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Unsteady Magnetohydrodynamic Flow of Second Grade Fluid due to Uniform Accelerating Plate

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ABSTRACT

New exact solutions for unsteady magnetohydrodynamic (MHD) flows of a generalized second-grade fluid due to uniform accelerating plate are derived. The generalized second-grade fluid saturates the porous space. Fractional derivative is used in the governing equation. The analytical expressions for velocity and shear stress fields are obtained by using Laplace transform technique for the fractional calculus. The obtained solutions are expressed in series form in terms of Fox H-functions. Similar solutions for ordinary second-grade fluid passing through a porous space are also recovered. Moreover, several figures are sketched for the pertinent parameters to analyze the characteristics of velocity field and shear stress.

Keywords: MHD flow; Generalized second-grade fluid; Fractional derivatives; Fox H-functions; Discrete Laplace transform.

1. INTRODUCTION

Interest and research activities regarding the flows of non-Newtonian fluids have increased in the last few decades. This is due to their industrial and engineering applications as well as the interesting mathematical challenges offered by the equations governing the flows. A large class of fluids is non-Newtonian in which the relation between the deformation rate and shear stress is non-linear. Since we have no model available which is considered to be a universal constitutive model and can also predict the behavior of all available non-Newtonian fluids. As a result, many constitutive models of non-Newtonian fluids have been developed. Rivlin and Ericksen (1955) introduced a subclass of non-Newtonian fluids known as second-grade fluids for which a possibility exist to obtain the exact solution. Exact solutions of second-grade fluids for start-up flows have been investigated by Bandelli and Rajagopal (1995) using integral transform technique. Tan and Xu (2002) discussed flow of a suddenly moved flat plate in a generalized second-grade fluid. Exact solutions of a generalized second grade fluid corresponding to oscillatory flow between two cylinders have been achieved by Mahmood *et al*. (2010). Tripathy (2011) discussed peristaltic motion of a generalized second grade fluid passing through a cylindrical tube. Tan and Masuoka (2004) obtained solutions for unsteady motion between two parallel plates of a generalized second grade fluid.

In the last few decades the study of fluid motion through porous media has received much attention due to its importance not only to the field of academic but also to the industry. Such motions have many applications in many industrial and biological processes such as food industry, irrigation problems, oil exploitation, motion of blood in the cardiovascular system (Zaman *et al*. 2012), chemistry and bio-engineering, soap and cellulose solutions and in biophysical sciences where the human lungs are considered as a porous layer. Unsteady MHD flows of viscoelastic fluids passing through porous space are of considerable interest. In the last few years a lot of work has been done on MHD flow, see (Kabir *et al*. 2015; Ramesh and DevakarSingh 2015; Das *et al*. 2014; Seth and Singh 2013; Ramakrishnan and Shailendhra 2013) and references therein.

Rheological constitutive equations with fractional derivatives (Podlubny 1999) have long played an important role in the description of the proper-ties of polymer solutions and melts. These equations are derived from well-known models (e.g., the Maxwell model) by substituting ordinary derivatives of the first, second, and higher orders with fractional derivatives of non-integer orders. In this way, the order of the derivative is related to a material parameter that can be associated with the degree of conversion as for the sol-gel transition. In some cases, it has been shown that constitutive equations employing fractional derivatives are linked to molecular theories. At least the modified viscoelastic models are appropriate to describe the behaviors of Xanthan gum and Sesbania gels.

According to the authors information up to yet no study has been done on the MHD flow of generalized second-grade fluid induced by uniform accelerating motion of the plate flowing through a porous space. Hence, our main objective in this note is to make a contribution in this regard. We take an incompressible MHD flow passing through porous space of a generalized second-grade fluid. Laplace transform method has been used for the fractional calculus to obtained exact solutions for the profiles of velocity field and the corresponding shear stress. The obtained solutions satisfies all the imposed initial and boundary conditions are expressed in terms of Fox-H function. Similar solutions for ordinary second-grade fluid are obtained as particular case of general solution. Finally, the effects of different parameters on the motion are analyzed graphically.

