Thermoslutal Convection in Walters' (Model B') Rotating Fluid Permeated with Suspended Particles and Variable Gravity Field in Porous Medium in Hydromagnetics

G. C. Rana

Department of Mathematics, NSCBM Govt. P. G. College, Hamirpur-177 005, Himachal Pradesh, India

Email: drgcrana15@gmail.com

ABSTRACT

The thermosolutal convection in Walters' (Model B') elastico-viscous rotating fluid permeated with suspended particles (fine dust) and variable gravity field in porous medium in hydromagnetics is considered. By applying normal mode analysis method, the dispersion relation has been derived and solved numerically. It is observed that the rotation, magnetic field, gravity field, suspended particles and viscoelasticity introduce oscillatory modes. For stationary convection, Walters' (Model B') elastico-viscous fluid behave like an ordinary Newtonian fluid and it is observed that rotation and stable solute gradient has stabilizing effects and suspended particles are found to have destabilizing effect on the system, whereas the medium permeability has stabilizing or destabilizing effect on the system under certain conditions. The magnetic field has destabilizing effect in the absence of rotation, whereas in the presence of rotation, magnetic field has stabilizing or destabilizing effect under certain conditions. The effect of rotation, suspended particles, magnetic field, stable solute gradient and medium permeability has also been shown graphically.
AMS subject classifications are 76A10, 76E07, 76E25 and 76S05.

Keywords: Walters' (Model B') elastico-viscous fluid, Thermosolutal convection, Suspended particles, Magnetic field, Variable gravity field, Porous medium.

NOMENCLATURE

p	pressure	κ'	solute diffusivity
g	acceleration due to gravity	P_l	dimensionless medium permeability
$\in$	medium porosity	μ_e	magnetic permeability
$\boldsymbol{\rho}$	fluid density	K	Stokes' drag coefficient
δp	perturbation in pressure	T	temperature
$\delta\boldsymbol{\rho}$	perturbation in density	α	thermal coefficient of expansion
μ	fluid viscosity	α'	solvent coefficient of expansion
μ'	fluid viscoelasticity	β	temperature gradient
υ	kinematic viscosity	β'	solute gradient
υ'	kinematic viscoelasticity	Θ	perturbation in temperature
k	medium permeability	ξ	z-component of current density
N	suspended particles number density	ζ	z-component of vorticity
η	particle radius	γ	perturbation in solute concentration
t	time coordinate		
κ	thermal diffusitivity		

1. INTRODUCTION

A detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics. has been given by Chandrasekhar (1981). Bhatia and Steiner (1973) have studied the thermal instability of a Maxwellian visco-elastic fluid in the presence of magnetic field while the thermal convection in Oldroydian visco-elastic fluid has been considered by Sharma (1975). Veronis (1965) has investigated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. The buoyancy forces can arise not only from density differences due to variations in solute concentration,

Thermosolutal convection problems arise in oceanography, limnology and engineering.

The medium has been considered to be non-porous in all the above studies. Lapwood (1948) has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding (1960) whereas Scanlon and Segel (1973) have considered the effect of suspended particles on the onset of Be'nard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by the particles. The suspended particles were thus found to destabilize the layer.

Sharma and Sunil (1994) have studied the thermal instability of an Oldroydian viscoelastic fluid with suspended particles in hydromagnetics in a porous medium. There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. One such class of fluids is Walters' (Model B') elastico-viscous fluid. Walters' (1962) reported that the mixture of polymethyl methacrylate and pyridine at 25^oC containg 30.5g of polymer per litre with density 0.98g per litre behaves very nearly as the Walters (Model B') elastico-viscous fluid. Such and other polymers are used in the manufacture of space crafts, aero planes, tyres, ropes, cushions, seats, foam, plastics, engineering equipments, adhesives, contact lenses etc. Recently, polymers are used in agriculture, communications appliances and in bio medical applications. Examples of these applications are filtration processes, packed bed reactors, insulation system, ceramic processing, enhanced oil recovery, chromatography etc.

Brakke (1955) explained a double-diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose.The problem of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat transfer mechanism in the development of young oceanic crust (Lister, 1972). Generally, it is accepted that comets consist of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice-versa. The physical properties of comets, meteorites and inter-planetary dust strongly suggest the importance of porosity in the astrophysical context (McDonnel , 1978).

Thermal instability of a fluid layer under variable gravitational field heated from below or above is investigated analytically by Pradhan and Samal (1987). Although the gravity field of the Earth is varying with height from its surface, we usually neglect this variation for laboratory purposes and treat the field as constant. However, this may not the case for large scale flows in the ocean, the atmosphere or the mantle. It can become imperative to consider gravity as a quantity varying with distance from the centre.

A porous medium is a solid with holes in it, and is characterized by the manner in which the holes are imbedded, how they are interconnected and the description of their location, shape and interconnection. However, the flow of a fluid through a homogeneous and isotropic porous medium is governed by Darcy's law which states that the usual viscous term in the equations of motion of Walters' (Model B') fluid is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu - \mu'\frac{\partial}{\partial t}\right)\right]\boldsymbol{q}$, where μ and μ' are the viscosity and viscoelasticity of the incompressible Walters' (Model B') fluid, k_1 is the medium permeability and q is the Darcian (filter) velocity of the fluid.

Sharma and Rana (2001) have studied Thermal instability of a Walters' (Model B') elastico-viscous in the presence of variable gravity field and rotation in porous medium. Sharma and Rana (2003) have also studied the thermosolutal instability of Rivlin-Ericksen rotating fluid in the presence of magnetic field and variable gravity field in porous medium. Recently, Sharma and Gupta (2010) have studied the stability of elastico-viscous Walters' (Model B') fluid in the presence of horizontal magnetic field and rotation, whereas thermal instability of Rivlin-Ericksen elastico-viscous rotating fluid permeated with suspended particles under varaiable gravity field in porous medium has been studied by Rana and Kumar (2010).

