

Mixed Convection Flow from Vertical Plate Embedded in Non-Newtonian Fluid Saturated Non-Darcy Porous Medium with Thermal Dispersion-Radiation and Melting Effects

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ABSTRACT

We analyzed in this paper the problem of mixed convection along a vertical plate in a non-Newtonian fluid saturated non-Darcy porous medium in the presence of melting and thermal dispersion-radiation effects for aiding and opposing external flows. Similarity solution for the governing equations is obtained for the flow equations in steady state. The equations are numerically solved by using Runge-Kutta fourth order method coupled with shooting technique. The effects of melting, thermal dispersion, radiation, temperature ratio, inertia and mixed convection on velocity distribution and temperature are examined for aiding and opposing external flows.

Keywords: Porous medium, Non-Newtonian fluid, Melting, Thermal dispersion, Radiation.

1. INTRODUCTION

Convection heat transfer in porous media in the presence of melting effect has received some attention in recent years. This stems from the fact that this topic has significant direct application in permafrost melting, frozen ground thawing, casting and bending processes as well as phase change metal. The study of melting effect is considered by many researchers in Newtonian fluids. Non-Newtonian power law fluids are so wide spread in industrial process and in the environment. The melting phenomena on free convection from a vertical front in a non-Newtonian fluid saturated porous matrix are analyzed by Poulidakos and Spatz (1988). Nakayama and Koyama (1991) studied the more general case of free convection over a non-isothermal body of arbitrary shape embedded in a porous medium. Rastogi and Poulidakos (1995) examined the problem of double diffusive convection from a vertical plate in a porous medium saturated with a non-Newtonian power law fluid. Shenoy (1994) presented many interesting applications of non-Newtonian power law fluids with yield stress on convective heat transport in fluid saturated porous media. Considering geothermal and oil reservoir engineering applications, Nakayama and Shenoy (1992)

studied a unified similarity transformation for Darcy and non-Darcy forced, free and mixed convection heat transfer in non-Newtonian inelastic fluid saturated porous media. Later Shenoy (1993) studied non-Darcy natural, forced and mixed convection heat transfer in non-Newtonian power law fluid saturated porous media.

Effect of melting and thermo-diffusion on natural convection heat mass transfer in a non-Newtonian fluid saturated non-Darcy porous medium was studied by Kairi and Murthy (2009). It is noted that the velocity, temperature and concentration profiles as well as the heat and mass transfer coefficients are significantly affected by the melting phenomena and thermal-diffusion in the medium. The non-linear behavior of non-Newtonian fluids in a porous matrix is quite different from that of Newtonian fluids in porous media. The prediction of heat or mass transfer characteristics for mixed and natural convection of non-Newtonian fluids in porous media is very important due to its practical engineering applications such as oil recovery and food processing. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the effect of thermal radiation and mass diffusion. On the other hand it is worth

mentioning that heat transfer simultaneous radiation and convection is very important in the context of space technology and processes involving high temperatures.

Recently, Bakier *et al.*(2009) studied Group method analysis of melting effect on MHD mixed convection flow from a radiative vertical plate embedded in saturated porous medium for Newtonian fluids. They developed linear transformation group approach to simulate problem of hydro magnetic heat transfer by mixed convection along vertical plate in a liquid saturated porous medium in the presence of melting and thermal radiation effects for opposing external flow. They studied the effects of the pertinent parameters on the rate of the heat transfer in terms of the local Nusselt number at the solid-liquid interface. More recently Melting and radiation effects on mixed convection from a vertical surface embedded in a non-Newtonian fluid saturated non-Darcy porous medium for aiding and opposing external flows is analyzed by Chamkha *et al.* (2010). They obtained representative flow and heat transfer results for various combinations of physical parameters. The present paper is aimed at analyzing the effect of melting and thermal dispersion-radiation on steady mixed convective heat transfer from a vertical plate embedded in a non-Newtonian power law fluid saturated non-Darcy porous medium for aiding and opposing external flows.

2. MATHEMATICAL FORMULATION

A mixed convective heat transfer in a non-Darcy porous medium saturated with a homogeneous non-Newtonian fluid adjacent to a vertical plate, with a uniform wall temperature is considered. This plate constitutes the interface between the liquid phase and the solid phase during melting inside the porous matrix at steady state. The plate is at a constant temperature T_m at which the material of the porous matrix melts. Fig.1 presents the geometry of the problem. The x-coordinate is measured along the plate and the y-coordinate normal to it. The solid phase is at temperature $T_0 < T_m$. A thin boundary layer exists close to the right of vertical plate and temperature changes smoothly through this layer from T_m to T_∞ ($T_m < T_\infty$) which is the temperature of the fluid phase.

Taking into account the effect of thermal dispersion the governing equations for steady non-Darcy flow in a non-Newtonian fluid saturated porous medium can be written as follows.