2. GOVERNING EQUATION

The equation of continuity and momentum of MHD flow passing through porous space is given by Tan and Masuoka (2004)

$$\nabla.\mathbf{V}=0; \quad \rho\left(\frac{d\mathbf{V}}{dt}\right)=div\mathbf{T}-\sigma B_0^2\mathbf{V}+\mathbf{R}, \qquad (1)$$

where V = (u,v,w) represents velocity vector, electrical conductivity and density of the fluid are represented by σ and ρ respectively, B_0 is the magnitude of a uniform magnetic field, material time derivative is denoted by d/dt, Cauchy stress tensor is rep-resented by T, and R is the Darcy's resistance of the porous space.

For an incompressible generalized second-grade fluid the Cauchy stress tensor T is given by

$$\mathbf{T}=\mathbf{S}-p\mathbf{I}, \quad S=\mu\mathbf{W}_1+\alpha_1\mathbf{W}_2+\alpha_2\mathbf{W}_1^2, \qquad (2)$$

where $\mathbf{S}$ and $p\mathbf{I}$ represents the extra stress tensor and the indeterminate spherical stress, the dynamic viscosity is denoted by μ, normal stress moduli are represented by α_1 and α_2 and the kinematic tensors are $\mathbf{W}_1$ and $\mathbf{W}_2$ defined as

$$\mathbf{W_1}=\mathbf{L}+\mathbf{L^T}, \quad \mathbf{W_2}=D_t^{\beta}\mathbf{W_1}+\mathbf{W_1}\mathbf{L}+\mathbf{L^T}\mathbf{W_1}, \qquad (3)$$

where L is the velocity gradient and D_t^{β} represents the operator for fractional differentiation whose order is β and is based on the Riemann-Liouville definition (Podlubny, 1999).

$$D_a^b[g(a)]=\frac{1}{\Gamma(1-b)}\frac{d}{da}\int_0^a\frac{g(t)}{(a-t)^b}dt, 0\le b<1, \qquad (4)$$

where the Gamma function is denoted by $\Gamma(\cdot)$. The model for an ordinary second-grade fluid can be obtained by putting β = 1. For the compatibility of this model with thermodynamics it is required that the material moduli should obey the following conditions

$$\alpha_1+\alpha_2=0, \quad 0\le\alpha_1 \text{ and } \mu\ge 0. \qquad (5)$$

For a second-grade fluid Darcy's resistance satisfies the following equation

$$R=-\frac{\phi}{\kappa}(\mu+\alpha_1\frac{\partial}{\partial t})\mathbf{V}, \qquad (6)$$

where κ > 0 and $\phi(0<\phi<1)$ are the permeability and the porosity of the porous medium. For the following problem we consider the velocity field and an extra stress of the form

$$\mathbf{V}=(u(y,t),0,0), \quad \mathbf{S}=S(y,t)$$
(7)

where u is the velocity taken in the x-direction. Substituting Eq. (7) into Eq. (2) and taking into account the initial condition

$$S(y,0)=0, \ y>0, \qquad (8)$$

the fluid being at rest up to the time t = 0, we get

$$S_{xy}=(\mu+\alpha_1 D_t^{\beta})\partial_y u(y,t) \qquad (9)$$

where S_{yy} = S_{zz} = S_{xz} = S_{yz} = *0* and S_{xy} = S_{yx}. The balance of linear momentum in the absence of body forces and pressure gradient is given by

$$\partial_y S_{xy}-\sigma B_0^2 u-\frac{\phi}{\kappa}(\mu+\alpha_1\frac{\partial}{\partial t})=\rho\partial_t u(y,t), \qquad (10)$$

By putting S_{xy} from Eq. (9) into (10), we find the governing equation under the form

$$\rho\partial_t u(y,t)=(\mu+\alpha_1 D_t^{\beta})\partial_y^2 u(y,t)-\sigma B_0^2 u(y,t)- \frac{\phi}{\kappa}(\mu+\alpha_1\frac{\partial}{\partial t})u(y,t). \qquad (11)$$

