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Effects of Hall Current and Rotation on Unsteady MHD Couette Flow in the Presence of an Inclined Magnetic Field

G.S. Seth[1†], R. Nandkeolyar[2] and Md.S. Ansari[3]

[1] *Department of Applied Mathematics, Indian School of Mines, Dhanbad-826004, India*
[2] *School of Applied Sciences, KIIT University, Bhubaneswar-751024, India*
[3] *School of Petroleum Technology, Pandit Deendayal Petroleum University, Gandhinagar-382007, India*

†*Corresponding Author Email: gsseth_ism@yahoo.com*

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ABSTRACT

Unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system in the presence of an inclined magnetic field taking Hall current into account is studied. Fluid flow within the channel is induced due to impulsive movement of the lower plate of the channel. Exact solution of the governing equations is obtained by Laplace transform technique. The expression for the shear stress at the moving plate is also derived. Asymptotic behavior of the solution is analyzed for small and large values of time t to highlight (i) the transient approach to the final steady state flow and (ii) the effects of Hall current, magnetic field, rotation and angle of inclination of magnetic field on the flow-field. It is found that Hall current and rotation tend to accelerate fluid velocity in both the primary and secondary flow directions. Magnetic field has retarding influence on the fluid velocity in both the primary and secondary flow directions. Angle of inclination of magnetic field has accelerating influence on the fluid velocity in both the primary and secondary flow directions.

Keywords: MHD Couette flow, Hall current, Inclined magnetic field, Modified Ekman-Hartmann boundary layer, Rayleigh boundary layer, Spatial and inertial oscillations.

1. INTRODUCTION

The study of unsteady MHD Couette flow is of considerable importance from practical point of view because fluid transient may be expected at the start-up time of MHD devices viz. MHD generators, MHD pumps and accelerators, flow meters and nuclear reactors using liquid metal coolants. Taking into account this fact Katagiri (1962) investigated unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in the presence of a transverse magnetic field fixed relative to the fluid when the fluid flow within the channel is induced due to impulsive motion of one of the plates. In recent years, interest in the study of magnetohydrodynamic flow of rotating fluids is motivated by several important problems like maintenance and secular variation of earth's magnetic field, the internal rotation rate of sun, the structure of rotating magnetic stars, the planetary and solar dynamo problems, rotating hydromagnetic generators, vortex type MHD power generators and other centrifugal machines. An order of magnitude analysis shows that, in the basic field equations, the Coriolis force is predominant over inertial and viscous forces. Furthermore, it may be noted that the Coriolis and magnetohydrodynamic forces are comparable in magnitude. Taking into account these facts, unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system is investigated by Seth *et al.* (1982, 2010), Chandran *et al.* (1993), Hayat *et al.* (2004 a, 2004 b) and Das *et al.* (2009) to analyze the various aspects of the problem. In all these investigations magnetic field is applied parallel to the axis of rotation. However, in actual situations of interest, it may not be possible to have magnetic field always acting parallel to the axis of rotation. In many applications including astrophysics, MHD power generation and magnetic material processing flow control magnetic fields may act obliquely to the flow. Keeping in view this fact, Seth and Ghosh (1986) initiated the study of oscillatory Hartmann flow in a rotating channel in the presence of an inclined magnetic field whereas Guria *et al.* (2009) considered oscillatory MHD Couette flow in a rotating system in the presence of an inclined magnetic field when the upper plate is kept fixed and the lower plate oscillates non-torsionally. Unsteady hydromagnetic Hartmann or Couette flow in a rotating system in the presence of an inclined magnetic field considering different aspects of the problem are considered by Ghosh (1991, 1996, 2001). Steady MHD Couette flow in a rotating system in the presence of an inclined magnetic field taking induced magnetic field into account is investigated by Seth *et al.* (2009 a). In all these investigations the effects of Hall current is not taken into account. It is well known that, in an ionized fluid where the density is low and/or the magnetic field is strong, the effects of Hall current become significant. Taking into account this fact, Jana and Datta (1980), Mandal *et al.* (1982), Ghosh and Pop (2004), Hayat *et al.* (2004 c) and Seth *et al.* (2009 b) studied the effects of Hall current on MHD Couette flow of a viscous incompressible electrically conducting fluid in a rotating system considering different aspects of the problem.

