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Behaviour of Mean Velocity in the Turbulent Axisymmetric Outer Near Wake

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ABSTRACT

An asymptotic study of the outer near wake of a long slender body of revolution is carried out. The long slender body is a cylinder which is kept parallel to the flow and takes the shape of a simplified geometry such as that of an underwater vehicle, a rocket or hull form of a ship model. The wake flow is axisymmetric and the analysis has been carried out without any assumption on the eddy viscosity but utilizing the general behaviour of turbulent shear stress in the near wake. The governing equations are solved with appropriate boundary conditions. Similarity analysis for the mean velocity characteristics is carried out which shows the existence of a logarithmic region in the normal direction in the overlap region between the outer near wake and the inner near wake. Also shown is the exponential decay of the mean velocity defect as freestream velocity is reached. Validation of the results of the analysis is done using available experimental data.

Keywords: Axisymmetric; Outer near wake; Boundary layer; Turbulence.

1. INTRODUCTION

Understanding of flow past axisymmetric bodies is of considerable importance from the point of view of both aerospace engineering and hydrodynamics. It is a fundamental problem related to lift and drag hence posseses both practical and theoretical importance. Better understanding of flow behaviour past these bodies could drastically minimize cost of design by providing efficient and more optimal criteria in design modelling. Among many other practical problems encouraging this study, include dispersion of pollutants from an aircraft, drag reduction of autonomous air vehicles, detection of submarines and other underwater vehicles, just to name a few.

The axisymmetric near wake is the region produced immediately after the trailing edge of a body of revolution after the flow past the body (Agrawal and Prasad 2003). It is the region known to greatly affect the aerodynamic characteristics of a body. This zone has a significant influence on base drag, base heat transfer and the configuration of the far wake (Merz *et al.* 1977). The axisymmetric near wake is the most complex part of the flow field and its structure depends on many parameters such as the geometric form of the body, the Reynolds number and the Mach number (Atli 1989). Complexities of the turbulent axisymmetric flows include those with non-homogenous and non-isotropic fluctuation motion. The result of this complexity yields in very challenging ways of acquiring data and performing experiments. Conducting experimental studies of the axisymmetric flow have proven to be difficult because of the inherent difficulties associated with obtaining symmetric wake flow and existence of trailing edge tip vibration which is very difficult to suppress.

Most of the analytical work relating to the near wake available in the literature is focused on two dimensional turbulent near wake flows, for instance, (Alber (1980), Prabhu and Patel (1982), Bogucz and Walker (1988), Subaschandar and Prabhu (1999) and Subaschandar (1988)), or are focused in the far wake field of the axisymmetric flow (Bevilaqua and Lykoudis (1978), Townsend 1956, Carmody (1964), Gibson *et al.* (1968) and Chevray (1968)). For instance, Alber (1980) considered the two dimensional near wake problem of a flat plate in which the wake was considered to be made of several regions. Alber's (1980) analogy, which relies on a specific closure assumption on the equations of motion indicates existence of a logarithmic relation of the centerline velocity. Prabhu and Patel (1982) also applied a closure assumption which was more general than the one employed by Alber (1980) to study the near wake. Bogucz and Walker (1988), Subaschandar and Prabhu (1999) and Subaschandar (1988) studied the two dimensional near wake problem in a more rigorous way. Analysis using asymptotic expansions were carried out prior to adoption of a specific closure turbulence model from which they established that the logarithmic behaviour mean velocity profile in the overlap layer between inner and outer near wake in the normal direction.