3. STATEMENT OF THE PROBLEM

We take an unsteady incompressible flow of homogenous and electrically conducting second-grade fluid bounded by a rigid plate at y = 0. The plate is taken normal to y-axis and the fluid saturates the porous medium y > 0. The electrically conducting fluid is stressed by a uniform magnetic field B0 parallel to the y-axis, while the induced magnetic field is neglected by choosing a small magnetic Reynolds number. Initially, both the plate and the fluid are at rest, and after time t = 0, it is suddenly set into motion by translating the flat plate in its plane, with a constant velocity A. The initial and boundary conditions of velocity field are

$$u(y,0)=0, \quad y>0,$$
$$u(0,t)=At, \quad t>0, \tag{12}$$
$$u(y,t),\ \partial_y u(y,t)\to 0 \quad as \quad y\to\infty \quad and \quad t>0.$$

4. CALCULATION OF VELOCITY FIELD

Employing the non-dimensional quantities

$$u^*=\frac{u}{U}, y^*=\frac{yU}{\nu}, t^*=\frac{tU^*}{\nu}, \alpha^*=\frac{\alpha_1 U^2}{\rho\nu^2},$$
$$A^*=\frac{A}{U}, \tau=\frac{S}{\rho U^2}, K=\frac{\kappa U^2}{\phi \nu^2}, M^2=\frac{\sigma \nu B_0^2}{\rho U^2}, \tag{13}$$

The dimensionless mark * is omitted here for simplicity. Thus, the governing equations of dimensionless motion become

$$\partial_t u(y,t)=(1+\alpha D_t^{\beta})\partial_y^2 u(y,t)-\frac{1}{K}(1+\alpha\frac{\partial}{\partial t})u(y,t)$$
$$M^2 u(y,t), \tag{14}$$

$$\tau(y,t)=(1+\alpha D_t^{\beta})\partial_y u(y,t) \tag{15}$$

with the given conditions as

$$u(y,0)=0, \quad y>0,$$
$$u(0,t)=At, \quad t>0, \tag{16}$$
$$u(y,t),\ \partial_y u(y,t)\to 0 \quad as \quad y\to\infty, \ and\ t>0.$$

We apply the Laplace transform to Eq. (14) and using the Laplace transform formula for sequential fractional derivatives (Sneddon 1951)

$$\bar{u}(y,q)=\int_0^{\infty} u(y,t)e^{-qt}dt, \quad q\ge 0. \tag{17}$$

Taking into account the corresponding initial and boundary conditions (16), we get the following differential equation

$$\partial_y^2 \bar{u}(y,t)-\left(\frac{1+\alpha q}{K(1+\alpha q^{\beta})}+\frac{q+M^2}{1+\alpha q^{\beta}}\right)\bar{u}(y,q)=0, \tag{18}$$

$$\bar{u}(0,q)=\frac{A}{q^2}, \quad q>0, \tag{19}$$

$$\bar{u}(y,q),\ \partial_y u(y,q)\to 0 \quad as \quad y\to\infty, \ and\ q>0. \tag{20}$$

The solution of Eq. (18) satisfying the boundary conditions (19) is of the following form

$$\bar{u}(y,q)=\frac{A}{q^2}\exp\left(-y\sqrt{\frac{((1+aq)+K(q+M^2))}{K(1+aq^{\beta})}}\right) \tag{21}$$

To get the analytical solution for the velocity field and to avoid difficult calculations of contour integrals and residues, we apply the discrete inverse Laplace transform method [21], but first we need to express Eq. (21) in series form as

$$\bar{u}(y,q)=\sum_{e_1=0}^{\infty}\sum_{f_1=0}^{\infty}\sum_{g_1=0}^{\infty}\sum_{h_1=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{A\alpha^{h_1+r+s}}{e_1!f_1!g_1!h_1!}$$
$$\times\frac{\Gamma(f_1-e_1/2)\Gamma(g_1-f_1)\Gamma(h_1+f_1)\Gamma(r+e_1/2)}{r!s!\Gamma(e_1/2)\Gamma(-e_1/2)\Gamma(e_1/2)\Gamma(f_1)\Gamma(-f_1)} \tag{22}$$
$$\times\frac{\Gamma(s-e_1/2)(-1)^{e_1+f_1+g_1+h_1+r+s}M^{2g_1}y^{e_1}}{K^{e_1/2-f_1}q^{-f_1-h_1-\beta r-s+2}}$$