Keeping in mind the importance in various applications mentioned above, our interest, in the present paper is to study the thermosolutal convection in Walters' (Model B') elastico-viscous rotating fluid permeated with suspended particles and variable field in porous medium in hydromagnetics.

2.   FORMULATION OF THE PROBLEM

Consider an infinite horizontal layer of an electrically conducting Walters' (Model B') elastico-viscous fluid of depth d in a porous medium bounded by the planes z = 0 and z = d in an isotropic and homogeneous medium of porosity $\in$ and permeability k_1, which is acted upon by a uniform rotation $\boldsymbol{\Omega}(0, 0, \boldsymbol{\Omega})$, uniform vertical magnetic field $\boldsymbol{H}(0,0,H)$ and variable gravity $\boldsymbol{g}(0, 0, -g)$, g = λg_0, $g_0(> 0)$ is the value of g at z = 0 and λ can positive or negative as gravity increases or decreases upward from its value g_0. This layer is heated and soluted from below such that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ and a uniform solute gradient $\beta'\left(=\left|\frac{dC}{dz}\right|\right)$ are maintained. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.

The equations expressing the conservation of momentum, mass, temperature, solute mass concentration and Maxwell's equations of Walters' (Model B') elastico-viscous fluid in porous medium are (Chandrasekhar, 1981; Walters, 1962; Sharma and Rana, 2003):

$$\frac{1}{\epsilon}\left[\frac{\partial q}{\partial t} + \frac{1}{\epsilon}(q.\nabla)q\right] = -\frac{1}{\rho_0}\nabla p + g\left(1 + \frac{\delta\rho}{\rho_0}\right) -$$
$$\frac{1}{k_1}\left(v - v'\frac{\partial}{\partial t}\right)q + \frac{2}{\epsilon}(q \times \Omega) + \frac{K'N}{\rho_0 \epsilon}(q_d - q) +$$
$$\frac{\mu_e}{4\pi\rho_0}(\nabla \times H) \times H, \tag{1}$$

$$\nabla.q = 0, \tag{2}$$

$$E\frac{\partial T}{\partial t} + (q.\nabla)T + \frac{mNC_{pt}}{\rho_0 C_f}\left[\epsilon\frac{\partial}{\partial t} + q_d.\nabla\right]T = \kappa\nabla^2 T, \tag{3}$$

$$E'\frac{\partial C}{\partial t} + (q.\nabla)T + \frac{mNC_{pt}'}{\rho_0 C_f'}\left[\epsilon\frac{\partial}{\partial t} + q_d.\nabla\right]T = \kappa'\nabla^2 T \tag{4}$$

$$\nabla.H = 0, \tag{5}$$

$$\epsilon\frac{\partial H}{\partial t} = \nabla \times (q \times H) + \epsilon\eta\nabla^2 H, \tag{6}$$

where $E = \epsilon + (1-\epsilon)\left(\frac{\rho_s c_s}{\rho_0 c_f}\right)$, ρ_s, c_s; ρ_0, c_f denote the density and heat capacity of solid (porous) matrix and fluid respectively and E' is a constant analogous to E but corresponding to solute rather than heat; κ, κ' are the thermal diffusivity and solute diffusivity respectively.

The equation of state is (Chandrasekhar, 1981)

$$\rho = \rho_0\left[1 - \alpha(T - T_0) + \alpha'(C - C_0)\right], \tag{7}$$

where the suffix zero refers to values at the reference level z = 0. Here ρ, v, v', p, ϵ, T, C, μ_e, α, α', q (0, 0, 0) and $H(0,0,H)$ stand for density, kinematic viscosity, kinematic viscoelasticity, pressure, medium porosity, temperature, solute concentration, magnetic permeability, thermal coefficient of expansion, an analogous solvent coefficient of expansion, velocity of the fluid and magnetic field. Here $q_d(\bar{x}, t)$ and $N(\bar{x}, t)$ denote the velocity and number density of the particles respectively, $K = 6\pi\eta\rho v$, where η is particle radius, is the Stokes drag coefficient, $q_d = (l, r, s)$ and $\bar{x} = (x, y, z)$.

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are

$$mN\left[\frac{\partial q_d}{\partial t} + \frac{1}{\epsilon}(q_d.\nabla)q_d\right] = K'N(q - q_d), \tag{8}$$

$$\epsilon\frac{\partial N}{\partial t} + \nabla.(Nq_d) = 0. \tag{9}$$

The presence of particles adds an extra force term proportional to the velocity difference between particles and fluid and appears in the equation of motion (1). Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles (8). The buoyancy force on the particles is neglected. Interparticles reactions are not considered either since we assume that the distance between the particles are quite large compared with their diameters. These assumptions have been used in writing the equations of motion (8) for the particles.

The initial state of the system is taken to be quiescent layer (no settling) with a uniform particle distribution number. The initial state is

$$q = (0,0,0), \quad q_d = (0,0,0),$$
$$T = -\beta z + T_0, C = -\beta' z + C_0,$$
$$\rho = \rho_0(1 + \alpha\beta z - \alpha'\beta' z), \text{N}_0\text{=constant} \tag{10}$$

is an exact solution to the governing equations.

3. THE PERTURBATION EQUATIONS

Let $q(u,v,w)$, $q_d(l,r,s)$, θ, γ, δp and $\delta\rho$ denote, respectively, the perturbations in fluid velocity $q(0,0,0)$, the perturbation in particle velocity $q_d(0,0,0)$, temperature T, solute concentration C, pressure p and density ρ.

