Intermittency in the Transition to Turbulence for a Shear-Thinning Fluid in Hagen-Poiseuille Flow

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ABSTRACT

Since Reynolds's original experiments in 1883, very few studies have been carried out on the transition to turbulence for shear thinning fluid in Hagen-Poiseuille (pipe) flow. Compared to the Newtonian fluids, the results showed two interesting phenomenon: a delay in transition to turbulence and the appearance of an asymmetry in the mean axial velocity profiles. In this study, we measured experimentally the friction factor and axial velocity profiles using Laser Doppler Velocimetry system (LDV) at different Reynolds numbers. It is shown an axisymmetric in the mean axial velocity profiles for laminar and turbulent regimes and an increasing asymmetry profile with increasing Re_w for transition regime. Two different stages are clearly identified during transition to turbulence. The first stage which we called it pre-transition, took place only in the non Newtonian fluids while the second stage correspond to the appearance of turbulent puffs. Another objective of this study is the statistical analysis of the axial velocity fluctuation. In the transition regime, power spectra identified the existence of a weak turbulence. It is generated by the shear-thinning character of the fluid, which follows a power law with an exponent close to -3 (f^{-3}).

1. INTRODUCTION

The mechanism of transition from laminar to turbulence for shear thinning fluid in Hagen-Poiseuille flow through a cylindrical pipe is still unrecognized despite the importance of this problem in the design and control in several industrial process such as cementation in the upstream oil and gas industries, extrusion of molten polymers, paper coating and etc. The main difficulty of this scenario lies on the linear stability of Hagen-Poiseuille flow Meseguer and Trefethen (2003).

Concerning the transitional regime for non-Newtonian fluid flows, the existing literature reveals two interesting phenomenon with yet unexplained effect: a delay in transition to turbulence Escudier *et al.* (1999) and the appearance of an asymmetry in the mean axial velocity profiles Güzel *et al.* (2009); Esmael and Nouar (2008); Peixinho *et al.* (2005).

According to the work of Esmael and Nouar (2008), in the transition regime, a new state with a robust coherent structure characterized by two weakly modulated counter-rotating longitudinal vortices was reported for a yield-stress-thinning fluid in a pipe. In this nonlinear asymmetric state, time-averaged axial velocity profiles, exhibited increasing asymmetry with increasing Reynolds number. Recently, Escudier *et al.* (2009) suggested that both upstream and downstream flow geometry have no influence on the symmetry.

Dissipative nonlinear systems such as fluid dynamical systems can reach a chaotic state when the parameter measuring the nonlinearity is large. For instance, parallel shear flows of Newtonian fluids are turbulent when the ratio of the nonlinear inertial term and the viscous dissipative term, defined by the Reynolds number is sufficiently important. In non-Newtonian fluid flows, an additional nonlinearity is introduced via the constitutive equation (Eq. 1). For viscoelastic fluids, this nonlinearity can give rise to a turbulent flow at low Reynolds number Larson (2000); Groisman and Steinberg (2000).
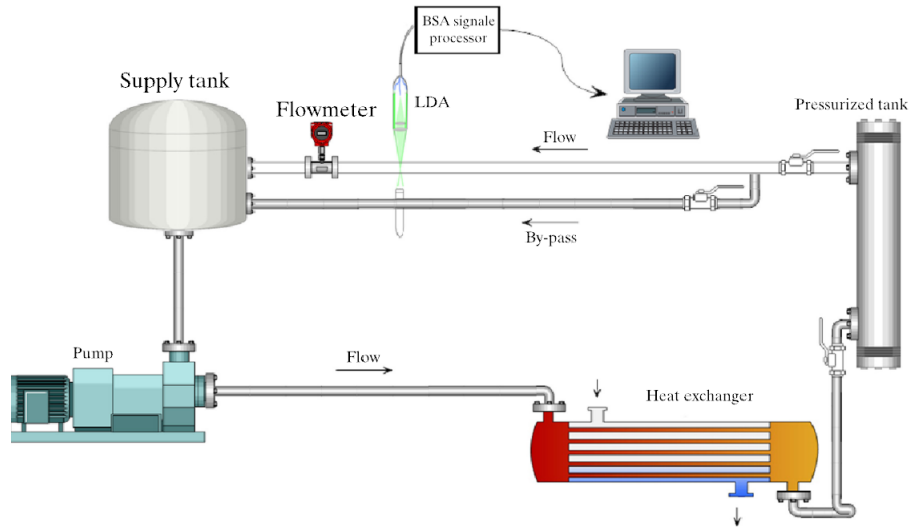
Fig. 1. Schematic of experimental set-up.

$$\frac{\partial V}{\partial t} + \overbrace{(V.\nabla)V}^{nonlinear\ inertial} = -\frac{1}{\rho}\nabla P + \underbrace{\frac{\mu_s}{\rho}\Delta V}_{viscous\ dissipative} + \frac{1}{\rho}\nabla\tau_p \quad (1)$$

Where τ_p is the parietal stress and μ_s, the fluid viscosity.

