Effect of Non-Uniform Temperature Gradients on the Onset of Convection in a Couple-Stress Fluid-Saturated Porous Medium

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ABSTRACT

The effect of different forms of basic temperature gradients on the criterion for the onset of convection in a layer of an incompressible couple-stress fluid-saturated porous medium is investigated. It is shown that the principle of exchange of stability is valid, and the eigenvalue problem is solved numerically using the Galerkin technique. The parabolic and inverted parabolic basic temperature profiles have the same effect on the onset of convection and similar is the case between piecewise linear temperature profiles heating from below and above. Amongst the various basic temperature profiles, the linear temperature profile is found to be more stabilizing on the onset of instability. In addition, the influence of thermal depth on the criterion for the onset of convection is assessed in the case of piecewise linear temperature profiles. Moreover, an increase in the value of couple-stress parameter is found to delay the onset of convection and to increase the width of convection cells. It is also noted that the critical wave numbers are slightly affected by the nature of basic temperature profiles.

NOMENCLATURE

a	overall horizontal wave number		w'	vertical component of perturbed velocity
A_h	ratio of heat capacities		W	amplitude of perturbed vertical velocity
D	differential operator		***Greek symbols***	
$f(z)$	non-dimensional temperature gradient		α	thermal expansion coefficient
K	permeability of the porous medium		ε_t	thermal depth
ℓ, m	horizontal wave numbers in the x and y directions respectively		Θ	amplitude of the perturbed temperature
			κ	effective thermal diffusivity
M	non-dimensional group		ρ	fluid density
p	pressure		ρ_0	density at reference temperature, $T = T_0$
Pr_D	modified Darcy- Prandtl number		Λ_c	couple-stress parameter
$\vec{q}$	seepage velocity		μ	fluid viscosity
R_D	Darcy- Rayleigh number		μ_c	couple-stress viscosity
t	time		φ	porosity
T	temperature		ω	growth rate

1. INTRODUCTION

Thermal convective instability in a layer of porous medium has merited extensive attention over the years and is now emerged as an important field of study in the general area of fluid dynamics and heat transfer. The growing volume of work pertaining to this field is well documented by Ingham and Pop (1998), Vafai (2000, 2005) and Nield and Bejan (2006). In many of the heat transfer problems, suppress or augment of convection plays a vital role. There are several mechanisms that can be used effectively to either suppress or augment

convectCion namely, by applying a magnetic/electric field externally or by Coriolis force due to rotation or by maintaining non-uniform temperature gradient across the porous layer. A non-uniform temperature gradient can arise in various ways, notably by (i) transient heating or cooling at a boundary, (ii) volumetric distribution of heat sources, (iii) radiative heat transfer, (iv) thermal modulation, and (v) vertical throughflow. A large cross section of the fundamental research carried out on these topics has been covered extensively in Nield and Bejan (2006).

Although several studies have been undertaken in the past to understand convective instability using non-uniform basic temperature gradients, all the investigations have been limited to Newtonian fluid-saturated porous media. Nevertheless, the growing importance of non-Newtonian fluids in modern technology has attracted researchers for the consideration of such fluids in the investigation of convective instability problems because the traditional Newtonian fluids cannot precisely describe the characteristics of the fluid flow encountered in many practical problems such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plates in a bath, exotic lubricants and colloidal fluids to mention a few. These fluids deform and produce a spin field due to the microrotation of suspended particles forming a micropolar fluid. The theory of micropolar fluids was developed by Eringen (1966) which takes care of local effects arising from the microstructure and as well as the intrinsic motions of microfluidics. The spin field due to microrotation of freely suspended particles sets-up an antisymmetric stress, known as couple stress, and thus forming couple stress fluid. Thus couple-stress fluid, according to Eringen (1966), is a particular case of micropolar fluid when microrotation balances with the natural vorticity of fluid. The couple-stress fluid has distinct features, such as polar effects and whose microstructure is mechanically significant. The constitutive equations for couple-stress fluids are given by Stokes (1966). The theory proposed by Stokes is the simplest one for micro-fluids, which allows polar effects such as the presence of couple-stress, body couple, and non-symmetric tensors.

