Journal of Applied Fluid Mechanics, Vol. 4, No. 4, pp. 53-58, 2011.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.36884/jafm.4.04.11946

Chemically Reactive Solute Distribution in a Steady MHD Boundary Layer Flow over a Stretching Surface

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(Received June 27, 2009; accepted June 27, 2010)

ABSTRACT

The paper is concerned to find the distribution of the chemically reactant solute in the MHD flow of an electrically conducting viscous incompressible fluid over a stretching surface. The first order chemical reaction and the variable solute distribution along the surface are taken into consideration. The governing partial differential equations along with appropriate boundary conditions for flow field and reactive solute are transformed into a set of non-linear self-similar ordinary differential equations by using scaling group of transformations. An exact analytic solution is obtained for the velocity field. Using this velocity field, we obtain numerical solution for the reactant concentration field. It reveals from the study that the values of concentration profile enhances with the increase of the magnetic field and decreases with increase of Schmidt number as well as the reaction rate parameter. Most importantly, when the solute distribution along the surface increases then the concentration profile decreases.

Keywords: MHD boundary layer, Stretching surface, Chemically reactive solute, Scaling group of transformations.

NOMENCLATURE

a	stretching constant
$\mathbf{B}$	magnetic field
C	concentration
C_0	positive solute constant
C_w	solute distribution along the stretching surface
C_∞	constant solute in the free stream
D	diffusion coefficient
$\mathbf{E}$	electric field
f	non-dimensional stream function
$\bar{f}$	variable
G,H	absolute invariants
$\mathbf{J}$	current density
k	reaction rate of the solute
M	magnetic parameter
n	a power-law exponent
R_M	magnetic Reynolds number
Sc	Schmidt number
u, v	velocity components
$\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4\ \alpha_5\ \alpha_6$	transformation parameters
$\alpha', \alpha'', \alpha''', \beta', \beta'', \beta'''$	transformation parameters
β	reaction rate parameter
η	similarity variable
$\bar{\eta}$	variable
Γ	scaling group transformations
υ	kinematic viscosity
$\phi,\bar{\phi}$	variables
ρ	density of the fluid
ψ	stream function
ψ^*	variable

1. INTRODUCTION

The flows due to stretching sheet in presence of electromagnetic fields are relevant to many practical applications in the metallurgy industries, polymer processing industries, paper production, filaments drawn through a quiescent electrically conducting fluid subject to a magnetic field and the purification of molten metals from nonmetallic inclusions.

The boundary layer equations play a central role in many aspects of fluid mechanics because they describe the motion of a viscous fluid close to a surface. These equations are especially very important since they have the capacity to admit a large number of invariant solutions. Lie-group analysis, also called symmetry analysis was developed by Sophius Lie to find point-transformations that map a given differential equation to itself. This method unifies almost all known exact integration techniques for both ordinary and partial differential equations (Pakdemirli and Yurusoy 1990). Group analysis is the only rigorous mathematical

method to find all symmetries of a given differential equation and no adhoc assumptions or a prior knowledge of the equation under investigation is needed.

The non-linear character of the partial differential equations governing the motion of the fluid produces difficulties in solving the equations. In fluid mechanics, researchers try to obtain the similarity solutions in such cases. In case of scaling group of transformations, the group-invariant solutions are nothing but the well known similarity solutions (Mukhopadhyay *et al.* 2005). A special form of Lie-group of transformations, known as scaling group transformations, is used in this work to find out the full set of symmetries of the flow problem (Mukhopadhyay *et al.* 2005).

Sakiadis (1961a, b) was the first person to study the laminar boundary layer flow caused by a rigid surface moving in its own plane. Crane (1970) extended the work of Sakiadis in stretching sheet. The heat and mass transfer problem associated with the Newtonian boundary layer flow past a stretching sheet was studied by Gupta and Gupta (1977). Chakrabarti and Gupta (1979) analyzed the magnetohydrodynamic (MHD) flow of Newtonian fluid initially at rest, over a stretching sheet at a different values of parameter related with uniform temperature. Anjali Devi and Ganga (2010) exhibited dissipation effects on MHD nonlinear flow and heat transfer past a stretching porous surface embedded in a porous medium.

