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Joule Heating Effects on MHD Natural Convection Flows in Presence of Pressure Stress Work and Viscous Dissipation from a Horizontal Circular Cylinder

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ABSTRACT

The effects of joule heating on MHD natural convection flow from a horizontal circular cylinder along the outer surface from the lower stagnation point to the upper stagnation point in presence of pressure stress work and viscous dissipation is investigated. The results have been obtained by transforming the governing boundary layer equations into a system of non-dimensional equations and by applying implicit finite difference method together with Newton's linearization approximation. Numerical results for different values of the magnetic parameter, joule heating parameter and Prandtl number have been obtained. The velocity profiles, temperature distributions, skin friction co-efficient and the rate of heat transfer have been presented graphically for the effects of the aforementioned parameters. Results are compared with previous investigation.

Keywords: Natural convection, Viscous dissipation, Pressure stress work, MHD, Joule heating.

NOMENCLATURE

Cf_x	local skin friction coefficient	x, y	dimensionless Cartesian coordinates
c_p	specific heat at constant pressure	$\bar{x}, \bar{y}$	dimensional Cartesian coordinates
f	dimensionless stream function	υ	kinematics viscosity
Gr	local Grashof number	μ	viscosity of the fluid
g	acceleration due to gravity	θ	dimensionless temperature function
J	joule heating parameter	λ	viscous dissipation parameter
M	magnetic parameter	ε	pressure stress work parameter
Nu_x	local Nusselt number coefficient	β	co-efficient of thermal expansion
Pr	Prandtl number	β_0	magnetic field strength
T_w	temperature at the surface of the cylinder	Ψ	stream function
T_∞	temperature of the ambient fluid	ρ	density of the fluid
T	temperature of the fluid in the boundary	σ	electric conduction
u, v	the dimensionless x and y component of the velocity		
$\bar{u}, \bar{v}$	the dimensional $\bar{x}$ *and* $\bar{y}$ component of the velocity		

1. INTRODUCTION

The influence and importance of viscous dissipation effects in free convection flows have been examined by Gebhar (1962). Zakerullah (1972) has been investigated the viscous dissipation and pressure work effects in axisymmetric natural convection flows. Ackroyd (1974) studied the stress work effects in laminar flat plate natural convection flow. Takhar and Soundalgekar (1980) have studied the effects of viscous and joule heating on the problem posed by Sparrow and Cess (1961), using the series expansion method of Gebhart. But they investigated generally not in a particular case of study. Natural convection flow from a horizontal cylinder due to thermal buoyancy was analyzed by a number of researchers Merkin *et al.* (1988), Kuehn *et al.* (1980) and Wang *et al.* (1990). Joshi and Gebhart (1981) have shown the effect of pressure stress work and viscous dissipation in some natural convection flows. Effects of pressure stress work and viscous dissipation in natural convection flow along a vertical flat plate with heat conduction have been investigated by Alam *et al.* (2006). Recently, Hye *et al.* (2007) have considered the effects of heat and mass transfer on natural convection flows across an isothermal horizontal circular cylinder with chemical reaction.

MHD flow and heat transfer process are now an important research area due to its potential application in engineering and industrial fields. A considerable amount of research has been done in this field. Wilks *et al.* (1976) studied MHD free convection about a semi-infinite vertical plate in a strong cross field. Takhar and Soundalgekar (1980) investigated dissipation effects on MHD free convection flow past a semi-infinite vertical plate. Hossain (1992) studied viscous and Joule heating effects on MHD free convection flow with variable plate temperature. Aldoss *et al.* (1996) analyzed MHD mixed convection from a horizontal circular cylinder. El-Amin (2003) found out the combined effect of viscous dissipation and Joule heating on MHD forced convection over a non-isothermal horizontal circular cylinder embedded in a fluid saturated porous medium. He observed that both the velocity profiles and temperature profiles shifted down for increasing value of magnetic parameter and that are rise up for increasing value of joule heating parameter. Recently, Molla *et al.* (2012) studied the effect of temperature dependent viscosity on MHD natural convection flow from an isothermal sphere.

However, the joule heating effects on MHD natural convection flow in presence of pressure stress work and viscous dissipation has received little attention. Hence, the present study is attempted.