The literature in the far wake includes both experimental and theoretical work, (Bevilaqua and Lykoudis (1978), Townsend (1956), Carmody (1964), Gibson *et al.* (1968) and Chevray (1968)). In the studies mentioned, research on the self similarity behaviour of mean velocity profiles behind different axisymmetric geometries is presented. From these works, it is observed that the self similarity solution is obtained for different geometries at different streamwise distances. Bevilaqua and Lykoudi's (1978) also made very significant observations when they studied the self preservation of the axisymmetric wake on a sphere and a porous disc with the same drag. They observed that, self preservation is a process which develops gradually with downstream distances, and first to preserve is the mean velocity profile, then the Reynolds stresses and later the turbulent moments. Bevilaqua and Lykoudi (1978) further challenge the common principle of Townsend (1956) that, turbulence forgets how it was created. Results from Bevilaqua and Lykoudi (1978) indicate that the self similarity of mean velocity and Reynolds stress profiles are different for different geometries. Other works in the axisymmetric wake, include those concerned with the wake produced by axisymmetric bodies with a blunt, round or bluff trailing edges. A laminar wake developing from a slender blunt-based axisymmetric body is studied by Bohorques *et al.* (2011). Bohorques *et al.* (2011) investigate the stability properties and flow regimes of laminar wakes behind slender cylindrical bodies with a blunt trailing edge at zero angle of attack by experiment, Direct numerical simulation and local/ global linear stability analyses. Biau (2011) presented an analytical solution of the axisymmetric wakes produced by the flow past fixed axisymmetric bodies such as spheres, disks or bullet shaped bodies valid in the development region before the equilibrium state. The analysis is carried out on the axisymmetric boundary layer equations and focuses on the far wake since the region close to the trailing edge of the geometries hosted counter rotating vortices. Grandemange *et al.* (2012), experimentally studied the sensitivity of a 3D blunt body with a fixed axisymmetric flow separation at a Reynolds number of 2.1×10^4. The study carried by Grandemange (2012) was focused on the wake developing from blunt body. However, in the current study, the axisymmetric body of interest, is characterized by a sharp trailing edge where flow mixes smoothly with relatively no flow separation.

From a numerical point of view, Lu and Sirviente (2005), solved RANS numerically in conjuction with a second order Reynolds stress closure to calculate the turbulent flow of a swirling axisymmetric momentumless wake. The results were compared with various experimental and numerical data sets.

The mathematical problem posed here is that of a turbulent flow at a trailing edge of along slender cylinder that is aligned to a constant pressure uniform mainstream. The primary goal of this study is to describe the salient features of the flow structure downstream of the trailing edge in the outer near wake without adoption of a turbulence model. The outer near wake may be described as the near wake region normal from the centerline. A method of asymptotic expansions as the Reynolds number goes to infinity is exploited of which results are obtained and validated using available experimental data. Reasonable assumptions based on the physical behaviour of the Reynolds stress have been useful and hence are applied using mathematical expansions to describe this behaviour. The following results also offer support for the Millikan principle of matching of the inner and outer near wake solutions at a sufficiently large Reynolds number for a turbulent boundary layer problem can also hold for a wake problem.

Pertaining to memory retained by the boundary layer at the trailing edge, it is expected that the geometrical requirement of the logarithmic relation that is observed in the boundary layer between the inner layer and the outer layer should also sustain in the near wake region for some more distance (Prabhu and Patel (1982) and Bogucz and Walker (1988)). The equations governing the flow in this region mimics the outer layer equations of a boundary layer growing on a cylinder of constant radius except that the boundary conditions applied for the outer layer of the boundary layer and that of the outer wake are different.

2. THE AXISYMMETRIC TURBULENT BOUNDARY LAYER

The development of the near wake is dependent upon the flow conditions approaching the trailing edge. Hence it is necessary to give a brief discussion on initial conditions for the wake development upstream of the trailing edge which are highly influenced by the turbulent boundary layer at the trailing edge. In this section we give a brief summary of the turbulent boundary layer and some relevant background studies on the subject. The axisymmetric turbulent boundary layer has been studied extensively both analytically and experimentally, for instance Afzal and Narasimha (1976), White (1972), Cebeci (1970) and Denli and Landweber (1979). In these works, it is reported that the boundary layer consists of two distinct layers, an inner layer and an outer layer. Very close to the wall, is a very thin viscous layer and between the wall and the inner layer is a buffer region, and on extending into the inner layer is the logarithmic region which eventually graduates into the defect layer (defect from the free stream) of the outer layer. Basically, the logarithmic layer is the overlap region between the outer and the inner layers of the boundary layer.