The present paper deals with the study of the effects of Hall current on unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system in the presence of a uniform magnetic field applied in a direction which makes an angle θ with the positive direction of axis of rotation. The fluid flow within the channel is induced due to the impulsive movement of the lower plate of the channel. The present model has applications in astrophysical fluid dynamics where this flow regime is sometimes referred to as magnetic mirror regime and in hybrid MHD energy generator exploiting variable orientation magnetic field. In fact, the problem is formulated for the general case of moving plate with velocity $F(t)$ (t being time variable). Exact solution of the governing equations is obtained when $F(t)=\Delta H(t)$ (Δ and $H(t)$ are, respectively, a constant and Heaviside unit step function) by Laplace transform technique. The expression for the shear stress at the moving plate is also derived. The solution, in dimensionless form, contains three pertinent flow parameters viz. M^2 (square of Hartmann number), K^2 (rotation parameter which is reciprocal of Ekman number) and m (Hall current parameter) and one geometric parameter, namely, θ (angle of inclination of magnetic field). Asymptotic behavior of the solution is analyzed for both small and large values of time t to highlight (i) the transient approach to the final steady state flow and (ii) the effects of Hall current, magnetic field, rotation and angle of inclination of magnetic field on the flow-field. It is found that, for small values of time t, the primary flow is independent of rotation whereas secondary flow has considerable effects of Hall current, magnetic field and rotation. The fluid flow in both the directions has significant effects of angle of inclination of magnetic field. However, in the absence of Hall current, the secondary flow is unaffected by magnetic field and angle of inclination of magnetic field whereas primary flow has considerable effects of angle of inclination of magnetic field. For large values of time t, the fluid flow is in quasi-steady state. The steady state flow is confined within a modified Ekman-Hartmann boundary layer of thickness of $O(\alpha_1^{-1})$ which becomes thinner with the increase in either M^2 or K^2 or both and has considerable effects of Hall current and angle of inclination of magnetic field. Also steady state flow exhibits spatial oscillations in the flow-field affected by Hall current, magnetic field, rotation and angle of inclination of magnetic field. The unsteady state flow presents inertial oscillations in the flow-field excited by Hall current and rotation which have considerable effects of magnetic field and angle of inclination of magnetic field due to the presence of Hall current. The inertial oscillations in the flow-field damp out effectively in dimensionless time of $O(\alpha_2^{-1})$ when the final steady state is developed. In the absence of Hall current, the inertial oscillations in the flow-field is generated by rotation and it damp out effectively in dimensionless time of $O\left((M^2\cos^2\theta)^{-1}\right)$ when the final steady state is developed. The time of decay of inertial oscillations in this case is less than that when Hall current and rotation both are present. It is noticed that, even in the absence of rotation, the inertial oscillations in the flow-field exist which is excited by Hall current. This is due to the established fact that either Hall current or rotation or both induce secondary flow. In the absence of Hall current ($m=0$) and rotation ($K^2=0$) there exist no inertial oscillations in the flow-field. To study the effects of Hall current, rotation, magnetic field and angle of inclination of magnetic field on the flow-field the numerical values of fluid velocity are depicted graphically versus channel width variable η and the numerical values of the shear stress at the moving plate due to the primary and secondary flows are presented in figures, for various values of m, K^2, M^2 and θ taking $\Delta=1$.

2. Formulation of the Problem and its Solution

Consider unsteady flow of a viscous incompressible electrically conducting fluid between two parallel plates $y=0$ and $y=h$ of infinite length in x and z directions. The fluid and channel rotate in unison with uniform angular velocity Ω about y axis. The fluid is permeated by a uniform magnetic field B_0 applied in a direction which makes an angle θ with the positive direction of $y-$axis in $xy-$plane. At time $t'\le 0$, both the fluid and plates are assumed to be at rest. At time $t'>0$ the lower plate ($y=0$), which coincides with the $xz-$plane, starts moving with time dependent velocity $U(t')$ in $x-$direction while the upper plate ($y=h$) is kept fixed. It is assumed that no applied or polarization voltages exist (i.e., $\vec{E}=0$, $\vec{E}$ being electric field). This corresponds to the case where no energy is being added or extracted from the fluid by electrical means. Since magnetic Reynolds number is very small for liquid metals and partially ionized fluids so the induced magnetic field can be neglected in comparison to the applied one. Therefore, the fluid velocity $\vec{q}$ and magnetic field $\vec{B}$ are given by

$$\vec{q}\equiv\left(u',0,w'\right),\ \vec{B}\equiv\left(B_0\sin\theta,B_0\cos\theta,0\right) \tag{1}$$

which are compatible with the fundamental equations of Magnetohydrodynamics in a rotating frame of eference.

Under the above assumptions the governing equations for the flow of a viscous incompressible electrically conducting fluid, taking Hall current into account, in a rotating frame of reference (Seth *et al*. 2009 b) are

$$\frac{\partial u'}{\partial t'}+2\Omega w'=\upsilon\frac{\partial^2 u'}{\partial y^2}-\frac{\sigma B_0^2\cos^2\theta}{\rho\left(1+m^2\cos^2\theta\right)}\left(u'+mw'\cos\theta\right), \tag{2}$$

$$0 = -\frac{1}{\rho}\frac{\partial p'}{\partial y} + \frac{\sigma B_0^2 \sin 2\theta}{2\rho\left(1+m^2\cos^2\theta\right)}\left(u' + mw'\cos\theta\right), \quad (3)$$