Based on this formulation, convective instability in either a couple-stress fluid layer or in a couple-stress fluid-saturated porous layer heated from below has been investigated in the recent past. Sharma and Thakur (2000) have investigated thermal instability of an electrically conducting couple-stress fluid-saturated porous layer in the presence of a uniform magnetic field. They have reported that the couple-stress delays the onset of stationary convection. A layer of couple-stress fluid saturating a porous medium heated from below in the presence of rotation has been studied by Sharma *et al.* (2000) and condition for the onset of convection is obtained. Sunil *et al.* (2002) have investigated the effect of magnetic field and rotation on a layer of couple-stress fluid heated from below in a porous medium, while Sunil *et al.* (2004) have considered the effect of suspended particles in a couple-stress fluid layer heated and soluted from below in a porous medium. The linear and non-linear double

diffusive convection with Soret effect in couple-stress liquids has been considered by Malashetty *et al.* (2006). Gaikwad *et al.* (2007) have studied linear and non-linear double diffusive convection with Soret and Dufour effects in couple-stress liquids. Recently, Malashetty *et al.* (2009) have studied convective instability in a layer of couple-stress fluid-saturated porous medium heated from below using a thermal non-equilibrium model.

All these previous investigators have considered the effect of only uniform temperature gradient on the onset of convection in a couple-stress fluid-saturated porous medium. Nevertheless, the effect of non-uniform basic temperature gradients concerning this problem has not been given any attention in the literature despite its importance in understanding convective instability encountered in many scientific and technological problems. The novelty of the present work is to understand which basic temperature profile leads to the least critical Darcy-Rayleigh number. The answer is not trivial and it is not answered even in the case of Newtonian fluids for isothermal boundaries and the available results are restricted to only insulating boundaries since this case is amenable for analytical treatment (Shivakumara 2009). The intent of the present study is, therefore, to analyze the influence of different forms of nonuniform basic temperature profiles, namely (i) linear temperature profile, (ii) parabolic temperature profile, (iii) inverted parabolic temperature profile, (iv) piecewise linear temperature profile heating from below and (v) piecewise linear temperature profile cooling from above and make clear their effects on the onset of convection in a couple-stress fluid-saturated isotropic porous medium for isothermal boundaries.

To achieve the above objectives, the paper is organized as follows. The mathematical formulation is given in section 2 and it is shown that the principle of exchange of stabilities is valid irrespective of the form of basic temperature profile. The method of solution to solve the eigenvalue problem is discussed in section 3. The results are discussed in section 4 and conclusions drawn are presented in section 5.
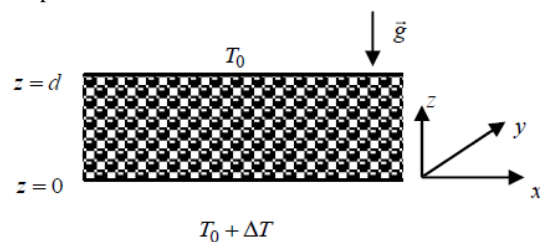
Fig. 1. Physical configuration

2. MATHEMATICAL FORMULATION

We consider an infinite horizontal layer of an incompressible couple-stress fluid-saturated porous medium heated from below, where the vertical distance between the top and bottom boundaries is d and a fixed temperature difference ΔT is maintained across the porous layer(see Fig.1). A Cartesian coordinate system (x, y, z) is used with the origin at the bottom of the porous layer and the z-axis vertically upward in the gravitational field.

