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Chemically Reacting MHD Boundary Layer Flow of Heat and Mass Transfer over a Moving Vertical Plate in a Porous Medium with Suction

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ABSTRACT

A mathematical model is presented for a two-dimensional, steady, incompressible electrically conducting, laminar free convection boundary layer flow of a continuously moving vertical porous plate in a chemically reactive and porous medium in the presence of a transverse magnetic field. The basic equations governing the flow are in the form of partial differential equations and have been reduced to a set of non-linear ordinary differential equations by applying suitable similarity transformations. The problem is tackled numerically using shooting techniques with the forth order Runga-Kutta method. Pertinent results with respect to embedded parameters are displayed graphically for the velocity, temperature and concentration profiles and were discussed quantitatively.

Keywords: Free convection, Moving vertical plate, Chemical reaction, Heat and mass transfer, Magnetic field, Porous medium.

NOMENCLATURE

M Magnetic parameter $\left(=\sigma B_0^2 / \rho B\right)$
Gt Local temperature Grashof number $\left\{=(g\beta_T (T_w - T_\infty))/(xB^2)\right\}$
Gc Local concentration Grashof number $\left\{=(g\beta_c (C_w - C_\infty))/(xB^2)\right\}$
Pr Prandtl number $(=\upsilon/\alpha)$
Sc Schmidt number $(=\upsilon/D_m)$
B constant
a,b stratification rate of the gradient of ambient temperature and concentration profiles
K Permeability parameter $\{=\upsilon/(K'xB)\}$
u,v velocity components along x-and y- axes respectively
x,y Cartesian coordinates along x-,y-axes respectively
T fluid temperature
C fluid concentration
k the thermal conductivity
D_m mass diffusivity
B_0 magnetic induction
g gravitational acceleration
C_∞ free stream concentration
K' permeability of the porous medium
f dimensionless stream function
C_f skin-friction coefficient
Fw dimensionless suction velocity $\left(=V/\sqrt{B\nu}\right)$

Greek Symbols

ν kinematics viscosity
ρ density
σ electric conductivity
β_T , β_c thermal and concentration expansions respectively
α the thermal diffusivity
θ dimensionless temperature $\left\{=(T - T_\infty)/(T_w - T_\infty)\right\}$
ϕ dimensionless concentration $\left\{=(C - C_\infty)/(C_w - C_\infty)\right\}$
η similarity variable
ψ stream function

1. INTRODUCTION

Convective flow and heat transfer in a saturated porous medium has gained growing interest. This fact has been motivated by its importance in many engineering applications such as building thermal insulation, geothermal systems, food processing and grain storage, solar power collectors, contaminant transport in groundwater, casting in manufacturing processes, drying processes, nuclear waste, just to name a few. A theoretical and experimental work on this subject can be found in the recent monographs by Ingham and Pop (1998) and Nield and Bejan (1998). Suction/blowing on convective heat transfer over a vertical permeable surface embedded in a porous medium was analyzed by Cheng (1977). In that work an application to warm water discharge along the well or fissure to an aquifer of infinite extent is discussed. Kim and Vafai (1989) have analyzed the buoyancy driven flow about a vertical plate for constant wall temperature and heat flux. Raptis and Singh (2010) studied flow past an impulsively started vertical plate in a porous medium by a finite difference method.

The effect of radiation on free convection flow of fluid with variable viscosity from a porous plate is discussed Anwar and Hossain et al (2001). The fluid considered in that paper is an optically dense viscous incompressible fluid of linearly varying temperature dependent viscosity. Salem (1985) discussed coupled heat and mass transfer in Darcy-Forchheimer Mixed convection from a vertical flat plate embedded in a fluid saturated porous medium under the effects of radiation and viscous dissipation. Paresh Vyas & Ashutosh Ranjan (2010) discussed the dissipative MHD boundary- layer flow in a porous medium over a sheet stretching nonlinearly in the presence of radiation. Seddeek and Almushigeh (2006) studied the Effects of radiation and variable viscosity on MHD free convective flow and mass transfer over a stretching sheet with chemical reaction.

