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Dynamic Analysis of Pig through Two and Three Dimensional Gas Pipeline

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ABSTRACT

This paper deals with the dynamic analysis and simulation of Pipeline Inspection Gage (PIG) through the two and three dimensional gas pipelines. Continuity, momentum and the state equations are employed to achieve the gas flow parameters like density, velocity and pressure along the pipeline since the dynamic behavior of the pig depends on the flow field characteristics. Also, a differential equation which governs the dynamic behavior of the pig is derived. The pig is assumed to be a small rigid body with a bypass hole in its body. The variation of the diameter of the bypass port, which is controlled by a valve, is considered in this research. The path of the pig or geometry of the pipeline is assumed to be 2D and 3D curve. 2D and 3D simulations of the pig motion are performed individually using Rung-Kutta method and a case has been solved and discussed for each of them. The simulation results show that the derived equations are valid and effective for online estimating of the position, velocity and forces acting on the pig in gas pipelines at any time of the motion.

Keywords: Pig, Dynamic, Simulation, Gas pipeline.

NOMENCLATURE

A	area cross section of the pipeline	F_f	friction force
D	pipeline diameter	f	coefficient of friction loss in pipeline
D_h	hydraulic diameter	P_{tail}	pressure at the tail of the pig
g	acceleration of gravity	P_{nose}	pressure at the nose of the pig
$f(x)$	function of centerline of the pipe in 2D	A_h	area cross section of the valve
sgn(x)	sign function of x	V_h	velocity of fluid at the valve
F_μ	dry friction force	p	wet pipeline perimeter
L_{pig}	length of the pig	λ	a time dependent parameter in 3D
m	mass of the pig		
N	normal force acting on the pig		
K_{SC}, K_V, K_{SE}	coefficients of pressure losses		

1. INTRODUCTION

Various kinds of petroleum products are yet transported in worldwide using pipelines. During these transportation, the pipeline will deposit debris or residual products such as scale, wax, and gas hydrates as well as exposed to physical damages such as dent and internal corrosion. Pipeline engineers have to know the internal status of these pipelines because after sometimes, they loose their healthy and may explode.
Pigging operation is a standard and accepted method to deal with these emerging challenges. Thus, the pipelines have to be cleaned and inspected periodically using the suitable pigs. The smart or intelligent pigs are those advanced pigs used in this area. They are equipped with a number of transducers which are installed on them circumferentially for detection of surface defects such as cracks and corrosions.

Engineers have to consider many parameters for designing a pigging operation such as the effects of velocity, upstream pressure, temperature and etc. The dynamic analysis of a pig in an arbitrary 2 and

3-dimensional pipelines can estimate these important parameters for the designers. This analysis can also specify the required injected pressure for pigging operations and approximate the total time of it. The dynamic behavior of the pig depends on the difference pressure across its body and height variations of its center of gravity. Determination of kinematics parameters of a pig and acting forces on it can be obtained by solving the governing equations of fluid flow together with the pig dynamic equations.

Results of research on the motion of the pig in pipeline are scarcely found in the literatures. Most of results are commercially based on field experience. It seems that the first research in modeling of the pig motion has been introduced by McDonal et. al. (1964). The computation errors were generated in this work due to the used assumption and numerical approach. This modeling was modified and improved by removing some limitations by Barua (1982). The first pigging model was based on full two-phase transient flow formulation proposed by Kohda et. al. (1988). This model is composed of correlations for pressure drop across the pig, slug holdup, pigging efficiency, pig velocity model and a gas and liquid mass flow boundary condition applied to the slug front. Some other complementary researches for pigging simulation in two-phase flow straight pipelines were also reported by (Minami and Shoham 1991; Taitel et. al. 1989;Scoggins and M. W. 1977; Xiao-Xuan and Gong 2005).Transient pig motion through gas and liquid pipelines was presented by Nieckele et. al. (2001). Modeling and simulation for pig flow control in natural gas pipeline was studied by Nguyen et. al. (2001). One type of pig using bypass flow in natural gas pipeline was considered by some investigators such as Nguyen et. al.(2001). They proposed a simple nonlinear controller for controlling the pig velocity when it flows in natural gas straight pipeline. Also, to provide an efficient tool to assist in the control and design of pig operations through pipelines, a numerical code has been developed by Esmaeilzadeh et. al.(2009). The results obtained with the code in this research were compared with experimental results and a good agreement between the two was obtained.

