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MHD Flow and Heat Transfer of an Exponential Stretching Sheet in a Boussinesq-Stokes Suspension

P. G. Siddheshwar[1], G. N. Sekhar[2] and A. S. Chethan[3†]

[1] *Department of Mathematics, Bangalore University, Central College Campus, Bangalore 560001,India*
[2] *Department of Mathematics, BMS College of Engineering, Bangalore, 560 019, Karnataka, India*
[3] *Department of Mathematics, BMS Institute of Technology, Bangalore, 560 064 Karnataka, India*

†Corresponding Author Email: as.chethan@gmail.com

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ABSTRACT

An analysis is carried out to study the flow and heat transfer due to an exponentially stretching sheet in a Boussinesq-Stokes suspension. Two cases are studied in heat transfer, namely (i) the sheet with prescribed exponential order surface temperature (PEST-case) and (ii) the sheet with prescribed exponential order heat flux (PEHF-case). The governing coupled, non-linear, partial differential equations are converted into coupled, non-linear, ordinary differential equations by a similarity transformation and are solved numerically using shooting method. The classical explicit Runge-Kutta-Fehlberg 45 method is used to solve the initial value problem by the shooting technique. The effects of various parameters such as the couple stress parameter, Reynolds number and Prandtl number on velocity and temperature profiles are presented and discussed. The results have possible technological applications in the liquid-based systems involving stretchable materials.

Keywords: Exponential stretching, Couple stress parameter, Shooting method.

NOMENCLATURE

A_0, A_1	parameters of temperature distribution
C_p	specific heat at constant pressure
H_0	applied magnetic field
k	thermal conductivity
l	reference length
Pr	Prandtl number
Q	Chandrasekhar number
Q^*	heat source
Re	Reynolds number
T	temperature
T_w	temperature at the wall
T_∞	temperature outside the dynamic region
U_0	constant
U_w	stretching velocity of the boundary
u, v	velocity in x, y directions

Greek symbols

η	similarity variable
λ	heat source parameter
μ_m	magnetic permeability
ν	kinematic coefficient of viscosity
ν'	couple stress viscosity
ρ	density
σ	electric conductivity of the fluid
ψ	non-dimensional stream function
Ψ	dimensionless stream function
θ	non-dimensional temperature in PEST case
ϕ	non-dimensional temperature in PEHF case

1. INTRODUCTION

Boundary layer flow on continuous moving surface is an important type of flow occurring in a number of engineering processes. Aerodynamic extrusion of plastic sheets, cooling of an infinite metallic plate in a cooling path, the boundary layer along a liquid film in condensation process and a polymer sheet of filament extruded continuously from a die are examples of practical applications of continuous moving surfaces. Gas blowing, continuous casting and spinning of fibers also involve the flow due to a stretching surface.

Sakiadis (1961 a, b, c) initiated the study of the boundary layer flow over a continuous solid surface moving with constant speed. Erickson *et al.* (1969) extended the work of Sakiadis to account for mass transfer at the stretching sheet surface. Tsou *et al.* (1967) reported both analytical and experimental results for the flow and heat transfer aspects developed by a continuously moving surface. Crane (1970) studied the steady two dimensional boundary layer flow caused by the stretching sheet, which moves in its own plane with a velocity which varies linearly with the axial distance. Several researchers considered various aspects of momentum and heat transfer characteristics in boundary layer flow over a stretching boundary (Gupta and Gupta (1977), Rajagopal *et al.* (1984), Siddappa and Abel (1985), Andersson (1992), Kumaran and Ramanaiah (1996) and Cortell (2007)).

Magyari and Keller (1999) studied the heat and mass transfer on the boundary layer flow due to an exponentially stretching surface. Elbashbeshy (2001) added new dimension to the study on exponentially stretching surface. Partha *et al.* (2005) have examined the mixed convection flow and heat transfer from an exponentially stretching vertical surface in quiescent liquid using a similarity solution. Heat and mass transfer in a viscoelastic boundary layer flow over an exponentially stretching sheet were investigated by Khan and Sanjayanand (2005, 2006). Sajid and Hayat (2008) considered the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. The constitutive equations for couple stress fluids are given by Stokes (1966). The present work analyses the flow and heat transfer due to an exponentially stretching continuous surface in the presence of Boussinesq-Stokes suspension.