The change in density $\delta\rho$ caused by perturbation θ and γ in temperature and solute concentration is given by

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma) \tag{11}$$

The linearized perturbation equations governing the motion of fluids are

$$\frac{1}{\epsilon}\frac{\partial q}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p - g(\alpha\theta - \alpha'\gamma) - \frac{1}{k_1}\left(v - v'\frac{\partial}{\partial t}\right)$$
$$+ \frac{K'N}{\epsilon}(q_d - q) + \frac{2}{\epsilon}(q \times \Omega)$$
$$+ \frac{\mu_e}{4\pi\rho_0}(\nabla \times h) \times H \tag{12}$$

$$\nabla.q = 0, \tag{13}$$

$$\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)q_d = q \tag{14}$$

$$(E + b \epsilon)\frac{\partial\theta}{\partial t} = \beta(w + bs) + \kappa\nabla^2\theta, \tag{15}$$

$$(E' + b' \epsilon)\frac{\partial\theta}{\partial t} = \beta'(w + b's) + \kappa'\nabla^2\gamma \tag{16}$$

$$\nabla.h = 0, \tag{17}$$

$$\epsilon\frac{\partial H}{\partial t} = (H.\nabla)q + \epsilon\eta\nabla^2 H \tag{18}$$

where $b = \frac{mNC_{pt}}{\rho_0 C_f}$, $b' = \frac{mNC_{pt}'}{\rho_0 C_f'}$ and w, s are the vertical fluid and particles velocity.

In the Cartesian form, equations (12)-(18) can be expressed as

$$\frac{1}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial}{\partial x}(\delta p) -$$
$$\frac{1}{k_1}\left(v - v'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)u - \frac{mN}{\epsilon\rho_0}\frac{\partial u}{\partial t} +$$
$$\frac{\mu_e H}{4\pi\rho_0}\left(\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x}\right) + \frac{2}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\Omega v, \tag{19}$$

$$\frac{1}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial v}{\partial t} = -\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial}{\partial y}(\delta p) -$$
$$\frac{1}{k_1}\left(v - v'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)v - \frac{mN}{\epsilon\rho_0}\frac{\partial v}{\partial t} +$$
$$\frac{\mu_e H}{4\pi\rho_0}\left(\frac{\partial h_y}{\partial z} - \frac{\partial h_z}{\partial y}\right) + \frac{2}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\Omega u \tag{20}$$

$$\frac{1}{\epsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial w}{\partial t} = -\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial}{\partial z}(\delta p) -$$
$$\frac{1}{k_1}\left(v - v'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)w - \frac{mN}{\epsilon\rho_0}\frac{\partial w}{\partial t} +$$
$$g\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\alpha\theta, \tag{21}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{22}$$

$$(E + b \in)\frac{\partial \theta}{\partial t} = \beta(w + bs) + \kappa\nabla^2\theta, \tag{23}$$

$$(E' + b' \in)\frac{\partial \theta}{\partial t} = \beta'(w + b's) + \kappa'\nabla^2\gamma \tag{24}$$

$$\in \frac{\partial h_x}{\partial t} = H\frac{\partial u}{\partial z} + \in \eta\nabla^2 h_x, \tag{25}$$

$$\in \frac{\partial h_y}{\partial t} = H\frac{\partial v}{\partial z} + \in \eta\nabla^2 h_y, \tag{26}$$

$$\in \frac{\partial h_z}{\partial t} = H\frac{\partial w}{\partial z} + \in \eta\nabla^2 h_z, \tag{27}$$

$$\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + \frac{\partial h_z}{\partial z} = 0, \tag{28}$$

Operating equations (19) and (20) by $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively, adding and using equation (25)-(28), we get

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right) = \frac{1}{\rho_0}\left(\frac{m}{K}\frac{\partial}{\partial t} + 1\right)\left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)\delta p - \frac{1}{k_1}\left(v - v'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\left(\frac{\partial w}{\partial z}\right) - \frac{mN}{\in\rho_0}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right) + \left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\mu_e H}{4\pi\rho_0}\nabla^2 h_z - \frac{2}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\Omega\zeta, \tag{29}$$

where $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the z-component of vorticity.

Operating equations (21) and (29) by $\left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)$ and $\frac{\partial}{\partial z}$ respectively and adding to eliminate δp between equations (21) and (29), we get

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial}{\partial t}(\nabla^2 w) = -\frac{1}{k_1}\left(v - v'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\nabla^2 w + g\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\alpha\theta - \frac{mN}{\in\rho_0}\frac{\partial}{\partial t}(\nabla^2 w) + \frac{\mu_e H}{4\pi\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial}{\partial t}\nabla^2 h_z - \frac{2}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\Omega\frac{\partial\zeta}{\partial z}, \tag{30}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

Operating equations (19) and (20) by $-\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial x}$ respectively and adding, we get

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial\zeta}{\partial t} = -\frac{1}{k_1}\left(v - v'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right) \\ - \frac{mN}{\in\rho_0}\frac{\partial\zeta}{\partial t} + \frac{2}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\Omega\frac{\partial w}{\partial z} \\ +$$

$$\frac{\mu_e H}{4\pi\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial\xi}{\partial t}. \tag{31}$$

where $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ is the z-component of current density.

Operating equations (25) and (26) by $-\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial x}$ respectively and adding, we get

$$\frac{1}{\in}\frac{\partial\xi}{\partial t} = H\frac{\partial\xi}{\partial t} + \in \eta\nabla^2\xi. \tag{32}$$

4. DISPERSION RELATION AND DISCUSSION

Analyzing the disturbances into normal modes, we assume that the perturbation quantities have x, y and t dependence of the form

$$[w, s, \theta, \gamma, \zeta, h_z, \xi] = \\ [W(z), S(z), \Theta(z), Z(z), \Gamma(z), K(z), X(z)]exp(ik_x x + ik_y y + nt), \tag{33}$$

where k_x and k_y are the wave numbers in the x and y-directions, $k = \left(k_x{}^2 + k_y{}^2\right)^{1/2}$ is the resultant wave number and n is the frequency of the harmonic disturbance, which is, in general, a complex constant.