In all the mentioned studies, researchers assumed that the pig moves in a straight line in the plane. Simulation of small pig in space pipeline was studied firstly by Saeidbakhsh et. al.(2009). This study was based on some simplifying assumptions such as: the pig is small, the pig/wall friction coefficient is constant and the driving force is time-dependent. In this research, the influence of the flow field is modeled only by time dependent driving force acting on the pig. This assumption was made only for simplicity. The effect of flow field was considered by Lesani et. al. (2012) for a bypass flow pig through two and three dimensional liquid pipeline. This extension is based on some simplifying assumptions such as: the pig is small, the pig/ wall friction coefficient is constant and fluid is incompressible. Speed control of the bypass flow pig using the QFT method was done by Mirshamsi and Rafeeyan (2012) to keep the pig velocity near a constant value. In this study fluid was considered incompressible and the pipeline is two dimensional. Considering the influence of flow field on the pig's trajectory through gas pipelines can be a logical extension for further work.

The objective of the present work is the dynamic analysis of a pig motion in the two and three dimensional gas pipelines. In this study, fluid is assumed compressible. The differential equations of the pig in pipelines are derived by Newton's second law for pig and momentum, continuity and state equation for fluid. Two differential equations are solved. First, the differential equation of fluid is solved to get fluid properties such as velocity, pressure and density along the pipelines. Then, the differential equation of pig motion is solved to get the pig velocity and position in pipeline. Rung-Kutta method is used for solving both equations. The 2D and 3D test cases are chosen to illustrate the application of this new formulation for these cases.

1. MODELING

2.1 Case 1: Two-Dimensional Path

2.1.1 Pig dynamics in 2-D pipeline

Figure 1 shows a typical small pig moving inside a two-dimensional pipeline and its free body diagram. The dynamic equations of the pig, derived from Newton's second law along the tangential and normal directions, are as follows:

$$N - mg\cos\theta = ma_n \tag{1}$$

$$F_1 - F_2 - mg\sin\theta - \text{sgn}(\dot{x})F_\mu = ma_t \tag{2}$$

Where θ is the angle of the tangent to the centerline curve of the pipeline with respect to the x-axis at any point; i.e. if $f(x)$ is assumed to be the function of the centerline of the pig, thus we can write

$$\theta = \tan^{-1} f'(x),$$

$$\sin\theta = f'(x)/\sqrt{1+f'(x)^2}, \tag{3}$$

$$\cos\theta = 1/\sqrt{1+f'(x)^2}$$

If S measures along the pig's path and the radius of curvature of the path is R, then we can derive both accelerations of the pig as follows:

$$V_{pig} = \dot{s} = \sqrt{1+f'(x)^2}\ \dot{x} \tag{4}$$

$$a_n = \frac{V_{pig}^2}{R} = \frac{f''(x)}{\sqrt{1+f'(x)^2}}\dot{x}^2 \tag{5}$$

$$a_t = \frac{d^2s}{dt^2} = \frac{f''(x)f'(x)}{\sqrt{1+f'(x)^2}}\dot{x}^2 + \ddot{x}^2\sqrt{1+f'(x)^2} \tag{6}$$

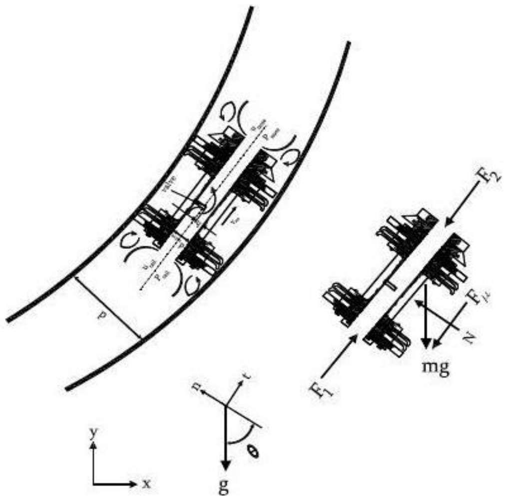


Fig. 1. Schematic view of a pig inside a planer pipeline.

