



The Effects of Variable Viscosity and Thermal Conductivity on MHD Oscillatory Free Convective Flow past a Vertical Plate in Slip Flow Regime with Variable Suction and Periodic Plate Temperature

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ABSTRACT

The effects of variable viscosity and thermal conductivity on free convective oscillatory flow of a viscous incompressible and electrically conducting fluid past a vertical plate in slip flow regime with periodic plate temperature when suction velocity oscillates in time about a constant mean is discussed. The fluid viscosity and thermal conductivity are assumed to be inverse linear functions of temperature. The problem is governed by a coupled non-linear system of partial differential equations. Explicit finite difference method is employed to solve the equations. The effects of viscosity variation parameter, thermal conductivity variation parameter, magnetic parameter on the velocity distribution and temperature distribution for $Pr_\infty=0.7$ fluid and for rarefaction parameter $h=0$ and $h=0.4$ are discussed and shown graphically. Also the effects of these parameters on the skin friction coefficient and on the rate of heat transfer are calculated.

Keywords: MHD, natural convection, variable viscosity, variable thermal conductivity.

NOMENCLATURE

a	constants	u_1, u_2	real and imaginary parts of u
A	suction parameter	v_0	steady suction velocity
B_0	uniform magnetic field strength	x, y	axis in the direction along and normal to the surface respectively
c	constants	α	surface wave amplitude
C_f	local skin-friction coefficient	β	coefficient of thermal expansion [K ⁻¹]
c_p	specific heat at constant pressure	γ	constant
E_c	Eckert number	σ	electrical conductivity
g	acceleration due to gravity [ms ⁻²]	μ	coefficient of dynamic viscosity [Nsm ⁻²]
Gr	Grashof number	ζ	constant
h	rarefaction parameter	ρ	density [kgm ⁻³]
k	thermal conductivity [Wm ⁻¹ K]	ω	frequency parameter
M	Hartman number	ε	amplitude
Nu	local Nusselt number	ν	the kinematic viscosity [m ² s ⁻¹]
p	fluid pressure [kPa]	θ	dimensionless temperature
Pr	Prandtl number	θ_k	thermal conductivity variation parameter
t	time [s]	θ_r	viscosity variation parameter
T	temperature [K]	∞	ambient free-stream values
T_k	constant	w	wall surface conditions
T_r	constant	*	dimensional quantity
u, v	velocity components in the x - and y -directions [ms ⁻¹]		

1. INTRODUCTION

Unsteady oscillatory free convective flows play an important role in chemical engineering, turbo machinery and aerospace technology. Such flows arise due to either unsteady motion of a boundary or boundary temperature. The unsteadiness may also be due to oscillatory free stream velocity or temperature. One important class of time dependent viscous flow problems initiated by Lighthill (1954) deals with the effects of unsteady fluctuations of the main stream velocity on the flow of an incompressible fluid past two dimensional bodies.

Many researchers have worked on fluctuating flows of viscous incompressible fluid past an infinite plate. In all these papers the plate was assumed to be maintained at constant temperature. In many practical applications the particle adjacent to a solid surface no longer takes the velocity of the surface. The particle at the surface has a finite temperature of the surrounding stationary fluid. But in many applications quite often the plate temperature starts oscillating about a non-zero mean temperature. The free-convection flow is enhanced by superimposing oscillating temperature on the mean plate temperature. The transient free convection flow past an infinite vertical plate with periodic temperature variation was investigated analytically by Das *et al.* (1998) Soundalgekar and Wavre (1977) and (1977) studied the unsteady free convection flow past an infinite vertical plate and mass transfer with constant / variable suction.

In many practical applications the particle adjacent to a solid surface no longer takes the velocity of the surface. The particle at the surface has a finite tangential velocity. It slips along the surface. The flow regime is called the slip flow regime and this effect cannot be neglected (Sharma 2003) When the gas is slightly rarefied, results arguing with the observed physical phenomena can be obtained by solving the usual Navier-Stokes equations together with modified boundary conditions allowing for a velocity slip and temperature jump at the surface. This scheme of theoretical investigation of the so called slip-flow regime is particularly suitable for studying the effects of rarefaction on any classical viscous flow problem. Reddy (1964) extended the work of Stuart (1955) for the viscous fluctuating flow over an infinite plate with suction by introducing the slip flow boundary condition in place of the no-slip boundary condition. He observed that the slip boundary condition exhibited a subduing influence on the response of the skin-friction and heat transfer at the wall to the main stream fluctuation. Using this assumption Sharma and Chaudhary (2003) had discussed the free convection flow past a vertical plate in slip-flow regime and also discussed various applications for engineering purposes. Ahmed and Kalita (2008) investigated the effect of the transverse magnetic field and suction parameter on the oscillatory free convective flow past a vertical plate in slip flow regime with variable suction and periodic plate temperature. Very recently Singh and Kumar (2011) investigated the effects of radiation and first order chemical reaction on an unsteady MHD free convective heat and mass transfer flow past a vertical porous flat