$$\frac{\partial w'}{\partial t'} - 2\Omega u' = \upsilon\frac{\partial^2 w'}{\partial y^2} - \frac{\sigma B_0^2 \cos^2\theta}{\rho\left(1+m^2\cos^2\theta\right)}\left(w' - mu'\cos\theta\right), \quad (4)$$

where υ, σ, ρ, p' and $m = \omega_e \tau_e$ (ω_e and τ_e being cyclotron frequency and electron collision time) are, respectively, kinematic coefficient of viscosity, fluid electrical conductivity, fluid density, fluid pressure including centrifugal force and Hall current parameter. Since the fluid motion is induced by the movement of the lower plate $y = 0$ in x direction so the pressure gradient term is not taken into account in Eq. (2). The absence of pressure gradient term in Eq. (4) implies that there is a net cross flow in z direction.

The initial and boundary conditions for the problem are

$$u' = w' = 0\ ;\ 0 \le y \le h \text{ at } t' \le 0, \quad (5)$$

$$\left.\begin{aligned} &u' = U(t');\ w' = 0 \text{ at } y = 0;\ t' > 0,\\ &u' = w' = 0 \text{ at } y = h;\ t' > 0.\end{aligned}\right\}. \quad (6)$$

Introducing non-dimensional variables $\eta = y/h$, $(u,w) = (u',w')h/\upsilon$, $t = t'\upsilon/h^2$, $p = p'h^2/\rho\upsilon^2$, in Eqs. (2) to (4), we obtain

$$\frac{\partial u}{\partial t} + 2K^2 w = \frac{\partial^2 u}{\partial \eta^2} - \frac{M^2\cos^2\theta}{\left(1+m^2\cos^2\theta\right)}\left(u + mw\cos\theta\right), \quad (7)$$

$$0 = -\frac{\partial p}{\partial \eta} + \frac{M^2 \sin 2\theta}{2\left(1+m^2\cos^2\theta\right)}\left(u + mw\cos\theta\right), \quad (8)$$

$$\frac{\partial w}{\partial t} - 2K^2 u = \frac{\partial^2 w}{\partial \eta^2} - \frac{M^2\cos^2\theta}{\left(1+m^2\cos^2\theta\right)}\left(w - mu\cos\theta\right), \quad (9)$$

where $K^2 = \Omega h^2/\upsilon$ is rotation parameter which is reciprocal of Ekman number and $M^2 = \sigma B_0^2 h^2/\rho\upsilon$ is magnetic parameter which is square of Hartmann number.

The initial and boundary conditions (5) and (6), in dimensionless form, become

$$u = w = 0\ ;\ 0 \le \eta \le 1 \text{ at } t \le 0, \quad (10)$$

$$\left.\begin{aligned} &u = F(t);\ w = 0 \text{ at } \eta = 0;\ t > 0,\\ &u = w = 0 \text{ at } \eta = 1;\ t > 0.\end{aligned}\right\}, \quad (11)$$

where $F(t) = U(t')h/\upsilon$.

Combining Eqs. (7) and (9), we obtain

$$\frac{\partial f}{\partial t} - 2iK^2 f = \frac{\partial^2 f}{\partial \eta^2} - \frac{M^2\cos^2\theta\left(1 - im\cos\theta\right)}{\left(1+m^2\cos^2\theta\right)} f, \quad (12)$$

where $f(\eta,t) = u(\eta,t) + iw(\eta,t)$ and $i = \sqrt{-1}$.

The initial and boundary conditions, in combined form, are

$$f = 0\ ;\ 0 \le \eta \le 1 \text{ at } t \le 0, \quad (13)$$

$$\left.\begin{aligned} &f = F(t) \text{ at } \eta = 0\ ;\ t > 0,\\ &f = 0 \text{ at } \eta = 1\ ;\ t > 0.\end{aligned}\right\}, \quad (14)$$

Taking Laplace transform of Eqs. (12) and (14) and using (13), we obtain

$$\frac{d^2\tilde{f}}{d\eta^2} - (g+s)\tilde{f} = 0, \quad (15)$$

$$\left.\begin{aligned} &\tilde{f} = \tilde{F}(s) \text{ at } \eta = 0,\\ &\tilde{f} = 0 \text{ at } \eta = 1,\end{aligned}\right\}. \quad (16)$$

where $\tilde{f}(\eta,s) = \int_0^\infty e^{-st} f(\eta,t)\,dt$ and $\tilde{F}(s) = \int_0^\infty e^{-st} F(t)\,dt$ are Laplace transforms of $f(\eta,t)$ and $F(t)$ respectively, $g = \left\{\frac{M^2\cos^2\theta\left(1 - im\cos\theta\right)}{\left(1+m^2\cos^2\theta\right)} - 2iK^2\right\}$ and $s > 0$ is the Laplace transform parameter.

