Boundary Layer Flow and Heat Transfer over a Permeable Exponentially Shrinking Sheet in the Presence of Thermal Radiation and Partial Slip

R. Sharma[1], A. Ishak[1†], R. Nazar[1], I. Pop[2]

[1] School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia
[2] Department of Mathematics, Babeş-Bolyai University, 400084 Cluj-Napoca, Romania

†Corresponding Author Email*: anuar_mi@ukm.my*

ABSTRACT

The steady boundary layer flow of a viscous fluid with heat transfer over an exponentially shrinking sheet in the presence of thermal radiation with mass suction is studied. Velocity and temperature slip is considered on the boundary. Using a similarity transformation, the governing boundary layer equations are transformed into a system of nonlinear ordinary differential equations, which are then solved numerically using MATLAB routine solver. Dual solutions exist for a certain range of mass suction parameter. It is also found that the range of mass suction parameter for obtaining the steady solution is enhanced with the increase of velocity slip parameter and is independent of the thermal slip parameter as well as the radiation parameter.

Keywords: Velocity slip, Thermal slip, Shrinking sheet, Heat transfer, Thermal radiation.

NOMENCLATURE

C_f	skin friction coefficient		U	shrinking velocity
C_p	specific heat at constant pressure		v_w	mass flux velocity
D	thermal slip factor		x, y	Cartesian coordinates along the surface and normal to it, respectively
f	dimensionless stream function			
N	velocity slip factor		α	thermal diffusivity
Nu_x	local Nusselt number		δ	thermal slip parameter
$\Pr$	Prandtl number		η	similarity variable
q_r	radiation flux		θ	dimensionless temperature
R	radiation parameter		λ	velocity slip parameter
Re_x	local Reynolds number		μ	dynamic viscosity
s	suction/blowing parameter		ν	kinematic viscosity
T	fluid temperature		ρ	fluid density
T_w	surface temperature		τ_w	surface shear stress
T_∞	ambient temperature		ψ	stream function
u, v	velocity components along the $x-$ and $y-$ directions, respectively			

1. INTRODUCTION

The heat transfer in the viscous boundary layer flow on a stretching/shrinking sheet has many practical applications in industrial manufacturing processes such as in the polymer industry, where one deals with production of plastic sheet. The prime aim is to generate better quality sheet, which depends upon the rate of cooling. To achieve a better control on the rate of cooling, considerable efforts have been made in recent years. Among other methods, it has been proposed that it may be advantageous to alter flow kinematics in such a way as to ensure a slower rate of solidification. The fluid flow past a stretching plate was

first investigated by Crane (1970), where an exact analytical solution to the Navier-Stokes equations was reported. This problem was then extended to a permeable surface by Gupta and Gupta (1977). Grubka and Bobba (1985) considered a more general case with power law surface temperature variation. They reported a series solution to the energy equation in terms of Kummer's functions and presented several closed-form analytical solutions for specific conditions. The stagnation-point flow past a porous stretching sheet was investigated by Kazem *et al.* (2011), while Hussain *et al.* (2013) investigated the boundary layer flow towards a permeable stretching sheet. Further, the unsteady flow past a stretching sheet was investigated by Andersson *et al.* (2000), Ali and Mehmood (2008), Ishak *et al.* (2006, 2009) and Hayat *et al.* (2008a), among others.

Wang (1990) first investigated the unsteady shrinking sheet film and gave only little information on this type of flow. Later, Miklavčič and Wang (2006) analysed shrinking sheet problem and established the existence and uniqueness criteria that if adequate suction on the surface is applied to confine vorticity, there may be similarity solutions for this problem. Further, the shrinking sheet flow problems were investigated by Hayat *et al.* (2007,2008b), Fang and Zhang (2009), Fang *et al.* (2009b), Cortell (2010), Merkin and Kumaran (2010), and many others. Recently, Bhattacharyya (2011) studied the boundary layer flow and heat transfer over an exponentially shrinking sheet. It is worth mentioning that this new type of shrinking sheet flow is essentially a backward flow and it shows physical phenomena quite distinct from the forward stretching flow (Fang *et al.* 2009b).

The non-adherence of the fluid to a solid boundary, also known as velocity slip, is a phenomenon that has been observed under certain circumstances (Yoshimura and Prudhomme 1998). It is a well-known fact that, a viscous fluid normally sticks to the boundary. But, there are many fluids, e.g. particulate fluids, rarefied gas etc., where there may be a slip between the fluid and the boundary (Shidlovskiy 1967). Beavers and Joseph (1967) proposed a slip flow condition at the boundary. Of late, there has been a revival of interest in the flow problems with partial slip (Andersson 2002; Ariel 2008). Wang (2002) undertook the study of the flow of a Newtonian fluid past a stretching sheet with partial slip and purportedly gave an exact solution. He reported that the partial slip between the fluid and the moving surface may occur in particulate fluid situations such as emulsions, suspensions, foams and polymer solutions. Fang *et al.* (2009a) gave a closed form solution for slip MHD viscous flow over a stretching sheet. Sajid et al. (2010) analyzed the stretching flow with general slip condition. Sahoo and Poncet (2011) investigated the flow and heat transfer solution for third grade fluid with partial slip boundary condition. Das (2012) examined the effects of partial slip, thermal buoyancy and heat generation/absorption on the flow and heat transfer of nanofluids over a permeable stretching surface. Noghrehabadi *et al.* (2012) analyzed the effect of partial slip on the flow and heat transfer of nanofluids past a stretching sheet. Zheng *et al.* (2012) analysed the effect of velocity slip with temperature jump on MHD flow and heat transfer over a porous shrinking sheet. We mention to this end, the papers by Mukhopadhyay and

Andersson (2009) on the effects of slip and heat transfer analysis of flow over an unsteady stretching surface and by Harris *et al.* (2009) on steady mixed convection boundary-layer flow near the stagnation point on a vertical surface in a porous medium with slip.