2. Mathematical formulation

We consider a steady, two-dimensional MHD boundary layer flow of an incompressible Boussinesq-Stokes suspension flow due to an exponentially stretching sheet .The flow is assumed to be generated by stretching of the sheet from a slit with a velocity which varies exponentially in the direction of x-axis. In this situation the governing boundary layer equations for momentum and heat transfer are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (2.1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\nu'\frac{\partial^4 u}{\partial y^4}-\frac{\mu_m{}^2\sigma H_0{}^2}{\rho}u, \qquad (2.2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+Q^*\left(T-T_\infty\right), \qquad (2.3)$$

subject to the boundary conditions:

$$u=U_w(x)=U_0e^{\frac{x}{l}},\ v=0,\ \frac{\partial^2 u}{\partial y^2}=0,\ \left\{\begin{array}{ll} T=T_w=T_\infty+A_0e^{\frac{x}{l}} & \text{in PEST case}\\ -k\left(\frac{\partial T}{\partial y}\right)_w=A_1e^{\frac{3x}{2l}} & \text{in PEHF case}\end{array}\right\}\ at\ \ y=0,$$

$$u\to 0,\ \ \frac{\partial^2 u}{\partial y^2}\to 0, T\to T_\infty\ \ as\ \ y\to\infty. \qquad (2.4)$$

where u and v are the velocity components of the fluid in x and y directions, ν is the kinematic coefficient of viscosity, ν' is the couple stress viscosity, μ_m is the magnetic permeability, ρ is the density, σ is the electric conductivity of the fluid, H_0 is the applied magnetic field, U_w stands for stretching velocity of the boundary, U_0 is a constant, l is the reference length, k is the thermal conductivity, T is the temperature, T_w is the temperature at the wall, T_∞ is the temperature outside the dynamic region, Q^* is the heat source and C_p is the specific heat at constant pressure. Here A_0 and A_1 are parameters of the temperature distribution on the stretching surface.

We introduce the stream function $\psi(x, y)$ defined by

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x} \qquad (2.5)$$

The above set of partial differential equations is converted in to ordinary differential equations using the following similarity transformation.

$$X=\frac{x}{l}, Y=\frac{y}{l}, \Psi(X,Y)=\frac{\psi(x,y)}{\nu}=\sqrt{2Re}\,f(\eta)e^{\frac{X}{2}}, \eta=Y\sqrt{\frac{Re}{2}}e^{\frac{X}{2}},$$

$$\left\{\begin{array}{ll}\theta(\eta)=\dfrac{T-T_\infty}{T_w-T_\infty} & \text{in PEST case}\\ \phi(\eta)=\dfrac{T-T_\infty}{\frac{A_1}{k}\sqrt{\frac{2}{Re}}e^{2X}} & \text{in PEHF case}\end{array}\right\}. \qquad (2.6)$$

where η is the similarity variable and $Re=\frac{U_0 l}{\nu}$ is the Reynold's number.

Using the similarity transformation given by Eq. (2.6) in the Eq. (2.2), one immediately obtains

$$C^2 Re\, f^{v}-2f'''-2f\,f''+4\left(f'\right)^2+4Q\,f'=0, \qquad (2.7)$$

The boundary conditions Eq. (2.4) for velocity can be written as:

$$f(0)=0,\ \ f'(0)=1,\ \ f'''(0)=0,$$
$$f'(\infty)\to 0,\ \ f'''(\infty)\to 0. \qquad (2.8)$$

Using Eq. (2.6) in Eqs. (2.3) and (2.4), we get:

(i) PEST:

$$\theta'' - Pr\left(2f'\theta - f\theta' - \frac{2\lambda}{Re}\theta\right) = 0, \tag{2.9}$$

$$\theta(0) = 1, \theta(\infty) \to 0. \tag{2.10}$$

(ii) PEHF:

$$\phi'' - Pr\left(2f'\phi - f\phi' - \frac{2\lambda}{Re}\phi\right) = 0, \tag{2.11}$$

$$\phi'(0) = -1, \phi(\infty) \to 0. \tag{2.12}$$

where

$C^2 = \frac{\nu'}{\nu l^2}$ is the couple stress parameter, $Q = \frac{\mu_m^2 \sigma H_0^2 l}{\rho U_w}$ is the Chandrasekhar's number and

$\lambda = \frac{Q^* l^2}{\rho C_p \nu}$ is the heat source parameter.