The solution of Eq. (15) subject to the boundary conditions (16) may be expressed in the form

$$\tilde{f}(\eta,s) = \tilde{F}(s)\sum_{r=0}^{\infty}\left(e^{-a\sqrt{g+s}} - e^{-b\sqrt{g+s}}\right), \quad (17)$$

where $a = 2r + \eta$ and $b = 2r + 2 - \eta$.

Taking inverse Laplace transform of (17), we obtain the solution of the problem after simplification, as

$$f(\eta,t) = \sum_{r=0}^{\infty}\left[\frac{a}{2\sqrt{\pi}}\int_0^t F(t-\lambda)\lambda^{-3/2}e^{-\left(g\lambda + a^2/4\lambda\right)}d\lambda - \frac{b}{2\sqrt{\pi}}\int_0^t F(t-\lambda)\lambda^{-3/2}e^{-\left(g\lambda + b^2/4\lambda\right)}d\lambda\right]. \quad (18)$$

We shall now discuss a particular case of interest of the solution (18) considering impulsive movement of the lower plate $\eta = 0$.

Setting $F(t) = \Delta H(t)$, where Δ is a constant and $H(t)$ is unit step function defined by

$$H(t) = \begin{cases} 0 \text{ for } t \le 0;\\ 1 \text{ for } t > 0,\end{cases} \quad (19)$$

we obtain the solution, in this case, from the general solution (18) as

$$\begin{aligned} f(\eta,t) &= u(\eta,t) + iw(\eta,t)\\ &= \frac{1}{2}\Delta H(t)\sum_{r=0}^{\infty}\left[e^{a\sqrt{g}}erfc\left(\frac{a}{2\sqrt{t}} + \sqrt{gt}\right) + e^{-a\sqrt{g}} \times\right.\\ &\quad \times erfc\left(\frac{a}{2\sqrt{t}} - \sqrt{gt}\right) - e^{b\sqrt{g}}erfc\left(\frac{b}{2\sqrt{t}} + \sqrt{gt}\right) -\\ &\quad \left. e^{-b\sqrt{g}}erfc\left(\frac{b}{2\sqrt{t}} - \sqrt{gt}\right)\right], \end{aligned} \quad (20)$$

where

$$\left.\begin{aligned}&\sqrt{g}=\alpha_1-i\beta_1,\ \alpha_1,\beta_1=\frac{1}{\sqrt{2}}\left[\left(\alpha_2^2+\beta_2^2\right)^{\frac{1}{2}}\pm\alpha_2\right]^{\frac{1}{2}},\\&\alpha_2=\frac{M^2\cos^2\theta}{\left(1+m^2\cos^2\theta\right)},\ \beta_2=\left[\frac{mM^2\cos^3\theta}{\left(1+m^2\cos^2\theta\right)}+2K^2\right].\end{aligned}\right\}\tag{21}$$

The solution (20) exhibits a unified representation of the initial MHD Couette flow induced due to impulsive movement of the lower plate, the final modified steady Ekman-Hartmann boundary layer affected by Hall current and angle of inclination of magnetic field, and decaying oscillations excited by the interaction of magnetic field, Coriolis force, Hall current, angle of inclination of magnetic field and initial impulsive motion. In the absence of Hall current ($m=0$) and angle of inclination of magnetic field ($\theta=0$), the solution (20) is in agreement with the solution obtained by Seth *et al.* (1982). On the other hand, in the absence of Hall current ($m=0$), rotation ($K^2=0$) and angle of inclination of magnetic field ($\theta=0$) it agrees with the solution obtained by Katagiri (1962).

In order to gain further insight into the flow pattern, we shall examine the solution (20) for small and large values of time t. When t is small i.e. $t<<1$, we obtain from solution (20) as

$$\begin{aligned}u(\eta,t)=\Delta\sum_{r=0}^{\infty}\Bigg[&erfc\left(\frac{a}{2\sqrt{t}}\right)-erfc\left(\frac{b}{2\sqrt{t}}\right)+\frac{\alpha_2}{2}\times\\&\times\left\{a^2erfc\left(\frac{a}{2\sqrt{t}}\right)-b^2erfc\left(\frac{b}{2\sqrt{t}}\right)-2\left(\frac{t}{\pi}\right)^{\frac{1}{2}}\times\right.\\&\left.\times\left(ae^{-a^2/4t}-be^{-b^2/4t}\right)\right\}\Bigg],\end{aligned}\tag{22}$$

$$\begin{aligned}w(\eta,t)=\frac{\Delta}{2}\beta_2\sum_{r=0}^{\infty}\Bigg[&2\left(\frac{t}{\pi}\right)^{\frac{1}{2}}\left(ae^{-a^2/4t}-be^{-b^2/4t}\right)-\\&-a^2erfc\left(\frac{a}{2\sqrt{t}}\right)+b^2erfc\left(\frac{b}{2\sqrt{t}}\right)\Bigg],\end{aligned}\tag{23}$$

It is evident from the Eqs. (22) and (23) that the initial impulsive movement of the lower plate ($\eta=0$) develops Rayleigh layer near the plates unaffected by Hall current, magnetic field, rotation and angle of inclination of magnetic field. It is noticed from (22) and (23) that the primary velocity $u(\eta,t)$ upto $O(\sqrt{t})$ is independent of rotation while secondary velocity $w(\eta,t)$ has considerable effects of Hall current, magnetic field and rotation. This is due to the fact that the Hall current as well as rotation induces secondary flow. The fluid flow in both the directions has significant effects of angle of inclination of magnetic field. Upto this stage there are no inertial oscillations in the flow field.