The effects of chemically reactive solute distribution on fluid flow due to a stretching sheet also bear equal importance in engineering researches. The chemical reaction effects were studied by many researchers on several physical aspects. The diffusion of a chemically reactive species in a laminar boundary layer flow over a flat plate was demonstrated by Chambre and Young (1958). The effect of transfer of chemically reactive species in the laminar flow over a stretching sheet explained by Andersson *et al.* (1994). Takhar *et al.* (2000) analyzed the flow and mass transfer on a stretching sheet with a magnetic field and chemically reactive species with n-th order reaction. Afify (2004) explicated the MHD free convective flow of viscous incompressible fluid and mass transfer over a stretching sheet with chemical reaction. Liu (2005) studied the momentum, heat and mass transfer of a hydromagnetic flow past a stretching sheet in the presence of a uniform transverse magnetic field. Akyildiz *et al.* (2006) obtained a solution for diffusion of chemically reactive species in a flow of a non-Newtonian fluid over a stretching sheet immersed in a porous medium. Cortell (2007) investigated the motion and mass transfer for two classes of viscoelastic fluid over a porous stretching sheet with chemically reactive species. Recently, Kandasamy *et al.* (2010) investigated the effects of temperature-dependent fluid viscosity and chemical reaction on MHD free convective heat and mass transfer with variable stream conditions.

In the present investigation, we have studied the Newtonian MHD boundary layer flow and reactive solute transfer with first order reaction past a stretching surface. The variable initial solute distribution along the surface is taken into account. The scaling group of transformation is applied into the governing equations without adopting any adhoc assumption and finally set of self-similar ordinary differential equations are obtained. Then the transformed self-similar equations are solved. Exact analytical solution of MHD boundary layer flow is obtained and then solution of concentration distribution is obtained numerically. The results are discussed physically in various contexts.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a steady MHD flow of an electrically conducting viscous incompressible fluid undergoing a first order chemical reaction over a stretching surface. The continuity, momentum and reactive concentration equations for governing the flow and concentration distribution in the boundary layer region along the stretching surface may be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+\frac{1}{\rho}\left(\mathbf{J}\times\mathbf{B}\right)_x \tag{2}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k\left(C-C_\infty\right), \tag{3}$$

where u and v are velocity components in x- and y-directions respectively, υ is the kinematic viscosity, ρ is the density of the fluid, **J** is the current density and **B** is the magnetic field. C is the concentration, D is the diffusion coefficient k denotes the reaction rate of the solute and C_∞ is constant solute in the free stream. One may note that in writing Eq. (2), we have neglected the induced magnetic field since the magnetic Reynolds number R_M for the flow is assumed to be small.

The magnetic field **B** having components $(0,B_0,0)$ with B_0 non-negative constant, the relation $\nabla\cdot\mathbf{B}=0$ is automatically satisfied. It is noted that the electric current in the flow acts parallel to z-axis (i.e. normal to the plane of the flow). Hence from Ohm's law we get the components of **J** as

$$j_x=0,\ j_y=0,\ j_z=\sigma[E_z+(\mathbf{q}\times\mathbf{B})_z]=\sigma[E_z+uB_0], \tag{4}$$

where σ is the constant electrical conductivity of the fluid and E_z is the component of electric field along the z-direction and **q** is the velocity vector. Now as the flow is steady, Maxwell's equation gives

$$\nabla\times\mathbf{E}=0 \tag{5}$$

where **E** is the electric field which is along the z-axis. This gives from Eq. (5) $\partial E_z/\partial y=0$ and $\partial E_z/\partial x=0$ so that E_z is a function of z only.