However, the viscous fluid flow with heat transfer due to an exponentially shrinking sheet with slip effect is still unknown. In the present paper, we investigate the boundary layer flow and heat transfer over an exponentially shrinking sheet with velocity and thermal slip effects. The mathematical model of the problem is non-linear whose analytical solution is very hard to find out, so the only choice left is approximate numerical solution. Therefore, in this study, MATLAB routine solver is used as a tool for the numerical simulation and the flow characteristics are discussed.

2. PROBLEM FORMULATION

Consider the steady two-dimensional forced convection boundary layer flow of a viscous and incompressible fluid past a permeable exponentially shrinking sheet coinciding with the plane $y = 0$, the flow being confined in the region $y \geq 0$ as shown in Fig. 1. Two equal and opposite forces are applied along the $x-$ axis towards the origin O of the coordinate system, so that the wall shrinks keeping the origin fixed. It is assumed that the mass flux velocity is $v_w(x)$ with $v_w(x) < 0$ for suction and $v_w(x) > 0$ for injection or withdrawal of the fluid. It is also assumed that a radiation flux q_r is applied normal to the surface of the shrinking sheet. Under the assumption of boundary layer approximation, the governing equations of continuity, motion and energy are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} \tag{3}$$

where u and v are the components of velocity in the x and y directions respectively, T is the fluid temperature, α is the thermal diffusivity, ν is the kinematic viscosity, ρ is the density and C_p is the specific heat of the fluid at constant pressure.

We assume that the boundary conditions of these equations are given by

$$v = v_w(x), \quad u = U + N\nu \frac{\partial u}{\partial y}, \quad T = T_w + D \frac{\partial T}{\partial y} \quad \text{at} \quad y = 0$$

$$u \to 0, \quad T \to T_\infty \quad \text{as} \quad y \to \infty \tag{4}$$

where $U = -U_0 \exp(x/L)$ is the shrinking velocity, $T_w = T_0 \exp(x/2L) \; v_0$ is the variable temperature at the sheet and $v_w(x) = V_0 \exp(x/2L)$. Here L, U_0, T_0 and V_0 are the length, velocity, temperature and mass flux velocity characteristics, respectively, with $V_0 < 0$

for suction and $V_0 > 0$ for injection or withdrawal of the fluid. Further, we assume that the velocity slip factor N and the thermal slip factor D change with x and are given by $N = N_1 \exp(-x/2L)$ and $D = D_1 \exp(-x/2L)$, where N_1 is the initial value of velocity slip factor and D_1 is the initial value of the thermal slip factor (see Mukhopadhyay and Andersson 2009). The no-slip case is recovered for $N = D = 0$.

Following Bataller (2008), we adopt the Rosseland approximation for the radiation flux q_r, namely

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

where σ^* is the Stefan-Boltzman constant and k^* is the mean absorbtion coefficient. We assume that the difference between the fluid temperature T and the free stream temperature T_∞ is small, so that expanding in Taylor series T^4 about T_∞ and neglecting the second and higher order terms, we have

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

Substituting Eq. (5) into Eq. (3) we get

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(1 + \frac{16\sigma^* T_\infty^3}{3k_m k^*}\right)\frac{\partial^2 T}{\partial y^2} \tag{7}$$

We introduce now the following similarity variables (see Mukhopadhyay and Gorla 2012),

$$u = U_0 \exp(x/L)f'(\eta), \quad v = -\sqrt{\frac{\nu U_0}{2L}}\left[f(\eta) + \eta f'(\eta)\right]$$

$$T = T_0 \exp(x/2L)\theta(\eta), \quad \eta = y\sqrt{\frac{U_0}{2\nu L}}\exp(x/2L) \tag{8}$$

where prime denotes differentiation with respect to η.
Substituting Eq. (5) and Eq. (8) into Eq. (2) and Eq. (3), we obtain the following ordinary differential equations (Mukhopadhyay and Gorla, 2012)

$$f''' + f f'' - 2f'^2 = 0 \tag{9}$$

$$(1 + \frac{4}{3}R)\theta'' + \Pr(f\theta' - f'\theta) = 0 \tag{10}$$

and the boundary conditions, Eq. (4) becomes:

$$f(0) = s, \quad f'(0) = -1 + \lambda f''(0), \quad \theta(0) = 1 + \delta\theta'(0)$$
$$f'(\eta) \to 0, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty \tag{11}$$

Here $\lambda = N_1\sqrt{U_0\nu/2L}$ (>0) is the velocity slip parameter, $\delta = D_1\sqrt{U_0/2\nu L}$ (>0) is the thermal slip parameter, $\Pr = \nu/\alpha$ is the Prandtl number,

$s = -V_0\sqrt{U_0/2\nu L}$ is the suction ($s > 0$) or blowing ($s < 0$) parameter and $R = 4\sigma^* T_\infty^3/(k_m k^*)$ is the radiation parameter.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{\mu}{\rho[U_0\exp(x/L)]^2}\left(\frac{\partial u}{\partial y}\right)_{y=0},$$

$$Nu_x = \frac{L}{T_0\exp(x/2L)}\left(-\frac{\partial T}{\partial y} + q_r\right)_{y=0} \tag{12}$$

Substituting Eq. (5) and Eq. (8) into Eq. (12), we get

$$(2\,\mathrm{Re}_x)^{1/2}C_f = f''(0),$$
$$(2/\mathrm{Re}_x)^{1/2}Nu_x = -(1+R)\theta'(0) \tag{13}$$

where $\mathrm{Re}_x = (U_0 L/\nu)\exp(x/L)$ is the local Reynolds number.