plate in slip-flow regime when suction velocity oscillates in time about a non-zero constant mean.

Fluid viscosity and thermal conductivity (hence thermal diffusivity) play an important role in the flow characteristic of laminar boundary layer problems. Fluid properties are significantly affected by the variation of temperature. The increase in temperature leads to a local increase in the transport phenomena by reducing the viscosity across the momentum boundary layer and so the heat transfer rate at the wall is affected. In the cooling of electronic equipments, it is relatively frequent to find circumstances in which variable property effects are significant and cannot be neglected. Eswara and Bommai (2004) established that like other thermo-physical properties, temperature dependent viscosity vital role in surface friction and heat transfer rate near the wall. Hazarika and Lahkar (1997) observed that a significant variation takes place in velocity and temperature distribution with the variation of the viscosity and thermal conductivity parameters. Hossain *et al.* (2002) studied natural convection of fluid with variable viscosity from a heated vertical wavy surface.

In all the above works, the Prandtl Number which is a function of viscosity and thermal conductivity is assumed to be constant across the boundary layer. Pantokratoras (2005) and (2007) and Rahman *et al.* (2009) found that this assumption lead to unrealistic result in modeling the thermal boundary layer flow with a temperature-dependent viscosity. It must be treated as a variable rather than a constant within the boundary layer. The results obtained by Pantokratoras (2005) are different from those existing in the literature, which have been obtained with the assumption of constant Prandtl number.

The objective of this paper is to study the effects of variable viscosity and thermal conductivity on free convective oscillatory flow of a viscous incompressible and electrically conducting fluid past a vertical plate in slip flow regime with periodic plate temperature when suction velocity oscillates in time about a constant mean. The fluid viscosity and thermal conductivity are assumed to be inverse linear functions of temperature and hence considering the variation of Prandtl number across the boundary layer.

2. FORMULATION OF THE PROBLEM

An unsteady free-convective flow of a viscous incompressible fluid past an infinite vertical porous plate in slip flow regime with periodic temperature when variable suction velocity distribution fluctuating with time is of the form

$$v = -v_0 \left(1 + \varepsilon A e^{i\omega t} \right) \quad (\text{Ahmed and Kalita 2008}),$$

is considered. The x^* - axis is taken along the plate with direction opposite to the gravity and y^* - axis is taken to be normal to the surface. The velocity components of the fluid are u^* and v^* in the x^* and y^* directions respectively, taken parallel and perpendicular to the plate. A magnetic field $B_0(x)$ of uniform strength is imposed transversely to the direction of the flow. The magnetic Reynolds Number of the flow is taken to be small enough so that the induced magnetic field can be

neglected. The fluid property variations with temperature are limited to

- i. viscosity,
- ii. thermal conductivity and
- iii. density.

Following [Lai and Kulacki \(1990\)](#) the fluid viscosity is assumed to be inverse linear function of temperature as

$$\frac{1}{\mu} = \frac{1}{\mu_\infty} [1 + \gamma(T - T_\infty)] = a(T - T_r) \quad (1)$$

$$a = \frac{\gamma}{\mu_\infty}, T_r = T_\infty - \frac{1}{\gamma} \quad (2)$$

a and T_r are constants and their values depend on the reference state and thermal property of the fluid. In general $a > 0$ for liquids and $a < 0$ for gases. γ is a constant based on thermal property of the fluid. For $\gamma \rightarrow 0$, $\mu = \mu_\infty$ (constant).

