New Permeability Model for Gel Coated Porous Media with Radial Flow

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ABSTRACT

Gel polymer has been widely used to reduce water production in mature oil reservoirs. One of the challenges in this area is evaluation of permeability of media after the gel treatment. Darcy’s law has been used for this purpose while this equation has been developed for rigid porous media. In this study, a new mathematic model was introduced to calculate the permeability of gel coated porous media. For this purpose, a modified version of Brinkman equation was used. This model showed that permeability of gel impregnated porous media is a function of pressure drop, fluid viscosity, and gel viscoelastic properties. In order to obtain performance of new permeability model, several experiments were carried out in a porous media with radial flow. A copolymer of 2-acrylamido-2-methyl-propanesulfonic-acid sodium salt (AMPS) and acrylamide (AcA) gelant was used to form the gel in situ. Finally, to investigate the applicability of polymer gel treatment to water shut-off in porous media (sandpack), residual resistant factors (RRF) were calculated based on new permeability model.

Keywords: Polymer gels; Brinkman equation; Radial flow; Permeability model; Darcy equation; Residual Resistant Factor.

NOMENCLATURE

- \( k \): permeability of media
- \( P \): pressure
- \( P_r \): pressure at the reservoir boundary
- \( P_w \): pressure at the wellbore
- \( Q \): flow rate
- \( r \): radius
- \( r_e \): radius of reservoir boundary
- \( r_w \): radius of wellbore
- \( \mu \): viscosity of fluid
- \( u_f \): velocity of fluid

1. INTRODUCTION

A usual problem of mature oil reservoir is water production. Based on global statistics, about 75% of production of a mature well is water (Lullo et al. 2002; Hajilary et al. 2015). This huge amount of water causes several problems. It reduces the well productivity by increasing pressure drop between the bottom and top of the wells and also increases the costs of oil treatment such as water separation from crude oil, effluent disposal and oil transportation (Simjoo et al. 2009).

Many techniques have been developed to control water production in oil reservoirs. These techniques are known as conformance control technology (Daneshy 2006; Nourani et al. 2011), which classified as chemical and mechanical methods. Some example of mechanical methods are: Horizontal drilling, using a central pipe in the stream of water production, drilling deviated wells, and applying wellhead isolation devices like Hydro-cyclones (Prado et al. 2009). Operation and maintenance costs of mechanical methods are very high and in some purpose chemical methods have been used instead (Sengupta et al. 2012). Due to thermal and PH stability of cross-linked polymer gel, they are more desired for water shut off in industrial and experimental applications (Nguyen et al. 2006).

Several theoretical and experimental analyses on water shut off ability of polymer gel were carried out in literatures (Liang et al. 2001; Al-Muntasheri et al. 2007; Salehi et al. 2012; Ganguly 2009). Viscoelastic properties of
polymer gel were considered by different authors (Al-Muntasheri et al. 2007; Seright et al. 2002; Ghosha et al. 2012; Srisuriyachai 2008) and a power law relation between permeability of gel filled capillary tube (or micro model) and Darcy velocity have been reported in different papers (Al-Muntasheri et al. 2007; Ganguly 2011; Ghosha et al. 2012; Seright 2003; Liang et al. 2001; Willhite 2001). A mathematical model for fluid flow through gel filled capillary tube has been developed by Yang et al. (2000). They assumed a fourth order polynomial for velocity of fluid flow through a gel-filled channel. They also developed a new mathematical model based on modification of Brinkman equation (Yang et al. 2002). The new velocity model is derived by substituting permeability term in Brinkman equation with a power law function of velocity (Yang et al. 2002). Al-Sharji et al. (1999) investigated the water and oil movement through polymer gel in a capillary tube and micro model. Their results have been shown that water flows through polymer gel as if it passes into real porous media while oil passes through polymer gel as immiscible droplets. Grattoni et al. (2001) indicated that rheological properties of gel depend on velocity. Ganguly (2009; 2011) investigated gel rupture, and its deformation under pressure gradient. He used visual tubes, which filled with polymer gel and introduced a new model for variation of permeability of oil and water with respect to the pressure gradient. Ganguly (2011) also reported that a power law relation exists between Darcy permeability and velocity.