The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

The momentum equation is

$$\frac{\partial u^n}{\partial y} + \frac{C\sqrt{K}}{v} \frac{\partial u^2}{\partial y} = \mp \frac{K\beta g}{v} \frac{\partial T}{\partial y} \tag{2}$$

The energy equation is

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left(\alpha \frac{\partial T}{\partial y} \right) - \frac{1}{\rho C_p} \frac{\partial q}{\partial y} \tag{3}$$

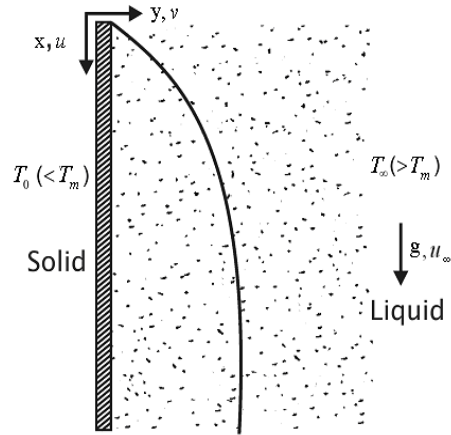


Fig. 1. Schematic diagram of the problem

In the above equations, the term which represents the buoyancy force effect on the flow field has \mp signs. The plus sign indicates aiding buoyancy flow where as the negative sign stands for buoyancy opposed flow. Here u and v are the velocities along x and y directions respectively, n is the power-law fluid viscosity index, T is Temperature in the thermal boundary layer, K is Permeability, C is Forchheimer empirical constant, β is coefficient of thermal expansion, ν is Kinematics viscosity, ρ is Density, C_p is Specific heat at constant pressure, g is acceleration due to gravity, and thermal diffusivity $\alpha = \alpha_m + \alpha_d$, where α_m is the molecular diffusivity and α_d is the dispersion thermal diffusivity due to mechanical dispersion. As in the linear model proposed by Plumb (1983), the dispersion thermal diffusivity α_d is proportional to the velocity component i.e $\alpha_d = \gamma u d$, where γ is the dispersion coefficient and d is the mean particle diameter. The radiative heat flux term q is written using the Rosseland approximation (Sparrow and Cess (1978), Raptis (1998)) as

$$q = - \frac{4\sigma_R}{3a} \frac{\partial T^4}{\partial y} \tag{4}$$

Where σ_R is the Stefan - Boltzmann constant and 'a' is the mean absorption coefficient.

The physical boundary conditions for the present problem are

$$y = 0, T = T_m, k \frac{\partial T}{\partial y} = \rho [h_{sf} + C_s(T_m - T_0)]v \tag{5}$$

$$\text{and } y \rightarrow \infty, T \rightarrow T_\infty, u \rightarrow u_\infty \tag{6}$$

Where h_{sf} and C_s are latent heat of the solid and specific heat of the solid phases respectively and u_∞ is the assisting external flow velocity, $k = \alpha \rho C_p$ is the effective thermal conductivity of the porous medium. The boundary condition in Eq. (5) means that the temperature on the plate is constant and thermal flux of heat conduction to the melting surface is equal to the sum of the heat of melting and the heat required for raising the temperature of solid to its melting temperature T_m .

Introducing the stream function ψ with

$$u = \frac{\partial \psi}{\partial y}, \text{ and } v = -\frac{\partial \psi}{\partial x}$$

The continuity Eq. (1) will be satisfied and the Eq. (2) and Eq. (3) transform to

$$\frac{\partial}{\partial y} \left(\frac{\partial \psi}{\partial y} \right)^n + \frac{C\sqrt{K}}{v} \frac{\partial}{\partial y} \left(\frac{\partial \psi}{\partial y} \right)^2 = \mp \frac{K g \beta \theta T}{v} \frac{\partial T}{\partial y} \quad (7)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left[(\alpha_m + \gamma \frac{\partial \psi}{\partial y} d) \frac{\partial T}{\partial y} \right] + \frac{4\sigma_R}{3\rho C_p a} \frac{\partial}{\partial y} \left[\frac{\partial T^4}{\partial y} \right] \quad (8)$$

Introducing the similarity variables as

$$\psi = f(\eta)(\alpha_m u_\infty x)^{1/2}, \quad \eta = \left(\frac{u_\infty x}{\alpha_m} \right)^{1/2} \left(\frac{y}{x} \right), \quad \theta(\eta) = \frac{T - T_m}{T_\infty - T_m}$$

The momentum Eq. (7) and energy Eq. (8) are reduced to

$$n f^{11} f^{1n-1} + 2 F f^1 f^{11} = \mp \left(\frac{Ra_x}{Pe_x} \right)^n \theta^1 \quad (9)$$

$$\left(1 + D f^1 \right) \theta^{11} + \left(\frac{1}{2} f + D f^{11} \right) \theta^1 + \frac{4}{3} R \left[\theta + C_r \right]^3 \theta^{11} + 3 \theta^{12} \left(\theta + C_r \right)^2 = 0 \quad (10)$$

Where the prime symbol denotes the differentiation with respect to the similarity variable η and Ra_x/Pe_x is the mixed convection parameter,

$Ra_x = \frac{x}{a} \left(\frac{g \beta k (T_\infty - T_m)}{v} \right)^{1/n}$ is the local Rayleigh number, $Pe_x = \frac{u_\infty x}{\alpha}$ is the local pecllet number.