Since the induced magnetic field is neglected in view of the assumption $R_M<<1$ electric current in the flow is determined from Ohm's law and not from $\nabla\times\mathbf{B}=\mu_e\mathbf{J}$, μ_e being the magnetic permeability. But the consequence $\nabla\cdot\mathbf{J}=0$ of this equation must be satisfied (Shercliff, 1965). This readily gives from Eq. (4), E_z=constant since E_z is independent of x and y. Thus using Eq. (4), we find from Eq. (2),

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u-\frac{\sigma B_0 E_z}{\rho} \tag{6}$$

In the free stream one can write

$$\frac{\sigma B_0^2}{\rho}U-\frac{\sigma B_0}{\rho}E_z=0\,,$$

which gives $E_z=-B_0U$.

Here U is the free-stream velocity and according to this problem U=0.

Finally, the momentum Eq. (6) becomes

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{7}$$

The appropriate boundary conditions for the velocity components and reactant concentration are given by

$$\left.\begin{aligned}&u=ax,\ v=0 \text{ at } y=0\\&u\to 0 \text{ as } y\to\infty\end{aligned}\right\} \tag{8}$$

and

$$\left.\begin{aligned}&C=C_w=C_\infty+C_0x^n \text{ at } y=0\\&C\to C_\infty \text{ as } y\to\infty,\end{aligned}\right\} \tag{9}$$

where a is assumed to be stretching constant and we consider a variable solute distribution along the stretching surface i.e. $C_w=C_\infty+C_0x^n$, where C_0 is a positive solute constant, n is a power-law exponent, which signifies the change of amount of solute in the x-direction.

Introducing the stream function to this boundary layer flow we get the following relation as

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x} \tag{10}$$

and the concentration C is related by

$$C=C_\infty+\bar{C}\left(C_w-C_\infty\right) \tag{11}$$

The continuity Eq. (1) is satisfied clearly by the relations (10). In view of the relations (10) and (11), the Eqs. (7) and (3) reduce respectively to

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\upsilon\frac{\partial^3\psi}{\partial y^3}-\frac{\sigma B_0^2}{\rho}\frac{\partial\psi}{\partial y} \tag{12}$$

$$x\frac{\partial\psi}{\partial y}\frac{\partial\bar{C}}{\partial x}+n\frac{\partial\psi}{\partial y}\bar{C}-x\frac{\partial\psi}{\partial x}\frac{\partial\bar{C}}{\partial y}=xD\frac{\partial^2\bar{C}}{\partial y^2}-xk\bar{C} \tag{13}$$

and the boundary conditions become

$$\left.\begin{aligned}&\frac{\partial\psi}{\partial y}=ax \text{ and } \frac{\partial\psi}{\partial x}=0 \text{ at } y=0\\&\frac{\partial\psi}{\partial y}\to 0 \text{ at } y\to\infty\end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned}&\bar{C}=1 \text{ at } y=0\\&\bar{C}\to 0 \text{ at } y\to\infty.\end{aligned}\right\} \tag{15}$$

3. Invariant Solution through Scaling Group of Transformations

We now introduce the simplified form of Lie-group transformations, namely, the scaling group of transformations (Mukhopadhyay *et al.* 2005) as

$$\Gamma:\begin{cases}x^*=xe^{\varepsilon\alpha_1},\ y^*=ye^{\varepsilon\alpha_2},\ \psi^*=\psi e^{\varepsilon\alpha_3},\\u^*=ue^{\varepsilon\alpha_4},\ v^*=ve^{\varepsilon\alpha_5}\ and\ C^*=\bar{C}e^{\varepsilon\alpha_6}\end{cases} \tag{16}$$

The transformation (16) may be considered as a point transformation, which transformed the coordinates (x,y,ψ,u,v) to the coordinates (x^*,y^*,ψ^*,u^*,v^*).