Although there are several models for water flow through gel filled capillary tube, there is not any model for fluid flow through gel impregnated porous media in radial system. Models that have existed so far are derived in Cartesian coordinate and considering the one-dimensional fluid flow. These models are not appropriate for the fluid flows through porous media with radial flow. In a radial flow system, fluid moves in r and y directions (assuming there is no change in fluid velocity in θ direction). Therefore, a new model is required to describe the fluid flow through radial polymer gel coated porous media. Development of newly theoretical model for permeability of oil and water through gel coated porous media is the main objective of this paper. In this pursue, Brinkman equation in cylindrical coordinate is modified based on the assumption that permeability of gel-impregnated porous media varies as power low function of velocity. For evaluation of new model, several experiments are carried out in a radial flow system. To illustrate the ability of polymer gel in controlling water flow, the residual resistance factor (RRF) is calculated using new model.

### 2. Mathematical Formulation

A set of mass, momentum, energy conservative and constitutive equations are involved for fluid flow through porous media. For simplicity, the isothermal conditions are assumed and the energy conservation can be omitted from equations. Usually Navier-Stokes equation is used as momentum conservation, but in the case of low velocity flow, Brinkman and Darcy equations are applicable. Darcy equation for one-dimensional radial flow is given as follow:

\[ \nu_{darcy} = -\frac{\mu}{k} \frac{\partial p}{\partial r} \]

Where \( \mu \) viscosity of fluid, k is permeability of media, \( P \) is pressure and \( r \) is radius. Eq. (1) does not consider the viscous forces; consequently, combination of this equation and Navier-Stokes equation gives the Brinkman equation which involves both Darcy effects and viscous forces:

\[ \frac{\partial p}{\partial r} = -\frac{\mu}{k} \nu_r + \mu \frac{\partial^2 \nu_r}{\partial y^2} \]

Where \( \nu_r \) is velocity of fluid. To obtain the outlet flow rate with Darcy equation, a simple integration is performed:

\[ Q_{darcy} = \frac{2\pi l k}{\mu \ln \left( \frac{r_e}{r_w} \right)} \left( P_e - P_w \right) \]

Where \( Q \) is flow rate and \( l \) is the height of the porous medium. Solving the Brinkman equation with Yang et al. [15] method gives average velocity as follow:

\[ \nu_{r,brinkman} = \frac{-\mu}{k} \frac{\partial p}{\partial r} \left( 1 - \frac{\tanh(l/k)}{l/k} \right) \]

And the flow rate is given as:

\[ Q_{brinkman} = \frac{2\pi l k}{\mu \ln \left( \frac{r_e}{r_w} \right)} \left( P_e - P_w \right) \left( 1 - \frac{\tanh(l/k)}{l/k} \right) \]

It is clear that, difference between Eqs. (2) and (5) is \( \left( 1 - \frac{\tanh(l/k)}{l/k} \right) \). As Fig. 1 shows, the \( f(x) = \frac{\tanh(x)}{x} \), \( f(x) \) approaches to zero as \( x \) tends to infinity. In this study \( x = l/k \) therefore, due to the very low value of permeability \( k \leq 10^{-10} \text{ m}^2 \), value of \( l/k \) goes to infinity so \( x \to \infty \) and eventually, the value of \( \frac{\tanh(l/k)}{l/k} \ll 1 \). As demonstrates in Fig. 2, the Brinkman and Darcy equations are very close to each other in the case of one dimension radial flow.