$F = f_0 (Pe_d)^{2-n}$, $f_0 = \left(\frac{a}{d} \right)^{2-n} \left(\frac{C\sqrt{K}}{v} \right)$ is the non-darcian parameter. Pe_d is the pore diameter dependent pecllet number. $D = \frac{\gamma d u_\infty}{\alpha_m}$ is the dispersion parameter. $C_r = \frac{T_m}{T_\infty - T_m}$ is the temperature ratio,

$R = \frac{4\sigma_R (T_\infty - T_m)^3}{ka}$ is the radiation parameter.

Taking into consideration, the thermal dispersion effect together with melting, the boundary conditions in Eq. (5) and Eq. (6) take the form

$$\eta=0, \theta=0, f(0) + \{1 + D f^1(0)\} 2M \theta^1(0) = 0. \quad (11)$$

$$\text{and } \eta \rightarrow \infty, \theta=1, f^1=1. \quad (12)$$

Where $M = \frac{C_f (T_\infty - T_m)}{h_{sf} + C_s (T_m - T_0)}$ is the melting parameter. The local heat transfer rate from the surface of the plane is given by $q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}$.

The Nusselt number is $Nu_x = \frac{hx}{k} = \frac{q_w x}{k(T_m - T_\infty)}$, where h is the local heat transfer coefficient and k is the effective thermal conductivity of the porous medium, which is the sum of the molecular thermal conductivity k_m and the dispersion thermal conductivity k_d . The Nusselt number is obtained as

$$\frac{Nu_x}{(Pe_x)^{1/2}} = \left[1 + \frac{4}{3} R (\theta(0) + C_r)^3 + D f^1(0) \right] \theta^1(0) \quad (13)$$

3. SOLUTION PROCEDURE

The dimensionless equations Eq.(9) and Eq.(10) together with the boundary conditions in Eq. (11) and Eq. (12) are solved numerically by means of the fourth order Runge-Kutta method coupled with double shooting technique. The solution thus obtained is matched with the given values of $f^1(\infty)$ and $\theta(0)$. In addition the boundary condition $\eta \rightarrow \infty$ is approximated by $\eta_{max} = 8$ which is found sufficiently large for the velocity and temperature to approach the relevant free stream properties. Numerical computations are carried out for $F=0, 0.5, 1$; $D=0, 0.5, 1$; $Ra/Pe=1$; $M=0, 0.4, 0.8, 1.2, 1.6, 2$; $n=0.5, R=0, 0.5, 1$; $Cr=0.1, 0.5, 1$

4. RESULTS AND DISCUSSION

In order to get clear insight on the physics of the problem, a parametric study is performed and the obtained numerical results are displayed with the help of graphical illustrations. The parameters governing the physics of the present study are the melting (M), the mixed convection (Ra/Pe), the inertia (F), thermal dispersion (D), and Fluid viscosity index (n), radiation(R), temperature ratio(Cr). The numerical computations were carried out for the fixed value of buoyancy parameter $Ra/Pe = 1$ for both the aiding and opposing external flows. The results of the parametric study are shown in Fig. 2 to Fig. 21.

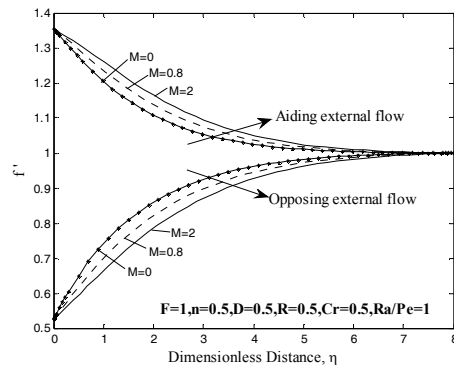
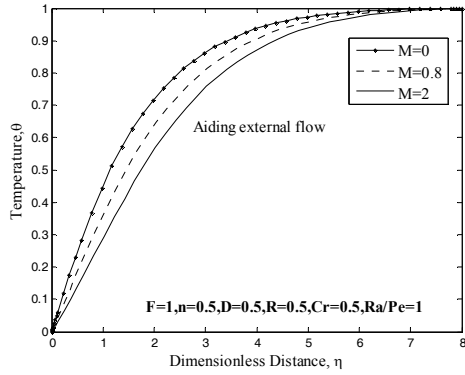


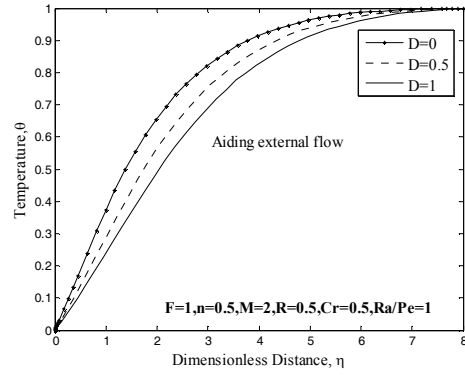
Fig. 2. Effect of melting parameter on fluid velocity

Figure. 2 shows the effect of melting parameter M on fluid velocity for both aiding and opposing external flows. It is observed that different behaviour exists between the fluid velocity in the presence of solid phase melting effect in the case of aiding and opposing external flows. The increase in the melting parameter M causes the increase of the fluid velocity for the case of aiding flow. Whereas the opposite behaviour for the fluid velocity as M increases is found in the case of opposing flow case.