Applying the discrete inverse Laplace transform to Eq. (22), we get

$$\bar{u}(y,q)=\sum_{e_1=0}^{\infty}\sum_{f_1=0}^{\infty}\sum_{g_1=0}^{\infty}\sum_{h_1=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{A\alpha^{h_1+r+s}}{e_1!f_1!g_1!h_1!}$$
$$\times\frac{\Gamma(f_1-e_1/2)\Gamma(g_1-f_1)\Gamma(h_1+f_1)\Gamma(r+e_1/2)}{r!s!\Gamma(e_1/2)\Gamma(-e_1/2)\Gamma(e_1/2)\Gamma(f_1)\Gamma(-f_1)} \tag{23}$$
$$\times\frac{\Gamma(s-e_1/2)(-1)^{e_1+f_1+g_1+h_1+r+s}M^{2g_1}y^{e_1}}{K^{e_1/2-f_1}\Gamma(-f_1-h_1-\beta r-s+2)}$$
$$\times t^{-f_1-g_1-h_1-r-s+1}.$$

To get Eq. (23) in a more compact form we use the Fox H-function (Mathai 2010)

$$u(y,t)=A\sum_{f_1=0}^{\infty}\sum_{g_1=0}^{\infty}\sum_{h_1=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{(-1)^{e_1+f_1+g_1+r+s}}{e_1!f_1!g_1!r!}$$
$$\times\frac{M^{2g_1}y^{e_1}t^{-f_1-\beta r-s+1}\alpha^{r+s}}{s!K^{e_1/2-f_1}}$$
$$\times H_{5,7}^{1,5}\left[\frac{\alpha}{t}\left|\begin{matrix}(1-g_1+f_1,0),(1-f_1,1)\\(1-s+e_1/2,0),(1-r-e_1/2,0)\\(1-f_1+e_1/2,0).\\(1-e_1/2,0),(1-f_1,0),(1+f_1,0)\\(1+e_1/2,0),(1-e_1/2,0)\\(0,1),(f_1,\beta r+s-1,-1).\end{matrix}\right.\right]. \tag{24}$$

To obtain Eq. (24), the following Fox H-function property is used

$$H_{s,t+1}^{1,s}\left[-\sigma\left|\begin{matrix}(1-a_1,A_1),...,(1-a_s,A_s)\\(1,0),(1-b_1,B_s),...,(1-b_t,B_t)\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{\Gamma(a_1+A_1r)...\Gamma(a_s+A_sr)}{r!\Gamma(b_1+B_1r)...\Gamma(b_t+B_tr)}\sigma^r.$$

5. CALCULATION OF SHEAR STRESS

To obtain the shear stress we apply the Laplace transform to Eq. (15), to obtain

$$\bar{\tau}(y,q)=(1+\alpha q^{\beta})\partial_y \bar{u}(y,q). \tag{25}$$

Substituting $\bar{u}(y,q)$ from Eq. (21), we get

$$\bar{\tau}(y,t)=-\frac{A(1+\alpha q)}{q^2}\exp(-\sqrt{B}y)\sqrt{B}. \tag{26}$$

where

$$B=\frac{(1+\alpha q)+K(q+M^2)}{K(1+\alpha q^{\beta})}.$$

To get a more compact form of $\overline{\tau}(y,q)$, we write Eq.

(26) in series form as

$$\overline{\tau}(y,q)=\sum_{e_1=0}^{\infty}\sum_{f_1=0}^{\infty}\sum_{g_1=0}^{\infty}\sum_{h_1=0}^{\infty}\sum^{**}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{A}{e_1!f_1!g_1!}$$
$$\times\frac{(-1)^{e_1+f_1+g_1+h_1+\zeta_1+r+s+1}\Gamma(f_1-e_1/2)}{h_1!i_1!j_1!k_1!l_1!m_1!r!s!K^{e_1/2-f_1-i_1+1/2}}$$
$$\times\frac{(-1)^{h_1+k_1+l_1+m_1+r+s}\Gamma(g_1-f_1)\Gamma(h_1+f_1)}{\Gamma(e_1/2)\Gamma(-e_1/2)\Gamma(e_1/2)\Gamma(f_1)\Gamma(-f_1)}$$
$$\times\frac{M^{2g_1+2j_1}y^{e_1}\Gamma(i_1+1/2)\Gamma(j_1+i_1)\Gamma(k_1+i_1)}{\Gamma(1/2)\Gamma(1/2)\Gamma(1/2)\Gamma(i_1)\Gamma(-i_1)}$$
$$\times\frac{\Gamma(l_1-1/2)\Gamma(m_1-1/2)\Gamma(r+e_1/2)\Gamma(s+e_1/2)}{q^{-f_1-h_1-i_1-k_1-\beta l_1-m_1-\beta r-s+2}},\tag{27}$$