Soundalgekar and Ramana (2010) have investigated the constant surface velocity case with a power-law temperature variation. Grubka and Bobba (1985) have analyzed the stretching problem for a surface moving with a linear velocity and with a variable surface temperature. Ali (1994) has reported flow and heat characteristics on a stretched surface subject to power-law velocity and temperature distributions. The flow field of a stretching surface with power-law velocity variations was discussed by Banks (1983). Ali (1995) and Elbashbeshy (1998) extended Banks's work for a porous stretched surface for 2 different values of the injection. Elbashbeshy (2000) have analyzed the stretching problem which was discussed be Elbashbeshy (1998) to include a uniform porous medium.

The present communication considers the effects of chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction as presented in Ibrahim and Makinde (2010). It investigates numerically the effects of heat and mass transfer in a hydromagnetic boundary layer flow of a moving vertical porous plate in a porous medium with uniform heat generation and chemical reaction. In the problem formulation, the continuity, momentum, energy and concentration equations are reduced to some parameter problem by introducing suitable transformation variables. The equations that govern the flow are coupled and solve numerically using shooting techniques with the forth order Runga-Kutta method. The effects of various governing parameters on the velocity, temperature, concentration are presented graphically and discussed quantitatively. The local skin-friction coefficient and the heat and mass transfer results are illustrated for representative values of the major parameters.

2. MATHEMATICAL ANALYSIS

Consider a two-dimensional free convection effects on the steady incompressible laminar MHD heat and mass transfer characteristics of a linearly started porous vertical plate, the velocity of the fluid far away from the plate surface is assumed zero for a quiescent state fluid. The flow configurations are linear.

All the fluid properties are assumed to be constant except for the density variations in the buoyancy force term of the linear momentum equation. The magnetic Reynolds number is assumed to be small, so that the induced magnetic field is neglected. The Hall effects, the viscous dissipation and the joule heating terms are also neglected. Under these assumptions, along with Boussinesq approximations, the boundary layer equations describing this flow as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u+g\beta_T(T-T_\infty)+g\beta_c(C-C_\infty)-\frac{\nu}{K'}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The boundary conditions at the plate surface and for into the cold fluid may be written as

$$v=V,\ u=Bx,\ T=T_w=T_\infty+ax,\ C=C_w=C_\infty+bx \quad \text{at } y=0,$$

$$u->0,T->T_\infty,C->C_\infty \text{ as } y->\infty \tag{5}$$

We introduce the following non-dimensional variables:

$$\eta=y\sqrt{B/\upsilon},\psi=x\sqrt{\upsilon B}\,f(\eta)\,. \tag{6}$$

The velocity components u and v are respectively obtained as follows:

$$u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x}\,. \tag{7}$$

With this new set of independent and dependent variables defined by equation (6), the partial differential equations (2) to (4) are transformed in to local similarity equations as follows:

$$f''' + ff'' - f'(f' + M) + Gt\theta + Gc\ \phi - Kf' = 0 \quad (8)$$

$$\theta'' + \Pr f\ \theta' - \Pr f'\theta = 0 \quad (9)$$

$$\phi'' + Scf\ \phi' - Scf'\phi = 0 \quad (10)$$

$$f'(0) = 1, f(0) = -Fw, \theta(0) = 1, \phi(0) = 1 \quad (11)$$

$$f'(\infty) = 0, \theta(\infty) = \phi(\infty) = 0. \quad (12)$$

Skin-friction: Skin-friction coefficient at the moving vertical plate is given by

$$C_f = \frac{\tau_w}{\rho c (c\nu)^{\frac{1}{2}}} = xf''(0)$$

Where $\tau_w = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)_{y=0}$ is the shear stress at the moving vertical plat,

3. SOLUTION OF THE PROBLEM

The set of coupled non-linear governing boundary layer equations (8)-(10) together with the boundary conditions (11&12) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. First of all, higher order non-linear differential Eqs (8)-(10) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique [Jain *et al.*(1985)]. The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size $\Delta\eta$ =0.05 is used to obtain the numerical solution with decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient, the Nusselt number and the Sherwood number, which are respectively proportional to $f''(0), -\theta'(0)$ and $-\phi'(0)$, are also sorted out and their numerical values are presented in a tabular form.