Under the Boussinesq approximation, the equation of continuity and the equation of state are, respectively

$$\nabla.\vec{q} = 0 \qquad (1)$$

$$\rho = \rho_0 \left\{ 1 - \alpha (T - T_0) \right\}. \qquad (2)$$

The equation of motion of an incompressible couple-stress fluid saturating a porous medium, following Malashetty *et al.* (2009), is given by

$$\frac{1}{\varphi}\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla p + \frac{\rho}{\rho_0}\vec{g} - \frac{1}{\rho_0 K}\left(\mu - \mu_c \nabla^2\right)\vec{q}$$
(3)

The time derivative term is taken into consideration in Eq. (3) in order to look into the possibility of oscillatory convection. When $\mu_c = 0$, Eq. (3) reduces to the usual modified Darcy equation which describes the flow of a Newtonian fluid saturating a porous medium and note that the presence of couple-stress is to alter the viscosity of the fluid.

The temperature equation is given by

$$A_h \frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \kappa \nabla^2 T \qquad (4)$$

Since the non-uniform basic temperature profile finds its origin in transient heating or cooling at a boundary, the temperature distribution in the basic state usually depends explicitly on position and time. Thus, the quiescent basic state is described by

$$\vec{q}_b = 0 , \; p = p_b(z,t) , \; \frac{\partial T_b}{\partial t} = \kappa \frac{\partial^2 T_b}{\partial z^2} \qquad (5a)$$

where the subscript b denotes the basic state. As propounded by Nield (1975), it is also not our intention to treat here the full problem of temperature profiles depending explicitly on time. Instead, we introduce a simplification in the form of a quasi-static approximation which consists of freezing the basic temperature distribution $T_b(z,t)$ at a given instant of time. This simplification is justified so long as the disturbances are growing faster than the basic profile is evolving (Nield and Bejan 2006). Under the circumstances, the basic state temperature distribution admits a solution of the form

$$-\frac{d}{\Delta T}\frac{dT_b}{dz} = f(z) \qquad (5b)$$

where $f(z)$ is a non-dimensional basic temperature gradient satisfies the condition

$$\int_0^1 f(z)dz = 1 . \qquad (6)$$

To study the stability of the basic state, we superimpose infinitesimally small perturbations $(\vec{q}\text{¢ } p\text{¢}q\text{¢}r\text{¢})$ on the basic state in the form

$$\vec{q} = \vec{q}' , \; p = p_b + p' , \; T = T_b + \theta' , \rho = \rho_b + \rho' . \qquad (7)$$

Following the standard linear stability analysis procedure, non-dimensionalizing the quantities by scaling (x, y, z) by d, t by $d^2\varphi/\kappa$, w' by κ/d, θ' by ΔT, and employing the normal mode analysis procedure in the form

$$(w', \theta') = (W, \Theta)(z)\exp\left[i(\ell x + my) + \omega t\right] \qquad (8)$$

we obtain the stability equations in the form

$$\left[\frac{\omega}{Pr_D} + 1 - \Lambda_c\left(D^2 - a^2\right)\right]\left(D^2 - a^2\right)W = -R_D a^2\Theta \quad (9)$$

$$\left[M\omega - \left(D^2 - a^2\right)\right]\Theta = f(z)W . \qquad (10)$$

In the above equations, $D = d/dz$ is the differential operator, $a = \sqrt{l^2 + m^2}$ is the overall horizontal wave number, $\Lambda_c = \mu_c/\mu d^2$ is the couple-stress parameter, $R_D = \alpha g \Delta T d K/\nu\kappa$ is the Darcy- Rayleigh number, $Pr_D = \nu\varphi^2 d^2/K\kappa$ is the modified Darcy- Prandtl number, and $M = A_h/\varphi$ is a non-dimensional group.