where

$$\sum^{**}=\sum_{i_1=0}^{\infty}\sum_{j_1=0}^{\infty}\sum_{k_1=0}^{\infty}\sum_{l_1=0}^{\infty}\sum_{m_1=0}^{\infty},$$
$$\zeta_1=i_1+j_1+k_1+l_1+m_1.$$

Taking the inverse Laplace to Eq. (27), we get

$$\overline{\tau}(y,q)=\sum_{e_1=0}^{\infty}\sum_{f_1=0}^{\infty}\sum_{g_1=0}^{\infty}\sum_{h_1=0}^{\infty}\sum^{**}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{A}{e_1!f_1!g_1!}$$
$$\times\frac{(-1)^{e_1+f_1+g_1+h_1+\zeta_1+r+s+1}\Gamma(f_1-e_1/2)}{h_1!i_1!j_1!k_1!l_1!m_1!r!s!K^{e_1/2-f_1-i_1+1/2}}$$
$$\times\frac{(-1)^{h_1+k_1+l_1+m_1+r+s}\Gamma(g_1-f_1)\Gamma(h_1+f_1)}{\Gamma(e_1/2)\Gamma(-e_1/2)\Gamma(e_1/2)\Gamma(f_1)\Gamma(-f_1)}$$
$$\times\frac{M^{2g_1+2j_1}y^{e_1}\Gamma(i_1+1/2)\Gamma(j_1+i_1)\Gamma(k_1+i_1)}{\Gamma(1/2)\Gamma(1/2)\Gamma(1/2)\Gamma(i_1)\Gamma(-i_1)}$$
$$\times\frac{\Gamma(l_1-1/2)\Gamma(m_1-1/2)\Gamma(r+e_1/2)\Gamma(s+e_1/2)}{\Gamma\left(-f_1-h_1-i_1-k_1-\beta l_1-m_1-\beta r-s+2\right)}\times t^{-f_1-h_1-i_1-k_1-\beta l_1-m_1-\beta r-s+1}.\tag{28}$$

Finally, using Fox H-function to get the stress field as

$$\overline{\tau}(y,q)=\sum_{e_1=0}^{\infty}\sum_{f_1=0}^{\infty}\sum_{g_1=0}^{\infty}\sum^{**}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{Ay^{e_1}}{e_1!f_1!g_1!i_1!j_1!}$$
$$\times\frac{(-1)^{e_1+f_1+g_1+\zeta_1+r+s+1}}{k_1!l_1!m_1!r!s!M^{e_1/2-f_1-i_1+1/2}}$$
$$\times\frac{t^{-f_1-i_1-k_1-\beta l_1-m_1-\beta r-s+1}}{\alpha^{-k_1-l_1-m_1-r-s}K^{e_1/2-f_1-i_1+1/2}}\times H_{10,12}^{1,10}$$
$$\left[\frac{\alpha}{t}\left|\begin{matrix}(-i_1+3/2,0),(1-j_1+i_1,0),(1-k_1-i_1,0),\\(1-f_1,1),(1-s+e_1/2,0),(1-r-e_1/2,0),\\(-m_1+3/2,0),(1-g_1+f_1,0),\\(1-f_1+e_1/2,0),(1-l_1+1/2,0),\\(1/2,0),(1-i_1,0),(1+i_1,0),(1-e_1/2,0),\\(1+f_1,0),(1-e_1/2,0),(1/2,0),(1/2,0),\\(1+e_1/2,0),(1-f_1,0),(0,1),\\(f_1+i_1+k_1+\beta l_1+m_1+\beta r+s-1,-1).\end{matrix}\right.\right]\tag{29}$$