Also the variation of thermal conductivity is considered as follows ([Hazarika and Lahkar 1997](#))

$$\frac{1}{k} = \frac{1}{k_\infty} [1 + \xi(T - T_\infty)] = c(T - T_k) \quad (3)$$

$$c = \frac{\xi}{k_\infty}, T_k = T_\infty - \frac{1}{\xi} \quad (4)$$

where c and T_k are constants and their values depend on the reference state and thermal property of the fluid. ξ is a constant based on thermal property of the fluid. $c > 0$ for liquids and $c < 0$ for gases.

The influence of variation of density with temperature is restricted to the body force term only, in accordance with the Boussinesq approximation. Since the plate is considered infinite in the x^* -direction, hence all physical quantities will be independent of x^* . Under this assumption, the physical variables are functions of y^* and t^* . Thus the equations which govern the free-convective unsteady hydromagnetic flow are:

$$\frac{\partial v^*}{\partial y^*} = 0 \Rightarrow v^* = -v_0 \left(1 + \varepsilon A e^{i\omega^* t^*} \right) \quad (5)$$

$$\begin{aligned} \frac{\partial u^*}{\partial t^*} - v_0 \left(1 + \varepsilon A e^{i\omega^* t^*} \right) \frac{\partial u^*}{\partial y^*} \\ = \frac{1}{\rho_\infty} \frac{\partial}{\partial y^*} \left(\mu \frac{\partial u^*}{\partial y^*} \right) - \frac{\sigma B_0^2 u^*}{\rho} + g\beta(T - T_\infty) \end{aligned} \quad (6)$$

$$\begin{aligned} c_p \left[\frac{\partial T^*}{\partial t^*} - v_0 \left(1 + \varepsilon A e^{i\omega^* t^*} \right) \frac{\partial T^*}{\partial y^*} \right] \\ = \frac{1}{\rho_\infty} \frac{\partial}{\partial y^*} \left(k \frac{\partial T^*}{\partial y^*} \right) + \frac{\mu}{\rho_\infty} \left(\frac{\partial u^*}{\partial y^*} \right)^2 \end{aligned} \quad (7)$$

The boundary conditions are

$$u^* = h^* \left(\frac{\partial u^*}{\partial y^*} \right), T = T_w + \varepsilon (T_w - T_\infty) e^{i\omega^* t^*} \text{ at } y^* = 0 \quad (8)$$

$$u^* \rightarrow 0, T \rightarrow T_\infty \text{ as } y^* \rightarrow \infty$$

where g is the acceleration due to gravity, T is the fluid temperature, σ is the electrical conductivity, ρ is the fluid density, c_p is the specific heat of the fluid at

constant pressure, μ is the fluid viscosity, k is the thermal conductivity of the fluid, β is the thermal expansion coefficient, ε ($\ll 1$) is the amplitude, v_0 is the steady suction velocity (normal) at the plate, h^* is the Rarefaction parameter, A is the suction parameter, ω^* is the frequency parameter, the subscripts ∞ denotes the free stream condition, and w denotes condition at the wall respectively.

We introduce the following non-dimensional quantities

$$\begin{aligned} y = \frac{y^* v_0}{\mathcal{G}_\infty}, t = \frac{t^* v_0^2}{4\mathcal{G}_\infty}, \omega = \frac{4\mathcal{G}_\infty \omega^*}{v_0^2}, \\ u = \frac{u^*}{v_0}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, G_r = g\beta \mathcal{G}_\infty \frac{T - T_\infty}{v_0^3}, \\ M = \frac{\sigma B_0^2 \mathcal{G}_\infty}{\rho_\infty v_0^2}, h = \frac{v_0 h^*}{\mathcal{G}_\infty}, E_c = \frac{v_0^2}{c_p (T_w - T_\infty)}, \\ Pr_\infty = \frac{\mu_\infty c_p}{k_\infty}, \mathcal{G}_\infty = \frac{\mu_\infty}{\rho_\infty} \end{aligned} \quad (9)$$