We now outline the procedure for solving two boundary value problems Eqs. (2.9)-(2.10) and Eqs. (2.11)-(2.12) which are coupled with Eqs.(2.7)-(2.8).

3. Method of Solution

We adopt the shooting method with Runge-Kutta Fehlberg 45 scheme to solve the initial value problems in PEST and PEHF cases mentioned in the previous section. The coupled non-linear Eqs. (2.7)-(2.10) in PEST case are transformed in to a system of seven first order ordinary differential equations as follows.

$$\frac{dy_1}{dY} = y_2,$$
$$\frac{dy_2}{dY} = y_3,$$
$$\frac{dy_3}{dY} = y_4,$$
$$\frac{dy_4}{dY} = y_5,$$
$$\frac{dy_5}{dY} = \frac{1}{C^2 Re}\left(2y_4 + 2y_1 y_3 - 4y_2^2 - 4Q y_2\right),$$
$$\frac{dy_6}{dY} = y_7,$$
$$\frac{dy_7}{dY} = Pr\left(2y_2 y_6 - y_1 y_7 - \frac{2\lambda}{Re} y_6\right), \tag{2.13}$$

The corresponding boundary conditions are

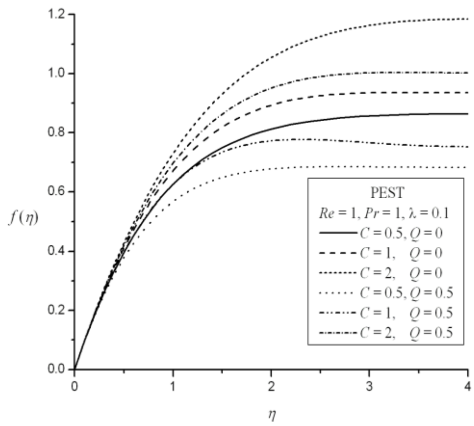
$$y_1(0) = 0,\ y_2(0) = 1,\ y_4(0) = 0, y_6(0) = 1,$$
$$y_2(\infty) = 0,\ y_4(\infty) = 0,\ y_6(\infty) = 0. \tag{2.14}$$

Here, $y_1 = f(\eta)$ and $y_6 = \theta(\eta)$.

Aforementioned boundary value problem is converted in to an initial value problem by choosing the values of $y_3(0)$ and $y_7(0)$ appropriately. The value of $y_5(0)$ can be obtained from the known initial conditions. Resulting initial value problem is integrated using Runge-Kutta-Fehlberg 45 method. Newton-Raphson method is used to correct the guess values of $y_3(0)$ and $y_7(0)$. In solving Eqs. (2.13) subjected to the boundary conditions given by Eq. (2.14) the appropriate '∞' is determined through actual computation. Same procedure is adopted to solve Eqs. (2.7)-(2.8) and (2.11)-(2.12). The results are presented in several graphs.

4. Results and Discussion

The MHD boundary layer flow and heat transfer of a stretching sheet in the presence of Boussinesq-Stokes suspension is analyzed.The effects of various parameters such as couple stress parameter, Reynolds number, Prandtl number are shown in several graphs in Fig 1.1 to 1.9.



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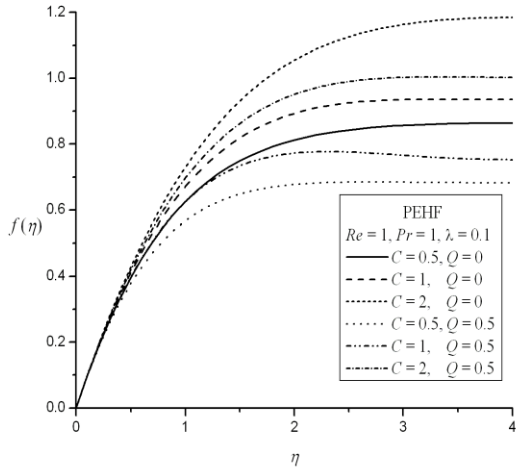

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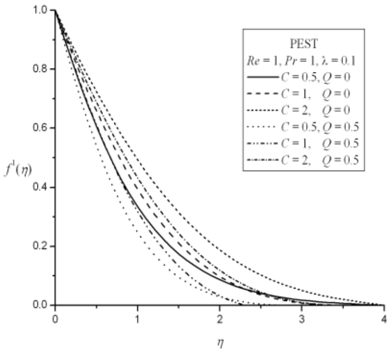
Fig. 1.1. Plot of $f(\eta)$ versus η for different values of couple stress Parameter (C) and Chandrasekhar number (Q).