Taking the relations (16) in to account in Eqs. (12) and (13), we obtain respectively

$$\begin{aligned}&e^{\varepsilon(\alpha_1+2\alpha_2-2\alpha_3)}\left(\frac{\partial\psi^*}{\partial y^*}.\frac{\partial^2\psi^*}{\partial x^*\partial y^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial^2\psi^*}{\partial y^{*2}}\right)\\&=e^{\varepsilon(3\alpha_2-\alpha_3)}\upsilon\frac{\partial^3\psi^*}{\partial y^{*3}}-e^{\varepsilon(\alpha_2-\alpha_3)}\frac{\sigma B_0^2}{\rho}\frac{\partial\psi^*}{\partial y^*}\end{aligned} \tag{17}$$

and

$$\begin{aligned}&e^{\varepsilon(\alpha_2-\alpha_3-\alpha_6)}x^*\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial C^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial C^*}{\partial y^*}\right)\\&+e^{\varepsilon(\alpha_2-\alpha_3-\alpha_6)}n\frac{\partial\psi^*}{\partial y^*}C^*\\&=e^{\varepsilon(2\alpha_2-\alpha_1-\alpha_6)}x^*D\frac{\partial^2C^*}{\partial y^{*2}}-e^{-\varepsilon(\alpha_1+\alpha_6)}kx^*C^*\end{aligned} \tag{18}$$

In order that, the system will remain invariant under the group of transformation Γ we then would have the following relations among the transformation parameters

$$\left.\begin{aligned}&\alpha_1+2\alpha_2-2\alpha_3=3\alpha_2-\alpha_3=\alpha_2-\alpha_3\\&\text{and } \alpha_2-\alpha_3-\alpha_6=2\alpha_2-\alpha_1-\alpha_6=-\alpha_1-\alpha_6\end{aligned}\right\} \tag{19}$$

From (19) we can obtain easily α_2=0 and $\alpha_1=\alpha_3$. The relation $u^*=\frac{\partial\psi^*}{\partial y^*}$ and $v^*=-\frac{\partial\psi^*}{\partial x^*}$ gives us $\alpha_3=\alpha_4$, α_5=0. In view of these, the boundary conditions (14) and (15) are transformed to

$$\left.\begin{aligned}&\frac{\partial\psi^*}{\partial y^*}=ax^* \text{ and } \frac{\partial\psi^*}{\partial x^*}=0 \text{ at } y^*=0\\&\frac{\partial\psi^*}{\partial y^*}\to 0 \text{ at } y^*\to\infty\end{aligned}\right\} \tag{20}$$

$$\left.\begin{aligned}&C^*=1 \text{ at } y^*=0\\&C^*\to 0 \text{ at } y^*\to\infty\end{aligned}\right\} \tag{21}$$

where the boundary condition C^*=1 gives α_6=0. Thus the set Γ finally reduces to a one-parameter group transformation

$$\Gamma:\begin{cases}x^*=xe^{\varepsilon\alpha_1}\ y^*=y,\ \psi^*=\psi e^{\varepsilon\alpha_1},\\u^*=ue^{\varepsilon\alpha_1},\ v^*=v \text{ and } C^*=\bar{C}\end{cases} \tag{22}$$

Firstly, we consider the absolute invariant, η which is a function of the independent variables and is taken as $\eta = y^* x^{*s}$.

Since the quantity η is absolute invariant, we get $y^* x^{*s} = y x^s$ .Now, $y^* x^{*s} = y x^s e^{\varepsilon\alpha_1 s} = y x^s$ if $s = 0$ (since α_1 cannot be 0). Hence, we get the first absolute invariant as $\eta=y^*$.

We now find the second absolute invariant, $G=f(\eta)$ which involves the dependent variable ψ^* and assume that $G = x^{*r}\psi^*$. Since G is an absolute invariant, we will find r such that $x^{*r}\psi^* = x^r\psi$. Now, $x^{*r}\psi^* = \left(xe^{\varepsilon\alpha_1}\right)^r \psi e^{\varepsilon\alpha_1} = \left(x^r e^{\varepsilon\alpha_1 r}\right)\psi e^{\varepsilon\alpha_1}$

$= e^{\varepsilon\alpha_1(r+1)} x^r\psi = x^r\psi$ if $r=-1$. Putting $r=-1$, the second absolute invariant G becomes $G = x^{*-1}\psi^*$ i.e. $f(\eta) = x^{*-1}\psi^*$.