2. MATHEMATICAL ANALYSIS

Let us consider a steady natural convection flow of a viscous incompressible fluid from an isothermal horizontal circular cylinder of radius a placed in a fluid of uniform temperature. A uniform magnetic field having strength B_0 is acting normal to the cylinder surface. The effects of pressure stress work, viscous dissipation and joule heating in the flow region and conduction from surface considered in the present study. The flow configuration and the coordinates system are shown in Fig.1. Under the balance laws of mass, momentum and energy and with the help of Boussinesq approximation for the body force term in the momentum equation, the equations governing this boundary-layer natural convection flow can be written as:

Continuity equation

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \qquad (1)$$

Momentum equation

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta(T - T_\infty)\sin\left(\frac{\bar{x}}{a}\right) - \frac{\sigma B_0^2 \bar{u}}{\rho} \qquad (2)$$

Energy equation

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{K}{\rho C_P}\frac{\partial^2 T}{\partial \bar{y}^2} + \frac{\mu}{\rho C_P}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \frac{T\beta}{\rho C_P}\bar{u}\frac{\partial P}{\partial \bar{x}} + \frac{\sigma B_0^2 \bar{u}^2}{\rho} \qquad (3)$$

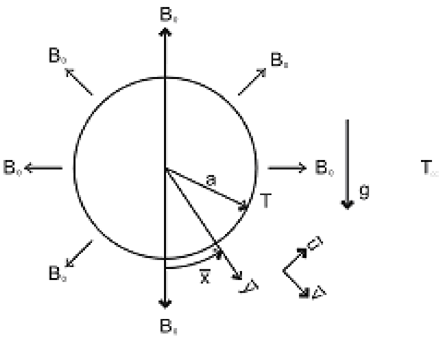

Fig. 1. The geometry of the problem

The physical situation of the system suggests the following boundary conditions

$$\left.\begin{aligned} &\bar{u} = \bar{v} = 0, T = T_w \text{ at } \bar{y} = 0, x > 0 \\ &\bar{u} \to 0, T \to T_\infty \text{ as } \bar{y} \to \infty \end{aligned}\right\} \qquad (4)$$

The governing equations and the boundary conditions Eqs. (1)-(4) can be made non-dimensional, using the Grashof number $Gr = \frac{g\beta a^3 (T_w - T_\infty)}{\nu^2}$ which is assumed large and the following non-dimensional variables:

$$\left.\begin{aligned} &x = \frac{\bar{x}}{a}, y = \frac{\bar{y}}{a}Gr^{\frac{1}{4}}, u = \frac{\bar{u}a}{\nu}Gr^{-\frac{1}{2}}, \\ &v = \frac{\bar{v}a}{\nu}Gr^{-\frac{1}{4}}, \theta = \frac{T - T_\infty}{T_w - T_\infty} \end{aligned}\right\} \qquad (5)$$

Where θ is the dimensionless temperature. The non dimensional forms of the Eqs. (1)-(3) are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (6)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+Mu=\frac{\partial^2 u}{\partial y^2}+\theta\sin x \quad (7)$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}-\frac{g\beta a}{c_p}\left\{\frac{T_\infty+\theta(T_w-T_\infty)}{T_w-T_\infty}\right\}u+Ju^2 \quad (8)$$

Where $M=(\sigma a^2 B_0^2)/(\nu\rho Gr^{1/2})$ is the magnetic parameter, $J=(\sigma\nu B_0^2 Gr^{1/2})/\{\rho c_p(T_w-T_\infty)\}$ is the joule heating parameter and $Pr=\mu c_p/\kappa$ is the Prandtl number.

The boundary condition Eq. (4) can be written as in the following dimensionless form:

$$u=v=0,\ \theta=1\ at\ y=0$$
$$u\to 0,\ \theta\to 0\ as\ y\to\infty, \quad (9)$$

To solve Eqs. (6)-(8), subject to the boundary condition Eq. (9), we assume following transformations

$$\psi=x\,f(x,y),\ \theta=\theta(x,y) \quad (10)$$

Where ψ is the stream function usually defined as

$$u=\partial\psi/\partial y,\ v=-\partial\psi/\partial x \quad (11)$$

Substituting Eq. (11) into the Eqs. (6)-(9), the new forms of the dimensionless Eq. (7) and Eq. (8) are

$$f'''+ff''-f'^2-Mf'+\theta\frac{\sin x}{x}=x\left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right) \quad (12)$$

$$\frac{1}{\Pr}\theta''+f\theta'+\lambda x^2 f''^2-\varepsilon xf'\left[\frac{T_\infty}{T_w-T_\infty}\right]-\varepsilon\theta f'$$
$$+Jx^2f'^2=x\left(f'\frac{\partial\theta}{\partial x}-\theta'\frac{\partial f}{\partial x}\right) \quad (13)$$