6. LIMITING CASES

By putting β → 1 in Eqs. (24) and (29), we get the velocity field and associated shear stress of an ordinary second-grade fluid ordinary second-grade fluid

$$\overline{\tau}(y,q)=\sum_{e_1=0}^{\infty}\sum_{f_1=0}^{\infty}\sum_{g_1=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{(-1)^{e_1+f_1+g_1+r+s}}{e_1!f_1!g_1!}$$
$$\times\frac{M^{2g_1}y^{e_1}t^{-f_1-r-s+1}\alpha^{r+s}}{s!k^{e_1/2-f_1}}$$
$$\times H_{5,7}^{1,5}\left[\frac{\alpha}{t}\left|\begin{matrix}(1-g_1+f_1,0),(1-f_1,1),\\(1-s+e_1/2,0),(1-r-e_1/2,0)\\(1-f_1+e_1/2,0)\\(1-e_1/2,0),(1-f_1,0),(1+f_1,0)\\(1+e_1/2,0),(1-e_1/2,0),\\(0,1),(f_1+r+s-1,-1).\end{matrix}\right.\right]\tag{30}$$

$$\overline{\tau}(y,q)=\sum_{e_1=0}^{\infty}\sum_{f_1=0}^{\infty}\sum_{g_1=0}^{\infty}\sum^{**}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{Ay^{e_1}}{e_1!f_1!g_1!i_1!j_1!}$$
$$\times\frac{(-1)^{e_1+f_1+g_1+\zeta_1+r+s+1}}{k_1!l_1!m_1!r!s!M^{-2g_1-2j_1}}$$
$$\times\frac{t^{-f_1-i_1-k_1-l_1-m_1-r-s+1}}{\alpha^{-k_1-l_1-m_1-r-s}K^{e_1/2-f_1-i_1+1/2}}\times H_{10,12}^{1,10}$$
$$\left[\frac{\alpha}{t}\left|\begin{matrix}(-i_1+3/2,0),(1-j_1+i_1,0),(1-k_1-i_1,0),\\(1-f_1,1),(1-s+e_1/2,0),(1-r-e_1/2,0),\\(-m_1+3/2,0),(1-g_1+f_1,0),\\(1-f_1+e_1/2,0),(1-l_1+1/2,0),\\(1/2,0),(1-i_1,0),(1+i_1,0),(1-e_1/2,0),\\(1+f_1,0),(1-e_1/2,0),(1/2,0),(1/2,0),\\(1+e_1/2,0),(1-f_1,0),(0,1),\\(f_1+i_1+k_1+l_1+m_1+r+s-1,-1).\end{matrix}\right.\right]\tag{31}$$

7. NUMERICAL RESULTS AND DISCUSSION

We investigate the effect of a transverse magnetic field on the unsteady flow of a generalized second-grade fluid through a porous medium past an infinite flat plate. Using Laplace transform technique for fractional partial differential equations, we are able to describe the velocity and stress fields of the flow. We obtain exact analytic solutions of these differential equations in terms of Fox H-function. Similar solutions for ordinary second-grade fluid are also obtained as a limiting case. Several graphs are presented here for the analysis of some important physical aspects of the obtained solutions.

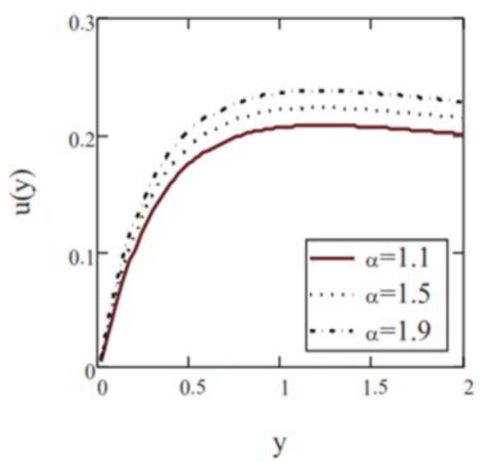

Fig. 1. Velocity $u(y,t)$ profile given by Eq. (24) for $K = 2$, $\beta = 0.6$, $t = 4s$, $M = 0.3$, $P = 1.2$, $A = 1$ and different values of α.