In the absence of Hall current ($m=0$), the Eqs. (22) and (23) reduce to

$$\begin{aligned}u(\eta,t)=\Delta\sum_{r=0}^{\infty}\Bigg[&erfc\left(\frac{a}{2\sqrt{t}}\right)-erfc\left(\frac{b}{2\sqrt{t}}\right)+\frac{M^2\cos^2\theta}{2}\times\\&\times\left\{a^2erfc\left(\frac{a}{2\sqrt{t}}\right)-b^2erfc\left(\frac{b}{2\sqrt{t}}\right)-\right.\\&\left.-2\left(\frac{t}{\pi}\right)^{\frac{1}{2}}\left(ae^{-a^2/4t}-be^{-b^2/4t}\right)\right\}\Bigg],\end{aligned}\tag{24}$$

$$\begin{aligned}w(\eta,t)=\Delta K^2\sum_{r=0}^{\infty}\Bigg[&2\left(\frac{t}{\pi}\right)^{\frac{1}{2}}\left(ae^{-a^2/4t}-be^{-b^2/4t}\right)-\\&-a^2erfc\left(\frac{a}{2\sqrt{t}}\right)+b^2erfc\left(\frac{b}{2\sqrt{t}}\right)\Bigg].\end{aligned}\tag{25}$$

Equations (24) and (25) reveal that, in the absence of Hall current, the secondary velocity $w(\eta,t)$ is unaffected by magnetic field and angle of inclination of magnetic field whereas primary velocity $u(\eta,t)$ has considerable effects of angle of inclination of magnetic field.

When t is large i.e. $t>>1$, using the asymptotic behavior of the complimentary error function, we obtain from (20) as

$$\begin{aligned}u(\eta,t)=\frac{\Delta}{2}\sum_{r=0}^{\infty}\Bigg[&2\left(e^{-a\alpha_1}\cos a\beta_1-e^{-b\alpha_1}\cos b\beta_1\right)-\\&-\frac{e^{-\alpha_2 t}}{\sqrt{\pi t}}\left\{\frac{ae^{-a^2/4t}}{\xi}\left(\left(\alpha_2 t-\frac{a^2}{4t}\right)\cos\beta_2 t-\beta_2 t\sin\beta_2 t\right)\right\}+\\&+\frac{e^{-\alpha_2 t}}{\sqrt{\pi t}}\left\{\frac{be^{-b^2/4t}}{\xi'}\left(\left(\alpha_2 t-\frac{b^2}{4t}\right)\cos\beta_2 t-\beta_2 t\sin\beta_2 t\right)\right\}\Bigg],\end{aligned}\tag{26}$$

$$\begin{aligned}w(\eta,t)=\frac{\Delta}{2}\sum_{r=0}^{\infty}\Bigg[&2\left(e^{-a\alpha_1}\sin a\beta_1-e^{-b\alpha_1}\sin b\beta_1\right)-\\&-\frac{e^{-\alpha_2 t}}{\sqrt{\pi t}}\left\{\frac{ae^{-a^2/4t}}{\xi}\left(\left(\alpha_2 t-\frac{a^2}{4t}\right)\sin\beta_2 t+\beta_2 t\cos\beta_2 t\right)\right\}-\\&-\frac{e^{-\alpha_2 t}}{\sqrt{\pi t}}\left\{\frac{be^{-b^2/4t}}{\xi'}\left(\left(\alpha_2 t-\frac{b^2}{4t}\right)\sin\beta_2 t+\beta_2 t\cos\beta_2 t\right)\right\}\Bigg],\end{aligned}\tag{27}$$

where

$$\left.\begin{aligned}&\xi=\left(\alpha_2 t-\frac{a^2}{4t}\right)^2+\beta_2^2t^2,\\&\xi'=\left(\alpha_2 t-\frac{b^2}{4t}\right)^2+\beta_2^2t^2.\end{aligned}\right\}\tag{28}$$

The expressions (26) and (27) reveal that, for large values of time t, the fluid flow is in quasi-steady state. The first term in the expressions (26) and (27) represents the final steady state flow. The steady state flow is confined within a modified Ekman-Hartmann boundary layer of thickness of $O\left(\alpha_1^{-1}\right)$. The modified

