conditions.

From the first part, following conclusions are drawn:

- The main controlling parameter Q, which represents the effect of magnetic field on the onset of convection, strengthening the stability of system.
- Inter-phase heat transfer coefficients (H) and DuFour parameter (Du) stabilize the system that is delay the onset of convection.
- Soret parameter (Sr) destabilizes the system, i. e. enhance the onset of convection, and system is most stable for negative value of Sr.
- From $R_c - \log H$ plane, we conclude that for $H \to 0$ and $H \to \infty$, system behaves like CLTE than LTNE, whereas for intermediate values of H, the effect of LTNE is reasonable.

From the second part, the conclusions are:

- We compared the results of mechanical anisotropic and thermally isotropic case and find the stability criteria as, most stable for $\xi < 1$ then for $\xi = 1$ (isotropic case) and least stable for $\xi > 1$.
- We also compared the results of thermally anisotropic and mechanically isotropic case and conclude stability criteria as: most stable when $\eta_s > \eta_f$ then $\eta_s = \eta_f$ and least stable $\eta_s < \eta_f$.
- It is observed that system is more stable for pure double diffusive convection than with

cross-diffusion case.

- We presented the stability criteria for different conditions based on values of Sr and Du.

ACKNOWLEDGEMENT

The authors are grateful to the referees for many useful comments.

REFERENCES

Banu, N. and D. A. S. Rees (2002). Onset of Darcy-Benard convection using a thermal non-equilibrium modal. *Int. J. Heat Mass Trans.* 45, 2221-2228.

Bhadauria, B. S. and A. Sherani (2008b). On- set of Darcy-convection in a magnetic-fluid saturated porous medium subject to temperature modulation of the boundaries. *Trans. in Porous Media* 73, 107-127.

Bhadauria, B. S. and A. K. Srivastava (2010). Magneto-double diffusive convection in an electrically conducting-fluid-saturated porous medium with temperature modulation of the boundaries. *Int. J. Heat and Mass Trans.* 53, 2530-2538.

Bhadauria, B. S., (2008a). Combined effect of temperature modulation and magnetic field on the onset of convection in an electrically conducting-fluid-saturated porous medium. *ASME J. Heat Trans*. 130.

Castinel, G. and M. Combarnous (1974). Ceitere d' apparition de la convection naturelle dans une couche poreuse anisotropic horizontal. *C. R. Acad. Sci*. 278, 701-704.

Epherre, J. F., (1975). Critere d' apparition de la convection naturelle dans une couche poreuse anisotrope. *Revue. Gen. Thermique* 168, 949-950.

Govender, S. and P. Vadasz (2007). The effect of mechanical and thermal anisotropy on the stability of gravity driven convection in rotating porous media in the presence of thermal non-equilibrium. *Trans. in Porous Media* 69, 55-66.

Hage, E. and A. Tilgner (2010). High Rayleigh number convection with double diffusive finger. *Phys. Fluids* 22, 1-7.

Knobloch, E. (1980) Convection in binary fluids. *Phys. Fluids* 23(9), 1918-1920.

Kuznetsov, A. V., D. A. Nield, A. Barletta, M. Celli (2015). Local Thermal Non-equilibrium and Heterogeneity Effects on the Onset of Double-Diffusive Convection in an Internally Heated and Soluted Porous Medium. *Trans. in Porous Media* 109, 393-409.

La Porta, A. and C. M. Surko (1998). Convective instability in a fluid mixture heated from above. *Phys. Rev. Lett.* 80(17), 3759-3762.

Malashetty, M. S., I. S. Shivakumara and K. Sridhar (2005b). The onset of convection in an anisotropic porous layer using a thermal non-equilibrium model. *Trans. in Porous Media* 60, 199-215.

Nield, D. A. and A. Bejan (2013*). Convection in Porous Media*. 4th ed., Springer, New York.

Patil, R. P. and N. Rudraiah (1973). Stability of hydromagnetic thermoconvective flow through porous medium. *Trans. ASME J. Appl. mech.* 40(E), 879-884.

Rees, D. A. S. and I. Pop (2005). Local thermal non-equilibrium in porous medium convection. *Trans. Phenomena in porous media* 3, 147-173.

Rees, D. A. S., A. P. Bassom and .P. G. Siddeshwar (2008). Local thermal non-equilibrium effects arising from the injection of a hot fluid into a porous medium. *J. Fluid Mech.* 594, 379-398.

Rudraiah, N. and M. S. Malashetty (1986). The influence of coupled molecular diffusion on the double diffusive convection in porous medium. *ASME J. Heat Trans.* 108, 872-876.

Saravanan, S. and R. Jegajothi (2010). Stationary fingering instability in a non-equilibrium porous medium with coupled molecular diffusion. *Trans. in porous media* 84(3), 755-771.

Saravanan, S. and H. Yamaguchi (2005) Onset of centrifugal convection in a magnetic-fluid-saturated porous medium. *Phys. Fluids* 17, 1-9.

Srivastava, A. K., B. S. Bhadauria and J. Kumar (2011) Magnetoconvection in an Anisotropic Porous layer using Thermal Non-equilibrium Model. *J. Special Topics and Reviews in Porous Med.* 2(1), 1-10.

Wallace, W. E., C. I. Pierce and W. K. Sawyer (1969). Experiments on the flow of mercury in porous media in a transverse magnetic field, Report, Bureau of Mines: Washington DC.