The degree of nonlinearity is expressed by the Weissenberg number which is a product of a characteristic rate of deformation and the relaxation time of the polymer. The shear-thinning behavior, non-linear decrease of the effective viscosity with the shear rate, is the most common property of non-Newtonian fluids. It is reasonable to inquire, whether an interplay between this nonlinearity and inertia can lead to a chaotic flow. This point has been addressed in Ashrafi and Khayat (2000) using low order dynamical system (generalized Lorenz system) in the Taylor-Couette flow of weakly-thinning fluid. It is shown that the additional nonlinearity gives rise to a Hopf bifurcation otherwise non-existent for Newtonian fluid.

In the present paper, we report the first experimental evidence of a weakly turbulent flow induced by the nonlinear dependence of the effective viscosity with the shear rate, resulting from an inter-play between inertia and the shear-thinning behaviour of the fluid. This chaotic flow was discovered in our study of transition to turbulence in a pipe of a non-Newtonian fluid. In this nonlinear state, time-averaged axial velocity profiles exhibit increasing asymmetry with increasing Reynolds number. A complete statistical analysis of the axial velocity fluctuations is provided with an attempt of interpretation.

2. EXPERIMENTAL SET-UP AND MEASUREMENTS OF FLUID RHEOLOGY

We carried out the tests to obtain a pressure drop, a "fluctuation and average" axial velocity in the downstream pipe (cylindrical conduit). The experimental set-up (Fig. 1) consists of a Plexiglas pipe with internal diameter of 30 mm and 4.20 m long, which is longer than the entry length necessary for the laminar flow. The velocity measurements can be performed by Laser Doppler Velocimetry (LDV, Dantec, Denmark) at horizontal (azimuthal) position.

The applicable fluid is a 0.2 wt.% aqueous solution of carbopol 940, which its stability and transparency has been widely used as a model yield stress. Flow behavior was characterized using the cone/plate geometry (6 cm/1 °) and all measurements were carried out in a controlled torque rheometer (TA Instrument AR2000). The Herschel-Bulkley model was found to satisfactory fit in the flow curves.

3. RESULTS AND DISCUSSIONS

The mean axial velocity profiles shown in Fig. 2a, cover a range of Reynolds numbers which span the three flow regimes at the azimuthal positions of $\theta = 0$ and the downstream of the pipe. The definition of the Reynolds number is as follows:

$$Re_w = \frac{\rho U_B D}{\mu_s} \quad (2)$$

Where the viscosity μ_s was determined from the Herschel-Bulkley model. The results show a zone called plug region, in which the rate of strain tensor is zero and $\tau < \tau_0$. This means that the plug region is moving with constant velocity. It's necessary to mention that this strange behavior will fade away in
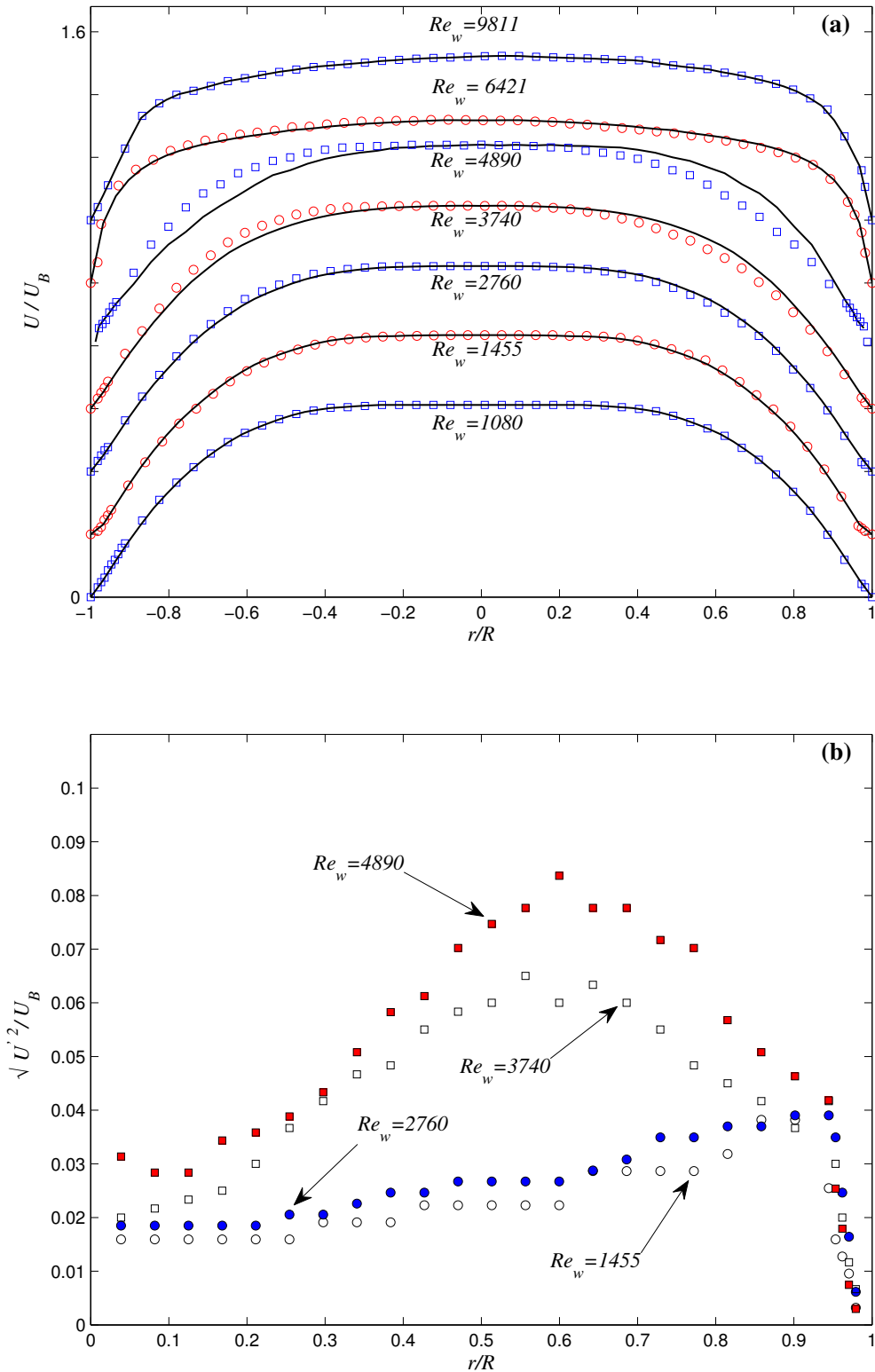
Fig. 2. (a) Mean and (b) fluctuation rate RMS axial velocity profiles of 0.2% Carbopol at the azimuthal positions of $\theta = 0$ and the downstream of the pipe, ($\tau_0 = 7.0$ Pa, K= 3.1 $Pa.s^n$, n= 0.46). The symbols denote the experimental data and the continuous line is an average velocity profile obtained by taking the average of the velocity data from either side of the centerline.
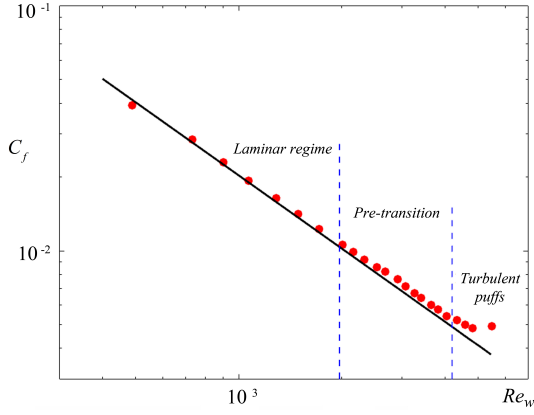
Fig. 3. Friction factor as a function of Reynolds number; The symbols are the experimental measurements and the continuous line is the theoretical laminar solution.
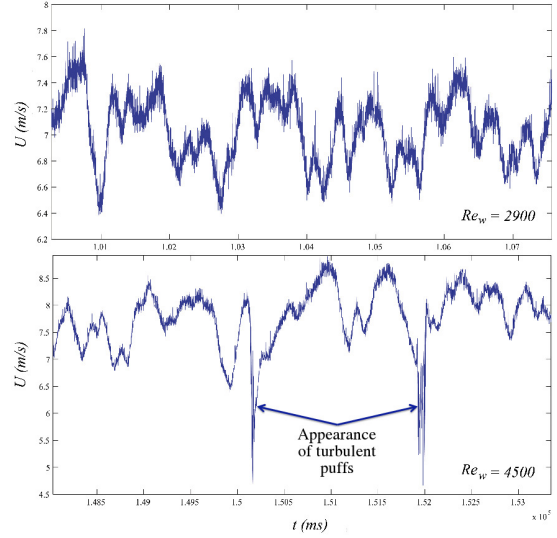