A real porous media includes free spaces with different sizes and lengths which are connected to each other and fluid flows through these spaces. In polymer gel treatment, these channels are filled with polymer and polymer blocks water pathway. In this case, the permeability changes
due to variation of pressure drop and velocity of fluid flow. Several authors (Al-Sharji 1999; Grattoni 2001; Ganguly 2011) mentioned that permeability has a power law relation with velocity. Based on this assumption, the permeability can be defined as Eq. (6):

$$ k = \beta u_r^b $$

(6)

Where $b$ and $\beta$ are unknown parameters and can be obtained by using experimental data. These parameters depend on inherent properties of polymer gel (Yang et al. 2002). Substituting the Eq. (2) in Eq. (6) gives:

$$ \beta \frac{\partial p}{\partial r} = \beta \frac{\partial^2 u_r}{\partial y^2} - u_r^{1-b} $$

(7)

For solution of Eq. (7), Yang et al. (2002) method has been used. Neglecting the variation of $u_r$ at $y$ direction gives:

$$ u_r^{1-b} = -\frac{\beta \partial p}{\mu \partial r} $$

(8)

Considering $n=1-b$ and $k_{gel} = -\frac{\beta \partial p}{\mu \partial r}$, then:

$$ u_r = (k_{gel})^{1/n} $$

(9)

Now, the $u_r(y)$ is:

$$ u_r = (k_{gel})^{1/n} + f(y) $$

(10)

And power to $n$ for both sides of Eq. (10):

$$ u_r^n = (k_{gel})^{1/n} + f(y) $$

(11)

The factorization of $k_{gel}$ gives:

$$ u_r^n = k_{gel} \left[ 1 + nk_{gel}^{-1/n} f(y) \right]^n $$

(12)

In the case of polymer gel, the second term in second in Eq. (12) is very small and the truncated Taylor with considering the two primarily sentences is valid:

$$ u_r^n = k_{gel} \left[ 1 + nk_{gel}^{-1/n} f(y) \right] $$

(13)

Substitution of Eqs. (10) & (13) in Eq. (7) yields:

$$ -k_{gel} = -k_{gel} \left[ 1 + nk_{gel}^{-1/n} f \right] + \beta \frac{\partial^2 f}{\partial y^2} $$

(14)

Rearranging Eq. (14):

$$ \frac{\partial^2 f}{\partial y^2} = \frac{n k_{gel}^{-1/n}}{\beta} f = 0 $$

(15)

Solution of Eq. (15) gives:

$$ f = C_1 \cosh(\xi y) + C_2 \sinh(\xi y) $$

(16)

$$ \xi = \frac{n k_{gel}^{-1/n}}{\beta} $$

Substitution of velocities at well boundaries yields the unknown parameters ($C_1$ and $C_2$) of Eq. (16). At $y = \pm 1 \rightarrow u_r = 0$ so $C_1 = C_2 = k_{gel}^{1/n}$

C1 and C2 in Eq. (16) are subsisted, then the velocity profile can be written as:

$$ u_r = k_{gel}^{1/n} \left[ 1 - \frac{\cosh(\xi^{1/2} y)}{\cosh(\xi^{1/2})} \right] $$

(17)

To get integral average at $y$ direction, the equation

$$ - u_r = \int_{-1}^{1} u_r dA $$

is used:

$$ - u_r = k_{gel}^{1/n} \left[ 1 - \frac{\tanh(\xi^{1/2} y)}{\xi^{1/2}} \right] $$

(18)

As mentioned before, $\frac{\tanh(\xi^{1/2} y)}{\xi^{1/2}}$ can be neglected and the Eq. (18) can be simplified as:

$$ - u_r = \left( \frac{\beta \partial p}{\mu \partial r} \right)^{1/(1-b)} $$

(19)

Eventually, integrating the Eq. (19) yields the flow rate:

$$ Q = 4\pi \left( \frac{b \beta}{\mu} \frac{P_e - P_w}{R_e - R_w} \right)^{1/(1-b)} $$

(20)
In this equation $\Delta P$ and $Q$ are based on $P_a$ and $m^3/s$, respectively. The dependence of velocity to the permeability is obtained using Eq. (21):

$$k = \beta u^b \tag{21}$$

Substituting the Eq. (19) in Eq. (21) gives:

$$k = \beta \left( \frac{\beta P_a - P_w}{\mu R_e - R_w} \right)^{b/(1-b)} \tag{22}$$