Using expression (33) in (30)-(32), (27), (23) and(24) become

$$\frac{n}{\in}\left[\frac{d^2}{dz^2} - k^2\right]W = -gk^2\alpha\Theta - \frac{1}{k_1}(v - v'n)\left(\frac{d^2}{dz^2} - k^2\right)W - \frac{mNn}{\in\rho_0\left(\frac{m}{K}n+1\right)}\left(\frac{d^2}{dz^2} - k^2\right)W - \frac{2\Omega}{\in}\frac{dZ}{dz} + \frac{\mu_e H}{4\pi\rho_0}\frac{d}{dz}\left(\frac{d^2}{dz^2} - k^2\right)K, \tag{34}$$

$$\frac{n}{\in}Z = -\frac{1}{k_1}(v - v'n) - \frac{mNn}{\in\rho_0\left(\frac{m}{K}n+1\right)}Z + \frac{2\Omega}{\in}\frac{dW}{dz} + \frac{\mu_e H}{4\pi\rho_0}DX, \tag{35}$$

$$\in nX = H\frac{dZ}{dz} + \in \eta\left(\frac{d^2}{dz^2} - k^2\right)X, \tag{36}$$

$$\in nK = H\frac{dW}{dz} + \in \eta\left(\frac{d^2}{dz^2} - k^2\right)K, \tag{37}$$

$$(E + b \in)n\Theta = \beta(W + bS) + \kappa\left(\frac{d^2}{dz^2} - k^2\right)\Theta, \tag{38}$$

$$(E' + b' \in)n\Gamma = \beta'(W + b'S) + \kappa'\left(\frac{d^2}{dz^2} - k^2\right)\Gamma. \tag{39}$$

Equations (34) – (39) in non dimensional form, become

$$\left[\frac{\sigma}{\in}\left(1 + \frac{M}{1+\tau_1\sigma}\right) + \frac{1-F\sigma}{P_l}\right](D^2 - a^2)W + \frac{ga^2 d^2\alpha\theta}{v} + \frac{2\Omega d^3}{\in v}DZ - \frac{\mu_e Hd}{4\pi v\rho_0}(D^2 - a^2)DK = 0, \tag{40}$$

$$\left[\frac{\sigma}{\in}\left(1 + \frac{M}{1+\tau_1\sigma}\right) + \frac{1-F\sigma}{P_l}\right]Z = \left(\frac{2\Omega d^2}{\in v}\right)DW + \frac{\mu_e Hd}{4\pi v\rho_0}DX, \tag{41}$$

$$[D^2 - a^2 - p_1\sigma]X = -\left(\frac{Hd}{\in\eta}\right)DZ, \tag{42}$$

$$[D^2 - a^2 - p_2\sigma]K = -\left(\frac{Hd}{\in\eta}\right)DW, \tag{43}$$

$$[D^2 - a^2 - E_1 p_1\sigma]\Theta = -\left(\frac{\beta d^2}{\kappa}\right)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \tag{44}$$

$$[D^2 - a^2 - E_1'p_1'\sigma]\Gamma = -\left(\frac{\beta' d^2}{\kappa'}\right)\left(\frac{B'+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \tag{45}$$

where we have put

$$a = kd, \sigma = \frac{nd^2}{v}, \tau = \frac{m}{K'}, \tau_1 = \frac{\tau v}{d^2}, M = \frac{mN}{\rho_0},$$

$E_1 = E + b \in$, B = b+1, $= \frac{v'}{d^2}$, $P_l = \frac{k_1}{d^2}$, is the dimensionless medium permeability, $p_1 = \frac{v}{\kappa}$, is the thermal Prandtl number, $p_1 = \frac{v}{\kappa'}$, is the Schmidt number, $p_2 = \frac{v}{\eta}$, is the magnetic Prandtl number and $D^* = d\frac{d}{dz}$ and the superscript * is suppressed.

Applying the operator $(D^2 - a^2 - p_2\sigma)$ to the equation (41) to eliminate X between equations (41) and (42), we get

$$\left\{\left[\frac{\sigma}{\epsilon}\left(1 + \frac{M}{1+\tau_1\sigma}\right) + \frac{1-F\sigma}{P_l}\right](D^2 - a^2 - p_2\sigma) + \frac{Q}{\epsilon}D^2\right\}W = \frac{2\Omega d^2}{v}(D^2 - a^2 - p_2\sigma)DW. \quad (46)$$

Eliminating K, Θ and Z between equations (40) – (46), we obtain

$$\left[\frac{\sigma}{\epsilon}\left(1 + \frac{M}{1+\tau_1\sigma}\right) + \frac{1-F\sigma}{P_l}\right](D^2 - a^2)(D^2 - a^2 - E_1p_1\sigma)(D^2 - a^2 - p_2\sigma)(D^2 - a^2 - E_1'p_1'\sigma)W -$$

$$Ra^2\lambda\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)(D^2 - a^2 - E_1'p_1'\sigma)(D^2 - a^2 - p_2\sigma)W + Sa^2\lambda\left(\frac{B'+\tau_1\sigma}{1+\tau_1\sigma}\right)(D^2 - a^2 - E_1p_1\sigma)(D^2 - a^2 - p_2\sigma)W +$$

$$\frac{Q}{\epsilon}(D^2 - a^2)(D^2 - a^2 - E_1'p_1'\sigma)(D^2 - a^2 - E_1p_1\sigma)W +$$

$$\left[\frac{\frac{T_A}{\epsilon^2}(D^2-a^2-E_1p_1\sigma)(D^2-a^2-E_1'p_1'\sigma)(D^2-a^2-p_2\sigma)^2}{\left[\frac{\sigma}{\epsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right](D^2-a^2-p_2\sigma)+\frac{Q}{\epsilon}D^2}\right]D^2W = 0, \quad (47)$$

where $R = \frac{g_0\alpha\beta d^4}{v\kappa}$, is the thermal Rayleigh number,

$S = \frac{g_0\alpha'\beta'd^4}{v\kappa'}$, is the analogous solute Rayleigh number,