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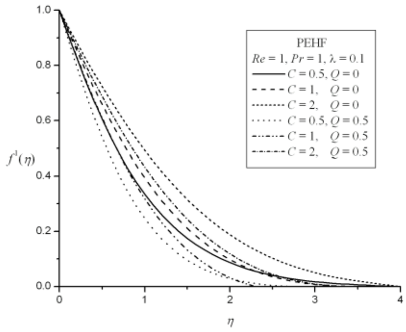

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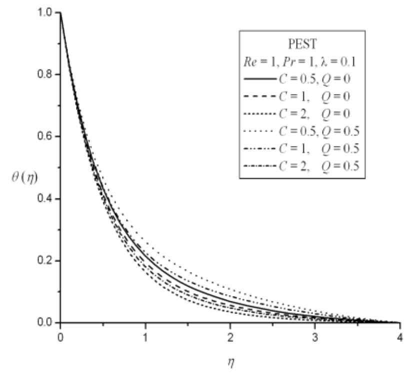
Fig. 1.2. Plot of $f'(\eta)$ versus η for different values of C and Q.



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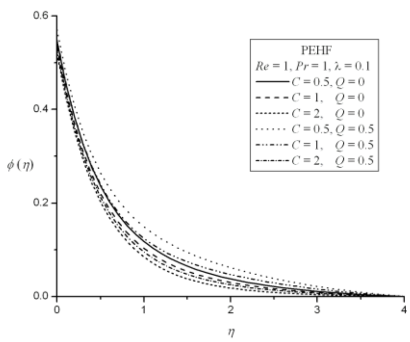

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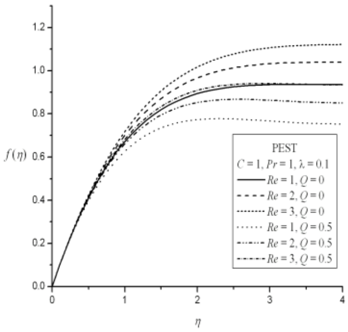
Fig. 1.3. Plot of temperature profiles for different values of C and Q.



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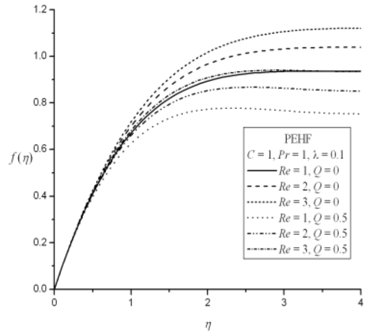

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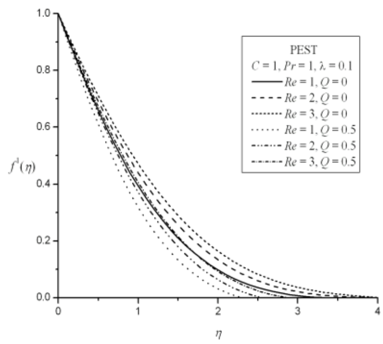
Fig. 1.4. Plot of $f(\eta)$ versus η for different values of Re and Q.



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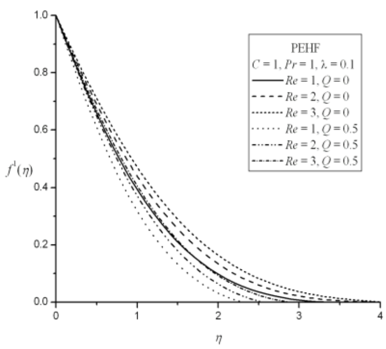

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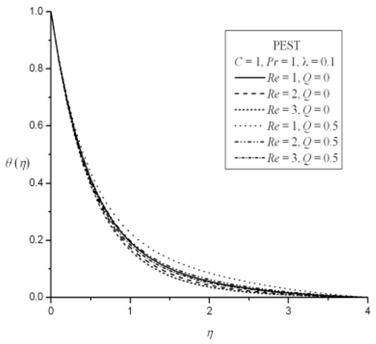
Fig. 1.5. Plot of $f'(\eta)$ versus η for different values of Re and Q.