3. EXPERIMENTAL SECTION

3.1 Materials

A copolymer of 2-acrylamido-2-methylpropanesulfonic-acid sodium salt (AMPS) and acrylamide (AcA) and chromium triacetate are used as polymer powder and cross-link, respectively. Polymer powder has average molecular weight of 2 million Dalton and sulfonation degree of 25%, which is provided by SNF Co. (France) under the trade name of AN125VLM. Chromium triacetate is fabricated by Carlo Erba Co. (Italy).

3.2 Gelant Solution Preparation

For gelant solution preparation, polymer powder was slowly added into distilled water and simultaneously stirred; the stirring process was continued for 6 hr. The solution was held at room temperature to obtain a homogenous mixture. Afterward, 0.12 weight ratio of Cr(III)-acetate was mixed in distilled water for 10 min and finally the polymer and cross-linker solutions were stirred together to prepare gelant solution. Usually, the bottle test is used to estimate the gelation time [16]; gelant solution is poured into a bottle and put in a water bath at 90 °C. Based on classification scheme of Sydansk (1995) gelation time of a solution with 26000 ppm concentration was recorded 9 hr as a G code.

3.3 Experimental Procedure

To study water movement through polymer gel impregnated porous media in radial flow a visual core holder with 10 cm diameter and 1 cm height was used. Coreflooding experiments were performed on a sandpack core in order to validate mathematical modeling. Sandpack was used as a porous media due to more homogeneity in compare with others and also the Brinkman equation is more applicable for this media. The schematic of the experimental set up is shown in Fig. 3. Temperature of the system was held at 90 °C by a water bath. Three different porosities (30%, 32%, and 35%) have been chosen and the gelant solutions with polymer concentration of 26000 ppm were injected into the each core.

Sand grains were placed in the visual core holder and saturated with distilled water. Darcy equation was used to calculate absolute permeability (shown in Fig. 4); and the porosities were measured as the volume of the injected water dividing to total empty volume. The values of R-square for three curves in Fig. 4 are more than 0.98; showing minimum experimental error.

The oil effective permeability of sandpack before gel treatment ($k_{eff\,before}$) at irreducible water saturation ($S_{irw}$), was measured by applying outlet flow rates versus pressure drop into Darcy equation.
The effective permeability of water before gel injection \( (k_{\text{effw,before}}) \) at residual oil saturation \( (S_o) \) was also obtained. Gelant solution, containing 26000 ppm co-polymer and a 0.12 weight ratio of crosslinker/co-polymer, was prepared and injected into the middle of the sandpack (well position) as it is illustrated in Fig. 3 and the system was shut-in at temperature of 90 °C for 9 hr for complete gelation. After the shut-in period, oil and then water at different rates were injected into the treated sandpack to calculate the effective oil and water permeability after treatment \( (k_{\text{effo,after}} \text{ and } k_{\text{effw,after}}) \).

4. RESULTS AND DISCUSSIONS

4.1. Measuring \( b \) and \( \beta \) using Experimental Data

The effective oil and water permeability before gel treatment for sandpack with porosities of 30, 32, and 35% were measured based on procedure mentions in 3.3 section (Fig. 5). R-square values in all curves in Fig. 5 are more than 0.98; it reveals the differences between the experimental data and the model's predicted values are small and unbiased.

After gel treatment, water and oil were injected into the core at different flow rates. Oil or water came out from the outlet region after a long. When the system reached at a steady state condition and there was single phase at the outlet, the outlet flow rates were recorded at different pressure drops. The varying of oil and water flow rates versus different pressure drop after gel treatment in three different sandpack (porosities of 30, 32, and 35%) are depicted in Fig. 6.

As demonstrated in Table 1, the unknown parameters of Eq. (20), is yielded by substituting the experimental data into Eq. (20).