3. NUMERICAL SOLUTION

The set of coupled nonlinear differential Eq. (9) and Eq. (10), along with the boundary conditions Eq. (11) form a two-point boundary value problem (BVP) and is solved numerically using MATLAB routine solver by converting it into an initial value problem (IVP). In this method we have to choose a suitable finite value of η_∞ (where η_∞ corresponds to $\eta \to \infty$). We run our computations with the value $\eta_\infty = 12$, which is sufficient to achieve and satisfy the far field boundary conditions asymptotically for all values of the parameters considered (see Pantokratoras 2009 and Ishak 2010).

We construct the following first order differential equations:

$$f' = p, \quad p' = q, \quad q' = 2p^2 - f q \tag{14}$$

$$\theta' = z, \quad z' = -(\Pr/(1 + \frac{4}{3}R))(f z - p\theta) \tag{15}$$

with the boundary conditions

$$f(0) = s, \quad p(0) = -1 + \lambda q(0), \quad \theta(0) = 1 + \delta z(0) \tag{16}$$

To solve Eq. (14) and Eq. (15) with Eq. (16) as an IVP we need the values of $q(0) = f''(0)$ and $z(0) = \theta'(0)$, but no such values are given. The initial guesses values for $f''(0)$ and $\theta'(0)$ are chosen to obtain the solution satisfying the boundary conditions Eq. (11).

4. RESULT AND DISCUSSION

Numerical solutions to the governing ordinary differential Eq. (9) and Eq. (10), along with the boundary conditions Eq. (11) were obtained using the method described in the previous section for various values of the velocity slip parameter, thermal slip

parameter, suction parameter and radiation parameter. The default values of parameters are taken as $s = 2.2, \ \lambda = 0.1, \ \delta = 0.1, \ Pr = 0.7$ and $R = 0.1$.

Comparison with the existing results from the literature shows a favorable agreement, as presented in Table 1.

Table 1 Values of $\theta'(0)$ for several values of Prandtl number with $s = Nr = \lambda = \delta = 0 \ \& \ f'(0) = 1$

Pr	Magyari and Keller (1999)	El-Aziz (2009)	Present results
1	-0.954782	-0.954785	-0.954789
2			-1.471461
3	-1.869075	-1.869074	-1.869073
5	-2.500135	-2.500132	-2.500125
10	-3.660379	-3.660372	-3.660350
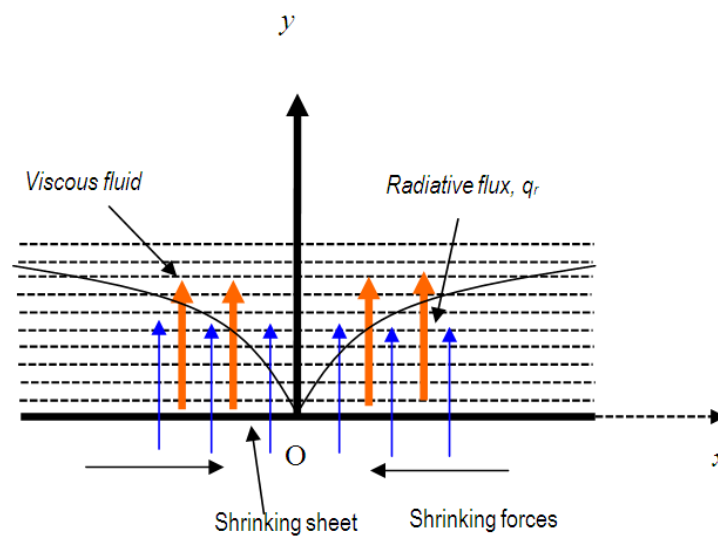
Fig. 1. Physical model and coordinate system

Variation of the reduced skin friction coefficient (or the surface shear stress) $f''(0)$ and the reduced local Nusselt number $-\theta'(0)$ with s for different values of the velocity slip parameter λ, thermal slip parameter δ and radiation parameter R are presented in Figs. 2-5. It can be seen that there are dual (upper branch and lower branch) solutions for each value of s under the same value of $\lambda, \ \delta$ and R. Figures 2-5 show that there are two solutions when $s > s_c$, unique solution when $s = s_c$ and no solution when $s < s_c$. These values of s_c are given in Figs. 2-5 and are dependent on the velocity slip parameter λ. It is seen that the value of s_c decreases as λ increases and does not depend upon the values of δ and R. Hence, the velocity slip parameter widens the range of s for which the solution exists. The reduced skin friction coefficient and the reduced local Nusselt number also change with the variations of λ, δ and R. For the similar problems for which dual solutions exist, Merkin (1985), Weidman *et al.* (2006), Postelnicu and

Pop (2011) and Rosca and Pop (2013) have shown that the first solutions are stable and physically realizable, whilst those of the second solutions are not. For the first solution which we expect to be the physically realizable solution, the reduced skin friction coefficient increases with an increase in s. This is due to the effect of suction at the boundary which slows down the fluid motion and thus increases the velocity gradient at the surface. It is worth mentioning that duality nature of the solution is consistent with the previous analysis for the shrinking sheet case done by Miklavčič and Wang (2006) and Bhattacharyya (2011). The reduced local Nusselt number shows the similar nature to the skin friction coefficient for the first solution, but for the second solution it initially decreases with increasing s, but for large value of s it starts to increase again. The variation with mass suction parameter shown in those figures are similar to the results reported by Bhattacharyya (2011).

In Figs. 2-3, for the first solution, $f''(0)$ decreases but $-\theta'(0)$ increases with an increase in λ. As expected, slip effect is to reduce the friction at the solid-fluid interface, and thus reduces the skin friction coefficient.