The boundaries of the porous layer are assumed to be impermeable with vanishing couple-stress and perfect conductors of heat. Accordingly, the boundary conditions are

$$W = D^2 W = \Theta = 0 \qquad \text{at } z = 0,1 . \qquad (11)$$

To prove the validity of principle of exchange of stability, the moment approach is used which does not suffer from the ambiguities of satisfying some of the higher boundary conditions (Khalili *et al.* 2002). To derive the moment equation, we multiply Eq. (9) by W^m and Eq. (10) by Θ^m, where m is a non-negative integer, and then integrate from $z = 0$ to 1. Many terms are integrated by parts and simplified using the boundary conditions, to get

$$\left(\frac{\omega}{Pr_D} + 1\right)< DW^m DW + a^2 W^m W > + \Lambda_c < D^2 W^m D^2 W >$$

$$+ < -2a^2 DW^m DW + a^4 W^m W > = R_D a^2 < W^m \Theta > \quad (12)$$

$$\left(M\omega + a^2\right)< \Theta^m \Theta > + < D\Theta^m D\Theta > = < f(z)\Theta^m W > \qquad (13)$$

Here $m=0$ gives the moment equation and $m=1$ gives the energy equation. For $m=0$, Eq. (12) and Eq. (13), respectively reduce to

$$\left[\frac{\omega}{Pr_D} + 1 + \Lambda_c a^2\right]< W > = R_D < \Theta > \qquad (14)$$

$$\left(M\omega + a^2\right)< \Theta > = < f(z)W > . \qquad (15)$$

Eliminating $< \Theta >$ from Eq. (14) by using Eq. (15) and rearranging the terms, we obtain

$$\frac{M}{Pr_D}\omega^2 + \left\{ M\left(1 + \Lambda_c a^2\right) + \frac{a^2}{Pr_D}\right\}\omega + \left(1 + \Lambda_c a^2\right)a^2 \qquad (16)$$

$$- R_D \frac{< f(z)W >}{< W >} = 0$$

Since we are interested in the occurrence of oscillatory convection, we set $\omega = i\omega_i$, where ω_i is real, in Eq. (16) and obtain

$$\frac{M}{Pr_D}\omega_i^2 - i\left\{M\left(1+\Lambda_c a^2\right)+\frac{a^2}{Pr_D}\right\}\omega_i - \left(1+\Lambda_c a^2\right)a^2 \tag{17}$$
$$+ R_D\frac{< f(z)W >}{< W >} = 0$$

Equating the real and imaginary parts of Eq. (17), we, respectively, note that

$$\frac{M}{Pr_D}\omega_i^2 - \left(1+\Lambda_c a^2\right)a^2 + R_D\frac{< f(z)W >}{< W >} = 0 \tag{18}$$

and

$$\left\{M\left(1+\Lambda_c a^2\right)+\frac{a^2}{Pr_D}\right\}\omega_i = 0 . \tag{19}$$

Equation (19) implies that $\omega_i = 0$, and hence, the principle of exchange of stability is valid. Therefore, we confine the analysis to stationary convection and take $\omega = 0$ in Eqs. (9) and (10) to arrive at the following system of ordinary differential equations:

$$\left[\Lambda_c\left(D^2-a^2\right)-1\right]\left(D^2-a^2\right)W = R_D a^2\Theta \tag{20}$$

$$\left(D^2-a^2\right)\Theta = -f(z)W . \tag{21}$$

The following five different forms of $f(z)$ are considered to investigate their effect on the criterion for the onset of convection:

(i)   Linear temperature profile

$$f(z) = 1 . \tag{22}$$

(ii)   Piecewise linear temperature profile heating from below

$$f(z) = \begin{cases} \dfrac{1}{\varepsilon_t}, & 0 \le z < \varepsilon_t \\ 0, & \varepsilon_t < z \le 1 \end{cases} . \tag{23}$$

(iii)   Piecewise linear temperature profile cooling from above

$$f(z) = \begin{cases} 0, & 0 \le z < 1-\varepsilon_t \\ \dfrac{1}{\varepsilon_t}, & 1-\varepsilon_t < z \le 1 \end{cases} \tag{24}$$

(iv)   Inverted parabolic temperature profile

$$f(z) = 2(1-z) . \tag{25}$$

(v)   Parabolic temperature profile

$$f(z) = 2z . \tag{26}$$

3.   METHOD OF SOLUTION

Equations (20) and (21) together with boundary conditions (11) constitute an eigenvalue problem with R_D as the eigenvalue. The resulting eigenvalue problem is solved using the Galerkin technique.