$Q = \frac{\mu_e H^2 d^2}{4\pi v\rho_0\eta}$, is the Chandrasekhar number

and $T_A = \left(\frac{2\Omega d^2}{v}\right)^2$, is the Taylor number.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries and adjoining medium is electrically non conducting. The boundary conditions appropriate to the problem are (Chandrasekhar, 1981; Veronis, 1965)

$W = D^2W = DZ = \Gamma = \Theta = 0$ *at z = 0* and *1* (48)

and the components of *h* are continuous. Since the components of the magnetic field are continuous and the tangential components are zero outside the fluid, we have

$DK = 0,$ (49)

on the boundaries. Using the boundary conditions (48) and (49), we can show that all the even order derivatives of W must vanish for z = 0 and z = 1 and hence, the proper solution of equation (47) characterizing the lowest mode is

$W = W_0 \sin\pi z$; W_0 is a constant. (50)

Substituting equation (50) in (47), we obtain the dispersion relation

$$R_1 x\lambda = \left[\frac{i\sigma_1}{\epsilon}\left(1 + \frac{M}{1+\tau_1\pi^2 i\sigma_1}\right) + \frac{1-F\pi^2 i\sigma_1}{P}\right](1 + x)(1 + x + E_1p_1 i\sigma_1)\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right) +$$

$$\frac{S_1 x\lambda(1+x+E_1p_1 i\sigma_1)}{(D^2-a^2-E_1'p_1'\sigma)}\left(\frac{B'+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right) +$$

$$\frac{Q_1}{\epsilon}\frac{(1+x)(1+x+E_1p_1 i\sigma_1)}{1+x+p_2 i\sigma_1}\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right) +$$

$$\frac{\frac{T_{A_1}}{\epsilon^2}(1+x+E_1p_1 i\sigma_1)}{\frac{i\sigma_1}{\epsilon}\left(1+\frac{M}{1+\tau_1\pi^2 i\sigma_1}\right)+\frac{1-F\pi^2 i\sigma_1}{P}}\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right) \quad (51)$$

where $R_1 = \frac{R}{\pi^4}$, $S_1 = \frac{S}{\pi^4}$, $T_{A_1} = \frac{T_A}{\pi^4}$, $x = \frac{a^2}{\pi^2}$, $i\sigma_1 = \frac{\sigma}{\pi^2}$, $P = \pi^2 P_l$, $Q_1 = \frac{Q}{\pi^4}$.

Equation (51) is required dispersion relation accounting for the effect of suspended particles, stable solute gradient, magnetic field, medium permeability, variable gravity field, rotation on thermosolutal convection in Walters' (Model B') elastico-viscous fluid in porous medium.

5. STABILITY OF THE SYSTEM AND OSCILLATORY MODES:

Here we examine the possibility of oscillatory modes, if any, in Walters' (Model B') elastico-viscous fluid due to the presence of suspended particles, stable solute gradient, rotation, magnetic field, viscoelasticity and variable gravity field. Multiply equation (40) by W^* the complex conjugate of W, integrating over the range of z and making use of equations (41)-(44) with the help of boundary conditions (48) and (49), we obtain

$$\left[\frac{\sigma}{\epsilon}\left(1 + \frac{M}{1+\tau_1\sigma}\right) + \frac{1-F\sigma}{P_l}\right]I_1 - \frac{\mu_e\epsilon\eta}{4\pi v\rho_0}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)(I_2 + p_2\sigma^*I_3) - \frac{\alpha a^2\lambda g_0\kappa}{v\beta}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)(I_4 + E_1p_1\sigma^*I_5) +$$

$$d^2\left[\frac{\sigma^*}{\epsilon}\left(1 + \frac{M}{1+\tau_1\sigma}\right) + \frac{1-F\sigma^*}{P_l}\right]I_6 +$$

$$\frac{\mu_e\epsilon\eta d^2}{4\pi v\rho_0}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)(I_7 + p_2\sigma^*I_8) +$$

$$\frac{\alpha'a^2\lambda g_0\kappa'}{v\beta'}\left(\frac{1+\tau_1\sigma^*}{B'+\tau_1\sigma^*}\right)(I_9 + E_1'p_1'\sigma^*I_{10}) = 0, \quad (52)$$

where $I_1 = \int_0^1(|DW|^2 + a^2|W|^2)\,dz$,

$I_2 = \int_0^1(|D^2K|^2 + a^4|K|^2 + 2a^2|DK|^2)\,dz$,

$I_3 = \int_0^1(|DK|^2 + a^2|K|^2)\,dz$,

$I_4 = \int_0^1(|D\Theta|^2 + a^2|\Theta|^2)\,dz$,

$I_5 = \int_0^1|\Theta|^2 dz$,

$I_6 = \int_0^1|Z|^2 dz$,

$I_7 = \int_0^1(|DX|^2 + a^2|X|^2)\,dz$,

$I_8 = \int_0^1|X|^2 dz$,

$I_9 = \int_0^1(|D\Gamma|^2 + a^2|\Gamma|^2)\,dz$,

$I_{10} = \int_0^1|\Gamma|^2 dz$.

