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Effect of External Constraints of Magnetic Field and Velocity Shear on the Propagation of Internal Waves in a Chiral Fluid

N. Rudraiah[1], M. L. Sudheer[2] and G. K. Suresh[3]

[1]*National Research Institute for Applied Mathematics, 4921/G, 7th Cross, 7th Block(West) Jayanagar, Bangalore 560070 and UGC – Center for Advanced Studies in Fluid Dynamics, Department of Mathematics ,Bangalore University, Bangaolre – 560001*
[2]*Department of Electronics Engineering., University Visvesvaraya College of Engineering, Bangalore University, Bangalore – 560001;*
[3]*Department of Telecommunication Engineering., Siddaganga Institute of Technology, Tumkur-572103.*

Email: rudraiahn@hotmail.com; profgksuresh@gmail.com

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ABSTRACT

The propagation of internal electromagnetic waves in an inviscid chiral fluid in the presence of the external constraint of transverse magnetic field is investigated. These waves are shown to be generated due to the stabilizing nature of the distribution of charge density. It is shown that the effect of the external constraint of magnetic field in a chiral fluid is analogous to the effect of viscosity in ordinary fluids. The wave equation, derived from the conservation of mass and momentum together with Maxwell's equations and suitable auxiliary equations for chiral materials, reveals the existence of a critical level (i.e., resonance level) at which the Doppler shifted frequency $\Omega_d = 0$, i.e., at the point where the basic fluid velocity matches with the phase velocity of the wave. The solution of this wave equation is obtained near and away from the critical levels from which the attenuation of waves is predicted using momentum flux. This is verified using group velocity approach.

KeyWords: wave propagation, chiral fluids, magnetic field, velocity shear.

NOMENCLATURE

$\vec{B}$	Magnetic Flux Density *v component of velocity in y direction*	W	Component of velocity in z direction
$\vec{D}$	Dielectric Field Strength / Electric Flux Density	***Greek Symboles***	
$\vec{E}$	*Electric Field Strength*	Ω_d	Brunt – Vaisala Frequency
$\vec{H}$	*Magnetic Field Strength*	α	Wave number
$\vec{J}$	Current Density electric	ε	Permittivity
		η	Wave Impedance
M	Momentum	γ	Chirality coefficient
c	Wave velocity	η	Wave impedance
h	Length scale	μ	Magnetic permeability
p	Pressure	ρ	Density of the fluid
q	The velocity	σ	Electrical conductivity
r	Roots of the equation		
t	*Time*	ρ_e	Electric Charge density
u	Component of velocity in x direction	ω	Angular frequency
v	Component of velocity in y direction		

1. INTRODUCTION

In recent years, considerable interest has been evinced (Rudraiah, 2003) in the development of new technologies like Information Technology, Bio-Technology, Nano – Technology , technologies involving Smart and Chiral Materials, Particles Reinforced Aluminum Matrix composite (PRAMCs), Fiber Reinforced Composite Materials (FRCMs) and technologies involving chiral materials, using improved and novel processing routes which will replace most of the existing technologies today. It is forecasted that these technologies are going to change every aspect of our lives and lead to generation of new capabilities, new materials and new products. They therefore can be described as enabling technologies that will pave the way for novelty in every stream of technologies. An important aspect, associated with these new technologies, is their multi disciplinary nature with applications in Science and Technology and their impact on society is expected to be wide spread and all pervasive. By definition, a three dimensional object is chiral if it cannot be brought into congruence with its mirror image by translation and rotation. An object of this type has the property of handedness and must be either right handed or left handed. Therefore, chirality is connected with handedness. The fluids like sugar solution, benzene, turpentine, bio-fluids and so on, exhibit chirality. Extensive literature is available on theoretical and experimental aspects of solid chiral materials (see Arago, 1811; Biot, 1812). Recently Jaggard (1979, 1988), Kritikos and Jaggard (1989), Varadan and Varadan (1989), and Lakhtakia (1985, 1986, 1994) have done extensive theoretical and experimental works on wave phenomenon in solid chiral materials. However, much attention has not been given to a detailed study of electromagnetic waves in chiral fluids (see Garel, 2003) in spite of their importance in, biomedical and chemical engineering problems, in the design of an efficient antenna, manufacturing devices like photonic display devices and so on. Most of these problems strongly involve interaction of electromagnetic waves with chiral fluids and hence require information regarding the propagational characteristics of electromagnetic waves in chiral fluids. The study of it is the main objective of the present paper. Such a study requires a proper theory and also designing an experiment to validate the theory. However, in the present paper we concentrate only on developing a proper theory for the propagation of internal electromagnetic waves in an inviscid, incompressible and homogeneous chiral fluid in the presence of distribution of charge density decreasing with height.