4. RESULTS AND DISCUSSION

The governing equations (8)-(10) subject to the boundary conditions (11)-(12) are integrated as described in section 3. Numerical results are reported in the tables 1-2. The prandtl number was taken to be Pr=0.72 which corresponds to air, the value of Schmidt number (Sc) were chosen to be Sc=0.24,0.62, 0.78,2.62, representing diffusing chemical species of most common interest in air like H_2 , H_2O, NH_3 and Propyl Benzene respectively. Attention is focused on positive value of the buoyancy parameters that is, local temperature Grashof number Gt>0(which corresponds to the cooling problem)and local concentration Grashof number Gc>0(which indicates that the chemical species concentration in the free stream region is less then the concentration at the boundary surface).In order to benchmark our numerical results, we have compared the plate surface temperature $\theta(0)$ and the local heat transfer rate at the plate surface $\theta'(0)$ in the absence of both magnetic field and buoyancy forces for various values of M with those of Ibrahim and Makinde (2010) and found them in excellent agreement as demonstrated in table 1.From table 2, it is important to note that the local skin friction together with the local heat and mass transfer rate at the moving plate surface increases with increasing intensity of buoyancy forces (Gt,Gc), the Schmidt number (Sc),However, an increase in the magnetic field (M),magnitude of fluid suction (Fw), Permeability parameter (K) causes a decrease in both skin friction and surface heat transfer rate and an increase in the surface mass transfer rate.

Case 1: Effects of Parameter Variation on Velocity Profiles:

The effects of various parameters on velocity profiles in the boundary layer are depicted in Figures 1-6. It is observed from Figures 1-6, that the velocity starts from a zero value at the moving vertical plate surface and increase to the free stream value far away from the moving vertical plate surface satisfying the far field boundary condition for all parameter values.

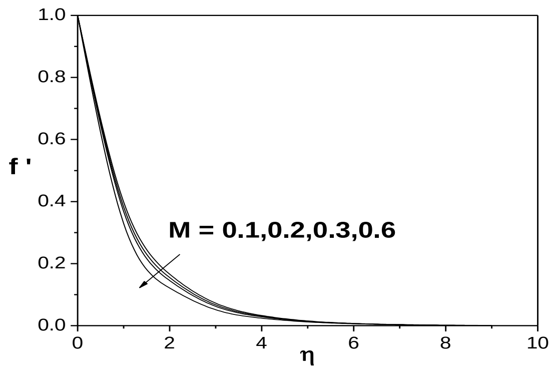

Fig. 1. Variation of the velocity component f' with M forPr=0.72, Sc=0.62, Gt=Gc=Fw=0.1, K=0.1.

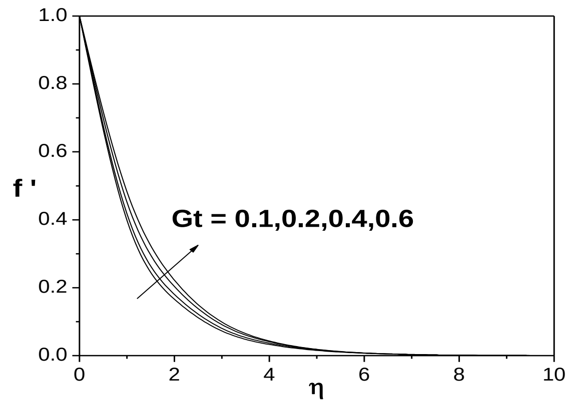

Fig. 2. Variation of the velocity component f' with *Gt For Pr=0.72, Sc=0.62, Gc=M=Fw=K=0.1.*

Table 1 Variation of $F''(0)$, $-\theta'(0)$, $-\phi'(0)$ at the plate with Gr, Gc, M, F_w, Sc for *Pr=0.72*.