The second solution shows more complicated and quite different behaviors compared to the first solution, which are consistent with the findings reported by Postelnicu and Pop (2011) and Rosca and Pop (2013), among others, that the second solution is unstable. For the second solution, both $f''(0)$ and $-\theta'(0)$ decrease with the increase in λ, while for $s > 2.54$ the pattern is reversed. In Figs. 4-5, it is seen that for the first

solution, the value of $-\theta'(0)$ is consistently higher for lower values of δ and R, while reverse pattern is observed for the second solution. For the first solution, which we expect to be the physically realizable solution, the temperature gradient at the surface decreases (due to the thermal slip), and thus decreases the local Nusselt number, which represents the heat transfer rate at the surface.
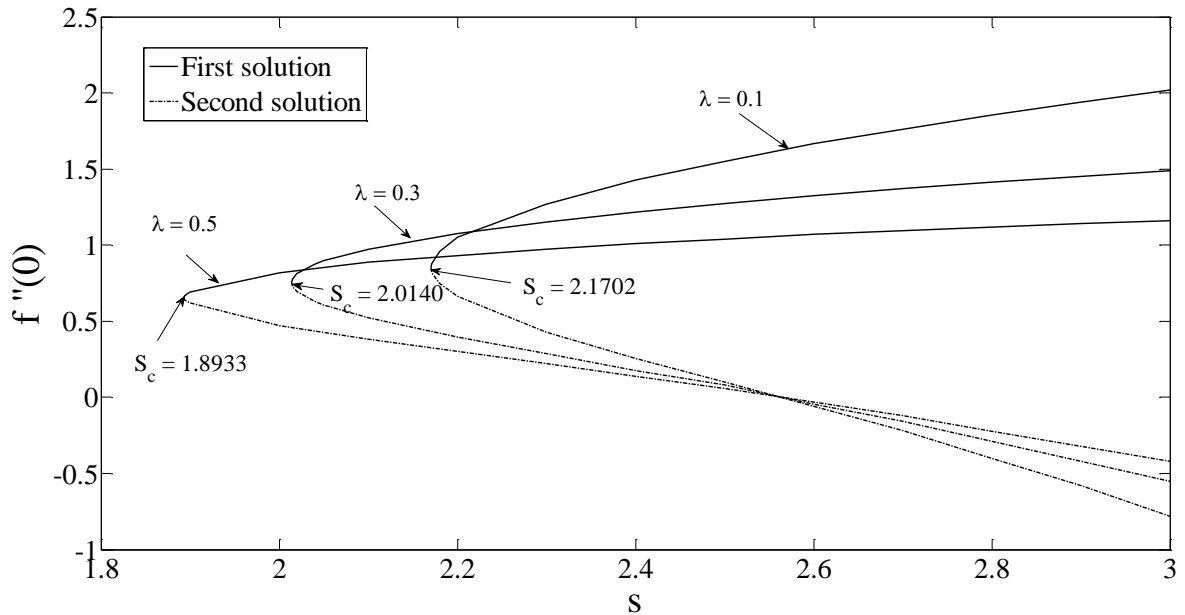
Fig. 2. Variation of $f''(0)$ with s for various values of λ
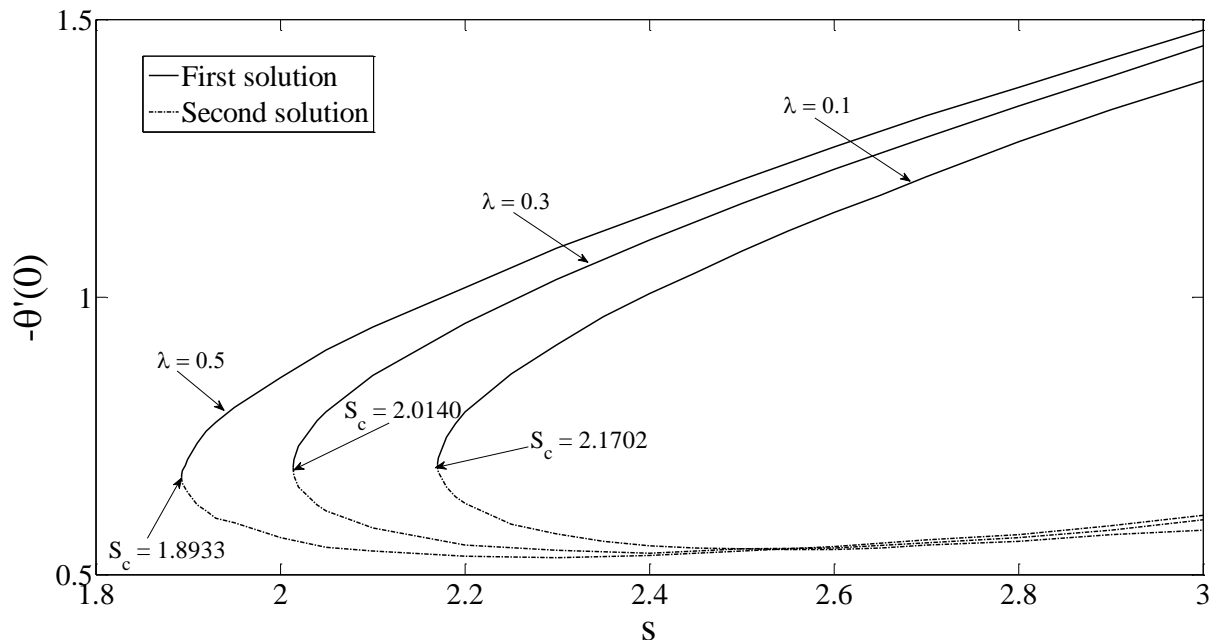
Fig. 3. Variation of $-\theta'(0)$ with s for various values of λ
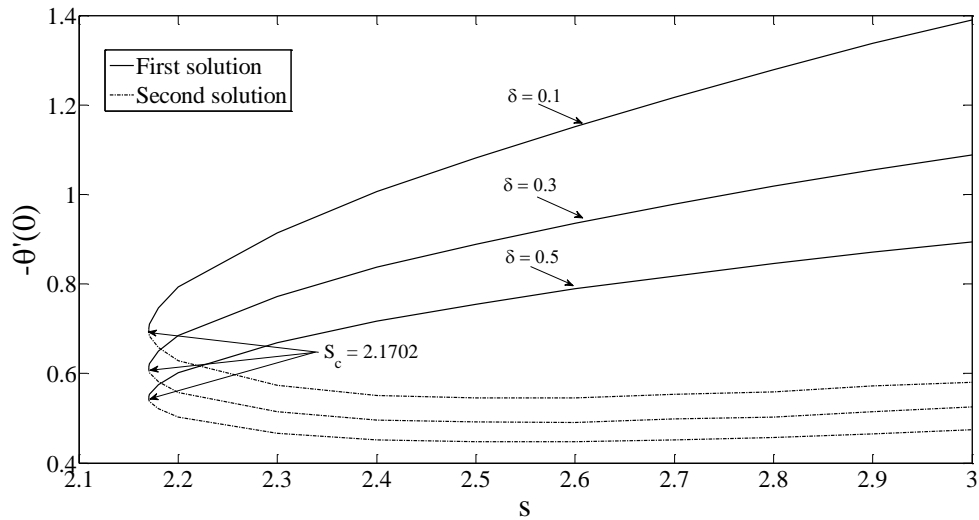
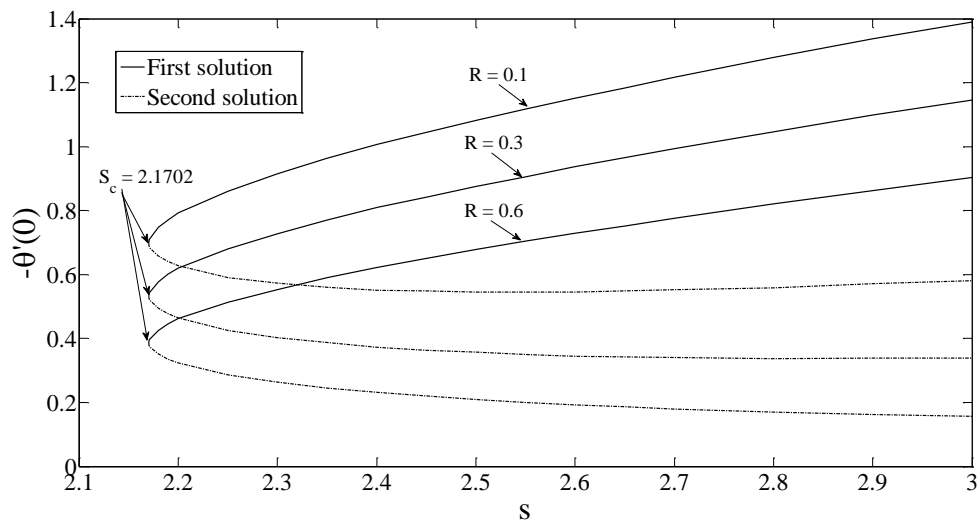**Fig. 4.** Variation of $-\theta'(0)$ with s for various values of δ