The integral part I_1-I_{10} are all positive definite. Putting $\sigma = i\sigma_i$ in equation (52), where σ_i is real and equating the imaginary parts, we obtain

$$\sigma_i\left\{\left[\frac{1}{\epsilon}\left(1 + \frac{M}{1+\tau_1^2\sigma_i^2}\right) - \frac{F}{P_l}\right](I_1 - d^2I_4) - \frac{\mu_e\epsilon\eta}{4\pi v\rho_0}\left[\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}\right)I_2 + \frac{B+\tau_1^2\sigma_i^2}{B^2+\tau_1^2\sigma_i^2}p_2I_3\right] + \frac{\alpha a^2\lambda g_0\kappa}{v\beta}\left[\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}\right)I_4 +$$

$\frac{B+\tau_1{}^2\sigma_i{}^2}{B^2+\tau_1{}^2\sigma_i{}^2}E_1 p_1 I_5 \Big] \frac{\alpha'a^2\lambda g_0\kappa'}{\nu\beta'}\Big[\Big(\frac{\tau_1(B'-1)}{B'^2+\tau_1{}^2\sigma_i{}^2}\Big)I_9 +$

$\frac{B'+\tau_1{}^2\sigma_i{}^2}{B'^2+\tau_1{}^2\sigma_i{}^2}E_1'p_1'I_{10}\Big] + \frac{\mu_e\in\eta d^2}{4\pi\nu\rho_0}\Big[\Big(\frac{\tau_1(B-1)}{B^2+\tau_1{}^2\sigma_i{}^2}\Big)I_6 +$

$\frac{B+\tau_1{}^2\sigma_i{}^2}{B^2+\tau_1{}^2\sigma_i{}^2}p_2 I_8\Big]\Big\} = 0.$ (53)

Equation (53) implies that $\sigma_i = 0$ or $\sigma_i \neq 0$ which mean that modes may be non oscillatory or oscillatory. The oscillatory modes introduced due to presence of rotation, stable solute gradient, magnetic field, suspended particles, viscoelasticity and variable gravity field.

6. THE STATIONARY CONVECTION:

For stationary convection putting $\sigma = 0$ in equation (51) reduces it to

$R_1 = \frac{1+x}{\lambda xB}\Big[\frac{1+x}{P} + \frac{Q_1}{\in} + \frac{T_{A_1}(1+x)P}{\{\in(1+x)+Q_1 P\}\in}\Big] + \frac{S_1 B'}{B},$ (54)

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters T_{A_1}, B , P, Q_1 and Walters' (Model B') elastico-viscous fluid behave like an ordinary Newtonian fluid since elastico-viscous parameter F vanishes with σ

To study the effects of suspended particles, rotation and medium permeability, we examine the behavior of $\frac{dR_1}{dB}$, $\frac{dR_1}{dT_{A_1}}$, $\frac{dR_1}{dQ_1}$, $\frac{dR_1}{dS_1}$ and $\frac{dR_1}{dP}$ analytically.

Equation (54) yields

$\frac{dR_1}{dB} = -\frac{1+x}{\lambda xB^2}\Big[\frac{1+x}{P} + \frac{Q_1}{\in} + \frac{T_{A_1}(1+x)P}{\{\in(1+x)+Q_1 P\}\in}\Big] - \frac{S_1 B'}{B^2},$ (55)

which is negative implying thereby that the effect of suspended particles is to destabilize the system when the gravity increases upward from its value g_0 (i.e., $\lambda > 0$).

From equation (54), we get

$\frac{dR_1}{dT_{A_1}} = \Big(\frac{1+x}{\lambda xB}\Big)\frac{(1+x)P}{\{\in(1+x)+Q_1 P\}\in},$ (56)

which shows that rotation has stabilizing effect on the system when gravity increases upwards from its value g_0 (i.e., $\lambda > 0$). This stabilizing effect is an agreement of the earlier work of Sharma and Rana (2010).

From equation (54), we get

$\frac{dR_1}{dQ_1} = \frac{1+x}{\lambda xB}\Big[\frac{1}{\in} - \frac{T_{A_1}(1+x)P^2}{\{\in(1+x)+Q_1 P\}^2\in}\Big],$ (57)

which implies that magnetic field stabilizes the system, if

$\{\in(1+x)+Q_1 P\}^2 > T_{A_1}(1+x)P^2,$

and destabilizes the system, if

$\{\in(1+x)+Q_1 P\}^2 < T_{A_1}(1+x)P^2,$

when gravity increases upwards from its value g_0 (i.e., $\lambda > 0$).

In the absence of rotation, magnetic field has destabilizing effect on the system, when gravity increases upwards from its value g_0 (i.e. $\lambda > 0$).

From equation (54), we get

$\frac{dR_1}{dS_1} = \frac{B'}{B},$ (58)

which is positive implying thereby that the stable solute gradient has a stabilizing effect.

It is evident from equation (54) that

$\frac{dR_1}{dP} = -\frac{(1+x)^2}{\lambda xB}\Big[\frac{1}{P^2} - \frac{T_{A_1}(1+x)}{\{\in(1+x)+Q_1 P\}^2}\Big],$ (59)

From equation (58), we observe that medium permeability has destabilizing effect when

$\{\in(1+x)+Q_1 P\}^2 < T_{A_1}(1+x)P^2$ and medium permeability has a stabilizing effect when $\{\in(1+x)+Q_1 P\}^2 > T_{A_1}(1+x)P^2$, when gravity increases upwards from its value g_0 (i.e., $\lambda > 0$).

In the absence of rotation and for constant gravity field $\frac{dR_1}{dP}$ is always negative implying thereby the destabilizing effect of medium permeability. This stabilizing effect is an agreement of the earlier work of Sharma and Rana (2010).