To achieve the objective of this paper, the required basic equations for two dimensional flows involving the conservation of mass, momentum and density of charges together with Maxwell's equations for chiral fluids are given in section 2. Using these basic equations, the required internal electromagnetic wave equation in the presence of transverse magnetic field and stable charge density decreasing with height subject to linear theory is derived in section 3 using normal mode analysis. The solution of this wave equation is obtained in section 4 using Frobenius technique. To know the nature of the internally propagating electromagnetic waves, we make use of the group velocity method. The momentum flux method is used to determine whether there is any momentum transfer in the vertical direction. This is verified using the group velocity and energy approaches.
The solutions obtained reveal that the external constraint of magnetic field acting on the chiral fluid is analogous to the effect of viscosity in ordinary fluids. The results obtained may also be useful for an effective design of computer monitor screen which is going to be a new generation display device.

2. Formulation of the Problem

The physical configuration considered in this paper is shown in Fig. 1. It consists of a channel bounded by rigid boundaries at y = ± h/2, with x-axis parallel to the plates, y and z axes perpendicular to them. We consider a two dimensional flow in the x and y directions with applied uniform magnetic field B_0 in the z direction, u and v are the components of velocities in the x and y directions respectively.

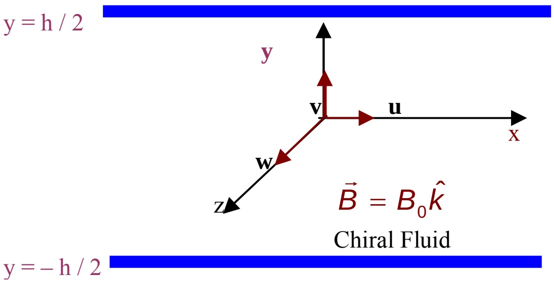

Fig 1. Physical configuration

We assume that the chiral fluid is incompressible and non-viscous because chiral fluids, such as the sugarcane solution used in the process of manufacturing of sugar, particularly at the solidification stage offer negligible resistance to the flow and hence has negligible viscosity.
The required basic equations are the equations of interaction of Maxwell's equations with fluid equations. The Maxwell's equations, for our formulation mentioned above in Fig. 1, are,

Gauss's law: $$\frac{\partial D_x}{\partial x}+\frac{\partial D_y}{\partial y}=\rho_v \qquad (2.1)$$

The Faraday's law, with negligible induced magnetic field, because of negligible conductivity and uniform applied magnetic field, becomes:

$$\frac{\partial E_y}{\partial x}-\frac{\partial E_y}{\partial y}=0 \qquad (2.2)$$

which implies that the electrostatic field is conservative. The current, in the absence of conduction current, is

$$\vec{J}=\rho_e\vec{q}+\frac{\partial\vec{D}}{\partial t} \qquad (2.3)$$

where $\rho_e\vec{q}$ is the convective current and $\frac{\partial\vec{D}}{\partial t}$ is the displacement current. Further, we note that the chiral materials like sugar solutions and turpentine mentioned above have low relaxation frequencies where the convective current in Eq. (2.3) dominates over the displacement current. Therefore, in the present paper, we deal only with convective current and neglect the displacement current. We also note that these chiral fluids have non – polar molecules so that the convective current dominates over the displacement current and have negligible conductivity. In this case the Lorentz' force, for our formulation with only convective current is

$$\vec{J}\times\vec{B} = \rho_e B_0 v\ \hat{i} - \rho_e B_0 u\ \hat{j} \qquad (2.4)$$