Accordingly, the unknown variables are written in a series of basis functions as

$$W = \sum_{i=1}^{N} A_i\, W_i \ , \ \ \Theta = \sum_{i=1}^{N} B_i\, \Theta_i \tag{27}$$

where A_i and B_i are constants and the basis functions W_i and Θ_i will be represented by the power series satisfying the respective boundary conditions. Substituting Eq. (27) into Eqs. (20) and (21), multiplying the resulting momentum equation by $W_j(z)$, energy equation by $\Theta_j(z)$, performing the integration by parts with respect to z between z = 0 and z = 1 and using the boundary conditions (11), we obtain the following system of linear homogeneous algebraic equations:

$$C_{ji}\, A_i + D_{ji}\ B_i = 0$$
$$\tag{28}$$
$$E_{ji}\, A_i + F_{ji}\ B_i = 0.$$

The coefficients C_{ji} to F_{ji} involve inner products of the base functions and are given by

$$C_{ji} = \Lambda_c < D^2W_j D^2W_i + a^4W_jW_i + 2a^2DW_jDW_i > + < DW_jDW_i + a^2W_jW_i >$$

$$D_{ji} = -R_D a^2 < W_j\Theta_i >$$

$$E_{ji} = < f(z)\Theta_j W_i >$$

$$F_{ji} = - < D\Theta_j D\Theta_i + a^2\Theta_j\Theta_i >$$

$$\tag{29}$$

where the inner product is defined as $< \cdots > = \displaystyle\int_0^1 (\cdots)dz$.

The above set of homogeneous algebraic equations given by Eq. (29) will have a non-trivial solution if and only if

$$\begin{vmatrix} C_{ji} & D_{ji} \\ E_{ji} & F_{ji} \end{vmatrix} = 0 . \tag{30}$$

We select trial functions satisfying the appropriate boundary conditions. The following polynomial trial functions are chosen:

$$W_i = (z^{i+1} - 2z^{i+2} + z^{i+3})T_{i-1}^*$$
$$\tag{31}$$
$$\Theta_i = (z^i - z^{i+1})T_{i-1}^*$$

where T_i^*'s are the modified Chebyshev polynomials. The inner products involved in Eq. (30) are evaluated analytically rather than numerically to avoid errors in the numerical integration. For a fixed value of Λ_c, ε_t and for chosen basic temperature profile $f(z)$, Eq. (30) gives a relation between R_D and a which enables us to plot a locus in the (R_D, a) - plane. The Darcy-Rayleigh numbers for basic temperature profiles (i) to (v) are respectively denoted through R_{D1} to R_{D5}. The minimum point of R_D as a function of a as well as with respect to ε_t (in the case of piecewise linear

temperature profiles), gives the critical Darcy-Rayleigh number R_{Dic} $(i = 1$ to $5)$ and the corresponding critical wave number a_{ic}, as well as critical thermal depth depending on the choice of basic temperature profile.

4. RESULTS AND DISCUSSION

The effect of different forms of basic temperature gradients on the criterion for the onset of convection in a layer of couple-stress fluid-saturated porous medium is investigated. The resulting eigenvalue problem is solved numerically using the Galerkin technique. The convergence of the results is achieved, in general, by considering six terms in the series expansion Eq. (27) and the results obtained for this order (i.e., $N = 6$) are presented graphically in Figs. 2 to 5. However, for a linear temperature profile (i.e., $f(z) = 1$) the resulting eigenvalue problem can be solved exactly with the eigen functions $W(z) = A_1 \sin \pi z$ and $\Theta(z) = A_2 \sin \pi z$, where A_1 and A_2 are constants. In this case, an analytic expression for the Rayleigh number, denoted by R_{D1}, is obtained in the form

$$R_{D1} = \frac{\left(\pi^2 + a^2\right)^2 \left\{\Lambda_c\left(\pi^2 + a^2\right) + 1\right\}}{a^2}. \tag{32}$$