Figure. 3 shows the effect of the melting parameter M on temperature distributions for aiding and opposing external flows. It is observed that as increasing the value of the melting parameter M , the temperature distributions decrease for both cases of aiding and opposing external flow conditions.

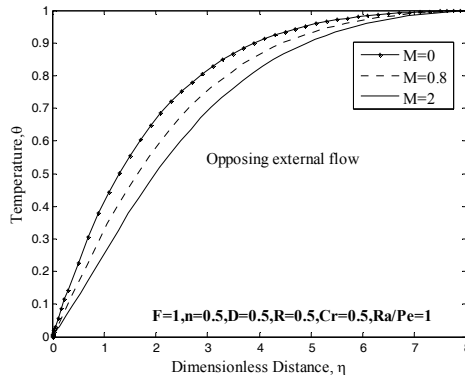


(a)

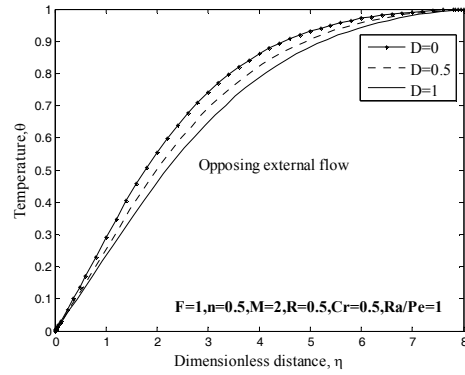


(b)

Fig. 3. Effect of M on fluid temperature.



(a)



(b)

Fig. 5. Effect of thermal dispersion parameter on fluid temperature for aiding and opposing flows.

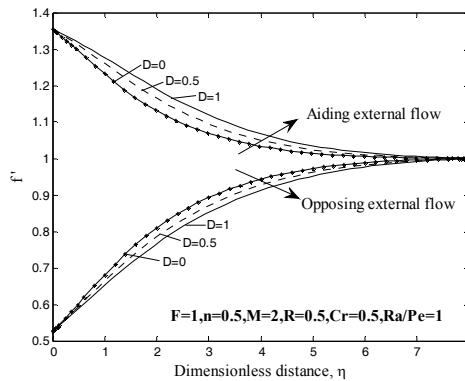


Fig. 4. Effect of thermal dispersion parameter on fluid velocity

The effects of thermal dispersion parameter D on the fluid velocity for both cases of aiding and opposing external flow conditions are plotted in Fig. 4. It is noted that for the case of aiding flow the fluid velocity increase with increase in the value of D . But this effect is found opposite in the case of opposing flow.

The effects of thermal dispersion parameter D on temperature distributions for aiding and external flow conditions are plotted in Fig. 5. It is noted that increasing the values of D leads to decrease in the liquid temperature distributions in both cases.

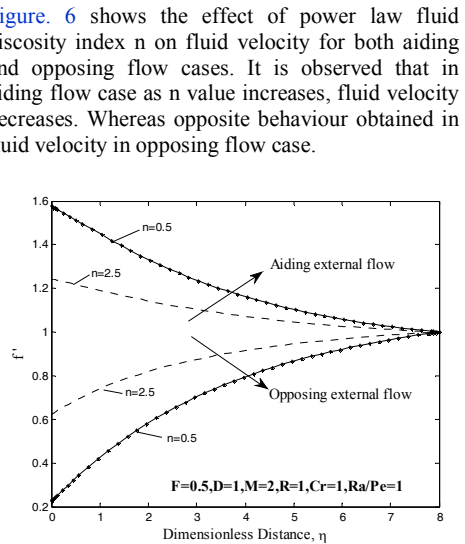


Fig. 6. Effect of viscosity index on fluid velocity

Figure 7 show the effects of power law fluid viscosity index n on temperature distributions for both aiding and opposing flow cases. Significant effect is not found. In aiding flow case as n increases, the temperature distributions decrease. Whereas opposite result is found in opposing flow case.

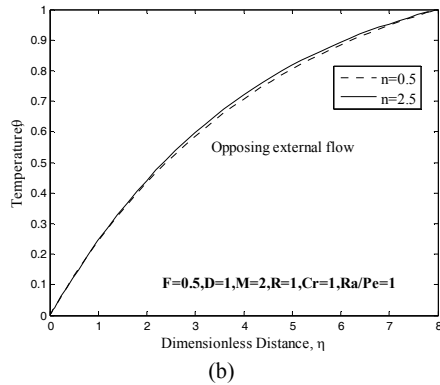
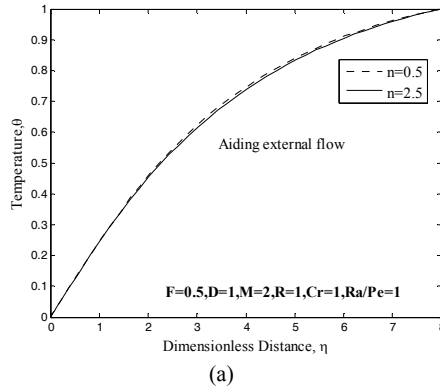


Fig. 7. Effect of viscosity index on fluid temperature for aiding and opposing flows.