As Fig. 6 shows, curve fitting the experimental data and Eq. (20) gives the \( b \) and \( \beta \). It is clear that polymer gel completely blocks all pore volumes and fluids (oil and water) should pass through the polymer gel. The overlap of model and experimental data shows the model can predict the permeability with high accurate.

Table 1 \( b \) and \( \beta \) at different porosities

<table>
<thead>
<tr>
<th>Porosity (%)</th>
<th>( \beta )</th>
<th>( b )</th>
<th>( \beta )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>( 2 \times 10^{-14} )</td>
<td>0.178</td>
<td>( 2 \times 10^{-15} )</td>
<td>0.127</td>
</tr>
<tr>
<td>32</td>
<td>( 2.9 \times 10^{-14} )</td>
<td>0.165</td>
<td>( 5 \times 10^{-15} )</td>
<td>0.184</td>
</tr>
<tr>
<td>35</td>
<td>( 5.8 \times 10^{-15} )</td>
<td>0.209</td>
<td>( 9 \times 10^{-15} )</td>
<td>0.219</td>
</tr>
</tbody>
</table>

Therefore, water and oil flow through gap of polymer molecules and this is similar to pass through free spaces in a real porous media.

The Fig. 7 illustrates variation of oil and water permeability with pressure drop after gel treatment.
The \( b \) and \( \beta \), which are shown in Table 1, are used to plot the Fig. 7 from Eq. (22).

It is clear that polymer gel has more effect on permeability of water compare to oil permeability. The increase in oil and water permeability with pressure drop when there is no additional displacement of any other phases suggests that gel permeability is not a constant function of pressure and fluid velocity.

However, Darcy’s law explains a constant permeability for porous media with linear function of fluid pressure and velocity. The change in permeability demonstrates the response of polymer gel to a reversible deformation. The effect of porosities is also clear; lower porosities yield lower permeability for both oil and water. This phenomenon is noticeable at higher pressure drop.

### 4.2. Residual Resistance Factor

RRF is a criteria of core flood experiments and defined as a ratio between the mobility before and after gel treatment (Seright 2006) with assuming constant viscosity of oil and water:

\[
RRF_o = \left( \frac{k_{\text{eff, before}} / \mu_o}{k_{\text{eff, after}} / \mu_o} \right) \frac{P_o - P_w}{\beta (\mu_R - \mu_o)}^{b(1-b)}
\]

\[
RRF_w = \left( \frac{k_{\text{eff, before}} / \mu_w}{k_{\text{eff, after}} / \mu_w} \right) \frac{P_o - P_w}{\beta (\mu_R - \mu_w)}^{b(1-b)}
\]

Figure 8 shows the variation of oil and water RRF with pressure drop. The results of Fig. 8 indicates polymer gel has been able to reduce the permeability of water more than oil.

So, the gel polymer can be an effective way in water shut off application. RRF is not same at different porosities and pressure drop. As pressure drop rises, the RRF reduces due to deformation of polymer gel at high pressure. Also in lower porosities, RRF is higher than other ones. Consequently, both pressure drop and porosity of media affect the RRF.
4. CONCLUSIONS

In this study, a new model was presented to calculate the permeability of gel coated porous media in a radial flow system. Several experiments were carried out to show the accuracy of model. The model showed that permeability of media was a function of pressure drop, gel, and fluid properties. A comparative analysis was carried out between new model and Darcy equation and the results demonstrated that Darcy predict lower values and was not applicable in gel coated media. Other conclusions can be described as follow:

- The permeability of gel treated porous media in a radial flow system is a power law function of fluid velocity. In addition, it is as a function of pressure drop, and fluid and polymer gel properties.
- Increasing the pressure drop enhances the permeability but the growth rate is not smooth and varied with media characteristics.
- Both Darcy effect and viscous forces affects the fluid flow through coated media. So the new mathematical model consider both of these factors.
- New model for permeability measurement of gel treated porous media in radial flow system is a reliable model.

REFERENCES


Sengupta, B., V. P. Sharma and G. Udayabhanu