It is noted that R_{D1} attains its critical value, R_{D1c} at $a^2 = a_{1c}^2$, where

$$a_{1c}^2 = \frac{-\left(\pi^2 \Lambda_c + 1\right) + \sqrt{\left(\pi^2 \Lambda_c + 1\right)\left(9\pi^2 \Lambda_c + 1\right)}}{4\Lambda_c}. \tag{33}$$

When $\Lambda_c = 0$, the critical Rayleigh number and the critical wave number are respectively found to be $R_{D1c} = 4\pi^2$ and $a_{1c} = \pi$ which are the known exact values. From the above equations, it is also evident that an increase in the value of couple-stress parameter Λ_c is to increase the Rayleigh number and to decrease the critical wave number. In fact, $a_{1c} \rightarrow \pi/\sqrt{2}$ as $\Lambda_c \rightarrow \infty$.

Figures 2a and 2b depict the neutral stability curves in the (R_{Di}, a)- plane for different temperature profiles for $\Lambda_c = 0$ and 1, respectively with $\varepsilon_t = 0.5$. The coordinates of the minimum point on these curves correspond to the critical values R_{Dic} and a_{ic} ($i = 1$ to 5). From these figures, it can be seen that the critical wave number is slightly affected by the nature of the basic temperature profiles. The numerical calculations of Darcy-Rayleigh numbers for different values of wave number revealed that $R_{D2} = R_{D3}$ as well as $R_{D4} = R_{D5}$. That is, parabolic and inverted parabolic temperature profiles have the same effect on the onset of convection and similar is the case between piecewise linear temperature profiles heating from below and cooling from above. This may be due to the symmetric boundary conditions considered. Furthermore, increasing Λ_c increases the critical Rayleigh number

and thus the presence of couple-stresses is to delay the onset of convection.
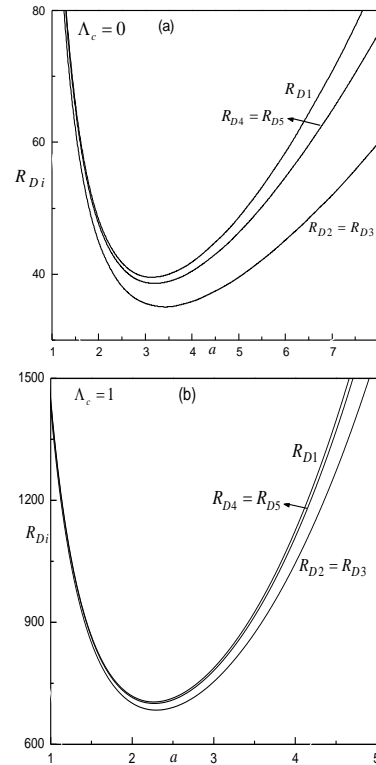