Using the transformation [Eq. \(9\)](#), [Eq. \(6\)](#) and [Eq. \(7\)](#)

reduces to the following non-dimensional form,

$$\begin{aligned} \frac{1}{4} \frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{i\omega t} \right) \frac{\partial u}{\partial y} = \frac{\theta_r}{(\theta_r - \theta)^2} \frac{\partial u}{\partial y} \frac{\partial \theta}{\partial y} \\ + G_r \theta + \frac{\theta_r}{\theta_r - \theta} \frac{\partial^2 u}{\partial y^2} - Mu \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{1}{4} \frac{\partial \theta}{\partial t} - \left(1 + \varepsilon A e^{i\omega t} \right) \frac{\partial \theta}{\partial y} = \frac{1}{Pr_\infty} \frac{\theta_k}{(\theta_k - \theta)^2} \left(\frac{\partial \theta}{\partial y} \right)^2 \\ + \frac{1}{Pr_\infty} \frac{\theta_k}{\theta_k - \theta} \frac{\partial^2 \theta}{\partial y^2} + E_c \frac{\theta_r}{\theta_r - \theta} \left(\frac{\partial u}{\partial y} \right)^2 \end{aligned} \quad (11)$$

where \mathcal{G}_∞ is the kinematic viscosity, G_r is the Grashof Number, M is the Hartmann Number, h is the dimensionless rarefaction parameter, E_c is the Eckert number, Pr_∞ is the ambient Prandtl number,

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} \text{ is the viscosity variation parameter and}$$

$$\theta_k = \frac{T_k - T_\infty}{T_w - T_\infty} \text{ is the thermal conductivity variation parameter.}$$

The Prandtl number $Pr = \frac{\mu c_p}{k}$ is a function of

viscosity as well as thermal conductivity and as both the viscosity and thermal conductivity varies across the boundary layer, the Prandtl number also varies. The assumption of constant Prandtl number may produce unrealistic results ([Rahman et al. 2009](#)) Therefore, the variable Prandtl number related to the variable viscosity and variable thermal conductivity is defined as

$$Pr = \frac{\mu c_p}{k} = \frac{\theta_r}{\theta_r - \theta} \frac{\theta_k - \theta}{\theta_k} Pr_\infty \quad (12)$$

Using [Eq. \(12\)](#), the non-dimensional temperature [Eq. \(11\)](#) can be written as

$$\begin{aligned} & \frac{1}{4} \frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \theta}{\partial y} \\ &= \frac{1}{Pr} \frac{\theta_r}{(\theta_r - \theta)(\theta_k - \theta)} \left(\frac{\partial \theta}{\partial y} \right)^2 \\ &+ \frac{1}{Pr} \frac{\theta_r}{\theta_r - \theta} \frac{\partial^2 \theta}{\partial y^2} + E_c \frac{\theta_r}{\theta_r - \theta} \left(\frac{\partial u}{\partial y} \right)^2 \end{aligned} \quad (13)$$

which is the corrected non-dimensional form of the energy equation for modeling thermal boundary layer flows with temperature dependent viscosity and thermal conductivity.

The corresponding boundary conditions in non-dimensional form

$$\begin{aligned} y=0 & ; \quad u = h \frac{\partial u}{\partial y}, \quad \theta = 1 + \varepsilon e^{i\omega t} \\ y \rightarrow \infty & ; \quad u = 0, \quad \theta = 0 \end{aligned} \quad (14)$$

3. SOLUTION OF THE PROBLEM

In order to solve the differential Eq. (10) and Eq. (13) we assume that:

$$u = u_1 + i u_2 \quad (15)$$

$$\theta = \theta_1 + i \theta_2 \quad (16)$$

By substituting Eq. (15) and Eq. (16) into Eq. (10) and Eq. (13) and equating real and imaginary parts we get the following system of differential equation:

$$\begin{aligned} & \frac{1}{4} \frac{\partial u_1}{\partial t} - (1 + \varepsilon A \cos \omega t) \frac{\partial u_1}{\partial y} + \varepsilon A \sin \omega t \frac{\partial u_2}{\partial y} \\ &= G_r \theta_1 - M u_1 \\ &+ \frac{\theta_r}{(\theta_r - \theta_1)^2 + \theta_2^2} \left((\theta_r - \theta_1) \frac{\partial^2 u_1}{\partial y^2} - \theta_2 \frac{\partial^2 u_2}{\partial y^2} \right) \\ &+ \frac{\theta_r}{((\theta_r - \theta_1)^2 + \theta_2^2)^2} \\ &\left\{ \begin{aligned} & ((\theta_r - \theta_1)^2 - \theta_2^2) \left(\frac{\partial u_1}{\partial y} \frac{\partial \theta_1}{\partial y} - \frac{\partial u_2}{\partial y} \frac{\partial \theta_2}{\partial y} \right) \\ & - 2\theta_2 (\theta_r - \theta_1) \left(\frac{\partial u_2}{\partial y} \frac{\partial \theta_1}{\partial y} + \frac{\partial u_1}{\partial y} \frac{\partial \theta_2}{\partial y} \right) \end{aligned} \right\} \end{aligned} \quad (17)$$