Fig. 5. Variation of $-\theta'(0)$ with s for various values of R

Velocity and temperature profiles are shown in Figs. 6-11. These figures satisfy the far field boundary conditions Eq. (11) asymptotically, which support the validity of the numerical results obtained. It is also seen that momentum as well as thermal boundary layer thickness for the first solution is less than that of the second solution, which indicates the stability of the first solution compared to the second solution. Figure 6 shows that the velocity increases with the increasing value of λ for the first solution throughout the boundary layer region, while for the second solution, the velocity increases with λ only near the sheet surface and reverse pattern is observed away from the sheet. We also observed that for the first solution, the momentum boundary layer become thinner due to increasing value of λ and reverse nature is noticed for the second solution. In Fig. 7, the temperature at a point in the boundary layer decreases for an increase in λ for the first solution and for the second solution, the temperature increases with s. Figure 8 shows that the temperature decreases with increasing thermal slip parameter for both first and second solutions. It is also observed from Fig. 8 that the change in temperature with thermal slip parameter is higher near the sheet, but very less away from the sheet. Since thermal slip parameter reduces the rate of heat transfer (see in Fig. 3) and thus temperature is found to decrease. Variation in velocity and temperature profiles shown in Fig. 9 and Fig. 10 respectively have essentially the same form as in the no-slip case ($\lambda = \delta = 0$). Figure 11 shows that temperature increases with increasing the radiation parameter.
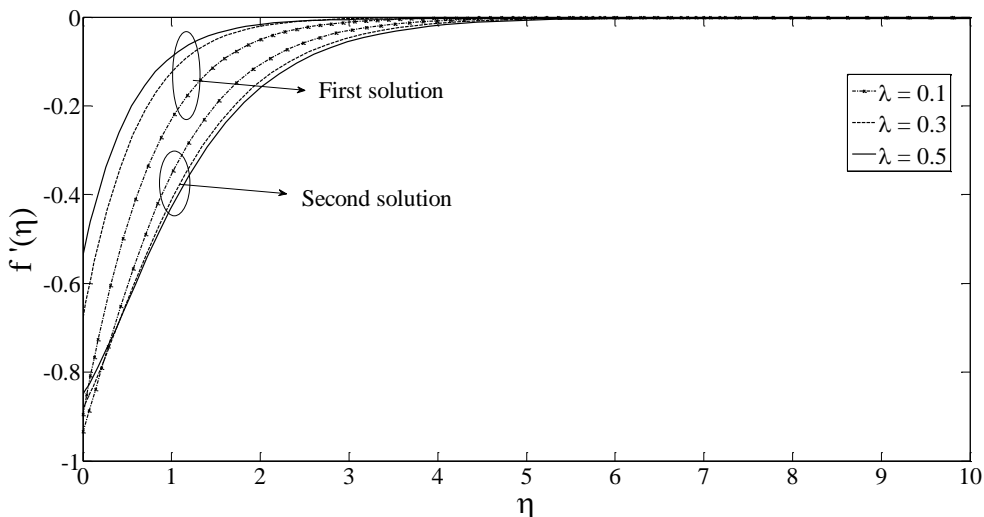
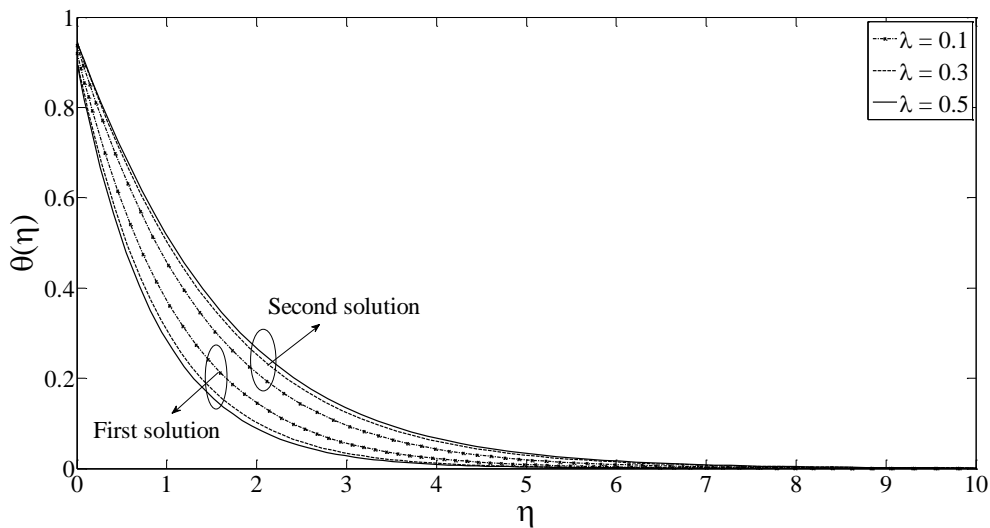**Fig. 6.** Dimensionless velocity profiles for various values of λ