Lastly, we want to find the third absolute invariant, $H=\phi(\eta)$ which involves the independent variables and the dependent variable C^* and is taken as $H = x^{*p}C^*$. H is an absolute invariant if $x^{*p}C^* = x^p\bar{C}$.

Now, $x^{*p}C^* = \left(xe^{\varepsilon\alpha_1}\right)^p \bar{C} = \left(x^p e^{\varepsilon\alpha_1 p}\right)\bar{C}$

$= e^{\varepsilon\alpha_1 p} x^p\bar{C} = x^p\bar{C}$ if $p=0$. Thus, the third absolute invariant is $H=C^*$ i.e. $\phi(\eta)=C^*$.

Finally, from three absolute invariants, we get the transformations as given below:

$$\eta=y^*,\ \psi^*=x^* f(\eta) \text{ and } C^*=\phi(\eta) \tag{23}$$

In view of the above relations, the Eqs. (17) and (18) become

$$\upsilon f''' + ff'' - f'^2 - \frac{\sigma B_0^2}{\rho} f' = 0 \tag{24}$$

$$D\phi'' + f\phi' - n f'\phi - k\phi = 0 \tag{25}$$

and the boundary conditions reduced to

$$\left.\begin{aligned} &f(\eta)=0,\ \ f'(\eta)=a \text{ at } \eta=0 \\ &f'(\eta)\to 0 \text{ as } \eta\to\infty \end{aligned}\right\} \tag{26}$$

$$\left.\begin{aligned} &\phi=1 \text{ at } \eta=0 \\ &\phi\to 0 \text{ as } \eta\to\infty \end{aligned}\right\} \tag{27}$$

Again, we introduce the following transformations for η, f and ϕ in Eqs. (24)-(27):

$$\eta = \upsilon^{\alpha'} a^{\beta'}\bar{\eta},\ \ f = \upsilon^{\alpha''} a^{\beta''}\bar{f} \text{ and } \phi = \upsilon^{\alpha'''} a^{\beta'''}\bar{\phi} \tag{28}$$

and we obtain $\alpha' = \alpha'' = \frac{1}{2}$, $\beta' = \frac{1}{2}$, $\beta'' = -\frac{1}{2}$ and $\alpha''' = \beta''' = 0$.

Finally, in view of the above transformations and taking $\bar{\eta} = \eta$, $\bar{f} = f$ and $\bar{\phi} = \phi$, the Eqs. (24) and (25) reduce to the following forms:

$$f''' + ff'' - f'^2 - Mf' = 0 \tag{29}$$

$$\phi'' + Sc\, f\phi' - Sc\left(n f' + \beta\right)\phi = 0 \tag{30}$$

where $M=\sigma B_0^2/a\rho$ is the magnetic parameter, $Sc=\upsilon/D$ is the Schmidt number and $\beta=k/a$ reaction rate parameter of the solute.

The boundary conditions (26) and (27) reduce to the following forms:

$$\left.\begin{aligned} &f(\eta)=0,\ \ f'(\eta)=1 \text{ at } \eta=0 \\ &f'(\eta)\to 0 \text{ as } \eta\to\infty \end{aligned}\right\} \tag{31}$$

$$\left.\begin{aligned} &\phi(\eta)=1 \text{ at } \eta=0 \\ &\phi(\eta)\to 0 \text{ as } \eta\to\infty. \end{aligned}\right\} \tag{32}$$

4. SOLUTION OF THE PROBLEM

The Eq. (29) along with the boundary condition (31) is solved analytically (Sarpkaya 1961) and the exact solution is given by

$$f(\eta) = \frac{1-\exp\left(-\sqrt{1+M}\,\eta\right)}{\sqrt{1+M}},\ \eta\geq 0. \tag{33}$$

After substitution of the function f and using finite-difference technique in the Eq. (30) along with the boundary conditions (32) is solved numerically. The expression for wall shear stress is given by $\left|f''(0)\right| = \sqrt{1+M}$ which increases with the increase of magnetic field M and consequently, the boundary layer thickness of the stretching surface decreases.