Figure 8 shows the effect of radiation parameter R on fluid velocity for both aiding and opposing external flows. The increase in the radiation parameter R causes the increase of the fluid velocity for aiding flow case. Whereas in the case of opposing flow the result is found opposite.

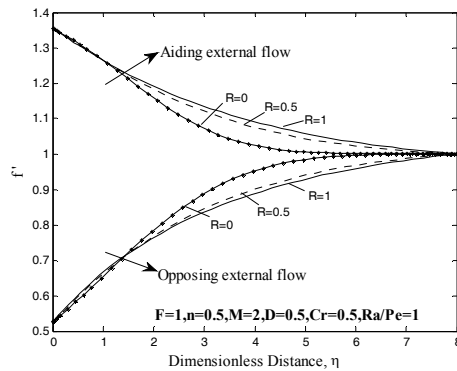


Fig. 8. Effect of radiation parameter on fluid velocity.

Figure 9 shows the effect of radiation parameter R on temperature distributions for aiding and opposing external flow conditions. It is observed that same phenomena exist in both cases. As R increases the temperature distributions decrease for both cases.

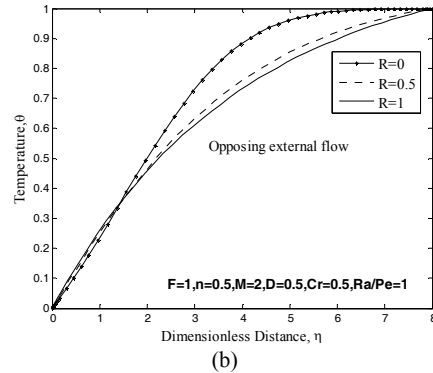
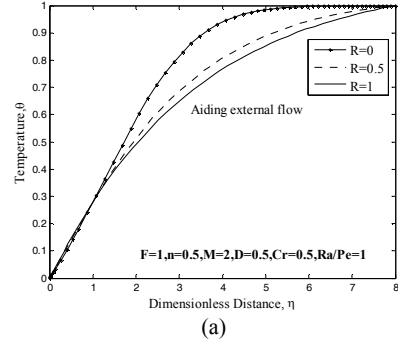


Fig. 9. Effect of radiation parameter on fluid temperature for aiding and opposing flows.

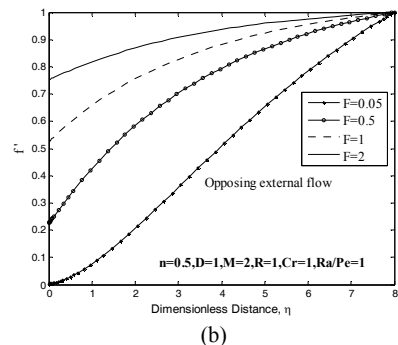
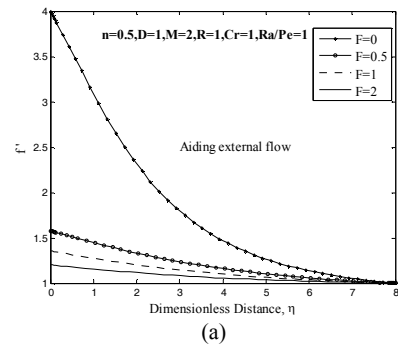


Fig. 10. Effect of non-Darcy porous medium parameter on fluid velocity for aiding and opposing flows.

Figure 10 shows the effect of non-Darcy porous medium parameter F on fluid velocity for aiding

and external flow conditions. In the aiding flow case the fluid velocity decrease with increasing the value of F . But in the opposing flow case the fluid velocity increase with the increase of F value.

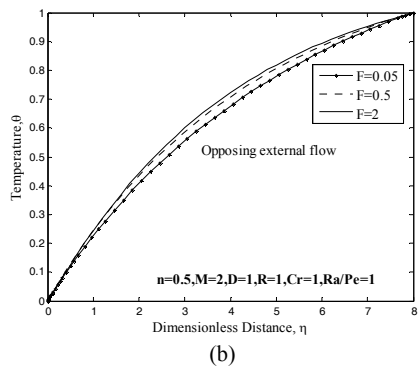
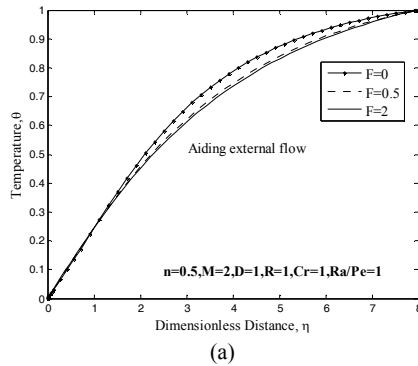


Fig. 11. Effect of non-Darcy porous medium parameter on fluid temperature for aiding and opposing flows.

Figure 11 show the effects of non-Darcy porous medium parameter F on temperature distributions for aiding and opposing flow cases. In aiding flow case the temperature distributions decrease with the increase of F value. Whereas in opposing flow case the temperature distributions increase with the increase of F value.