					Ibrahim and Makinde (2010)			Present work		
Gr	Gc	M	F_w	Sc	$F''(0)$	$-\theta'(0)$	$-\phi'(0)$	$F''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	0.1	0.1	0.1	0.62	0.888971	0.7965511	0.7253292	0.897945	0.805082	0.740192
0.5	0.1	0.1	0.1	0.62	0.695974	0.8379008	0.7658018	0.70684	0.841012	0.773803
1.0	0.1	0.1	0.1	0.62	0.475058	0.8752835	0.8020042	0.485886	0.876165	0.806773
0.1	0.5	0.1	0.1	0.62	0.686927	0.8421370	0.7701717	0.699552	0.843609	0.776285
0.1	1.0	0.1	0.1	0.62	0.457723	0.8818619	0.8087332	0.470894	0.880875	0.811303
0.1	0.1	1.0	0.1	0.62	1.264488	0.7089150	0.6400051	1.2696	0.734784	0.675272
0.1	0.1	3.0	0.1	0.62	1.868158	0.5825119	0.5204793	1.87194	0.63906	0.588803
0.1	0.1	0.1	1.0	0.62	0.570663	0.5601256	0.5271504	0.575823	0.566525	0.537923
0.1	0.1	0.1	3.0	0.62	0.275153	0.2955702	0.2902427	0.00301923	0.143197	0.143829
0.1	0.1	0.1	0.1	0.78	0.893454	0.7936791	0.8339779	0.901088	0.803798	0.841183
0.1	0.1	0.1	0.1	2.62	0.912307	0.7847840	1.6504511	0.918401	0.797633	1.64733

Table 2 Variation of $F''(0)$, $-\theta'(0)$, $-\phi'(0)$ at the plate with K for $Gr = Gc = M = F_w$ *,=0.1, Sc =0.62 , Pr=0.72.*

K	$F''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	0.944589	0.795952	0.531689
0.5	1.1165	0.762949	0.701131
1.0	1.30551	0.728382	0.669421
1.5	1.47358	0.699551	0.643201
2.0	1.6261	0.675081	0.621099
3.0	1.89742	0.635561	0.585681

with K for *Pr=0.72,Sc=0.62,Gt=Gc=Fw=M=0.1.*

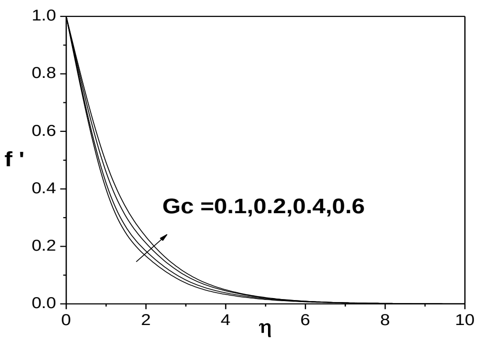

Fig. 3. Variation of the velocity component f' with *Gc* for *Pr=0.72, Sc=0.62,Gt =Fw=M=K=0.1.*

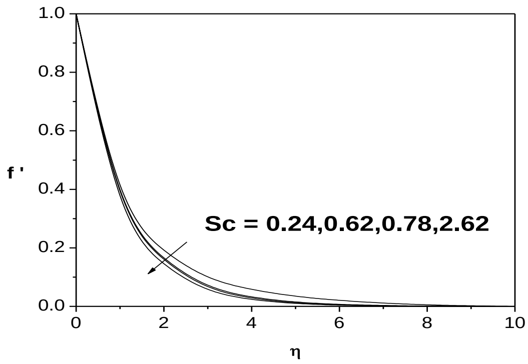

Fig. 5. Variation of the velocity component f' with *Sc* for *Pr=0.72, Gt=Gc=Fw=M=K=0.1.*

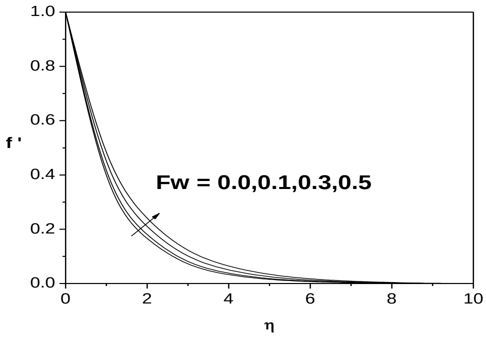

Fig. 4. Variation of the velocity component f' with *Fw* for *Pr=0.72, Sc=0.62,Gc=Gt =M=K=0.1.*

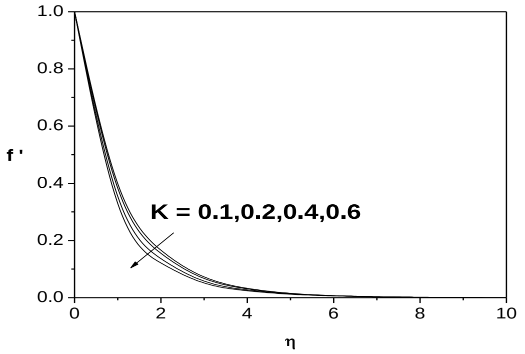

Fig. 6. Variation of the velocity component f'