Although the two dimensional turbulent boundary layer has generally been described well, however the axisymmetric turbulent boundary layer has its peculiarity which makes it necessary to give it a special treatment. Afzal and Narasimha (1976) studied the axisymmetric turbulent boundary layer at large values of the frictional Reynolds number based on the radius of the cylinder, with the boundary layer thickness being of the same order as the radius of the geometry. They used asymptotic expansions on the equations of mean flow to show that the flow can be described by the inner and outer layer concepts which are also applicable to the two dimensional turbulent boundary layers. It is also pointed out in studies concerning the turbulent axisymmetric boundary layer that for an axisymmetric turbulent boundary layer transverse curvature plays a vital role in shaping several characteristics of the turbulent axisymmetric boundary layer. White (1972) made an analysis of the properties of axisymmetric turbulent boundary layer over a long cylinder. Through an integral analysis, White (1972), showed that the curvature increased the skin friction, overall drag and the boundary layer thickness. Validation was done using several sources of data. A similar study was also done by Cebeci (1970) when he studied axisymmetric laminar and turbulent incompressible boundary layers on a slender body of revolution. Cebeci (1970) resolved the problem using finite difference method on equations of motion with an appropriate eddy viscosity assumption. Cebeci (1970) further made an analysis on the deviation of the cylinder skin friction from that of a flat plate. Denli and Landweber (1979) exploited the known fact that the stress moment in the law of the wall remain constant and apply their mixing length model which accounts for transverse curvature to obtain a law of the wall. They further report that skin friction on cylinder was greater than that of a flat plate corresponding to the same momentum thickness Reynolds number. It is appreciated in these studies that, for an axisymmetric boundary layer, transverse curvature is key, and the usual empirical relations which are applied in the two dimensional turbulent boundary layer theory tend to fail in the axisymmetric boundary layer, that is, in the two dimensional boundary layer problem, closure of Reynolds average Navier Stokes equations on τ (the Reynolds stress) is usually achieved by the law of the wall which is the empirical relation $\frac{U}{U_\tau} = \frac{1}{k}\ln y_+ + C$, where $y_+ = \frac{yU_\tau}{\nu}$. Afzal and Narasihma (1976) and Denli and Landweber (1979) point out that this classical relation of the two dimensional law of the wall needs modification, particularly in the argument of the similarity function, y_+, but not the function itself as refrenced in Afzal and Narasimha (1976), White (1972) and Denli and Landweber (1979). Furthermore, the constant of the logarithmic relation C, in the law of the wall was also debatably questioned whether it is a universal constant, as it has been in the case of a flat plate, and if it is not, what influences its value. Afzal and Narasimha (1976) deduced that the constant depends on the radius of the body and the boundary layer thickness. The studies pointed by Afzal and Narasimha (1976), including Rao (1967) who deduced a different modified law of the wall (the law of the wall deduced by Rao (1967) is applied by White (1972) in analyzing the axisymmetric turbulent flow past a long cylinder) also propose that the constants of the law of the wall depend on the radius of the body and the Reynolds number. However, with the modifications suggested by Rao (1967), the analysis of the boundary layer equations boil down to the logarithmic empirical relation as demonstrated by Afzal and Narasimha (1976). Hence it could be concluded that the logarithmic relation of the boundary layer is also observed in the axisymmetric boundary layer but the constants of the equation and the arguments of the function are subject to the special features that shape the axisymmetric boundary layer.