Ekman-Hartmann boundary layer may be viewed as classical Ekman-Hartmann boundary layer modified by Hall current and angle of inclination of magnetic field. It may be noted from Eq. (21) that α_1 increases with the increase in either magnetic parameter M^2 or rotation parameter K^2 or both. Thus we conclude that the thickness of the boundary layer decreases with the increase in either M^2 or K^2 or both. It is also observed from (26) and (27) that the steady state flow exhibits spatial oscillations in the flow field. The unsteady part in (26) and (27) presents inertial oscillations in the flow-field excited by Hall current and rotation which are affected by magnetic field and angle of inclination of magnetic field due to the presence of Hall current. The inertial oscillations in the flow-field damp out effectively in a dimensionless time of $O(\alpha_2^{-1})$ when the final steady state flow is developed. In the absence of Hall current ($m=0$) the inertial oscillations in the flow-field is excited by rotation only and it damp out effectively in dimensionless time of $O\left((M^2\cos^2\theta)^{-1}\right)$ when the final steady state is developed. It is interesting to note from (26) and (27) that, even in the absence of rotation, the inertial oscillations in the flow-field excited by Hall current damp out effectively in dimensionless time of $O(\alpha_2^{-1})$ when the final steady state flow is developed. This implies that, in the presence of Hall current, the time of decay of inertial oscillations in the flow-field are same in rotating and non-rotating systems and the inertial oscillations in the flow-field damp out quickly in the absence of Hall current.

3. Shear Stress at the Moving Plate

The non-dimensional shear stress components τ_x and τ_z at the moving plate $\eta=0$, due to primary and secondary flow respectively, are given by

$$\tau_x + i\tau_z\big|_{\eta=0} = \frac{\Delta H(t)}{2}\sum_{r=0}^{\infty}\left[\sqrt{g}\left\{e^{2r\sqrt{g}}erfc\left(\frac{r}{\sqrt{t}}+\sqrt{gt}\right)-\right.\right.$$
$$-e^{-2r\sqrt{g}}erfc\left(\frac{r}{\sqrt{t}}-\sqrt{gt}\right)+e^{2(r+1)\sqrt{g}}erfc\left(\frac{r+1}{\sqrt{t}}+\sqrt{gt}\right)-$$
$$\left.-e^{-2(r+1)\sqrt{g}}erfc\left(\frac{r+1}{\sqrt{t}}-\sqrt{gt}\right)\right\}-$$
$$\left.-\frac{2e^{-(\alpha_2-i\beta_2)t}}{\sqrt{\pi t}}\left\{e^{-r^2/t}+e^{-(r+1)^2/t}\right\}\right]. \tag{29}$$

4. Result and Discussion

To study the effects of Hall current, rotation, magnetic field and angle of inclination of magnetic field on the flow-field the numerical values of the fluid velocity are depicted graphically versus channel width variable η for various values of Hall current parameter m, rotation parameter K^2, magnetic parameter M^2 and angle of inclination of magnetic field θ in Figs. 1 to 4 considering $\Delta=1$ and $t=0.5$ and profiles of the shear stress components at the lower plate $\eta=0$ due to the primary and secondary flows (i.e. τ_x and τ_z) are drawn for different values of m, K^2, M^2 and θ in Figs. 7 and 8 taking $\Delta=1$ and $t=0.5$. To compare our results with already existing results of Seth *et al.* (1982) we have plotted the profiles of primary and secondary velocities and that of shear stress components at the lower plate $\eta=0$ due to primary and secondary flows in Figs. 5, 6 and 9 for various values of M^2 and K^2 taking $m=0$, $\theta=0$, $\Delta=1$ and $t=2$. These results are in agreement with the results obtained by Seth *et al.* (1982).

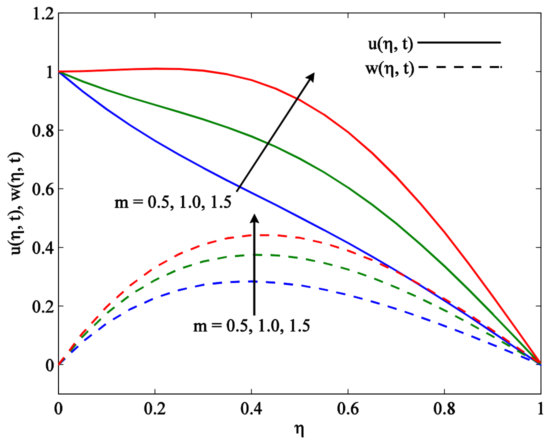

Fig. 1. Primary and secondary velocity profiles when M^2=6, K^2=3 and $\theta=\pi/4$