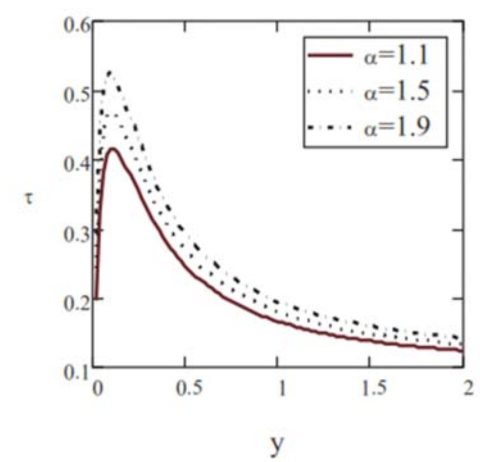

Fig. 2. Shear stress $\tau(y,t)$ profile given by Eq. (29) for $K = 2$, $\beta = 0.6$, $t = 4$s, $M = 0.3$, $P = 1.2$, $A = 1$ and different values of α.

In Figs 1 and 2 the effect of viscoelastic (second grade) parameter α on profiles of velocity and shear stress is shown. From these figures it is observed that the profiles of velocity and shear stress both amplifying with enlarging α. This is due to the fact that increasing the values of α would reduce the friction forces, and thus assists the flow of the fluid considerably; and hence, the fluid moves with greater velocity. Figs 3 and 4 shows the variation of the fractional parameter β. The velocity as well as the shear stress profiles changed its mono-tonicity by increasing β. The velocity increases first and then decreases when β is increased. Thus, it is obvious that the velocity field is influenced by the order of the time fractional derivative. Figs 5 and 6 shows the effect of the permeability K of the porous medium. It is noted that the velocity and boundary layer thickness increases with increasing permeability K of the porous medium. It may also be expected due to the fact that increasing values of K reduces the friction forces which assists the fluid considerably to move fast. Furthermore, the strongest shear stress occurs near the boundary and decreases rapidly with increasing distance from the plate. Figs 7 and 8 shows the variation of magnetic parameter M. It is observed that by increasing the magnetic parameter M the velocity decreases. The higher this value, the more prominent is the reduction in velocity. It is because of the fact that the introduction of a transverse magnetic field has a tendency to develop a resistive type force called Lorentz force, similar to drag force and upon in-creasing the values of M increases the drag force which leads to the deceleration of the flow. Also, it has been noticed that by increasing the trans-verse magnetic field results in thinning the boundary layer thickness. Thus the parameters M and K have opposite effects on the velocity profile. How-ever, the magnitude of the shear stress increases with increasing values of M. The magnitude of the shear stress in the close regime of the boundary is smaller as compared to the region away from the boundary.

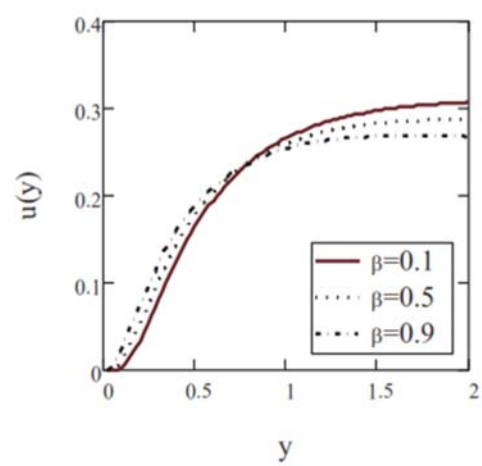

Fig. 3. Velocity $u(y,t)$ profile given by Eq. (24) for $K = 2$, $\alpha = 1.5$, $t = 4s$, $M = 0.3$, $P = 1.2$, $A = 1$ and different values of β.

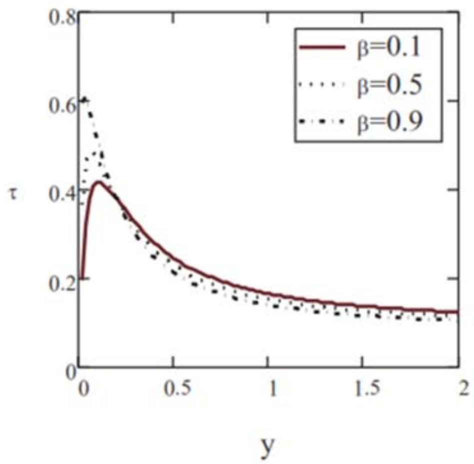

Fig. 4. Shear stress τ(*y*,*t*) profile given by Eq. (29) for *K* = 2, α = 1.5, *t* = 4*s*, *M* = 0.3, *P* = 1.2, *A* = 1 and different values of β.