3. ANALYSIS

Figure 1 depicts a schematic of various flow regions including the outer near wake. The same figure is also used in references Yane and Subaschandar (2016, 2017). However Yane and Subaschandar (2016) was focused on the inner near wake where the wake development was described in terms of a linear model of Alber (1980) to characterize the eddy viscosity. Yane and Subaschandar (2017) was focused on the inner near wake and employed a general description of the eddy viscosity using mathematical expansions. The equations of motion are presented in axisymmetric form, where x is the streamwise direction, y is the normal direction and the origin is at the trailing edge. It should be pointed that very close to trailing edge is a laminar sublayer wake like, which will be consumed by the mixing produced by the eddies of the viscous scale of the upstream turbulent boundary layer. The axisymmetric flow is of constant density and viscosity and the boundary layer is assumed to be fully developed with a very negligible pressure gradient. The relevant length scale for both streamwise and normal direction is υ/u_τ where υ is the kinematic viscosity and u_τ is the skin friction velocity. The governing equations for both the outer and inner near are obtained by applying the same limiting conditions that lead to the equation for the turbulent boundary layer. They are $\frac{u_\tau}{U_\infty} \to 0$ and $\frac{\delta u_\tau}{\upsilon} \to \infty$ where δ denotes the total wake thickness and U_∞ is the freestream velocity. In the outer layer, the variables $\frac{U_\infty - U}{u_\tau}, \tau/u_\tau^2, y/\delta, x/L$ are held fixed. L defines the length of the long slender body and U is the mean velocity in the streamwise direction. The steady governing equations of motion given in Townsend (1956), Denli and Landweber (1979) and White (2011) are given as

$$\frac{\partial(Uy)}{\partial x} + \frac{\partial(Vy)}{\partial y} = 0. \tag{1}$$

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho y}\frac{\partial(y\tau)}{\partial y}. \tag{2}$$

The boundary conditions for the entire near wake region are given by

$$\frac{\partial U}{\partial y} = 0, V(x,0) = 0, \tau = 0 \text{ at } y = 0$$

And

as $y \to \infty$ $U(x,y) = U_\infty$ and $\tau \to 0$.

U and V define the streamwise and normal mean velocities, x and y denotes the streamwise and normal coordinates while ρ is the fluid density and τ is the Reynolds stress. It is assumed that the turbulent velocity and length scales are still of the same order as in the logarithmic portion of the trailing edge boundary layer. The diffusion process is assumed dominated by turbulent rather than laminar.

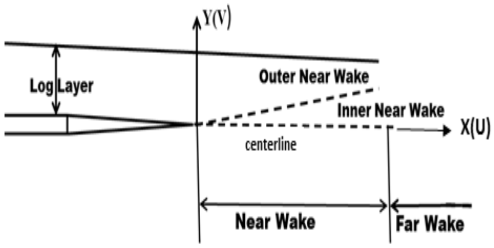

Fig. 1. Flow regions in the near wake.

The velocity defect in the outer layer is expected to be small with reference to the freestream (U_∞) and mean velocities. It is given by

$$U_d = U_\infty - U \tag{3}$$

Substituting these variables into the continuity and momentum equations, we get after simplification that,

$$V = \frac{-y}{2}\frac{d\,U_\infty}{dx}. \tag{4}$$

$$-U_\infty \frac{\partial\, U_d}{\partial x} - U_d \frac{d\,U_\infty}{dx} + \frac{y}{2}\frac{dU_\infty}{dx}\frac{\partial U_d}{\partial y} = \frac{1}{\rho y}\frac{\partial(\tau y)}{\partial y}. \tag{5}$$

We shall note that in the outer near wake $U_d \ll 1$, and $U_\infty \frac{dU_\infty}{dx} = -\frac{1}{\rho}\frac{dp}{dx}$. As the pressure gradient is assumed to be very small, hence we get,

$$-U_\infty \frac{\partial U_d}{\partial x} = \frac{1}{\rho y}\frac{\partial(\tau y)}{\partial y}. \tag{6}$$

The equation is then appropriately normalized as follows,

$$x_+ = \frac{x u_\tau}{\upsilon}, y_+ = \frac{y u_\tau}{\upsilon}, \overline{U_d} = \frac{U_\infty - U}{u_\tau},$$

after which we obtain

$$-U_\infty u_\tau \frac{\partial \overline{U}_d}{\partial x_+} = \frac{1}{\rho y_+}\frac{\partial(y_+\tau)}{\partial y_+}. \tag{7}$$