In the above equations primes denote differentiation with respect to y. The corresponding boundary conditions take the following form:

$$\left.\begin{array}{l}f(x,0)=f'(x,0)=0,\theta=1\ at\ y=0\\ f'(x,\infty)\to 0,\ \theta'(x,\infty)\to 0\ as\ y\to\infty\end{array}\right\} \quad (14)$$

3. Method of Solution

Equations (12) and (13) are solved numerically based on the boundary conditions as described in Eq. (14) using one of the most efficient and accurate methods known as implicit finite difference method with Keller box scheme.

4. Result and Discussion

Joule heating effects on magneto-hydrodynamic natural convection flow in presence of pressure stress work and viscous dissipation from a horizontal circular cylinder has been investigated. The velocity profiles, temperature distributions, local skin-friction and the local rate of heat transfer obtained by the finite difference method for various values of the governing parameters. The aims of the figures are to display how the profiles vary with the scaled stream wise coordinate.

From Fig. 2(a), it is observed that the velocity increases as the values of the joule heating parameter J increase. The velocity increases significantly along y and becomes maximum and then decreases slowly and finally approaches to zero, the asymptotic value. The maximum values of the velocity are 0.32931258, 0.33958319, 0.35539332 and 0.36033431 for J = 0.1, 0.3, 0.5 and 0.9 respectively which occur at y= 1.80 for first, second maximum values, at y = 1.45 for third and fourth maximum values. Here it is observed that the velocity increase by 15.23% as J increases from 0.1 to 0.9. From Fig. 2(b), it is seen that when the values of joule heating parameter J increase, the temperature also increases.

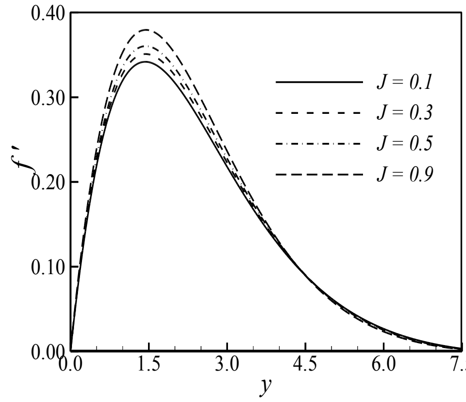

Fig. 2(a). Variation of velocity profile against *y* for varying of *J* with $M=0.1$, $\lambda=0.5$, $\varepsilon=0.5$ and $Pr=1.0$.

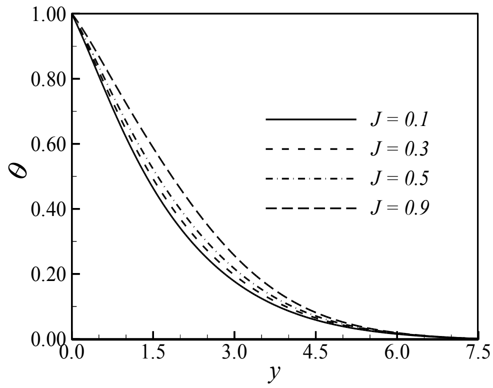

Fig. 2(b). Variation of temperature against y for varying of J with M = 0.1, $\lambda=0.5$, $\varepsilon=0.5$ and Pr=1.0.

Fig. 3(a) and Fig. 3(b) display results for the velocity and temperature profiles for different values of magnetic parameter M (M = 0.1, 0.3, 0.5, 0.9) having Prandtl number Pr = 1.0, J = 0.1 $\lambda=0.5$, $\varepsilon=0.5$. It is observed that, as the magnetic parameter M increases, the velocity profile decreases between $0\le y\le 5$ and then increases with very small difference and finally approaches to zero along y direction. The temperature profile increases with increasing magnetic parameter M. The maximum values of the velocity are recorded as 0.36033401, 0.32258730, 0.29037867 and 0.263928080 for M = 0.1, 0.3, 0.5 and 0.9 respectively which occur at y = 1.43 for 1st, 2nd, 3rd and 4th maximum values. It is found that the velocity decreases by 26.75% as the magnetic parameter M increases from 0.1 to 0.9.