Fig. 4. Instantaneous axial velocity plots at pre-transition $Re_w = 2900$ and turbulent puffs $Re_w = 4500$ transition regime ($\theta = 0$, $\frac{r}{R} = 0.7$ and the downstream of the pipe).

the beginning of a fully developed turbulence. The flow begins to present a stable asymmetry for the Reynolds number corresponding to the transition regime (Re_w = 2760, 3740 and 4890), while in the laminar and turbulent regimes the flow is axisymmetric. It's worth reminding that, this asymmetry is slight for the pre-transition stage (Re_w = 2760 and 3740) and becomes significant for the turbulent puffs stage (Re_w = 4890). The similar tendency has also been reported by Güzel *et al.* (2009); Esmael and Nouar (2008) and Peixinho *et al.* (2005). Fluctuation rate axial velocity profiles RMS, corresponding to the mean velocity profiles are shown in Fig. 2b. The profiles are shown normalized using the bulk velocity (U_B). The results show an increase of the fluctuation rate with increasing of Re_w outside the plug zone. The profile obtained for Re_w = 2760 correspond to the first stage of the transitional regime with a light increase in the fluctuation rate. Thus, the flow can be unstable. This flow instability becomes significant for second stage of transition at Re_w = 4890. The friction factor C_f is defined as follows:

$$C_f = \frac{2\tau_p}{\rho U_B^2} \tag{3}$$

Where the parietal stress τ_p is evaluated from the measurement of ΔP:

$$\tau_p = \frac{R}{2}\frac{\Delta P}{L} \tag{4}$$

ρ is the density and ΔP a pressure drop over the length L between the two pressure tappings along the pipe.

The Fig. 3 displays the variation of friction factor (C_f) as a function of Reynolds number. Two different stages are clearly identified during transition

to turbulence. The first stage, called pre-transition, took place only in the Reynolds number between 2000 and 4000, while the second stage correspond to the appearance of turbulent puffs (Re_w ¿ 4000). In the pre-transition stage, the experimental measurements of the friction factor are very close to the theoretical laminar solution. We measured axial velocity signals concerning the first stage (Re_w = 2900) and second stage (Re_w = 4500) of transitional regime at the $\theta = 0$, $\frac{r}{R} = 0.7$ and the downstream of the pipe as shown in Fig. 4. We observed an instantaneous deep fall in velocity at 1.502 and 1.52 ms, which corresponds to the appearance of turbulent puffs at the Reynolds number of 4500.

Elements analysis of these experimental results can be obtained by adopting the Reynolds decomposition for velocity, pressure, shear and effective viscosity. The Reynolds-averaged momentum equation is:

$$\frac{\partial V_i}{\partial t} + \frac{\partial}{\partial x_k}(V_i V_k) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_k}\left[\frac{1}{Re}\overline{\mu'\dot{\gamma}'_{ik}} - \overline{v'_i v'_k}\right] + \frac{1}{Re}\frac{\partial}{\partial x_k}(\overline{\mu}\Gamma_{ik}) \tag{5}$$