This equation is identical to the shallow wake equation for an axisymmetric flow of Townsend (1956), Prabhu and Narasimha (1972) and Denli and Landweber (1979). Substituting $\eta = \frac{y_+}{\delta_+}$ where $\delta_+ = \frac{\delta u_\tau}{\upsilon}$ and $\tau = \rho u_\tau^2\,\bar{\tau}$.

$$\frac{-U_\infty \delta_+}{u_\tau}\frac{\partial \overline{U}_d}{\partial x_+} = \frac{1}{\eta}\frac{\partial(\eta\bar{\tau})}{\partial \eta}. \tag{8}$$

We set $\overline{U}_d = q(x_+).F(\eta)$ and $\beta = \frac{U_\infty \delta_+}{u_\tau}$

And on differentiating Eq. (8) with respect to x_+, we get

$$\frac{\beta\delta'_+ q}{\delta_+}\left[\frac{q'\delta_+ F}{q\delta'_+} - \eta F'\right] = \frac{1}{-\eta}\frac{\partial\{\eta\bar{\tau}\}}{\partial\eta}. \tag{9}$$

Integrating the above equation we obtain

$$\frac{\beta\delta'_+ q}{\delta_+}\left[F\eta^2 + \left(\frac{q'\delta_+}{\delta'_+ q} + 2\right)\int_\eta^\infty F\eta d\eta\right] = -\eta\bar{\tau}. \tag{10}$$

And we take $\bar{\tau}$ as $\bar{\tau} = \check{\tau}\,(x_+)p(\eta)$. To obtain a similarity solution for Eq. (10), we should have

$$\frac{\check{\tau}\,\delta_+}{\beta\delta'_+ q} = C_1. \tag{11}$$

and

$$\frac{q'\delta_+}{\delta'_+ q} + 2 = C_2. \tag{12}$$

With these, Eq. (10) becomes

$$F\eta^2 + C_2\int_\eta^\infty F\eta d\eta = C_1\,\eta P(\eta). \tag{13}$$

Differentiating the above equation with respect to η we get

$$F'\eta^2 + (2 - C_2\,)\eta F = C_1\frac{\partial(\eta P(\eta))}{\partial\eta}. \tag{14}$$

3.1 Behaviour of Solution for Small η

In this section we analyse Eq. (14) for small η. We can take P as a series in η as

$$P = a_0 + a_1\,\eta + a_2\,\eta^2 + \cdots. \tag{15}$$

And substituting for P in Eq. (14) and simplifying gives

$$F' + (2 - C_2)\frac{F}{\eta} = -\frac{C_1}{\eta^2}[a_0 + 2a_1\,\eta + \cdots\,]. \tag{16}$$

The solution of the above equation is then given by

$$F = \frac{-C_1}{\eta^{2-C_2}}\int_0^\eta \eta^{-C_2}\,[a_0 + 2a_1\,\eta + \cdots]d\,\eta. \tag{17}$$

To obtain the logarithmic relation on evaluating the right hand side integral, C_2 has to equal 2, that is;

$$F = \frac{C_1\,a_0}{\eta} - 2a_1 c_1\,ln\,\eta + \cdots. \tag{18}$$

The solution given by Eq. (18) is dominated by the logarithmic term. The first term on the right hand side of equation shows a decay of order η^{-1}, however the behavior of the logarithmic term is more rapid and dominates over the contribution of the first term. Hence we can conclude that in the outer near wake, a logarithmic relation is exhibited for small η which matches with the logarithmic overlap