In Figure 1 the effect of increasing the magnetic field strength on the momentum boundary layer thickness is illustrated. It is now a well established fact that the magnetic field presents a damping effect on the velocity field by creating drag force that opposes the fluid motion, causing the velocity to decease. However, in this case an increase in the M only slightly slows down the motion of the fluid away from the moving vertical plate surface towards the free stream velocity, while the fluid velocity near the moving vertical plate surface increases. Figures 2, 3 & 4 shows the variation of the boundary-layer velocity with the buoyancy forces parameters (*Gr*, *Gc*), and magnitude of fluid suction (Fw). In both cases an upward acceleration of the fluid in the vicinity of the vertical wall is observed with increasing intensity of buoyancy forces.

Further downstream of the fluid motion decelerates to the free stream velocity. Figures 5 and 6 shows that a slight decrease in the fluid velocity with an increase in the Schmidt number (*Sc*) and the Permeability parameter (*K*).

Case 2: Effects of Parameter Variation on Temperature Profiles:

Generally, the fluid temperature attains its maximum value at the moving vertical plate surface and decreases exponentially to the free stream zero value away from the plate satisfying the boundary condition. This is observed in Figures 7-12. From these figures, it is interesting to note that the thermal boundary layer thickness decreases with an increase in the intensity of the buoyancy forces (Gr,Gc). Moreover, the fluid temperature increases with an increase in the Schmidt number (Sc), Magnetic field (M), Permeability parameter (K) and magnitude of fluid suction (Fw) leading to an increase in thermal boundary layer thickness.

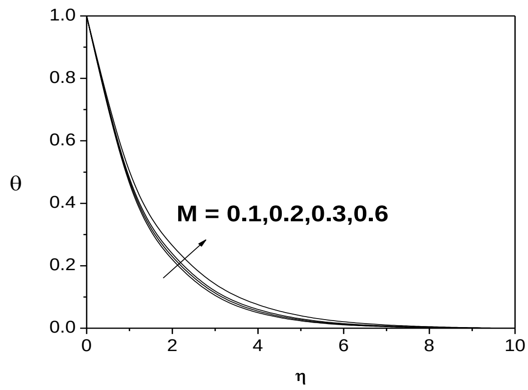

Fig. 7. Variation of the velocity component θ with *M* for *Pr=0.72, Sc=0.62, Gt=Gc=Fw=K=0.1.*

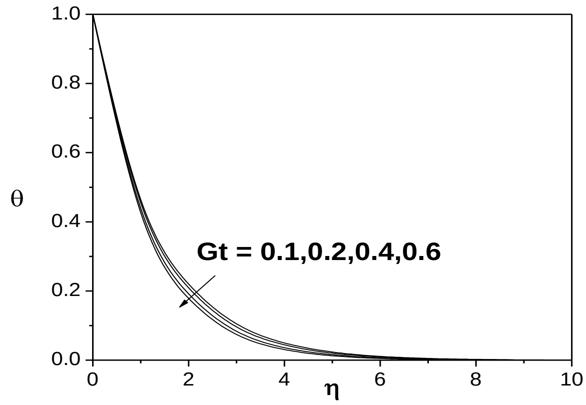

Fig. 8. Variation of the temperature θ with *Gt* for *Pr=0.72, Sc=0.62, Fw=Gc=M=K=0.1.*

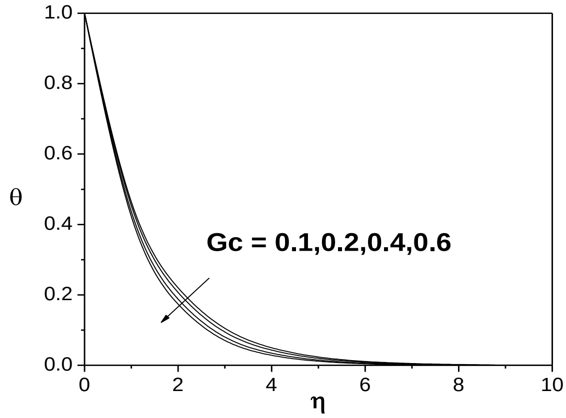

Fig. 9. Variation of the temperature θ with *Gc* for *Pr=0.72, Sc=0.62, Gt=Fw=M=K=0.1.*