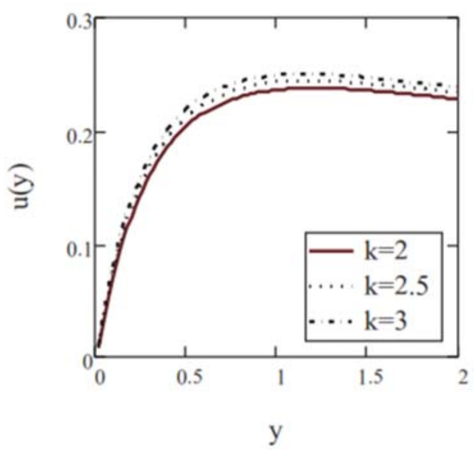

Fig. 5. Velocity *u*(*y*,*t*) profile given by Eq. (24) for α = 1.5, β = 0.6, *t* = 4*s*, *M* = 0.3, *P* = 1.2, *A* = 1 and different values of *K*.

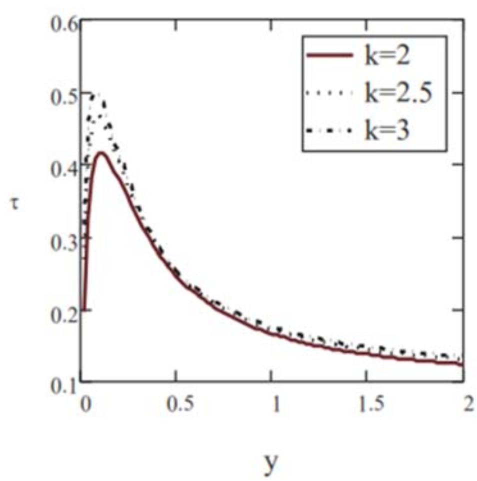

Fig. 6. Shear stress τ(*y*,*t*) profile given by Eq. (29) for α = 1.5, β = 0.6, *t* = 4*s*, *M* = 0.3, *P* = 1.2, *A* = 1 and different values of *K*.

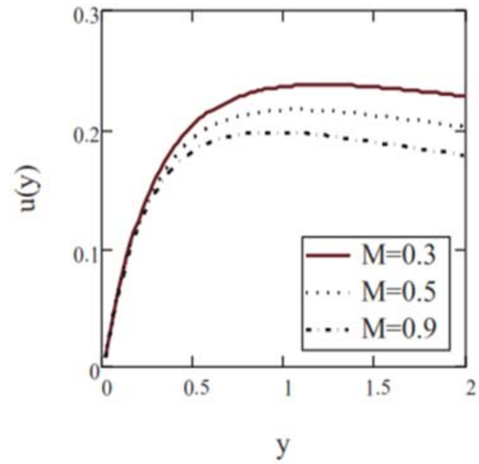

Fig. 7. Velocity *u*(*y*,*t*) profile given by Eq. (24) for α = 1.5, *K* = 2, β = 0.6, *t* = 4*s*, *P* = 1.2, *A* = 1 and different values of *M*.

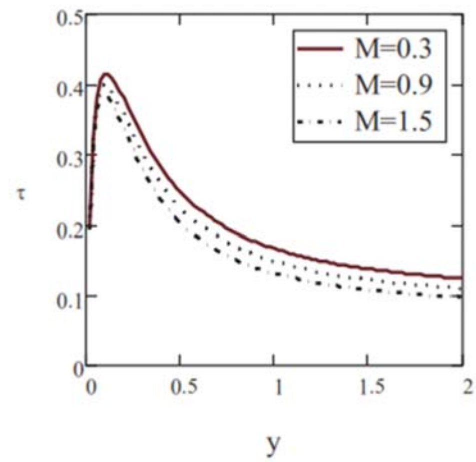

Fig. 8. Shear stress τ(*y*,*t*) profile given by Eq. (29) for α = 1.5, *K* = 2, β = 0.6, *t* = 4*s*, *P* = 1.2, *A* = 1 and different values of *M*.

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