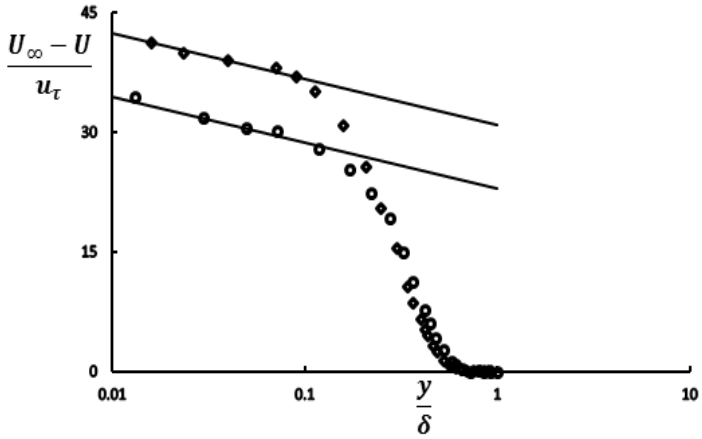

Fig. 2. Logarithmic relation of the velocity profiles for the data set of Jimenez *et al.* (2010).
◇ ; x/d = 6, and O ; x/d= 3 , —— logarithmic line.

condition between inner and outer layer.

Hence it can be concluded that the solution F behaves as

$$F = -K_1 \ln\eta + K_2. \tag{19}$$

Where K_1 and K_2 are constants.

3.2 Behaviour of Solution for Large η

The asymptotic behavior of the velocity defect on reaching the free stream velocity is expected to follow an exponential decay which tends to zero for large η. Hence for large η the function P should behave like $R_0 e^{-R_1\eta^{R_2}}$ (Subaschandar and Prabhu (1999) and Prabhu and Patel (1982)), where R_0, R_1, R_2 are constants. To facilitate the analysis we shall take $P = e^{-\eta}$. Substituting P into Eq. (14), we get

$$F'\eta^2 + (2 - C_2)\eta F = \frac{d}{d\eta}\{C_1 \eta e^{-\eta}\}. \tag{20}$$

The solution of the above equation is given by

$$F(\eta) = C_1 e^{-\eta}\left[\frac{1}{C_2 - 2} - \frac{1}{\eta(1 - C_2)}\right] - \frac{C_1}{\eta^{2-C_2}} \int_\eta^\infty e^{-\eta}\left(\frac{\eta^{-C_2+1}}{1 - C_2} - \frac{\eta^{C_2}}{2 - C_2}\right) d\eta. \tag{21}$$

The two terms on the right hand side of equation give the solution to the mean velocity defect for large values of the similarity variable η.

It is clear from the first term on the right hand side of Eq. (21) that, for large values of η, the expression will decay to zero. The exponential quantity $e^{-\eta}$ decays faster than any of the terms in the first expression and hence the expression will decay to zero for large values of η.

The second term on the right hand side of Eq. (21) has an integral that will go to zero for large values of the similarity variable η. The integral will decay faster to zero for large values of η due to the large contribution of the exponential expression in the expression. From the above solution it is clear that for large η , the mean velocity defect should decay exponentially. The exponential decays faster than any term in the above solution. The integrand in the solution will also decay faster to zero.

Hence we can conclude that the outer near wake solution behaves logarithmically for small values of the similarity variable η and decays exponentially to zero for large values of η.

4. RESULTS AND DISCUSSION

In this section we compare the results of the analysis with available experimental data of Jimenez *et al.* (2010), Swanson *et al.* (1974) and Patel and Lee (1977). Jimenez *et al.* (2010) carried out an experimental study of the axisymmetric wake behind a body of revolution for Reynolds number ranging between 1.1×10^6 and 67×10^6. The Reynolds number was based on the model length. In this study, only the data pertaining to Re= 1.1×10^6 is considered. Swanson *et al.* (1974) studies forebody shaped cylinders experimentally of pure drag body, self propelled by axial fluid injection and self propelled with a “well designed” propeller. The data set of Swanson *et al.* (1974) applied in this study is of pure drag body since it best describes the geometry currently analysed. The Reynolds number of Swanson *et al.* (1974) was 6.18×10^5 which was based on the diameter (d) of the geometry. Patel and Lee (1977) carried out experimental studies in the near wake behind an axisymmetric body in which the Reynolds number was 1.2×10^6, which was based on the body length (l) and the freestream velocity.