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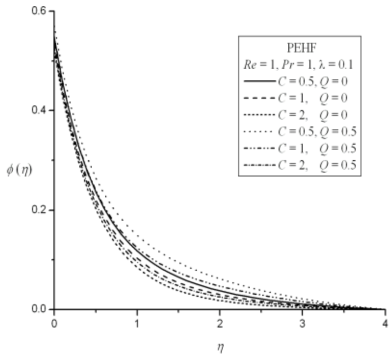

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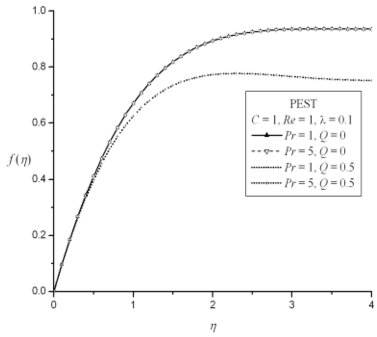
Fig. 1.6. Plot of temperature profiles for different values of *Re* and *Q*.

Figures 1.1-1.3 illustrates the effect of couple stress parameter *C* and the Chandrasekhar number *Q* on the flow and heat transfer in PEST and PEHF cases. It is observed from these plots that $f(\eta)$ and $f'(\eta)$ increases with increasing values of *C*, where as $\theta(\eta)$ decreases with increasing values of *C*. This means that the increasing values of *C* results in thickening of the momentum boundary layer and thinning of thermal boundary layer.

Figures 1.4-1.6 demonstrates the effect of Reynolds number *Re* and the Chandrasekhar number *Q* on the flow and heat transfer. The effect of *Re* is similar to that of *C* in both PEST and PEHF cases.

The impact of Prandtl number *Pr* and the Chandrasekhar number *Q* on the momentum and heat transfer is depicted in Figs. 1.7-1.9. Increasing values of *Pr* does not affect the profiles of $f(\eta)$ and $f'(\eta)$, whereas the temperature at a given point decreases with an increase in the Prandtl number *Pr*. This is in agreement with the physical fact that the thermal boundary layer thickness decreases with increasing Prandtl number.



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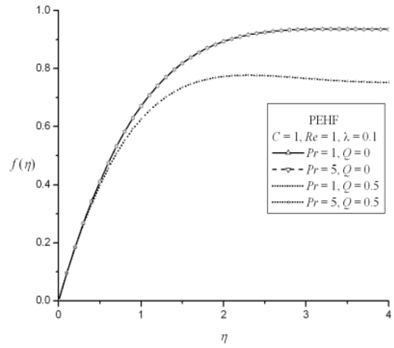

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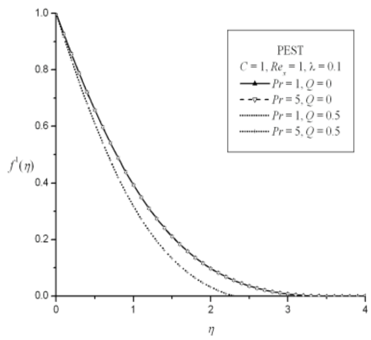
Fig. 1.7. Plot of $f(\eta)$ versus η for different values of *Pr* and *Q*.



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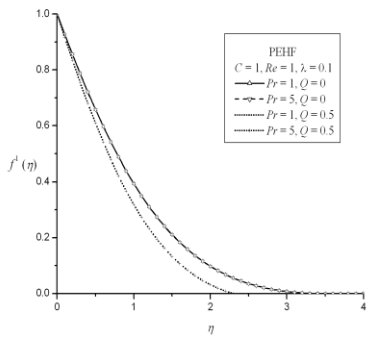

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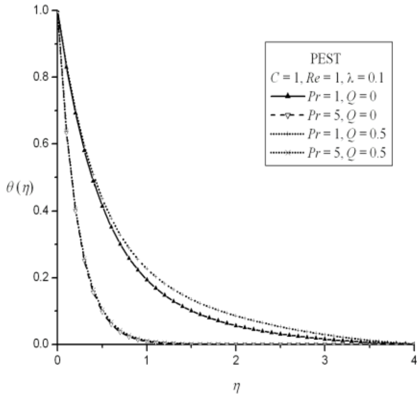
Fig. 1.8. Plot of $f'(\eta)$ versus η for different values of *Pr* and *Q*.