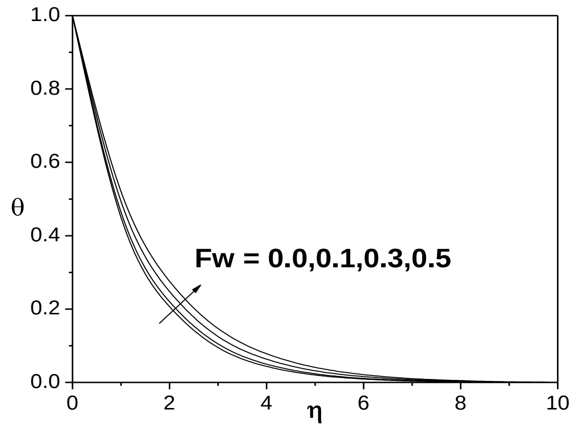

Fig. 10. Variation of the temperature θ with *Fw* for *Pr=0.72, Sc=0.62, Gc =Gt=M=K=0.1.*

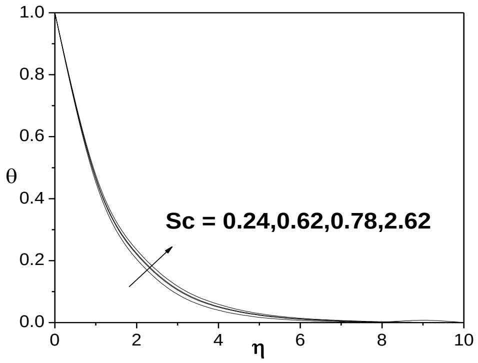

Fig. 11. Variation of the temperature θ with *Sc* for *Pr=0.72,Gt=Gc=Fw=M=K=0.1.*

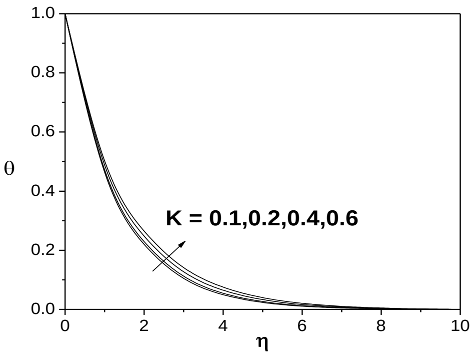

Fig. 12. Variation of the temperature θ with *K* for *Pr=0.72, Sc=0.62,Gt=Gc=Fw=M=0.1.*

Case 3: Effects of Parameter Variation on Concentration Profiles:

Figures 13-18 depict chemical species concentration profiles against span wise coordinate η for varying values physical parameters in the boundary layer. The species concentration is highest at the moving vertical plate surface and decrease to zero far away from the moving vertical plate satisfying the boundary condition. From these figures, it is noteworthy that the concentration boundary layer thickness decreases with an increase in, the buoyancy forces (Gr,Gc), Schmidt number (Sc) and Moreover, the fluid concentration increases with an increase in the Permeability parameter(K), the magnetic field (M) and magnitude of fluid suction (Fw) leading to an increase in thermal boundary layer thickness.

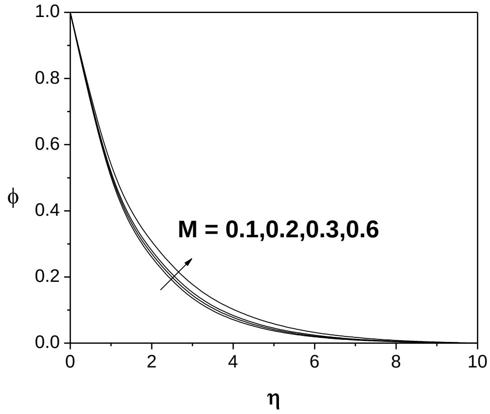

Fig. 13. Variation of the concentration ϕ with *M* for *Pr=0.72, Sc=0.62, Gt=Gc=Fw=K=0.1*