5. RESULTS AND DISCUSSIONS

The analytic solution of velocity has presented for various values of the magnetic parameter M. The reactant solute equation is solved numerically and the results are shown graphically.

The velocity profiles for various values of the magnetic parameter M have been plotted in Fig. 1. From the figure it is noted that with increase of M, the velocity for any fixed value of η decreases. Thus it is clear that the magnetic field opposes motion. This is due to the fact that variation of M leads to the variation of Lorenz force producing more resistance to the transport process. Consequently, the momentum boundary layer thickness reduces with the increase in M and this fact is also seen from wall shear stress behaviour.

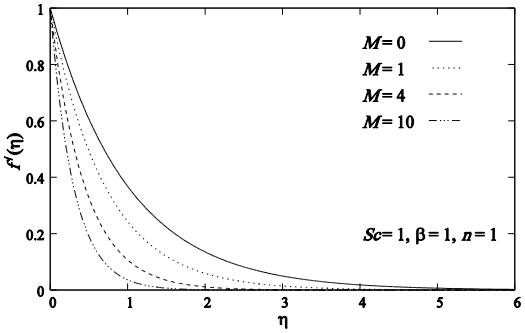

Fig. 1. Velocity profiles $f'(\eta)$ for various values of M

In order to assure the accuracy of applied numerical method, we have compared our obtained results for concentration gradient at the surface $-\phi'(0)$ which is related with the Local Sherwood number by the relation $Sh/Re^{1/2}=-\phi'(0)$ that of Takhar *et al.* (2000) and Andersson *et al.* (1994) for n=0 (i.e. with constant solute along the surface) in Table 1 and found in excellent agreement.

Table1 Comparison of the values of $-\phi'(0)$ with that of Takhar *et al.* (2000) and Andersson *et al.* (1994) for n=0.

Sc	β	Present Study	Takhar *et al.* (2000)	Andersson *et al.* (1994)
0.1	0.1	0.15057	0.15042	0.149
1.0	0.1	0.66873	0.67044	0.669
1.0	1.0	1.17679	1.17761	1.177
10	1.0	3.87347	3.87469	3.880
10	10	10.24535	10.24283	10.25

Figure 2 exhibits concentration profiles for various values of M. The value of contaminate solute at particular value of η increases with the increase of the magnetic parameter M and also the concentration boundary layer thickness increases. This implies that the magnetic field acts to enhance the distribution of the reaction solute on the stretching surface in case of an electrically conducting fluid subject to magnetic field. This result may be useful, in the situation where the enhancement of solute transfer from the surface is the prime important.

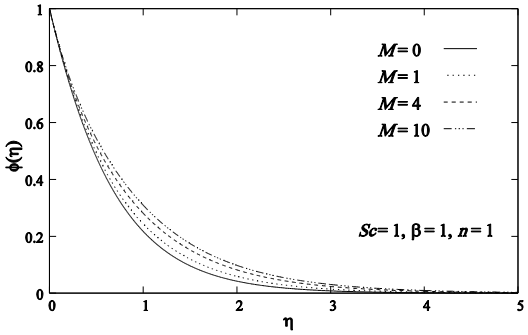

Fig. 2. Concentration profiles $\phi(\eta)$ for various values of M.

Now, we concentrate on variation the solute curves for different values of Schmidt number *Sc*. The curves are drawn in the Fig. 3. The Schmidt number has major effects on the distribution of solute. The concentration boundary layer thickness as well as the concentration at a fixed point decreases quickly with increasing values of *Sc*. This is due to the fact that the rate of solute transfer from the surface increases when the Schmidt number increases.

Figure 4 is the graphical representation of concentration profiles for various values of reaction rate parameter β. It has been found that the reaction rate parameter affect the solute profiles in similar way as that of the Schmidt number i.e., the increase of β reduces both the solute boundary layer thickness and value of the solute at fixed η. So, in case of the distribution of reactive solute, the reaction rate parameter is a decelerating agent.