The constitutive equations for the chiral fluids are

$$D_x = \varepsilon E_x;\ D_y = \varepsilon E_y\ \text{ and }\ D_z = \varepsilon E_z + i\gamma B_0 \qquad (2.5)$$

together with negligible induced magnetic field

$$\mu H_x - i\gamma\mu E_x = 0;\ \mu H_y - i\gamma\mu E_y = 0$$

$$(0,0,B_0) = \mu H_0 \qquad (2.6)$$

3. THE BASIC EQUATIONS OF MOTION AND THE WAVE EQUATION

To derive the required wave equation in the absence of true current and the displacement current and in the presence of the transverse magnetic field and convective current, we use the following assumptions (Rudraiah and Venkatachalappa, 1972):

1. The motion is two dimensional, in the x and y directions.
2. The chiral fluid, like sugar solution, is inviscid, incompressible and homogeneous.
3. The perturbation on velocities (u,v) are considered to be small compare to the basic state $u_b(y)$ in the x direction.
4. Since the electrical conductivity in chiral fluids is negligible, the induced magnetic field is negligible compared to the externally applied magnetic field B_0 in the z direction.

For the chosen physical configuration shown in Fig. 1, and using the above assumptions, the required basic equations, in Cartesian form, are:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\rho_e}{\rho}B_0\upsilon \qquad (3.1)$$

$$\frac{\partial \upsilon}{\partial t} + u\frac{\partial \upsilon}{\partial x} + \upsilon\frac{\partial \upsilon}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - \frac{\rho_e}{\rho}B_0 u \qquad (3.2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \qquad (3.3)$$

$$\frac{\partial \rho_e}{\partial t} + u\frac{\partial \rho_e}{\partial x} + \upsilon\frac{\partial \rho_e}{\partial y} = 0 \qquad (3.4)$$

together with Maxwell's equations and auxiliary equations given by Eqs. (2.4) to (2.8). Eqs. (3.1) and (3.2) are respectively the conservation of momentum for inviscid chiral fluid in x and y directions and Eq. (3.3) is the conservation of mass for incompressible chiral fluid.

3.1 Basic Flow

We consider the base flow to be steady and parallel to the plates in the x direction such that

$$u = u_b(y),\ \upsilon_b = 0,$$
$$\rho_e = \rho_{eb},\ p = p_b(y), \vec{B} = B_0\hat{k},$$
$$\frac{\partial p}{\partial x} = cons\tan t,\ \frac{\partial p}{\partial y} = \rho_{eb}B_0 u_b \qquad (3.5)$$

Then the basic flow is governed by

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (3.6)$$

and

$$0 = -\frac{1}{\rho}\frac{\partial p_b}{\partial y} - \frac{1}{\rho}\rho_{eb}B_0 u_b \qquad (3.7)$$

Eliminating **p** between Eqs. (3.6) and (3.7) by differentiating Eq. (3.6) with respect to y and Eq. (3.7) with respect to x and then subtracting and using $B_0 u_b \neq 0$, we get

$$0 = \frac{\partial \rho_{eb}}{\partial x} \qquad (3.8)$$

This implies ρ_{eb} is also a function of y .