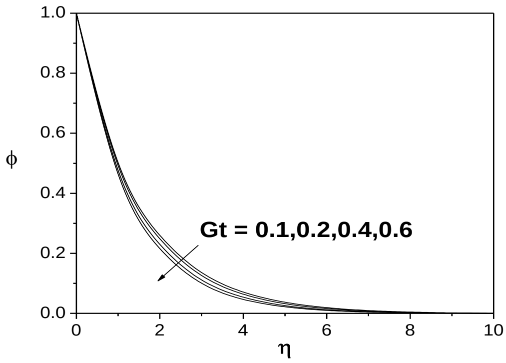

Fig. 14. Variation of the concentration ϕ *withGt* for *Pr=0.72, Sc=0.62, M=Gc =Fw=K=0.1.*

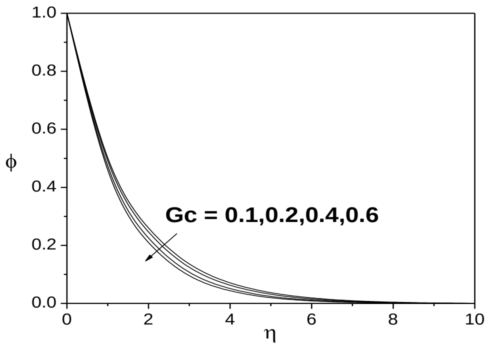

Fig. 15. Variation of the concentration ϕ with *Gc* for *Pr=0.72, Sc=0.62, Gt=Fw=M=K=0.1*.

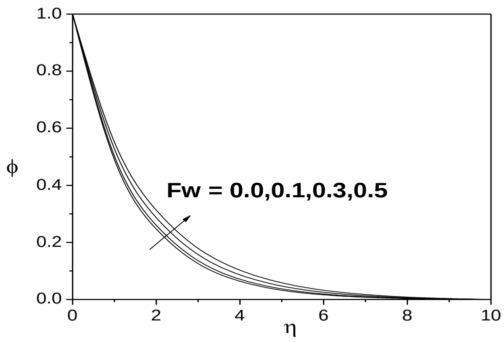

Fig. 16. Variation of the concentration ϕ with *Fw* for *Pr=0.72, Sc=0.62, Gt=Gc =M=K=0.1.*

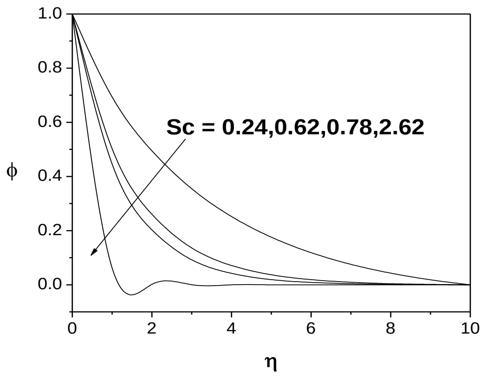

Fig. 17. Variation of the concentration ϕ with *Sc* for *Pr=0.72 Gt=Gc =Fw=M=K=0.1.*

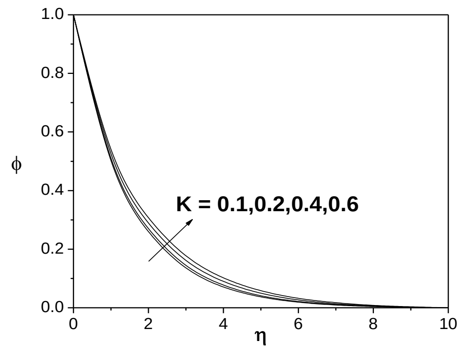

Fig. 18. Variation of the concentration ϕ with *K* for *Pr=0.72, Sc=0.62, Gt=Gc=Fw=M=0.1.*

5. CONCLUSION

This paper studied the combined effects of wall suction and magnetic field on boundary layer flow with heat and mass transfer over an accelerating vertical plate in a porous medium. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. Our results revealed that the momentum boundary layer thickness decreases, while both thermal and concentration boundary layer thicknesses increase with an increase in the magnetic field intensity. Furthermore, an increase in wall suction enhances the boundary layer thickness and reduce the skin friction together with the heat and mass transfer rate at the moving plate surface.

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