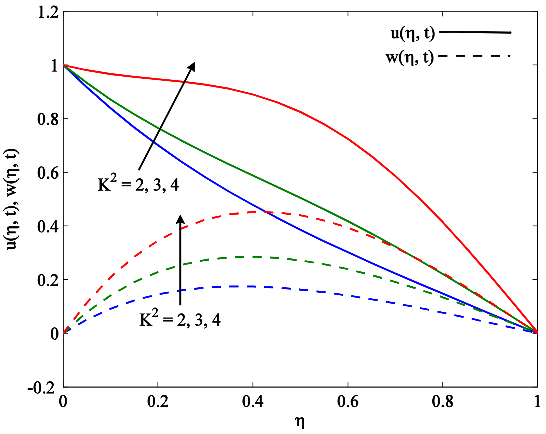

Fig. 2. Primary and secondary velocity profiles when M^2=6, m=0.5 and $\theta=\pi/4$

It is evident from Figs. 1 and 2 that the primary velocity u and secondary velocity w increase on increasing either m or K^2 which implies that Hall current and rotation tend to accelerate fluid velocity in both the primary and secondary flow directions.

Figure 3 reveals that primary and secondary velocities decrease on increasing M^2 which implies that magnetic field has retarding influence on the fluid velocity in both the primary and secondary flow directions. This is due to the established fact that the application of a magnetic field to an electrically conducting fluid gives rise to a force, known as Lorentz force, which tends to resist the fluid motion.

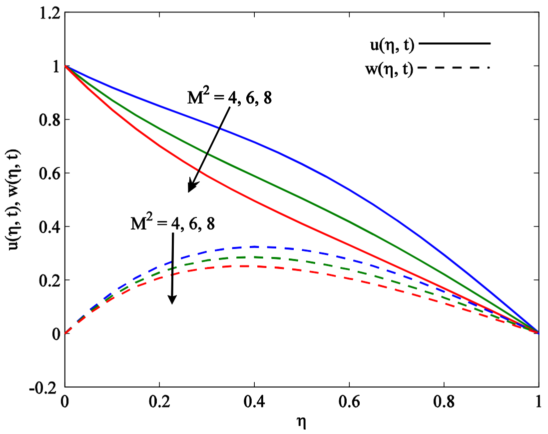

Fig. 3. Primary and secondary velocity profiles when K^2=3, m=0.5 and $\theta = \pi / 4$

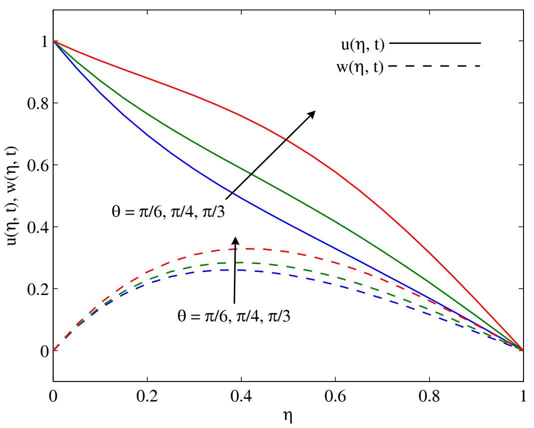

Fig. 4. Primary and secondary velocity profiles when M^2=6, K^2=3 and m=0.5

It is noticed from Fig. 4 that an increase in angle of inclination θ leads to an increase in both the primary and secondary velocities which implies that the angle of inclination of magnetic field has an accelerating influence on the fluid velocity in both the primary and secondary flow directions.

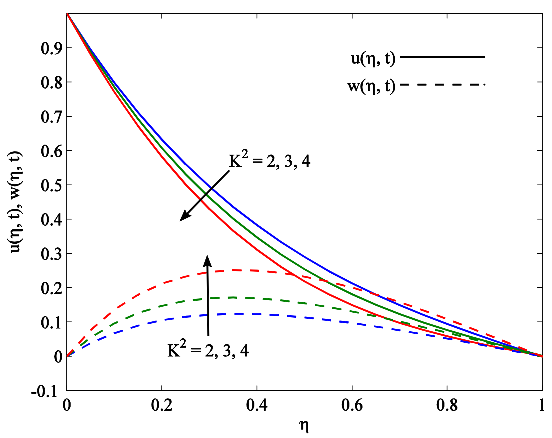

Fig. 5. Primary and secondary velocity profiles when M^2=4.

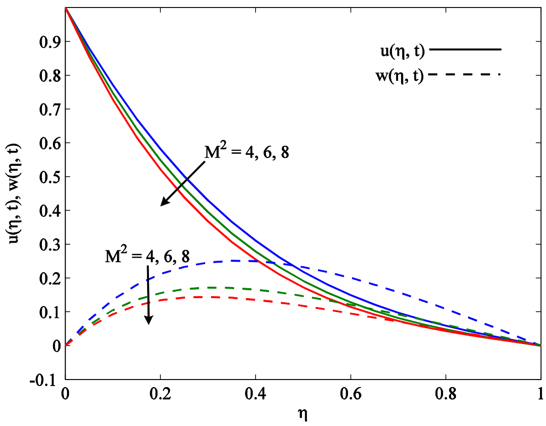

Fig. 6. Primary and secondary velocity profiles when K^2=4.