$$\begin{aligned} & \frac{1}{4} \frac{\partial u_2}{\partial t} - (1 + \varepsilon A \cos \omega t) \frac{\partial u_2}{\partial y} - \varepsilon A \sin \omega t \frac{\partial u_1}{\partial y} = G_r \theta_2 + \\ & \frac{\theta_r}{(\theta_r - \theta_1)^2 + \theta_2^2} \left((\theta_r - \theta_1) \frac{\partial^2 u_2}{\partial y^2} + \theta_2 \frac{\partial^2 u_1}{\partial y^2} \right) - M u_2 + \\ & \frac{\theta_r}{((\theta_r - \theta_1)^2 + \theta_2^2)^2} \\ & \left\{ \begin{aligned} & ((\theta_r - \theta_1)^2 - \theta_2^2) \left(\frac{\partial u_2}{\partial y} \frac{\partial \theta_1}{\partial y} + \frac{\partial u_1}{\partial y} \frac{\partial \theta_2}{\partial y} \right) \\ & + 2\theta_2 (\theta_r - \theta_1) \left(\frac{\partial u_1}{\partial y} \frac{\partial \theta_1}{\partial y} - \frac{\partial u_2}{\partial y} \frac{\partial \theta_2}{\partial y} \right) \end{aligned} \right\} \end{aligned} \quad (18)$$

$$\begin{aligned} & \frac{1}{4} \frac{\partial \theta_1}{\partial t} - (1 + \varepsilon A \cos \omega t) \frac{\partial \theta_1}{\partial y} + \varepsilon A \sin \omega t \frac{\partial \theta_2}{\partial y} \\ &= \frac{1}{Pr} \frac{\theta_r}{(\theta_r - \theta_1)^2 + \theta_2^2} \left((\theta_r - \theta_1) \frac{\partial^2 \theta_1}{\partial y^2} - \theta_2 \frac{\partial^2 \theta_2}{\partial y^2} \right) + \\ & \frac{1}{Pr} \frac{\theta_r}{((\theta_r - \theta_1)^2 + \theta_2^2) ((\theta_k - \theta_1)^2 + \theta_2^2)} \\ & \left\{ \begin{aligned} & ((\theta_r - \theta_1)(\theta_k - \theta_1) - \theta_2^2) \\ & \left(\left(\frac{\partial \theta_1}{\partial y} \right)^2 - \left(\frac{\partial \theta_2}{\partial y} \right)^2 \right) \\ & - 2\theta_2 (\theta_r + \theta_k - 2\theta_1) \\ & \frac{\partial \theta_1}{\partial y} \frac{\partial \theta_2}{\partial y} \end{aligned} \right\} \\ & + E_c \frac{\theta_r}{(\theta_r - \theta_1)^2 + \theta_2^2} \left\{ \begin{aligned} & (\theta_r - \theta_1) \left(\left(\frac{\partial u_1}{\partial y} \right)^2 - \left(\frac{\partial u_2}{\partial y} \right)^2 \right) \\ & - 2\theta_2 \frac{\partial u_1}{\partial y} \frac{\partial u_2}{\partial y} \end{aligned} \right\} \end{aligned} \quad (19)$$