Figures 2, 3 and 4 show the variation of mean velocity profiles in the outer near wake, in the outer near wake variables. These figures show the mean velocity profiles from the experimental studies of

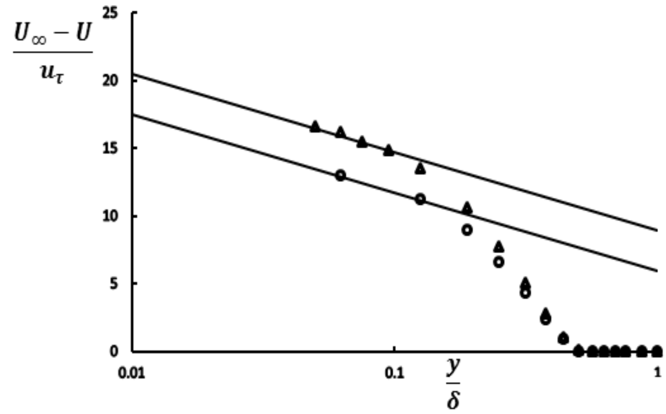

Fig. 3. Logarithmic relation of the velocity profile for the data of Swanson *et al.* (1974).
○ ; x/d = 5, △ ; x/d= 2, —— logarithmic line.

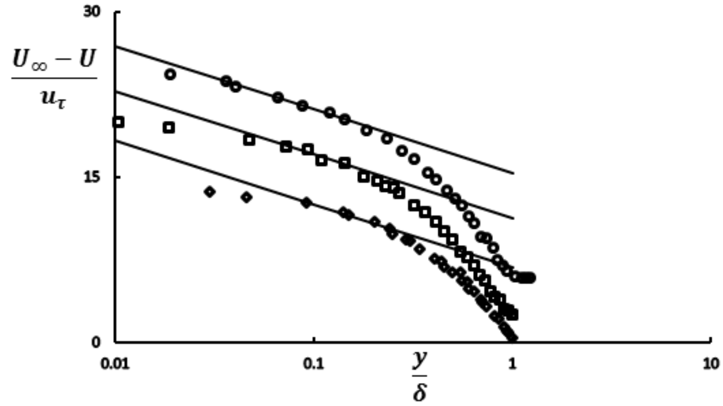

Fig. 4. Logarithmic relation of the velocity profile for the data of Patel and Lee (1977).
◇; x/l = 1.3,□; x/l= 1.06, ○ ;x/l= 1.02 ——logarithmic line.

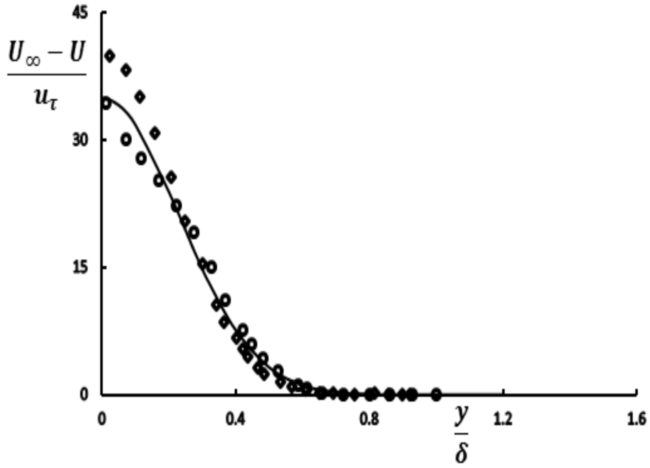

Fig. 5. Exponential decay of the velocity profiles for the data of Jimenez *et al.* (2010).
○ ; x/d = 6, ◇; x/d= 3 , —— exponential line.

Jimenez *et al.* (2010), Swanson *et al.* (1974) and Patel and Lee (1977) and show also the logarithmic line. These figures show that the mean velocity defect varies logarithmically with the outer near wake variable η for small values of η. It can be seen from these figures that the logarithmic behavior of mean velocity is gradually destroyed as the streamwise distance increases.