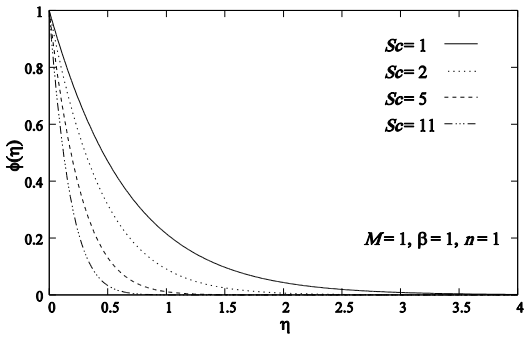

Fig. 3. Concentration profiles $\phi(\eta)$ for various values of *Sc*.

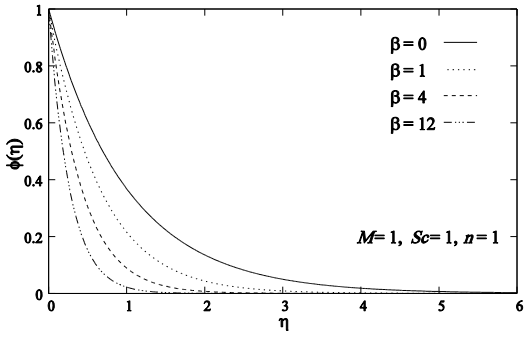

Fig. 4. Concentration profiles $\phi(\eta)$ for various values of β.

Finally, Figs. 5 and 6 exhibit the concentration profiles in the boundary layer flow region for different values of power-law exponent n. It is noticed from Fig. 5 that for the increasing values of n with n>0, the curve representing the distribution of solute for specific value of η decreases.

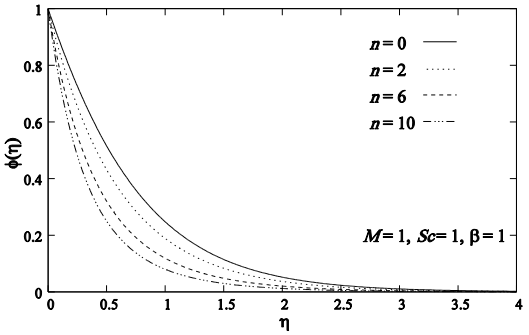

Fig. 5. Concentration profiles $\phi(\eta)$ for various values of $n(\geq 0)$.

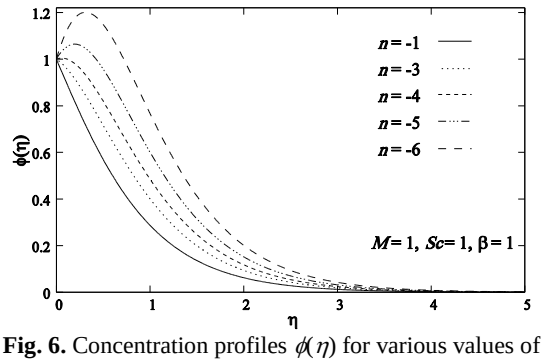

Fig. 6. Concentration profiles $\phi(\eta)$ for various values of $n(<0)$.

While, in Fig. 6 the concentration profile increases with increase in the magnitude of n with $n<0$ and for large negative values of n, the overshoot of solute is observed near the surface. Thus, the effect of increase of n when the surface concentration is $C_w=C_\infty+C_0x^n$ is completely opposite to the effect of increase n when the surface concentration is $C_w=C_\infty+C_0/x^n$. Note that, the wall concentration is constant when $n=0$.