$$\begin{aligned} & \frac{1}{4} \frac{\partial \theta_2}{\partial t} - (1 + \varepsilon A \cos \omega t) \frac{\partial \theta_2}{\partial y} - \varepsilon A \sin \omega t \frac{\partial \theta_1}{\partial y} \\ &= \frac{1}{Pr} \frac{\theta_r}{((\theta_r - \theta_1)^2 + \theta_2^2) ((\theta_k - \theta_1)^2 + \theta_2^2)} \\ & \left\{ \begin{aligned} & \theta_2 (\theta_r + \theta_k - 2\theta_1) \left(\left(\frac{\partial \theta_1}{\partial y} \right)^2 - \left(\frac{\partial \theta_2}{\partial y} \right)^2 \right) \\ & + 2((\theta_r - \theta_1)(\theta_k - \theta_1) - \theta_2^2) \frac{\partial \theta_1}{\partial y} \frac{\partial \theta_2}{\partial y} \end{aligned} \right\} \\ & + \frac{1}{Pr} \frac{\theta_r}{(\theta_r - \theta_1)^2 + \theta_2^2} \left((\theta_r - \theta_1) \frac{\partial^2 \theta_2}{\partial y^2} + \theta_2 \frac{\partial^2 \theta_1}{\partial y^2} \right) \\ & + E_c \frac{\theta_r}{(\theta_r - \theta_1)^2 + \theta_2^2} \left\{ \begin{aligned} & 2(\theta_r - \theta_1) \frac{\partial u_1}{\partial y} \frac{\partial u_2}{\partial y} \\ & + \theta_2 \left(\left(\frac{\partial u_1}{\partial y} \right)^2 - \left(\frac{\partial u_2}{\partial y} \right)^2 \right) \end{aligned} \right\} \end{aligned} \quad (20)$$

The boundary condition Eq. (14) reduces to

$$\begin{aligned} y=0 & ; \quad u_1 = h \frac{\partial u_1}{\partial y}, \quad u_2 = h \frac{\partial u_2}{\partial y}, \\ \theta_1 &= 1 + \varepsilon \cos \omega t, \quad \theta_2 = \varepsilon \sin \omega t \\ y \rightarrow \infty & ; \quad u_1 = 0, \quad u_2 = 0, \quad \theta_1 = 0, \quad \theta_2 = 0 \end{aligned} \quad (21)$$

Equations (17) to (20) are coupled non-linear partial differential equations which are to be solved by using the boundary conditions (Eq. (21)). However, exact or approximate solutions are not possible for this set of equations and hence we solve these equations by finite difference methods. The central difference is used for the diffusion terms and the forward difference scheme is used for the convection terms. After experimenting with a few set of mesh sizes, the mesh sizes were fixed at $\Delta y = 0.1$ and $\Delta t = 0.01$. The procedure is repeated until $t=1$. This gives sufficient accuracy as the boundary conditions are satisfied.

The important characteristics of the problem are the skin-friction co-efficient and heat transfer rate at the plate.

The coefficient of skin-friction at the surface is given by

$$C_f = \frac{\tau}{\rho_\infty v_0^2} = \text{real part} \frac{\theta_r}{\theta_r - \theta} \frac{\partial u}{\partial y} \Big|_{y=0} \quad (22)$$

where $\tau = \mu \frac{\partial u^*}{\partial y^*} \Big|_{y^*=0}$.

The rate of heat transfer in terms of Nusselt Number at the surface is

$$Nu = \frac{q_w \mathcal{G}}{k_\infty v_0 (T_w - T_\infty)} = -\text{real part} \frac{\theta_k}{\theta_k - \theta} \frac{\partial T}{\partial y} \Big|_{y=0} \quad (23)$$

where $q_w = -k \frac{\partial T}{\partial y^*} \Big|_{y^*=0}$.

4. RESULTS AND DISCUSSION

As a result of the numerical calculations the velocity and temperature distribution for the flow are obtained from Eq. (17) to Eq. (20) which are displayed in figures for different values of θ_r (Viscosity variation parameter), θ_k (Thermal conductivity variation parameter), M (Hartmann number) which are chosen arbitrarily for fluid with Prandtl number $P_r=0.71$ (Pantokratoras 2005) and Eckert number $E_c=0.01$. In the present analysis the values of Grashof number G_r are chosen arbitrarily, we take $G_r = 5 > 0$ correspond to the cooling of the plate. Throughout our investigation we take h (rarefaction parameter) = 0 and 0.4 (Sharma 2003), suction parameter $A=0.2$ (Ahmed and Kalita 2008), frequency parameter (dimensionless) $\omega=1$ (Ahmed and Kalita 2008), and amplitude oscillation $\varepsilon=0.1$ (Ahmed and Kalita 2008). Also the skin-friction coefficient C_f and the Nusselt number Nu are calculated from Eq. (22) and Eq. (23) and are tabulated in Table 1 to Table 3 for same set of parameters.

From Table 1 it is observed that an increasing value of the viscosity parameter ($\theta_r=3, 5 \dots 15$) leads to decrease in the values of skin friction coefficient C_f for rarefaction parameter $h=0.4$ and $h=0.0$.