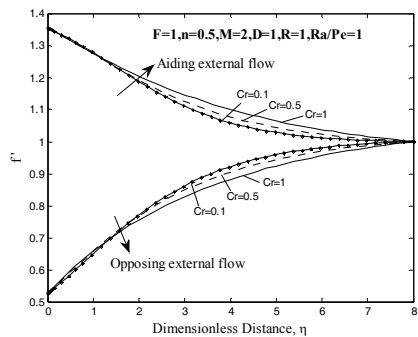


Fig. 12. Effect of temperature ratio parameter on fluid velocity.

The effects of temperature ratio parameter Cr on fluid velocity for aiding and opposing external flow cases are plotted in Fig.12. In aiding flow case, the increase in the temperature ratio parameter Cr

results increase of the fluid velocity. Whereas in the opposing flow case opposite effect is found in the fluid velocity.

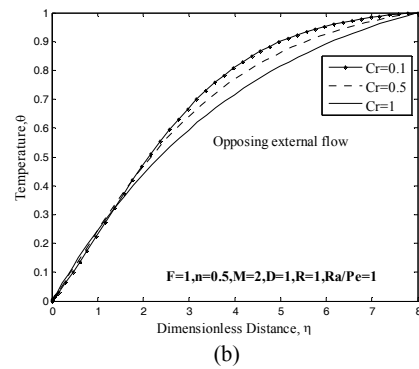
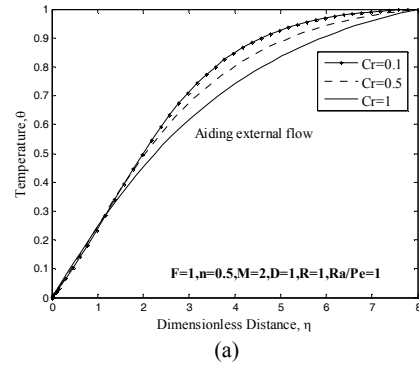


Fig. 13. Effect of temperature ratio parameter on fluid temperature for aiding and opposing flows.

The effects of temperature ratio parameter Cr on temperature distributions are plotted in Fig. 13 for aiding and opposing flow cases. Same effect is found in both cases. The increase in temperature ratio parameter Cr results decrease of the temperature distributions in both cases.

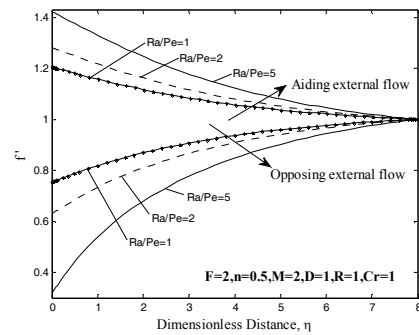


Fig. 14. Effect of mixed convection parameter on fluid velocity.

The effects of mixed convection parameter Ra/Pe on fluid velocity for aiding and opposing external flow cases are plotted in Fig. 14. In aiding flow case, the increase in the mixed convection parameter results increase of the fluid velocity.

Whereas in the opposing flow case opposite effect is found in the fluid velocity.

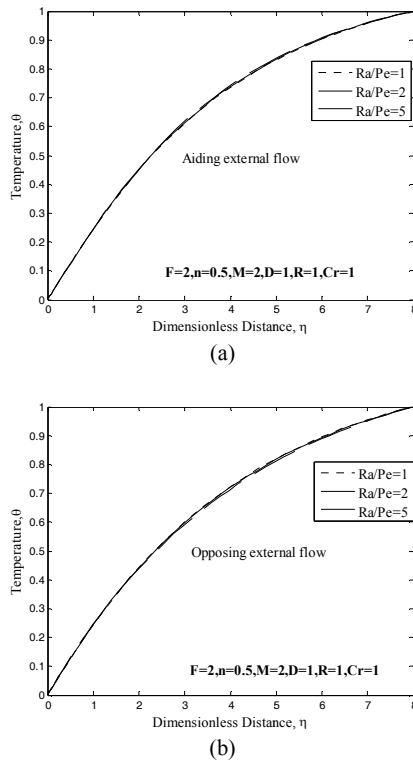


Fig. 15. Effect of mixed convection parameter on fluid temperature for aiding and opposing flows.

Figure 15 show the effects of mixed convection parameter on temperature distributions for both aiding and opposing flow cases. Significant effect is not found. In aiding flow case as Ra/Pe value increases, the temperature distributions increase. Whereas opposite result is found in opposing flow case.

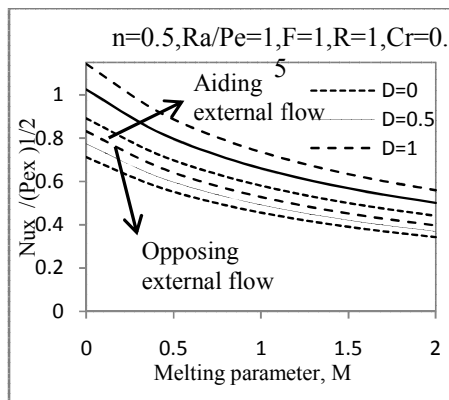


Fig. 16. Variation of Nusselt number with the melting parameter for different values of thermal dispersion parameter in aiding and opposing flows.