By comparison with the Newtonian case, there is a new diffusive term $\overline{\mu'\dot{\gamma}'_{ik}}$, which may be viewed as pseudo Reynolds-stress tensor. The influence of this term on the flow may be analyzed first through the friction factor C_f. Using an extension of the procedure used in Fukagata *et al.* (2002), an exact relationship for C_f is derived:
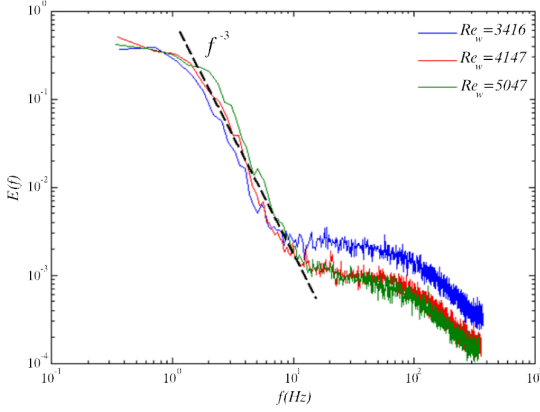
Fig. 5. Density energy spectra of axial velocity fluctuation at the azimuthal positions of $\theta = 0$, $\frac{r}{R} = 0.7$ and the downstream of the pipe, ($\tau_0 = 13.52$ Pa, K = 2.21 $Pa.s^n$, n= 0.52). The dashed lines represent the power law behavior f^{-3}.

$$C_f = C_L + \overbrace{8 \int_0^1 r^2 \overline{v'w'} dr}^{C_T} - \overbrace{\frac{8}{Re} \int_0^1 r^2 \overline{\mu' \dot{\gamma}'_{rz}} dr}^{C_{\mu'}}$$
$$- \underbrace{\frac{8}{Re} \int_0^1 r^2 \overline{\mu \Gamma_{rz}} dr}_{C_M} \quad (6)$$

This relation shows that the friction factor can be split into four contributing terms: the laminar C_L, the turbulent contribution C_T, the viscosity perturbation contribution $C_{\mu'}$, and an additional term C_M which arises from the modification of the mean velocity profile with respect to the laminar one. It can be shown straightforwardly that for a shearethinning fluid $C_{\mu'} < 0$. Thus, this term contributed to be drag reduction and probably compensates the turbulent contribution. This may explain why in this weakly turbulent flow, C_f is very close to that in laminar regime.

Power spectra E(f) of the axial velocity fluctuations at $\frac{r}{R} = 0.7$ and horizontal positions $\theta = 0$ (near the center of the high speed streak) for the three values of Reynolds, including the first and second stages of transitional regime, are shown in Fig. 5. The density energy spectra were calculated on 5×10^5 data points divided in segments of 2048 points and the obtained spectrum is an average overall segment in the full time series. As seen in Fig. 5, in the transitional regime, the power spectra, identifies the existence of a weak turbulence. It is generated by the shear-thinning character of the fluid, which follows a power law with an exponent close to -3 (f^{-3}) for a decade of $1 \le f \le 10$ Hz. The flattening of the curves at high frequency is due to an instrumental noise. Recently, Esmael et al. (2010) exhibited an exponent close to -5/3 ($f^{-5/3}$) at an early step of

transition with increasing Re_w. The spectra have a broad region of frequencies, where the fluctuation energy decays according to a power law with an exponent close to -3 (f^{-3}). The energy spectra in f^{-3} is very similar to that described in the literature for the case of a two-dimensional turbulence Tabeling (2002).

By adopting the hypothesis of Taylor, the spectra in time can be interpreted by the spectra in space by the relation between the frequency and the wave number given by:

$$k = \frac{2\pi f}{U} \quad (7)$$

This can indicate a multiplicity of spatial and temporal scale, which is a fundamental characteristic of a turbulent flow. A steep spectrum f^{-3} (k^{-3}) means that the energy is mainly concentrated in large-scale structures and the integral scale determines the velocity field. It is thus a weak turbulence which can be assimilated to a Batchelor regim flow Batchelor (1959). In this regim, the flow can be considered smooth in space and random in time and is appears in hydrodynamic turbulence below the dissipation scale Burghelea et al. (2007).

4. CONCLUSIONS

Two stages are clearly identified during the transition to turbulent. In the first stage called pretransition, a central zone (plug zone) remains in the laminar regime at the fluctuation rate. At this stage the profile of a mean axial velocity exhibits a slight asymmetry and the experimental measurements of the friction factor C_f are very close to the theoretical laminar solution, which can be explained by the new diffusive term $\overline{\mu' \dot{\gamma}'_{ik}}$ as the drag reduction. The second stage corresponds to the appearance of turbulent puffs with a significant asymmetry in the axial velocity profile. In the transitional regime, the power spectra identified the existence of a weak turbulence. This is generated by the shear-thinning character of the fluid, which follows a power law with an exponent close to -3 (f^{-3}).

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