3.2 Perturbed Flow

To derive the required wave equation, we superimpose on the basic state, given in Eq. (3.5), an infinitesimal perturbation of the form,

$$u = u_b + u' \quad \upsilon = \upsilon' \quad p = p_b + p' \quad \rho_e = \rho_{eb} + \rho_e'$$

$$B_x = B_x' \quad B_y = B_y' \quad B_z = B_0\hat{k} \qquad (3.9)$$

where the prime quantities are the perturbed quantities which are functions of x, y and t and are assumed to be infinitesimal compared to the basic state implying that we deal with only linear theory. The basic Eqs. (3.1) to (3.4), using Eq. (3.5), and linearising them by neglecting the product of prime quantities, become

$$\frac{\partial u}{\partial t} + u_b\frac{\partial u}{\partial x} + v\,Du_b = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\rho_{eb}}{\rho}v\,(\mu H_0 - i\mu\gamma E_0) \qquad (3.10)$$

$$\frac{\partial v}{\partial t} + u_b\frac{\partial v}{\partial x} = -\frac{1}{\rho}Dp - \left(\frac{\rho_e}{\rho}u_b + \frac{\rho_{eb}}{\rho}u\right)(\mu H_0 - i\mu\gamma E_0) \qquad (3.11)$$

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \qquad (3.12)$$

$$\frac{\partial \rho_e}{\partial t} + u_b\frac{\partial \rho_e}{\partial x} + \upsilon\,D\,\rho_{eb} = 0 \qquad (3.13)$$

where $D = \dfrac{d}{dy}$ and u_b is the basic velocity in the x direction. We now make the Eqs. (3.10) to (3.13) dimensionless using

$$u^* = \frac{u}{u_0};\ u_b^* = \frac{u_b}{u_0};\ v^* = \frac{v}{u_0};\ p^* = \frac{p}{\rho u_0^2};\ t^* = \frac{t}{(h/u_0)}$$

$$\rho_{eb}^* = \frac{\rho_{eb}h}{\varepsilon E_0 + i\gamma B_0} = \frac{\rho_{eb}h}{(\varepsilon + \mu\gamma^2)E_0 + i\gamma\mu H_0} \qquad (3.14)$$

where the asterisks denote the dimensionless quantities. Therefore the dimensionless form of equations from Eq. (3.10) to Eq. (3.13), using

Eq. (3.14) and for simplicity neglecting the asterisks, become

$$\frac{\partial u}{\partial t}+u_b\frac{\partial u}{\partial x}+v\,Du_b=-\frac{\partial p}{\partial x}+\rho_{eb}\left(R_1+i\gamma R_2\right)v \qquad (3.15)$$

$$\frac{\partial v}{\partial t}+u_b\frac{\partial v}{\partial x}=-Dp-\rho_e u_b\left(R_1+i\gamma\mu R_2\right)-\rho_{eb}v\left(R_1+i\gamma\mu R_2\right) \qquad (3.16)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (3.17)$$

$$\frac{\partial \rho_e}{\partial t}+u_b\frac{\partial \rho_e}{\partial x}+vD\rho_{eb}=0 \qquad (3.18)$$

where,

$$R_1=\mu\varepsilon\left(1+2\eta_0^2\gamma^2\right) \quad \text{and}$$

$$R_2=\mu H_0^2\left[1-\frac{\eta^2}{\eta_c^2}\right]$$

$$\eta_0=\sqrt{\frac{\mu}{\varepsilon}},\ \eta=\frac{E_0}{H_0}\ \text{and}\ \eta_c=\sqrt{\frac{\eta_0^2}{1+\eta_0^2\gamma^2}}$$

(3.19)

3.3 Derivation of the Wave Equation

We look for solutions of Eqs. (3.15) to (3.19) in the normal mode form

$$f(x,y,t)=f(y)\,e^{ia(x+ct)}=f(y)\,e^{i(ax+st)} \qquad (3.20)$$

where α is the dimensionless wave number k in the x direction,

$c = c_r + i\,c_i$ = wave velocity,

c_r = phase velocity of the wave,

c_i = frequency of the wave,

$\sigma = \alpha c$, the frequency of the perturbations, and

$\Omega_d = \alpha U_b + \sigma$ is the Brunt – Vaisala frequency such that $u_b + c = \dfrac{\Omega_d}{\alpha}$.