The effect of melting strength and thermal dispersion on heat transfer rate is shown in Fig. 16 for both aiding and opposing flows in terms of Nusselt number defined in Eq. (13). It is observed

that Nusselt number decreases significantly with the increase of melting strength M and increases with increase in thermal dispersion parameter D for both aiding and opposing flows.

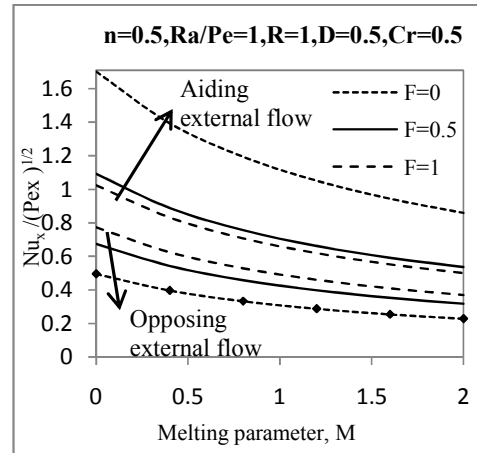


Fig. 17. Variation of Nusselt number with the melting parameter for different values of non-Darcy parameter in aiding and opposing flows.

The variation of Nusselt number with melting parameter for different values of non-Darcy parameter F is shown in Fig. 17 for both aiding and opposing flows. In aiding flow case the Nusselt number decreases as the non-Darcy parameter F increases. Whereas in the opposing flow case the effect is found opposite.

The variation of Nusselt number with melting parameter M for different values of radiation parameter R is shown in Fig. 18 for both aiding and opposing flows. It is observed that in both cases the Nusselt number increases as the radiation parameter R increases.

The variation of Nusselt number with melting parameter M for different values of temperature ratio parameter Cr are shown in Fig. 19 for both aiding and opposing flows. It is observed that the Nusselt number increases as the temperature ratio parameter increases in both flow conditions.

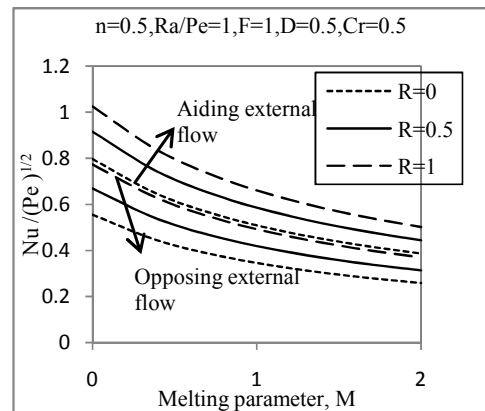


Fig. 18. Variation of Nusselt number with the melting parameter for different values of non-Darcy parameter in aiding and opposing flows.

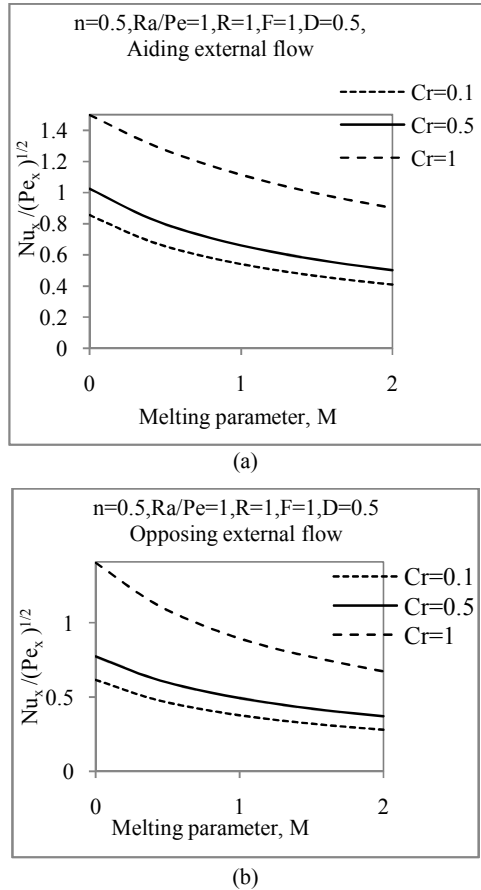


Fig. 19. Variation of Nusselt number with the melting parameter for different values of non-Darcy parameter in aiding and opposing flows.

The effect of melting strength and mixed convection on heat transfer rate is shown in Fig. 20 for both aiding and opposing flows in terms of Nusselt number defined in Eq (13). It is observed that Nusselt number decreases significantly with the increase of melting strength M in both flow cases and increases with increase in mixed convection parameter Ra/Pe in aiding flow. Whereas in the opposing flow case the nusselt number decreases as Ra/Pe value increases.