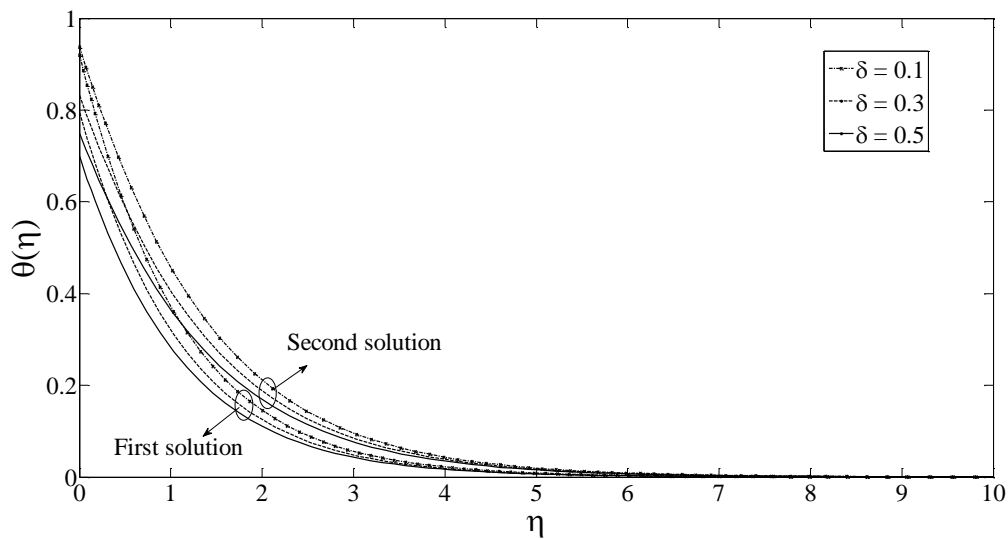**Fig. 7.** Dimensionless temperature profiles for various values of λ

Fig. 8. Dimensionless temperature profiles for various values of δ
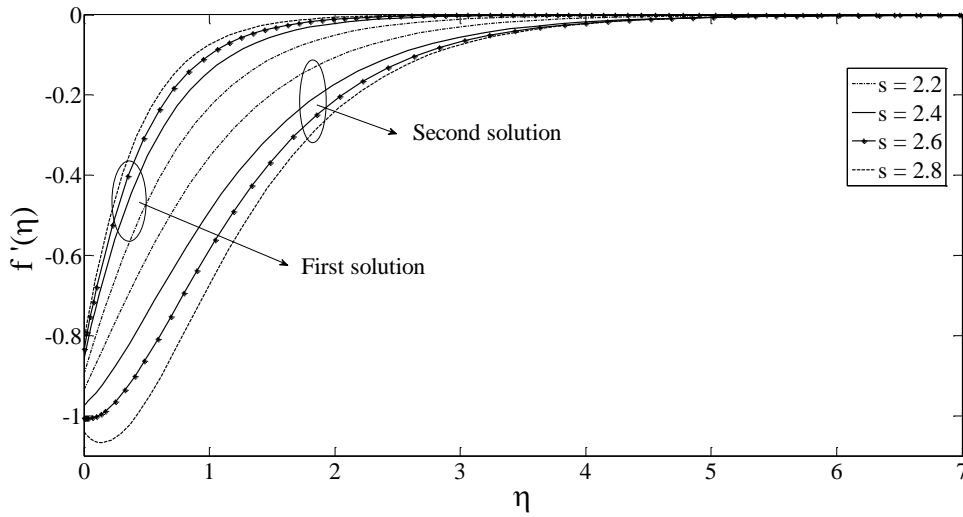
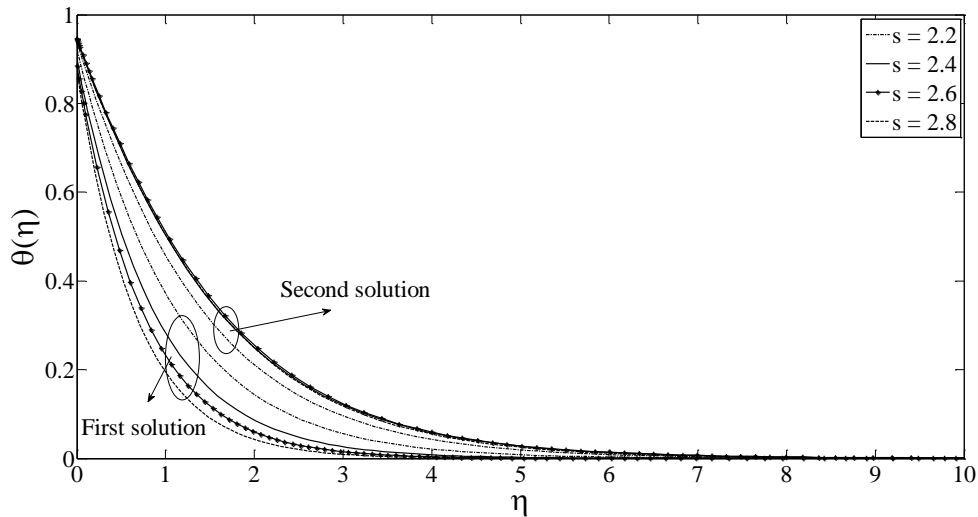**Fig. 9.** Dimensionless velocity profiles for various values of s