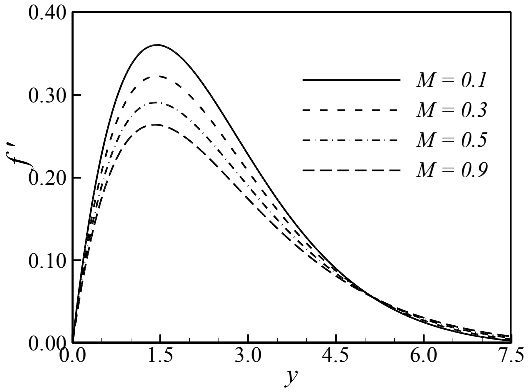

Fig. 3(a). Variation of velocity profile against y for varying of *M* with *J* = 0.1, $\lambda = 0.5$, $\varepsilon = 0.5$ and *Pr*=1.0.

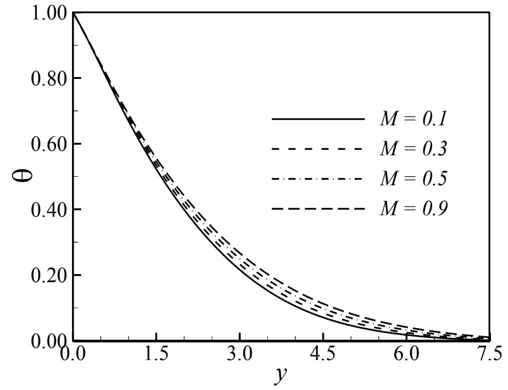

Fig. 3(b). Variation of temperature against y for varying of M with J = 0.1, $\lambda = 0.5$, $\varepsilon = 0.5$ and Pr=1.0.

Figures 4(a) and 4(b) indicate the effects of the Prandtl number Pr with M = 0.1, $\lambda = 0.5$ J = 0.1 and $\varepsilon = 0.5$ on the velocity profiles and the temperature profiles. From Fig. 4(a) it is observed that the increasing values of Prandtl number Pr leads to the decrease in the velocity profiles. The maximum values of the velocity are 0.34181852, 0.31031986, 0.27675142 and 0.26104378 for Pr = 0.72, 1.0, 1.44 and 1.74 respectively which occur at y = 1.43, y = 1.36, y = 1.30 and y = 1.26 for the first, second, third and fourth maximum value. Here it is depicted that the velocity decreases by 23.63% as Pr increases from 0.72 to 1.74. From Fig. 4(b) it is observed that the temperature profiles decreases with the increasing values of Prandtl number Pr.

It can easily be seen that the effect of the magnetic parameter M leads to a decrease in the local skin friction coefficient Cf_x and the local Nusselt number Nu_x in Fig. 5(a) and Fig. 5(b). This phenomenon can easily be understood from the fact that the magnetic parameter M opposes the flow, therefore decreases the velocity gradient and hence the local skin friction coefficient Cf_x decreases. Owing to increasing values of M in the presence of viscous dissipation and pressure stress work, the fluid temperature within the boundary layer increases and the associate thermal boundary layer becomes thicker. For increasing fluid temperature, the temperature difference between fluid and surface decreases and the corresponding rate of heat transfer decreases.

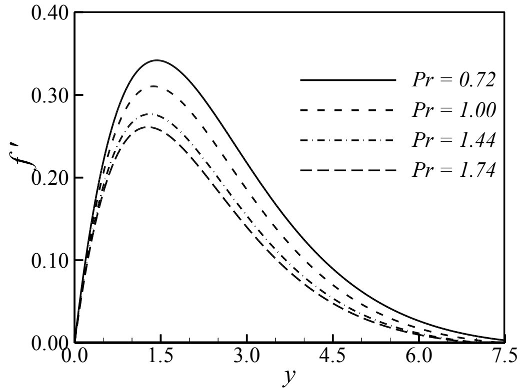

Fig. 4(a). Variation of velocity profile against *y* for varying of *Pr* with *M*= 0.1, $\lambda = 0.5$, $\varepsilon = 0.5$ and *J*=0.1.

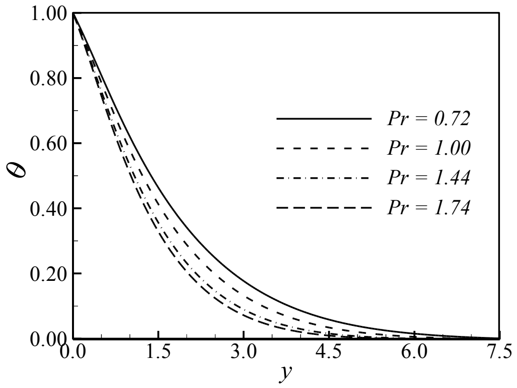

Fig. 4(b). Variation of temperature profile against *y* for varying of *Pr* with *M* =0.1, $\lambda = 0.5$, $\varepsilon = 0.5$ and *J*=0.1.