Fig. 2. Neutral stability curves for different temperature profiles

The critical Darcy-Rayleigh numbers obtained with respect to the wave number for three values of Λ_c (= 0, 0.1 and 0.2) are represented graphically in Fig. 3 as a function of thermal depth ε_t for piecewise linear temperature profiles. From the figure it is seen that the critical Darcy-Rayleigh numbers decrease at first to some minimum value and then increase steadily with further increase in ε_t. The critical value of ε_t is found to be 0.71 and it is independent of Λ_c values considered. Also, increasing Λ_c increases the critical Darcy-Rayleigh number and hence its effect is to reinforce stability of the system.
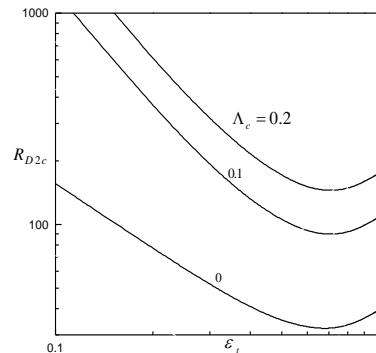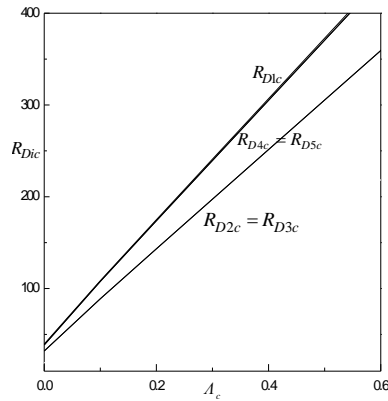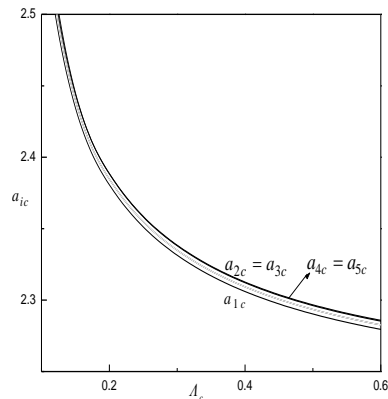

Fig. 3. Variation of R_{D2c} with ε_t for different values of Λ_c

Fig. 4. Variation of R_{Dic} ($i = 1$ to 5) as a function of Λ_c

Fig. 5. Variation of a_{ic} ($i = 1$ to 5) as a function of Λ_c

Figures 4 and 5, respectively show the variation of critical Darcy-Rayleigh numbers (R_{Dic}, i=1to 5) and the corresponding critical wave numbers (a_{ic}, i=1 to 5) as a function of Λ_c. The critical Darcy-Rayleigh numbers presented here are obtained with respect to the wave number as well as ε_t (in the case of piecewise linear temperature profiles). From Figure 4, it is observed that the critical Darcy-Rayleigh numbers increase monotonically with Λ_c, and it is further noted that

$$R_{D2c} = R_{D3c} < R_{D4c} = R_{D5c} < R_{D1c}.$$

This indicates that, as compared to non-uniform temperature profiles, the linear temperature profile has a more stabilizing effect on the system. In particular, it may be noted that the system is more unstable for piecewise linear temperature profiles as compared to parabolic/inverted parabolic basic temperature profiles. However, the difference in the critical Darcy-Rayleigh numbers for different basic temperature profiles is seen to be marginal.

The critical wave numbers for different temperature profiles decrease monotonically with an increase in Λ_c as may be seen from the results plotted in Fig. 5. Thus an increase in Λ_c is seen to increase the size of convection cells. Further inspection of the figure reveals that the deviation in the critical wave numbers among different temperature profiles is not so significant and in fact $a_{2c}(=a_{3c})$ values are almost same as $a_{4c}(=a_{5c})$.

5. CONCLUSIONS

From the foregoing discussions it can be concluded that (i) the effect of increasing the couple-stress parameter Λ_c is to stabilize the onset of instability and also (ii) the difference between critical values at each Λ_c for the various chosen basic temperature profiles is minimal. In this sense, there is very little influence that one can assert on the instability of the system by the choice of various basic temperature profiles different from linear. Therefore, one can simply use the analytic solution for the linear temperature profile to assess the strong effect of couple-stress parameter Λ_c on the onset of convection in a couple-stress fluid saturated porous medium. In addition, the parabolic or inverted parabolic temperature profiles and also piecewise linear temperature profiles heating from below or cooling from above have the same effect on the onset of convection in a couple-stress fluid-saturated porous medium. This may be attributed to symmetric boundary conditions. The critical thermal depth ε_{tc} is found to be same for piecewise linear temperature profiles and remains invariant with change in Λ_c values. The critical wave numbers are slightly affected by the nature of basic temperature profiles. Moreover, an increase in the value of Λ_c is to enlarge the size of convection cells.

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