Equations (3.15) to (3.18), using Eq. (3.20) and eliminating the pressure and expressing u in terms of v using Eq. (3.17), we get the following wave equation with variable coefficients:

$$\left[(D^2+\alpha^2)-\frac{D^2u_b}{(u_b+c)}+\frac{c(R_1+i\gamma R_2)D\rho_{eb}}{(u_b+c)^2}\right]v=0 \qquad (3.21)$$

where u_b is the basic chiral fluid velocity, c the velocity of the wave generated . This wave equation has a singularity at $u_b = -c$ which we call resonance effect and the level at which $u_b = -c$ is called the critical level (see Rudraiah and Venkatachalappa, 1972). In terms of Ω_d, the wave equation, Eq. (3. 21), may be written as

$$\left[(D^2+\alpha^2)-\frac{\alpha D^2u_b}{\Omega_d}+\frac{\alpha^2c(R_1+i\gamma R_2)D\rho_{eb}}{\Omega_d^2}\right]v=0 \qquad (3.22)$$

4. Solution of the Wave Equation

We try to find the solution of Eq. (3.21) using the Frobenius Method of the form

$$v=\sum_{n=0}^{\infty}\alpha_n\,(y-y_0)^{n+r} \qquad (4.1)$$

Substituting Eq. (4.1) into the wave equation Eq. (3.21) and then equating the coefficients of like powers of $(y-y_0)^r$, $(y-y_0)^{r+1}$, $(y-y_0)^{r+2}$ … to zero, we obtain the characteristic equation from which we get the two possible roots of the second order wave Eq. (3.21)

$$r=\frac{1\pm\sqrt{1-\dfrac{4c(R_1+i\gamma R_2)D\rho_{eb}}{u_{by}^2}}}{2} \qquad (4.2)$$

Using $r = r_1$ corresponding to the positive sign and $r = r_2$ corresponding to the negative sign in Eq. (4.2), we obtain the following constants:

$$\alpha_0=1 \quad \text{(assumed)} \qquad (4.3)$$

$$\alpha_1=\frac{u_b(y_0)\,u_{byy}(y_0)}{c(R_1+i\gamma R_2)D\rho_{eb}-r^2-r} \qquad (4.4)$$

$$\alpha_2=\frac{\alpha^2u_{by}^2(y_0)-\dfrac{u_{byy}^2(y_0)}{2}-u_b(y_0)u_{byyy}(y_0)-a_1u_b(y_0)u_{byy}(y_0)}{r^2+3r+2-c(R_1+i\gamma R_2)D\rho_{eb}} \qquad (4.5)$$

Equation (4.1), using Eq. (4.2), and letting $\alpha_n = a_n$ when $r = r_1$ and $\alpha_n = b_n$ when $r = r_2$, we get the solution of the wave Eq. (3.12) in the form

$$v=(y-y_0)^{r_1}\left\{1+a_1(y-y_0)+a_2(y-y_0)^2+\cdots\right\}+(y-y_0)^{r_2}\left\{1+b_1(y-y_0)+b_2(y-y_0)^2+\cdots\right\} \qquad (4.6)$$

Once a_n and b_n are known, the solution for v is known. Then, using this v in Eq. (3.17) we can find the solution for u. We next find the momentum flux to understand whether the waves are transmitted, reflected or absorbed at the critical level. This we determine using the group velocity approach. The results so obtained using this method will be confirmed using the energy method applied to the momentum equation, as has been done in (Rudraiah and Venkatachalappa, 1972).

5. Propagation of Thye Upward and Downward Traveling Waves

Equation (3.21) is now analysed to determine whether the waves generated in the chiral inviscid fluid is an upward traveling or downward traveling or absorbed at the critical level. This is done using the group velocity approach as well as the energy method as explained in the following sub – section.