Fig. 10. Dimensionless temperature profiles for various values of s
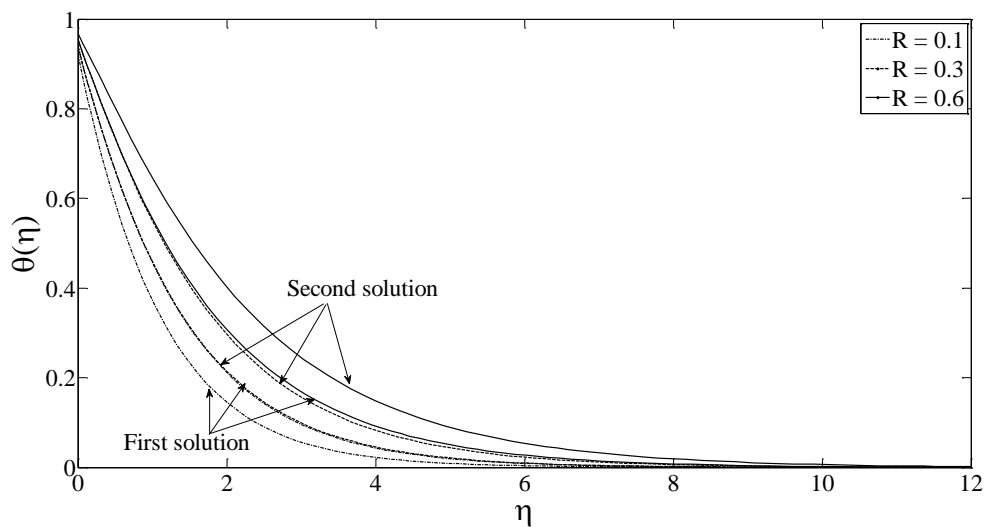
Fig. 11. Dimensionless temperature profiles for various values of R

5. CONCLUSION

In summary, the slip viscous flow with heat transfer over an exponentially shrinking sheet with wall mass transfer and thermal radiation has been solved numerically to exhibit the effects of velocity slip parameter λ, thermal slip parameter γ, radiation parameter R and mass suction parameter s. Dual solutions exist in the certain range of mass suction parameter. The velocity slip parameter widens the range of mass suction parameter for which the solution exists. For the first solution, momentum boundary layer become thinner with the increase of s and λ but reverse trend is observed for the second solution. It is also seen that momentum as well as thermal boundary layer thickness for the first solution is always thinner than that of the second solution. Velocity slip reduces the skin friction coefficient, while temperature slip reduces the heat transfer rate at the surface.

ACKNOWLEDGEMENTS

REFERENCES

Ali, A. and A. Mehmood (2008). Homotopy analysis of unsteady boundary layer flow adjacent to permeable stretching surface in a porous medium, Commun. *Nonlinear Sci. Numer. Simulat.*, 13, 340-349.

Andersson, H.I., J.B. Aarseth and B.S. Dandapat (2000). Heat transfer in a liquid film on an unsteady stretching surface, *Int. J. Heat Mass Transfer*, 43, 69-74.

Andersson, H.I. (2002). Slip flow past a stretching surface, *Acta. Mech.*, 158, 121–125.

Ariel, P.D. (2008). Two dimensional stagnation point flow of an elastico-viscous fluid with partial slip, *J. Appl. Math. Mech. (ZAMM)*, 88, 320–324.

Bataller, R.C. (2012). Radiation effects in the Blasius flow, *Appl. Math. Comp.*, 198, 333-338.

Beavers, G.S. and D.D. Joseph (1967). Boundary condition at a naturally permeable wall, *J. Fluid Mech.*, 30, 197–207.

Bhattacharyya, K. (2011). Boundary layer flow and heat transfer over an exponentially shrinking sheet, *Chin. Phys. Lett.*, 28, Paper ID 074701.

Cortell, R. (2010). On a certain boundary value problem arising in shrinking sheet flows, *Appl. Math. Comput.*, 217, 4086-4093.

Crane, L. J. (1970). Flow past a stretching plate, *J. Appl. Math. Phys. (ZAMP)*, 21, 645-647.

Das, K. (2012). Slip flow and convective heat transfer of nanofluids over a permeable stretching surface, *Comp. Fluids*, 64, 34-42.

El-Aziz, M.A. (2009). Viscous dissipation effect on mixed convection flow of a micropolar fluid over an exponentially stretching sheet, *Can. J. Phys.*, 87, 359-368.

Fang, T. and J. Zhang (2009). Closed-form exact solutions of MHD viscous flow over a shrinking sheet, Commun. *Nonlinear Sci. Numer. Simulat.*, 14, 2853-2857.