Table 2 depicts that the Nusselt number decreases due to increase in the thermal conductivity parameter ($\theta_k=3, 5 \dots 15$) for $h=0.4$. The results are qualitatively similar to the related study by Rahman *et al.* (2008) and show that increasing the thermal conductivity of the fluid leads to a decrease in the Nusselt number. This may partly be explained by the fact that that increasing thermal conductivity has the effect of accelerating and increasing the temperature of the fluid.

Table 3 depicts that skin friction coefficient C_f the Hartman number ($M=1, 2, 3, 4, 5$) for $h=(0, 0.4)$. It is observed that C_f increases (Muthucumaraswamy and Vadivel 2004) as M increases.

From Table 1 and Table 3, it is observed that C_f increases as $h=(0, 0.4)$ decreases which agrees with Sharma (2003).

Table 1 Effects of viscosity variation parameter θ_r on Skin-friction coefficient C_f for $P_r=0.71, G_r=5, M=0.5, E_c=0.01, \theta_k=15$

θ_r	$h=0.4$	$h=0.0$
3	1.578875	7.894374
5	1.282022	6.410112
7	1.186423	5.932116
9	1.139228	5.69614
11	1.111101	5.555506
13	1.092429	5.462145
15	1.07913	5.395649

Table 2 Effects of thermal conductivity variation parameter θ_k Nusselt number Nu for $P_r=0.71, G_r=5, M=0.5, E_c=0.01, \theta_r=15$

θ_k	$h=0.4$
3	1.580396
5	1.282624
7	1.18679
9	1.139491
11	1.111306
13	1.092596
15	1.079271

Table 3 Effects of Hartman Number M on Skin-friction coefficient C_f for $P_r=0.71, G_r=5, \theta_k=15, E_c=0.01, \theta_r=15$.

M	$h=0.4$	$h=0.0$
1	-2.43321	3.524447
2	-2.25446	3.932257
3	-2.0673	4.351734
4	-1.8716	4.783027
5	-1.66722	5.226292

There is no significant variation in Nusselt number Nu due to variation of viscosity variation parameter θ_r and Hartman number M .

The variation of velocity profiles are shown in Fig. 1 for $M=0.5, G_r=5$ where $\theta_r=5, 10, 15$ and $h=0.4$. Fig. 2 depicts the velocity profiles for same set of parameters and rarefaction parameter $h=0$.

Figure 3 and Fig. 4 represent temperature profiles for thermal conductivity variation parameter θ_k for $h=0.4$ and $h=0$ respectively. In both cases, temperature field increases with increasing θ_k . The temperature profiles are wider for rarefaction parameter $h=0.4$.

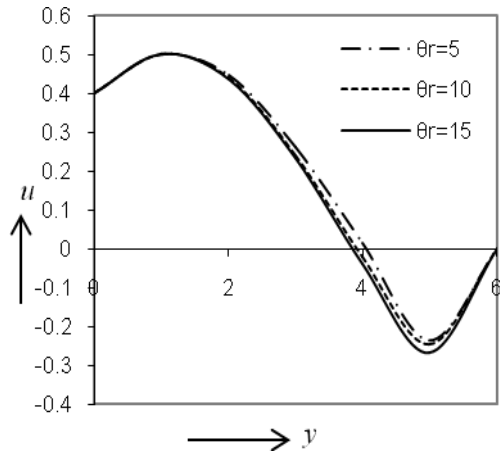


Fig. 1. Velocity Profiles for $M=0.5$, $G_r=5$, $\theta_k=15$, $h=0.4$

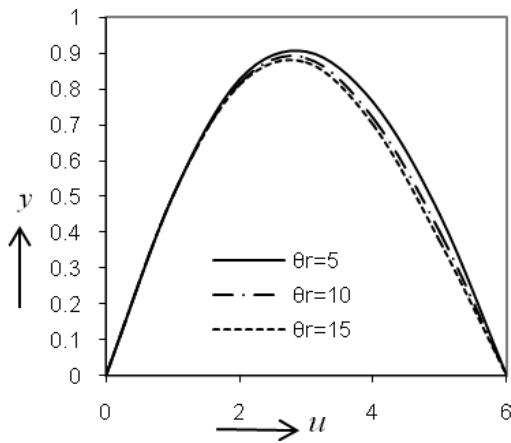


Fig. 2. Velocity Profiles for $M=0.5$, $G_r=5$, $\theta_k=15$, $h=0$.