5.1 Group Velocity Approach

The wave equation for the internal electromagnetic wave generated in a chiral fluid under the influence of velocity shear and transverse magnetic field is given by Eq. (3.21). We let $N=\sqrt{c(R_1+i\gamma R_2)D\rho_{eb}}$. Following Booker and Bretherton (1967), Rudraiah and Venkatachalappa (1972), Rudraiah *et al.* (2000) and Rudraiah (2003), we consider a uniform basic velocity u_b in a chiral fluid and a linear variation of charge distribution valid for infinitesimal stratification factor, with N as constant. Then the solution of Eq. (3.21) with horizontal wave number and wave velocity c has a vertical structure of the form

$$\upsilon = A\,e^{imy} + B\,e^{-imy} \tag{5.1}$$

where A and B are constants and

$$m = \left[\frac{N^2}{(u_b-c)^2} - \alpha^2\right]^{\frac{1}{2}} \tag{5.2}$$

For the sake of definiteness, we settle the branch for m by requiring that, If $c_i > 0$, $m_i > 0$. This implies that

$$\left.\begin{array}{lll} if\ \ \alpha^2 \ll \dfrac{N^2}{(u_b-c)^2}, & m \simeq \dfrac{N}{(u_b-c)} \\ if\ \ \alpha^2 \gg \dfrac{N^2}{(u_b-c)^2}, & m \simeq ik \end{array}\right\} \tag{5.3}$$

The complete spatial distribution of velocity associated with the first term in Eq. (5.1) is

$$\upsilon = \mathrm{Re}\,[A\ e^{i\alpha(x-ct)}] \tag{5.4}$$

This is a standard form of a plane wave with a phase front

$$\alpha(x-ct)=constant \tag{5.5}$$

at least for the range of frequencies αc for which m and c are real.

If $u_b - c$ is negative, i.e., the wave is propagating in the positive x direction relative to the chiral fluid, m is also negative and the wave front moves downwards. That is, the first term of the solution in Eq. (5.1) represents a wave propagating with a downward component of wave velocity. However, the influence of such a wave propagates upward since retardation in phase represents a forward traveling wave. Thus, the first part of the solution in Eq. (5.1), i.e., $A\,e^{imy}$, represents an upward traveling wave. Similarly, using the same argument, the second part of the Eq. (5.1) represents a downward traveling wave.

We observe from Eq. (5.2) that $\sigma = \alpha c$ is given by

$$\sigma = \alpha u_b \pm \frac{\alpha^2 N}{\sqrt{\alpha^2 + m^2}} \tag{5.6}$$

According to Eq. (5.3), we must take negative sign when m and $u_b - c$ are positive and positive sign when they are negative. In either case, for the first term of Eq. (5.1), we get the group velocity of the form

$$\frac{\partial\sigma}{dm} = \pm\frac{\alpha^2 Nm}{\left(\alpha^2+m^2\right)^{\frac{3}{2}}} = \frac{\alpha^2 m\left(u_b-c\right)^3}{N^2} \tag{5.7}$$

which is always positive. We know, from wave theory, that in a uniform medium, Eq. (5.7) is a standard form representing the upward component of group velocity. A slow modulation on a sinusoidal train of waves moves without change of shape with this group velocity. In a slowly varying medium the form of the modulation may change but it still moves essentially with the same group velocity as in the case of ordinary fluids discussed by Booker and Bretherton (1967) for uniform velocity. To verify the result obtained from the group velocity approach we study the energy method in the following section.