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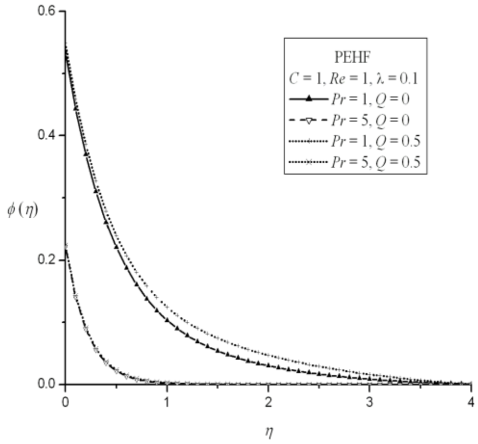

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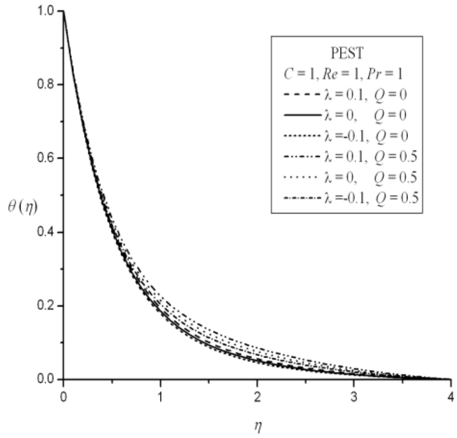
Fig. 1.9. Plot of temperature profiles for different values of *Pr* and *Q*.

From Figs 1.1-1.9 it is also clear that increasing values of Q results in flattening of $f(\eta)$ and $f'(\eta)$. The transverse contraction of the velocity boundary layer is due to the applied magnetic field which invokes the Lorentz force producing considerable opposition to the motion. The effect of transverse magnetic field on temperature profiles results in thickening of thermal boundary layer in both PEST and PEHF cases.

Figure 1.10 illustrates the influence of heat generation (thermal source) ($\lambda>0$) and heat absorption (thermal sink) ($\lambda<0$) parameter on dimensionless temperature profiles in PEST and PEHF cases. For $\lambda>0$, thermal boundary layer generates energy and this causes the temperature of the fluid to increase with increase in the value of λ, whereas for $\lambda < 0$ the temperature decreases.



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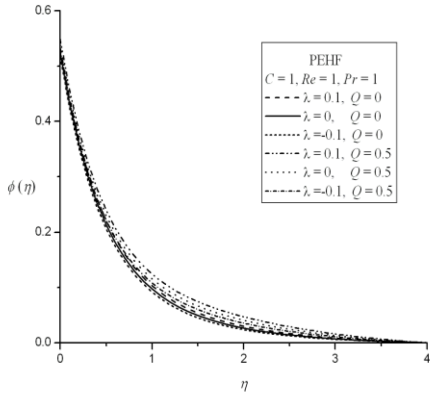

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Fig. 1.10. Plot of temperature profiles for different values of λ and *Q*.

In order to validate our results, we have compared the skin friction $-f''(0)$ and rate of heat transfer $-\theta'(0)$ in the absence of couple stress viscosity, Chandrasekhar number ($Q = 0$) and heat source/sink parameter ($\lambda = 0$) with the published results and found them to be in good agreement (see Tables 1 and 2).

Table1 Comparison of values of skin friction $-f''(0)$ with $C = 0$ and $Q = 0$.

$-f''(0)$		
Magyari and Keller (1999)	Elbashbeshy (2001)	Present study
1.28180	1.28181	1.281816

Table 2 Comparison of wall temperature gradient $-\theta'(0)$ in PST case with C = 0, Q = 0 and λ = 0 for different values of Pr.

Parameter		$-\theta'(0)$		
		Bidin and Nazar (2009)	Elbashbeshy (2001)	Present study
Pr	0.72	-	0.767778	0.767645
	1	0.9548	0.954779	0.954808
	2	1.4714	-	1.471455
	3	1.8691	1.869070	1.869069

5. CONCLUSIONS

1. Increasing values of couple stress parameter results in thickening of the momentum boundary layer and thinning of the thermal boundary layer.

2. Increasing values of Chandrasekhar number results in flattening of velocity profiles.

3. The temperature profiles are higher in the presence of magnetic field than in the absence of the same.

4. PEHF boundary condition is better suited for the effective cooling of the stretching sheet.

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