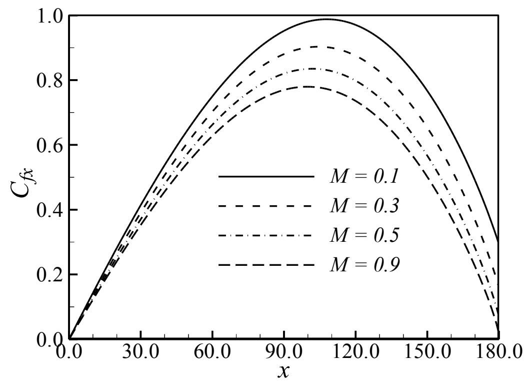

Fig. 5(a). Variation of skin friction against *x* for varying of *M* with *J* = 0.1, $\lambda = 0.5$, $\varepsilon = 0.5$ and *Pr*=1.0.

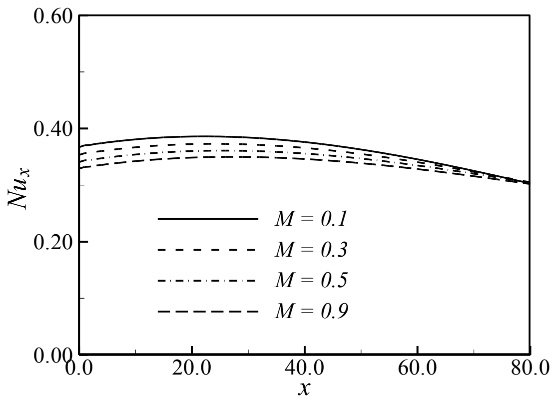

Fig. 5(b). Variation of heat transfer against x for varying of *M* with *J* = 0.1, $\lambda = 0.5$, $\varepsilon = 0.5$ and *Pr* = 1.0.

The variation of the reduced local skin friction coefficient and the local rate of heat transfer for different values of the joule heating parameter J (J = 0.1, 0.3, 0.5, 0.9) are illustrated in Fig. 6(a) and Fig. 6(b) with M = 0.1, $\lambda = 0.5$ and $\varepsilon = 0.5$ and Prandtl number Pr = 1.0. From the figures it can be seen that the increase of the joule heating parameter J leads to an increase in the local skin-friction coefficient Cf_x and a decrease in the local Nusselt number Nu_x. These are expected, since the joule heating mechanism in presence of viscous dissipation and pressure stress work creates a layer of hot fluid near the surface, and finally the resultant temperature of the fluid exceeds the surface temperature. For this reason the rate of heat transfer from the surface decreases. Owing to the enhanced temperature, the viscosity of the fluid increases and the corresponding local skin-friction coefficient increases.

In order to verify the accuracy of the present work, the numerical values of the local Nusselt number Nu_x for M = 0.0, J = 0.0, $\lambda = 0.0$, $\varepsilon = 0.0$ and Pr = 1.00 in different position of x are compared with those reported by Merkin (1976), Nazar *et al.* (2002) and Hye *et al.* (2007) as presented in table 1 The results are found to be in excellent agreement.

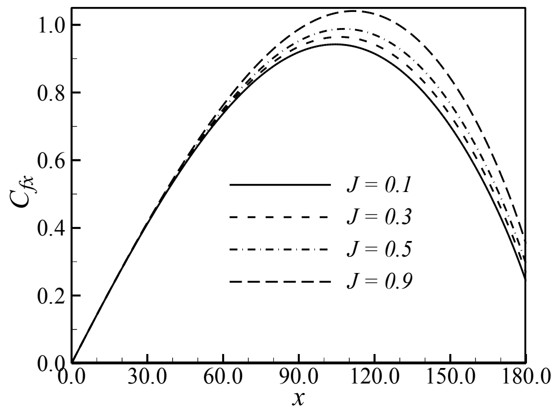

Fig. 6(a). Variation of skin friction against x for varying of *J* with *M* = 0.1, $\lambda = 0.5$, $\varepsilon = 0.5$ and *Pr* =1.0