Fang, T., J. Zhang and S. Yao (2009a). Slip MHD viscous flow over a stretching sheet-an exact solution, *Commun. Nonlinear Sci. Numer. Simulat.*, 17, 3731-3737.

Fang, T., J. Zhang and S. Yao (2009b). Viscous flow over an unsteady shrinking sheet with mass transfer, *Chin. Phys. Lett.*, 26, Paper ID 014703.

Gupta, P. S. and A. S. Gupta (1977). Heat and mass transfer on a stretching sheet with suction or blowing, *Can. J. Chem. Eng.*, 55, 744-746.

Grubka, L. J. and K. M. Bobba (1985). Heat transfer characteristics of a continuous, stretching surface with variable temperature, *Trans. ASME J. Heat Transfer*, 107, 248-250.

Harris, S.D., D.B. Ingham and I. Pop (2009). Mixed convection boundary-layer flow near the stagnation point on a vertical surface in a porous medium: Brinkman model with slip, Transp. *Porous Media*, 77, 267–285.

Hayat, T., Z. Abbas and M. Sajid (2007). On the analytic solution of magnetohydrodynamic flow of a second grade fluid over a shrinking sheet, *ASME J. Appl. Mech.*, 74, 1165-1171.

Hayat, T., Z. Abbas and T. Javed (2008a). Mixed convection flow of a micropolar fluid over a non-linearly stretching sheet, *Phys. Lett. A*, 372, 637-647.

Hayat, T., T. Javed and M. Sajid (2008b). Analytic solution for MHD rotating flow of a second grade fluid over a shrinking surface, *Phys. Lett. A*, 372, 3264-3273.

Hussain, M., M. Ashraf, S. Nadeem and M. Khan (2013). Radiation effects on the thermal boundary layer flow of a micropolar fluid towards a permeable stretching sheet, *J. Franklin Inst.*, 350, 194-210.

Ishak, A. (2010). Similarity solutions for flow and heat transfer over a permeable surface with a convective boundary condition, *Appl. Math. Comp.*, 217, 837-842.

Ishak, A., R. Nazar and I. Pop (2006). Unsteady mixed convection boundary layer flow due to a stretching vertical surface, *Arab J. Sci. Eng.*, 31, 165-182.

Ishak, A., R. Nazar and I. Pop (2009). Heat transfer over an unsteady stretching permeable surface with prescribed wall temperature, *Nonlinear Anal. Real World Appl.*, 10, 2909-2913.

Kazem, S., M. Shaban and S. Abbasbandy (2011). Improved analytical solutions to a stagnation-point flow past a porous stretching sheet with heat generation, *J. Franklin Inst.*, 348, 2044–2058.

Magyari, E. and B. Keller (1999). Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface, *J. Phys. D: Appl. Phys.*, 32, 577-585.

Merkin, J.H. (1985). On dual solutions occurring in mixed convection in a porous medium, *J. Eng. Math.*, 20, 171-179.

Merkin, J.H. and V. Kumaran (2010). The unsteady MHD boundary-layer flow on a shrinking sheet, *Eur. J. Mech. B Fluids*, 29, 357-363.

Miklavčič, M. and C.Y. Wang (2006). Viscous flow due to a shrinking sheet, *Quart. Appl. Math.*, 64, 283–290.

Mukhopadhyay, S. and H.I. Andersson (2009). Effects of slip and heat transfer analysis of flow over an unsteady stretching surface, *Heat Mass Transfer,* 45,1447–1452.

Mukhopadhyay, S. and R.S.R. Gorla (2012). Effects of partial slip on boundary layer flow past a permeable exponential stretching sheet in presence of thermal radiation, *Heat Mass Transfer*, 48, 1773-1781.

Noghrehabadi, A., R. Pourrajab, and M. Ghalambaz, (2012). Effect of partial slip boundary condition on the flow and heat transfer of nanofluids past stretching sheet prescribed constant wall temperature, *Int. J. Thermal Sci.*, 54, 253-261.

Pantokratoras A. (2009). A common error made in investigation of boundary layer flows, *Appl. Math. Model.* 33, 413–422.

Postelnicu, A. and I. Pop (2011). Falkner-Skan boundary layer flow of a power-law fluid past a stretching wedge, *Appl. Math. Comput.*, 217, 4359-4368.

Rosca, A.V. and I. Pop (2013). Flow and heat transfer over a vertical permeable stretching/shrinking sheet with a second order slip, I*nt. J. Heat Mass Transfer,* 60, 355-364.

Sahoo, B. and S. Poncet (2011). Flow and heat transfer of a third grade fluid past an exponentially stretching sheet with partial slip boundary condition, *Int. J. Heat Mass Transfer*, 54, 5010-5019.

Sajid, M., N. Ali, Z. Abbas, and T. Javed (2010). Stretching flows with general slip boundary condition, *Int. J. Mod. Phys. B*, 24, 5939-5947.

Shidlovskiy, V.P. (1967). *Introduction to the Dynamics of Rarefied Gases*. New York: American Elsevier Publishing Company Inc.

Wang, C.Y. (1990). Liquid film on an unsteady stretching sheet, *Quart. Appl. Math.*, 48, 601-610.

Wang, C.Y. (2002). Flow due to a stretching boundary with partial slip-an exact solution of the Navier-Stokes equations, *Chem. Eng. Sci.*, 57, 3745-3747.

Weidman, P.D., D.G. Kubitschek and A.M.J. Davis (2006). The effect of transpiration on self-similar boundary layer flow over moving surface, *Int. J. Eng. Sci.*, 44, 730-737.

Yoshimura, A. and R.K. Prudhomme (1998). Wall slip corrections for Couette and parallel disc viscometers, *J. Rheol.*, 32, 53–67.

Zheng, L., J. Niu, X. Zhang, and Y. Gao (2012). MHD flow and heat transfer over a porous shrinking surface with velocity slip and temperature jump, *Math. Comp. Model.*, 56, 133-144.