The dispersion relation (54) is analyzed numerically. Graphs have been plotted by giving some numerical values to the parameters, to depict the stability characteristics.
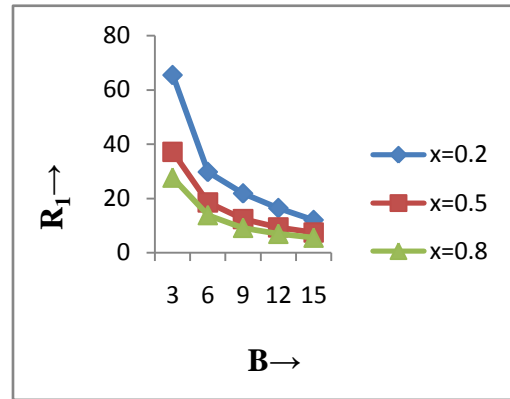

Fig. 1. Variation of Rayleigh number R_1 with suspended particles B *for* $\lambda = 2$, $T_{A_1} = 5$, $Q_{1 = 10}$, $\in = 0.2$, $P = 0.2$, $S_1 = 10$, $B' = 2$ for fixed non-dimensional wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$.
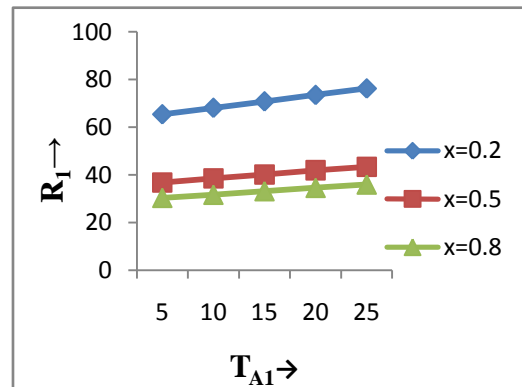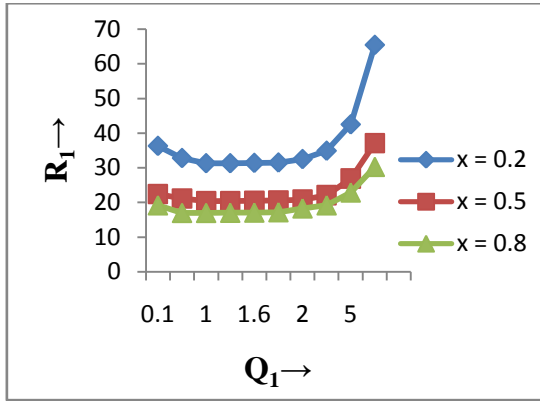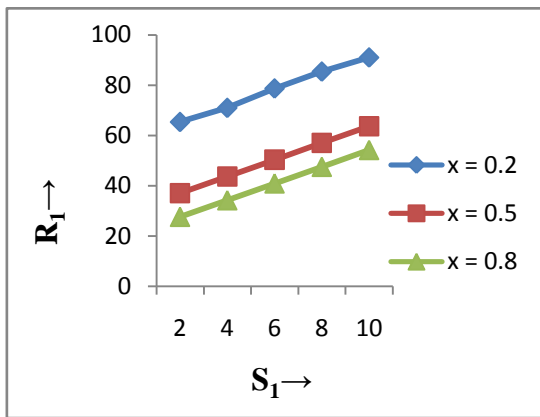

Fig. 2. Variation of Rayleigh number R_1 with rotation T_{A_1} for $B = 3$, $\lambda = 2$, $\in = 0.2$, $Q_{1 = 10}$, $P = 0.2$, $S_1 = 10$, $B' = 2$ for fixed non-dimensional wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$.

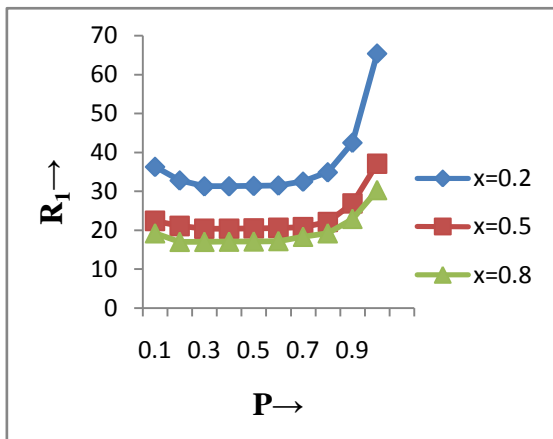**Fig. 3.** Variation of Rayleigh number R_1 with magnetic field Q_1 for $B = 3$, $\lambda = 2$, $\in = 0.2$, $P = 0.2$, $T_{A_1} = 5$, $S_1 = 10$, $B' = 2$ for fixed non-dimensional wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$.

Fig. 4. Variation of Rayleigh number R_1 with magnetic field S_1 for $B = 3$, $\lambda = 2$, $\in = 0.2$, $P = 0.2$, $T_{A_1} = 5$, $Q_{1 = 10}$ for fixed non-dimensional wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$.

Fig. 5. Variation of Rayleigh number R_1 with medium permeability P for $B = 3$, $= 2$, $Q_{1 = 2}$, $\in = 0.2$, $T_{A_1} = 5$, $S_1 = 10$, $B' = 2$ for fixed non-dimensional wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$.

In fig.1, Rayleigh number R_1 is plotted against suspended particles B for $\lambda = 2$, $T_{A_1} = 5$, $\in = 0.5$, P = 0.2, $Q_{1 = 10}$, $S_1 = 10$, $B' = 2$ for fixed non-dimensional wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$. For the non-dimensional wave numbers $x = 0.2$, x

$= 0.5$ and $x = 0.8$, suspended particles have a destabilizing effect. **In fig.2**, Rayleigh number R_1 is plotted against rotation T_{A_1} for B = 3, $\lambda = 2$, $\in = 0.2$, P = 0.2, $Q_{1 = 10}$, $S_1 = 10$, $B' = 2$ for fixed non-dimensional wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$. This shows that rotation has a stabilizing effect for fixed wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$.