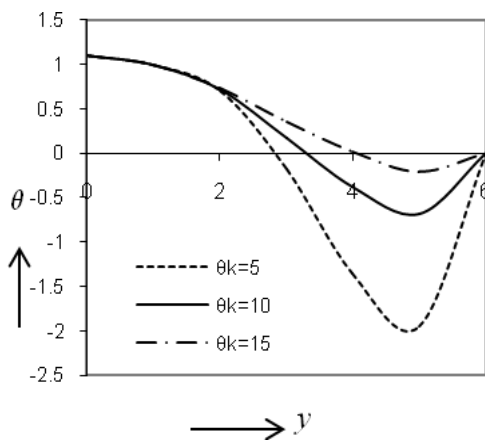


Fig. 3. Temperature Profiles for $\theta_r=15$, $G_r=5$, $M=0.5$, $h=0.4$

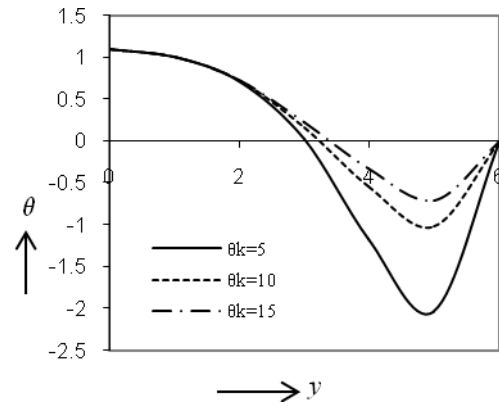


Fig. 4. Temperature Profiles for $\theta_r=15$, $G_r=5$, $M=0.5$, $h=0$

Figure 5 and Fig. 6 represent the variation in velocity distribution with the variation of Hartman number M for rarefaction parameter $h=0.4$ and $h=0$ respectively. These curves show that velocity decreases in the presence of magnetic field which agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow (Muthucumaraswamy and Vadivel 2004).

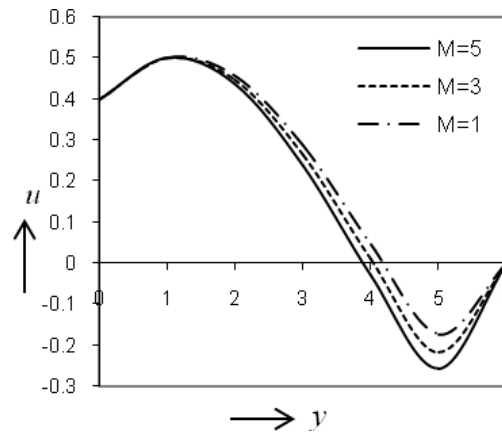


Fig. 5. Velocity Profiles for $\theta_r=15$, $G_r=5$, $\theta_k=15$, $h=0.4$.

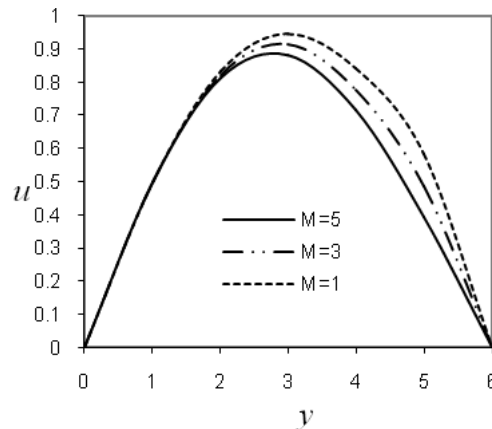


Fig. 6. Velocity Profiles for $\theta_r=15$, $G_r=5$, $\theta_k=15$, $h=0.0$

5. CONCLUSION

The theoretical solution of free convective oscillatory MHD past a vertical plate in slip flow regime with variable suction and periodic plate temperature is considered. Fluid viscosity and thermal conductivity are considered as inverse linear functions of temperature. The presented analysis has shown that for modeling thermal boundary layers with temperature dependent viscosity and thermal conductivity, the Prandtl number must be treated as variable inside the boundary layer. The flow field is appreciably influenced by the viscosity-temperature and thermal conductivity-temperature variation and also by the magnetic parameter.

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