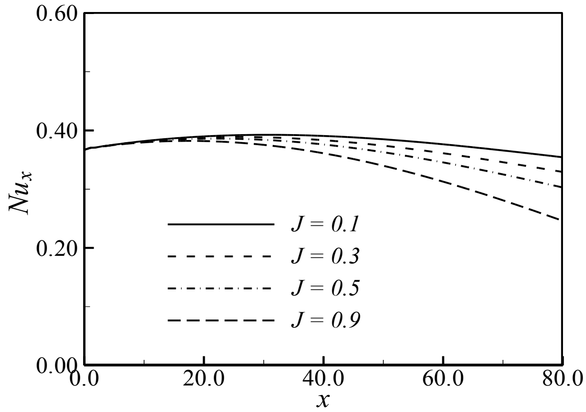

Fig. 6(b). Variation of heat transfer against x for varying of *J* with *M* = 0.1, $\lambda = 0.5$, $\varepsilon = 0.5$ and *Pr*=1.0.

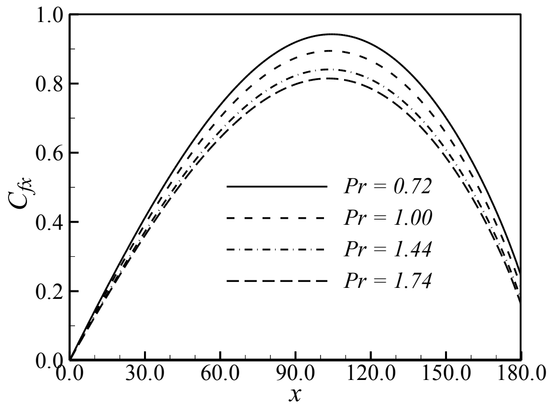

Fig. 7(a). Variation of skin friction against x for varying of Pr with M = 0.1, $\lambda = 0.5$, $\varepsilon = 0.5$ and J=0.1

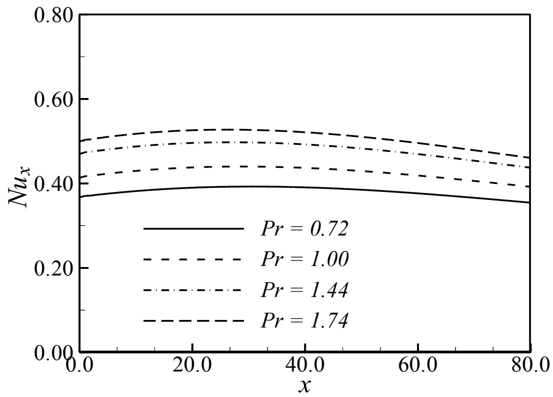

Fig.7(b). Variation of heat transfer against x for varying of Pr with M = 0.1, $\lambda = 0.5$, $\varepsilon = 0.5$, and J=0.1

Table 1 Numerical values of Nu_x for different values of x while Pr=1.0, M = 0.0, J=0.0, $\lambda = 0.0$ and $\varepsilon = 0.0$

x	Merkin (1976)	Nazar *et al.* (2002)	Hye *et al.* (2007)	Present
0.0	0.4214	0.4214	0.4241	0.4216
$\pi/6$	0.4161	0.4161	0.4161	0.4163
$\pi/3$	0.4007	0.4005	0.4005	0.4006
$\pi/2$	0.3745	0.3741	0.3741	0.3741
$2\pi/3$	0.3364	0.3355	0.3355	0.3355
$5\pi/6$	0.2825	0.2811	0.2811	0.2811
π	0.1945	0.1916	0.1916	0.1912

5. CONCLUSION

We have studied the joule heating effects on magneto-hydrodynamic (MHD) natural convection flow in presence of viscous dissipation and pressure stress work from a horizontal circular cylinder. The transformed non-similar boundary layer governing equations of the flow together with the boundary conditions were solved numerically using implicit finite difference method together with Keller box scheme. The coupled effect of natural convection that the temperature and the rate of heat transfer is continuous at the surface. From the present investigation, the following conclusions may be drawn:

- The local skin friction coefficients and the rate of heat transfer along the surface of the cylinder decrease for the increasing value of magnetic parameter M.
- An increase in values of M leads to decrease the velocity distribution but slightly increase the temperature distribution.
- For increasing values of joule heating parameter J, the skin-friction coefficient increases but the Nusselt number decreases significantly within the boundary layer.
- With the effect of joule heating parameter J, both the velocity and temperature distributions increase significantly the thickness of the thermal boundary layer.
- An increasing value of Prandtl number Pr leads to decrease in the velocity and the temperature distributions as expected.

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