In fig.3, Rayleigh number R_1 is plotted magnetic field Q_1 for B = 3, $\lambda = 2$, $\in = 0.2$, $T_{A_1} = 5$, P = 0.2, $S_1 = 10$, $B' = 2$ for fixed non-dimensional wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$. This shows that magnetic field has a destabilizing effect for $Q_1 = 0.1$ to 1.5 and has a stabilizing effect for $Q_1 = 1.5$ to 10. **In fig.4**, Rayleigh number R_1 is plotted against stable solute gradient B' for B = 3, $\lambda = 2$, $\in = 0.2$, P = 0.2, $Q_{1 = 10}$, $S_1 = 10$, for fixed wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$. This shows that the stable solute gradient has a stabilizing effect for fixed non-dimensional wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$.

In fig.5, Rayleigh number R_1 is plotted against medium permeability P for B = 3, $\lambda = 2$, $\in = 0.2$, $T_{A_1} = 5$, $Q_{1 = 2}$, $S_1 = 10$, $B' = 2$ for fixed non-dimensional wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$. This shows that medium permeability has a destabilizing effect for P = 0.1 to 0.8 and has a stabilizing effect for P = 0.8 to 2.0.

7. CONCLUSION

The thermosolutal convection in Walters' (Model B') elastico-viscous rotating fluid permeated with suspended particles and variable gravity field in porous medium in hydromagnetics has been investigated. For the stationary convection, Walters' (Model B') elastico-viscous fluid behave like an ordinary Newtonian fluid and it has been found that the rotation has stabilizing effect on the system as gravity increases upward from its value g_0 (i.e. for $\lambda > 0$). The stable solute gradient has stabilizing effect on the system and is independent of gravity field. The suspended particles are found to have destabilizing effect on the system as gravity increases upward from its value g_0 (i.e. for $\lambda > 0$) whereas the medium permeability has a stabilizing / destabilizing effect on the system for $\{\in (1 + x) + Q_1 P\}^2 < T_{A_1}(1 + x)P^2/ \{\in (1 + x) + Q_1 P\}^2 > T_{A_1}(1 + x)P^2$, as gravity increases upward from its value g_0 (i.e. for $\lambda > 0$). The magnetic field has stabilizing / destabilizing effect on the system for

$\{\in (1 + x) + Q_1 P\}^2 > T_{A_1}(1 + x)P^2/\{\in (1 + x) + Q_1 P\}^2 < T_{A_1}(1 + x)P^2$, as gravity increases upward from its value g_0 (i.e. for $\lambda > 0$). The presence of rotation, gravity field, suspended particles and viscoelasticity introduces oscillatory modes. The effects of rotation, suspended particles and medium permeability on thermal instability have also been shown graphically.

REFERENCES

Bhatia, P. K. and J. M. Steiner (1973). Thermal instability of a fluid layer in hydromagnetics. *J. Math. Anal. Appl.* 41, 271-289.

Brakke, M. K. (1955). Zone electrophoresis of dyes, proteins and viruses in density gradient columns of sucrose solutions. *Arch. Biochem. Biophys.* 55, 175-190.

Chandrasekhar, S. (1981) *Hydrodynamic and Hydromagnetic Stability.* Dover Publications, New York.

Lapwood, E. R. (1948). Convection of a fluid in porous medium. *Proc. Camb. Phil. Soc.* 44, 508-819.

Lister, C. R. B. (1972). On the thermal balance of a mid ocean ridge. *Geophysics J. Roy. Astr. Soc.* 26, 515-535.

McDonnel, J. A. M. (1978). *Cosmic Dust.* John Wiley and Sons, Toronto.

Pradhan, G. K. and P. C. Samal (1987). Thermal instability of a fluid layer under variable body forces. *J. Math. Anal. Appl.* 122, 487-498.

Rana, G. C. and S. Kumar (2010). Thermal instability of Rivlin-Ericksen elastico-viscous rotating fluid permeated with suspended particles under varaiable gravity field in porous medium. *Studia Geotechnica et Mechanica XXXII, 39-54*

Rivlin, R. S. and J. L. Ericksen (1955). Stress deformation relations for isotropic materials. *J. Rational Mech. Anal.* 4, 323-334.

Scanlon, J. W. and L. A. Segel (1973). Effect of suspended particles on onset of Be'nard convection. *Physics Fluids*, 16, 1573-1578.

Sharma, R. C. (1975). Thermal instability of viscoelastic fluid in hydromagnetics. *Acta Physica Hungaric,* 38, 293-298.

Sharma, R. C. and Sunil (1994). Thermal instability of an Oldroydian fluid with suspended particles in hydromagnetics in porous medium. *J. of Polymer Plastic Technology and Engineering* 33, 323-339.

Sharma, V. and U. Gupta (2010). Stability of stratified elastico-viscous Walters' (Model B') fluid in the presence of horizontal magnetic field and rotation. *Studia Geotechnica et Mechanica* XXXII, 41-53.

Sharma, V and G. C. Rana (2001). Thermal instability of Walters' (Model B') elastico-viscous rotating fluid in the presence of varaiable gravity field and rotation in porous medium. *J. of Non-Equilib. Thermodyn.* 26. 31-40.

Sharma, V. and G. C. Rana (2003). Thermosolutal instability of a Walters' ((Model B') elastico-viscous rotating fluid in the presence of magnetic field and variable gravity field in porous medium. *Proc. Nat. Acad. Sci. INDIA*, **73**, 93-111.

Veronis, G. (1967). On finite amplitude instability in thermohaline convection. *J. Marine Res.*, 23, 1-17.

Walters', K. (1962). The solution of flow problems in the case of materials with memory, *J. Mecanique*, 1, 469-778.

Wooding, R. A. (1960). Rayleigh instability of a thermal boundary layer in flow through porous medium. *J. Fluid Mech.* 9, 183-192.