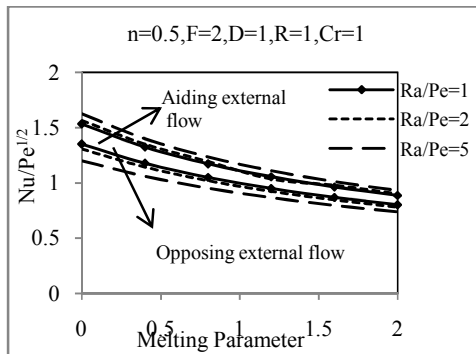


Fig. 20. Variation of Nusselt number with the melting parameter for different values of mixed convection parameter in aiding and opposing flows.

The effect of melting strength and fluid viscosity index ‘ n ’ on heat transfer rate is shown in Fig. 21 for both aiding and opposing flows in terms of Nusselt number defined in Eq (13). It is observed that Nusselt number decreases significantly with the increase of melting strength M in both flow cases and decreases with increase in fluid viscosity index in aiding flow. Whereas in the opposing flow case the nusselt number increases as ‘ n ’ value increases.

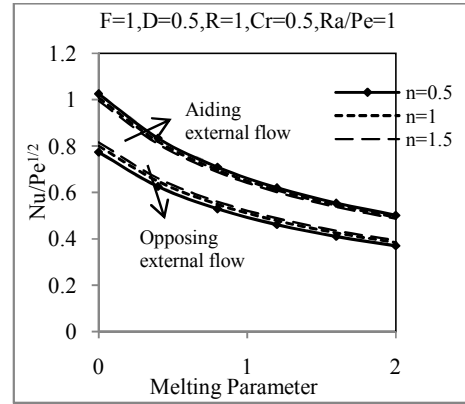


Fig. 21. Variation of Nusselt number with the melting parameter for different values of fluid viscosity index in aiding and opposing flows.

We compared our results with those of earlier published works of Chamkha *et al.* (2010) and Cheng and Lin (2007) for special cases of the problem under consideration. Table 1 to Table 3 show that our numerical values are in good agreement with the compared results.

Table 1 Comparison of $f'(0)$ with values obtained by Chamkha *et al.* (2010) and Cheng and Lin (2007) for Newtonian fluid ($n = 1.0$) with an aiding external flow for the case of $M = 2.0$

Ra/Pe	Chamkha <i>et al.</i> (2010)	Cheng and Lin (2007)	Present
0.0	1.000	1.000	1.000
1.4	2.400	2.400	2.400
3.0	4.000	4.000	4.000
8.0	9.008	9.000	9.000
10	11.00	11.00	11.00

Table 2 Comparison of $\theta^1(0)$ with values obtained by Chamkha *et al.* (2010) and Cheng and Lin (2007) for Newtonian fluid ($n = 1.0$) with an aiding external flow for the case of $M = 2.0$

Ra/Pe	Chamkha <i>et al.</i> (2010)	Cheng and Lin (2007)	Present
0.0	0.27060	0.27060	0.27056
1.4	0.38020	0.38010	0.38003
3.0	0.47470	0.47450	0.47449
8.0	0.69050	0.69020	0.69009
10	0.75980	0.75940	0.75929

Table 3 Comparison of $\theta^1(0)$ with values obtained by Chamkha *et al.* (2010) and Cheng and Lin (2007) for Newtonian fluid ($n = 1.0$) with an opposing external flow for the case of $M = 0.0$

Ra/Pe	Chamkha <i>et al.</i> (2010)	Cheng and Lin (2007)	Present
0.2	0.52720	0.52700	0.52694
0.4	0.48670	0.48660	0.48656
0.6	0.44210	0.44210	0.44204
0.8	0.39170	0.39170	0.39169
1.0	0.33200	0.33210	0.33205

5. CONCLUSION

In the present paper, the effect of melting on mixed convection flow from a vertical plate embedded in a Non-Newtonian fluid saturated non-Darcy porous medium in the presence of thermal dispersion - radiation is analyzed in aiding and opposing flow cases. The obtained non linear differential equations are solved by 4th order Runge-Kutta method coupled with shooting technique. The results are presented graphically. It is observed that the velocity of the fluid increases / decreases with the increase in the parameter values of melting, thermal dispersion, thermal radiation, temperature ratio parameter and mixed convection in aiding / opposing flow. It is also noted that the velocity decreases / increases with the increase in the parameter values of inertia and fluid viscosity index in aiding / opposing flow.

However, the temperature of the fluid decreases with the increase in the parameter values of melting, thermal dispersion, radiation and temperature ratio parameter in both aiding and opposing flow cases. In aiding flow, from Darcian to non-Darcian the temperature decreases. In opposing flow, the temperature increases from Darcian to non-Darcian. Also the temperature increases / decreases with the increase in the value of mixed convection parameter in aiding / opposing flow. Further the temperature decreases / increases with the increase in the value of fluid viscosity index in aiding / opposing flow.

Further, it is observed that the heat transfer rate increases with the increase of dispersion, radiation and temperature ratio parameter in both the flow cases. Furthermore, it is noticed that the Nusselt number decreases / increases with the increase in the parameter values of inertia and fluid viscosity index in aiding / opposing flow. The Nusselt number decreases significantly with the increase of melting parameter in both the flow cases. The heat transfer rate increases / decreases with the increase of mixed convection parameter value in aiding / opposing flow.

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