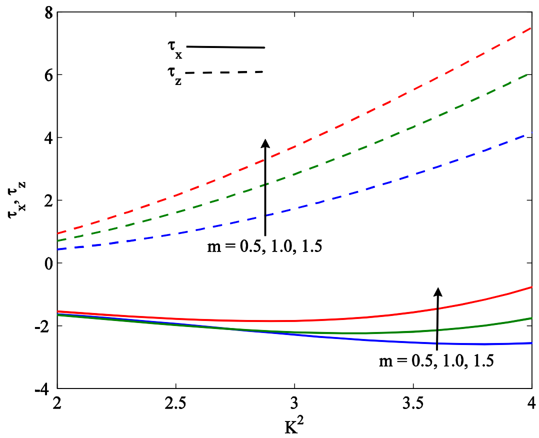

Fig. 7. Primary and secondary shear stress components when M^2=6 and $\theta = \pi / 4$

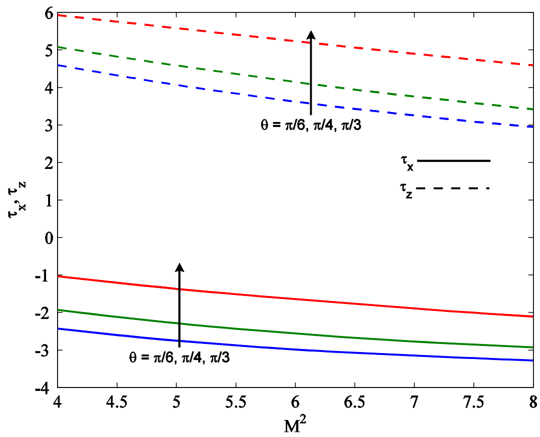

Fig. 8. Primary and secondary shear stress components when m=0.5 and $K^2 = 4$.

It is evident from Figs. 7 and 8 that, with an increase in either m or θ, primary shear stress τ_x at the lower plate decreases in magnitude whereas secondary shear stress τ_z at the lower plate increases. Primary and secondary shear stress components at the lower plate decrease on increasing M^2. Secondary shear stress τ_z at the lower plate increases on increasing K^2 and primary shear stress τ_x at the lower plate increases in magnitude, attains a maximum and then decreases in magnitude with the increase in K^2.

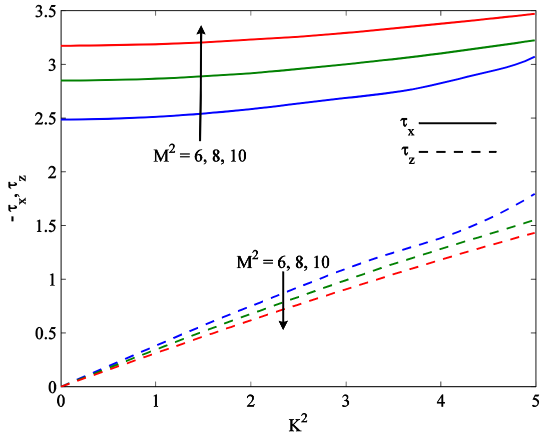

Fig. 9. Primary and secondary shear stress components.

This implies that Hall current and angle of inclination of magnetic field tend to reduce primary shear stress at the lower plate whereas these have reverse effect on the secondary shear stress at the lower plate. Magnetic field has tendency to enhance primary shear stress at the lower plate whereas it has reverse effect on the secondary shear stress at the lower plate. Rotation tends to increase secondary shear stress at the lower plate.

5. Conclusion

The effects of Hall current and rotation on unsteady MHD Couette flow of a viscous incompressible electrically conducting fluid in the presence of an inclined magnetic field is investigated. It is found that, for small values of time t, primary flow is independent of rotation whereas secondary flow has considerable effects of Hall current, magnetic field and rotation. The fluid flow in both the directions has significant effects of angle of inclination of magnetic field. For large values of time t, the fluid flow is in quasi-steady state. The steady state flow is confined within a modified Ekman-Hartmann boundary layer which becomes thinner with the increase in either magnetic parameter M^2 or rotation parameter K^2 or both and has considerable effects of Hall current and angle of inclination of magnetic field. Unsteady flow presents inertial oscillations in the flow-field excited by Hall current and rotation which have considerable effects of magnetic field and angle of inclination of magnetic field due to the presence of Hall current. Hall current and rotation tend to accelerate fluid velocity in both the primary and secondary flow directions. Magnetic field has retarding influence on the fluid velocity in both the primary and secondary flow directions. Angle of inclination of magnetic field has accelerating influence on the fluid velocity in both the primary and secondary flow directions.

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