6. CONCLUSIONS

In this investigation, an analysis is made to find the behaviour of the distribution of reactive solute undergo a first order reaction in steady MHD boundary layer flow of an electrically conducting incompressible fluid over a stretching surface taking variable surface concentration. Using the scaling group of transformation a set of self-similar equations is obtained from the governing equations. The analytical solution is found for the momentum equation and the equation of reactive solute is solved numerically. The results show that the magnetic field tends to reduce the rate of flow from the wall and is broadening the solute layer. The Schmidt number and the reaction rate parameter reduce the solute boundary layer thickness. Most, importantly, the effects of initial variable solute distribution over a stretching surface is interesting i.e. for the increase in magnitude of n, the concentration decreases when $n>0$ whereas increases when $n<0$.

ACKNOWLEDGEMENTS

The authors are thankful to the referee for his/her valuable comments and suggestions to improve the paper. One of the authors (K.B.) gratefully acknowledges the financial support of National Board for Higher Mathematics (NBHM), DAE, Mumbai, India for pursuing this work.

REFERENCES

Afify, A. (2004). MHD free convective flow and mass transfer over a stretching sheet with chemical reaction. *Heat and Mass Transfer* 40, 495-500.

Akyildiz, F.T., H. Bellout and K. Vajravelu (2006). Diffusion of chemically reactive species in a porous medium over a stretching sheet. *Journal of Mathematical Analysis and Application* 320, 322-339.

Andersson, H.I., O.R. Hansen and B. Holmedal (1994). Diffusion of a chemically reactive species from a stretching sheet. *International Journal of Heat and Mass Transfer* 37, 659-664.

Anjali Devi, S.P. and B. Ganga (2010). Dissipation effects on MHD nonlinear flow and heat transfer past a porous surface with prescribed heat flux. *Journal of Applied Fluid Mechanics* 3(1), pp. 1-6.

Chakrabarti, A. and A.S. Gupta (1979). Hydromagnetic flow and heat transfer over a stretching sheet. *Quarterly of Applied Mathematics* 37, 73-78.

Chambre, P.L. and J.D. Young (1958). On diffusion of a chemically reactive species in a laminar boundary layer flow. *Physics of Fluids* 1, 48-54.

Cortell, R. (2007). Toward an understanding of the motion and mass transfer with chemically reactive species for two classes of viscoelastic fluid over a porous stretching sheet. *Chemical Engineering and Processing* 46, 982-989.

Crane, L.J. (1970). Flow past a stretching plate. *Zeitschrift für Angewandte Mathematik und Physik* 21, 645-647.

Gupta, P. S. and A.S. Gupta (1977). Heat and Mass Transfer on a stretching sheet with suction and blowing. *The Canadian Journal of Chemical Engineering* 55, 744-746.

Kandasamy, R., M. Ismoen and H.B. Saim (2010). Lie group analysis for the effects of temperature-dependent fluid viscosity and chemical reaction on MHD free convective heat and mass transfer with variable stream conditions. *Nuclear Engineering and Design* 240, 39-46.

Liu, I-C (2005). A note on heat and mass transfer for a hydromagnetic flow over a stretching sheet. *International Communications in Heat and Mass Transfer* 32, 1075-1084.

Mukhopadhyay, S., G.C. Layek and Sk.A. Samad (2005). Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity. *International Journal of Heat and Mass Transfer* 48, 4460-4466.

Pakdemirli, M. and M. Yurusoy (1990). Similarity transformations for partial differential equations. *SIAM Review* 40, 96-101.

Sakiadis, B.C. (1961a). Boundary layer behaviour on continuous solid surface I: The boundary layer equation for two dimensional and asymmetric flow. *AIChE Journal* 7, 26-28.

Sakiadis, B.C. (1961b). Boundary Layer beheviour on continuous solid surface II: The boundary layer on a continuous flat surface. *AIChE Journal* 7, 221-225.

Sarpkaya, T. (1961). Flow of non-Newtonian fluids in a magnetic field. *AIChE Journal* 7, 324-328.

Shercliff, J.A. (1965). *A Textbook of Magnetohydnamics*, Pergamon Press.

Takhar, H.S., A.J. Chamkha, G. Nath (2000). Flow and mass transfer on a stretching sheet with a magnetic field and chemically reactive species. *International Journal of Engineering Science* 38, 1303-1314.