5.2 Energy Method

The equation for horizontal momentum for disturbance, from Eq. (3.15), treating u_b constant, is

$$\frac{\partial u}{\partial t} + u_b\frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} + (R_1 + i\gamma R_2)\rho_{eb}\upsilon \tag{5.8}$$

We look for, as before, the normal mode solution of Eq. (5.8) of the form given in Eq. (3.20). Substituting Eq. (3.20) into Eqs. (5.8) and (3.17) and solving for p, we get

$$p=-(u_b-c)u+\frac{(R_1+i\gamma R_2)\rho_{eb}\upsilon}{i\alpha} \tag{5.9}$$

$$i\alpha u + \frac{\partial\upsilon}{\partial y}=0 \tag{5.10}$$

Multiplying Eq. (5.9) on both sides by υ^*, the complex conjugate of υ, we get,

$$p\upsilon^*=-(u_b-c)u\upsilon^* + \frac{(R_1+i\gamma R_2)\rho_{eb}\upsilon\upsilon^*}{i\alpha} \tag{5.11}$$

The time average of Eq. (5.11) and replacing u in terms of v using Eq. (5.10) leads to

$$\overline{p\upsilon^*}=\frac{1}{2}\mathrm{Re}\left\{-i\frac{(u_b-c)}{\alpha}\upsilon^*\frac{\partial\upsilon}{\partial y} - i\frac{(R_1+i\gamma R_2)\rho_{eb}|\upsilon|^2}{\alpha}\right\} \tag{5.12}$$

where the over bar denotes the time average and we get

$$\overline{p\upsilon^*}=\frac{1}{2}\frac{(u_b-c)mA^2-\gamma R_2\rho_{eb}A^2}{\alpha}$$

$$=\frac{1}{2\alpha}[(u_b-c)mA^2+\gamma R_2\rho_{eb}A^2] \tag{5.13}$$

which is always positive as long as $(u_b - c) \geq 0$. This indicates that the first part of the solution in Eq. (5.1) represents an upward traveling wave, confirming the result obtained using group velocity method discussed in section 5.1

We now proceed to find whether any momentum transfer exists in y direction. For this purpose we use the momentum flux method in the following section.

5.3 The Momentum Flux Method

Multiplying both sides of the Eq. (5.10) by υ^* the complex conjugate of υ, we get,

$$u\upsilon^* = \frac{i}{\alpha}\upsilon^* \; D\upsilon \qquad (5.14)$$

To find the momentum flux, we substitute Eq. (5.4) into Eq. (5.14) and considering half of its real part, we get,

$$\frac{1}{2}\mathrm{Re}(u\upsilon^*) = -\frac{1}{2}\mathrm{Re}\left(\frac{\upsilon^* D\upsilon}{i\alpha}\right) \qquad (5.15)$$

Let $\frac{\upsilon^* D\upsilon}{i\alpha} = M$ be the momentum.

We compute the rate of vertical momentum transfer in the form

$$\frac{dM}{dy} = \frac{d}{dy}\left\{-\frac{1}{2}\mathrm{Re}\left(\frac{i}{\alpha}\upsilon^* D\upsilon\right)\right\} \qquad (5.16)$$

Equation (5.16), using Eq. (5.10), becomes

$$\frac{dM}{dy} = 0 \qquad (5.17)$$

This implies that there is no momentum transfer in the inviscid homogeneous chiral fluid in the vertical direction. This means that energy is not conserved in y direction.

6. Conclusion

Propagation of waves in an inviscid chiral fluid in the presence of a transverse magnetic field and convective current near the critical level given by Ω_d is studied using energy method and the group velocity approach. We find that, the effect of the transverse magnetic field is analogous to viscosity in ordinary fluids in introducing a critical level, namely the basic velocity of the fluid u_b balances the wave velocity of the induced wave, c. We have shown that when the waves approach the critical level, both upward and downward traveling waves exist. This result is verified by both the group velocity approach and the energy method. The solution of the wave equation governing the vertical component of velocity υ is determined using Frobenius method. Once υ is known, the horizontal component of velocity u can be determined form equation of conservation of mass. From these components of velocity, using momentum flux method, we have shown that transfer of momentum does not take place in the vertical (i.e. y) direction and hence energy is not conserved in this direction.

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