Figures 5, 6 and 7 show the mean velocity variation

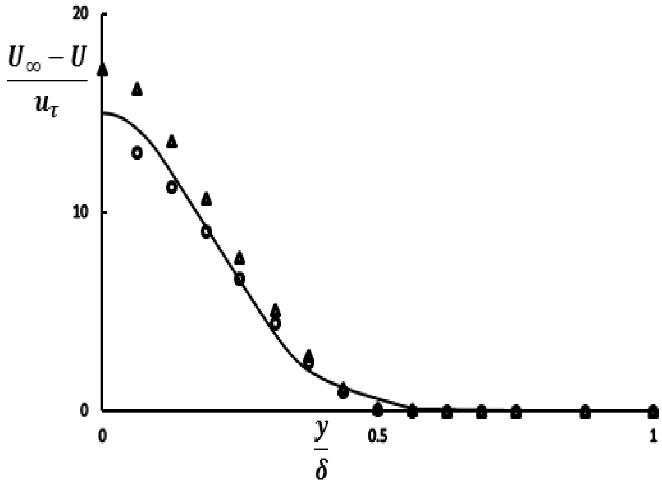

Fig. 6. Exponential decay of the velocity profiles for the data of Swanson *et al.* (1974).
○ ; x/d = 2, △ ; x/d= 5, — exponential line.

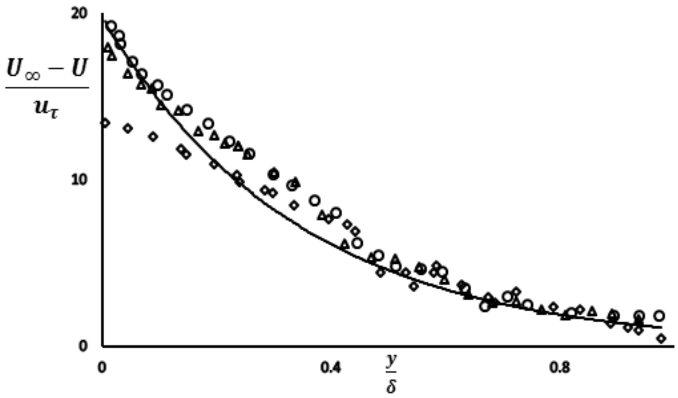

Fig. 7. Exponential decay of the velocity profiles for the data of Patel and Lee (1977).
◇ ; x/l = 1.30, △ ; x/l= 1.06, ○ ; x/l=1.02, — exponential line.

with the outer near wake variable η, for large η values. These figures also present the mean velocity profiles from the experimental studies of Jimenez *et al.* (2010), Swanson *et al.* (1974) and Patel and Lee (1977) and also the exponential curve. These figures show the exponential decay of mean velocity defect in the outer near wake for large values of η. From the above figures we can see that the mean velocity profile varies logarithmically for small values of the outer near wake similarity variable η and the mean velocity defect decays exponentially for large values of the outer near wake similarity variable η, thus validating the results of the analysis given earlier.

5. CONCLUSIONS

The turbulent outer near wake layer of a long slender cylinder developing from a fully developed zero pressure gradient turbulent axisymmetric boundary layers is analyzed using asymptotic expansions method. The focus is made on a specific region of the near wake called the outer near wake. The analysis has been carried out using the general characteristics and behavior of the turbulent shear stress near the centerline and near the wake edge. It is known that in the near wake when approaching the freestream, the eddy viscosity is expected to narrow down to a very small quantity while on approaching the center line from the outer layer of the wake, the Reynolds stress asymptotically approaches a very small quantity. Only mathematical expansions relying on the behavior of the stress term are applied without engaging a closure hypothesis on the mean equations. It is observed that the mean velocity profile varies logarithmically for small values of the outer near wake similarity variable η. The mean velocity defect decays exponentially as the outer near wake similarity variable increases. Both these characteristics are established on the basis of the dependence of the upstream conditions of the axisymmetric turbulent boundary layer at the trailing edge. Results of the analysis have been validated using available experimental data which gives credence to the analysis presented in this study.

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