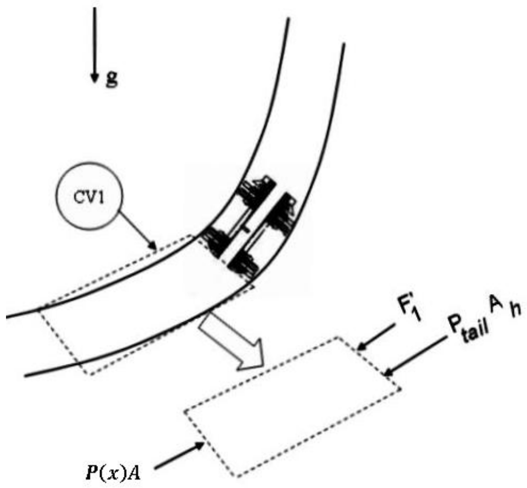


Fig. 2. The control volume No. 1

To derive the term $F_1 - F_2$ in the left side of equation (2), one can combine the momentum and energy equations of the fluid. To determine the term $F_1 - F_2$, two control volumes are employed as shown in Figs. 2 and 3; one behind the pig and the other in front of it. Also it is assumed that the lengths of control volumes are the same and equals to $2L_{pig}$, where L_{pig} is the length of the pig.

Equation (7) shows the momentum equation for an inertial control volume.

$$\vec{F} = \vec{F}_S + \vec{F}_B = \int_{CS} \vec{V}\rho\vec{V}.d\vec{A} + \frac{\partial}{\partial t}\int_{CV} \vec{V}\rho\, dV \tag{7}$$

In this equation, surface forces, body forces and velocity are denoted by $\vec{F}_S$, $\vec{F}_B$ and V respectively.

The different terms of the above equation in the tangential direction (t-direction) for the two control volumes are as follows:

$$F_{St1} = P(x)A - F_1' - P_{tail}A_h,$$

$$\int_{CS1} V_t\rho\vec{V}.d\vec{A} = \rho_h A_h (V_h - V_{pig})^2 - \rho(x)A(V(x) - V_{pig})^2,$$

$$\frac{\partial}{\partial t}\int_{CV1} V_t \rho dV \cong 0 \tag{8}$$

$$F_{St2} = -P(x + 5L_{pig}\cos\theta)A + F_2' + P_{nose}A_h,$$

$$\int_{CS2} V_t\rho\vec{V}.d\vec{A} = -\rho_h A_h (V_h - V_{pig})^2$$

$$+ \rho(x + 5L_{pig}\cos\theta)A(V(x + 5L_{pig}\cos\theta) - V_{pig})^2$$

$$, \frac{\partial}{\partial t}\int_{CV2} V_t \rho dV \cong 0 \tag{9}$$

$$F_{Bt1} = -2\rho(x + L_{pig}\cos\theta)gL_{pig}A\sin\theta,$$

$$F_{Bt2} = -2\rho(x + 4L_{pig}\cos\theta)gL_{pig}A\sin\theta \tag{10}$$

Where $V(x)$, $\rho(x)$ and $P(x)$ are the velocity, density and pressure of the fluid in position x, respectively that will be obtained in part 2.1.2, F_1', F_2' are reactions of the pig on the control volumes ($F_1 = -F_1', F_2 = -F_2'$), P_{tail} and P_{nose} are pressures of the fluid in behind and in front of the pig, A_h is the area cross section of the valve, ρ_h is the density of fluid at the valve. Substituting (8)-(10) in (7) for each control volumes, it leads to

$$F_1 - F_2 = P(x + 5L_{pig}\cos\theta) - P(x)]A$$
$$+ (P_{tail} - P_{nose})A_h + 2[\rho(x + L_{pig}\cos\theta)$$
$$+ \rho(x + 4L_{pig}\cos\theta)]gL_{pig}A\sin\theta +$$
$$[\frac{\rho^2(x)A^2}{\rho_h A_h} - \rho(x)A](V(x) - V_{pig})^2$$
$$+ [-\frac{\rho^2(x + 5L_{pig}\cos\theta)A^2}{\rho_h A_h}$$
$$- \rho(x + 5L_{pig}\cos\theta)A]$$
$$(V(x + 5L_{pig}\cos\theta) - V_{pig})^2 \tag{11}$$

In general, the pressure difference between the tail and the nose of the bypass hole in the pig $(P_{tail} - P_{nose})$ is established from three parts; i.e. pressure losses from a sudden contraction at the tail (K_{SC}), the valve inside the hole (K_V) and a sudden expansion at the nose (K_{SE}). The following relations are suggested for the pressure difference and the loss coefficients in the fluid mechanics books and papers, e.g. Nguyen *et al.* (2001):

$$P_{tail} - P_{nose} = \frac{K_{total}\rho(V_h - V_{pig})^2}{2} \tag{12}$$

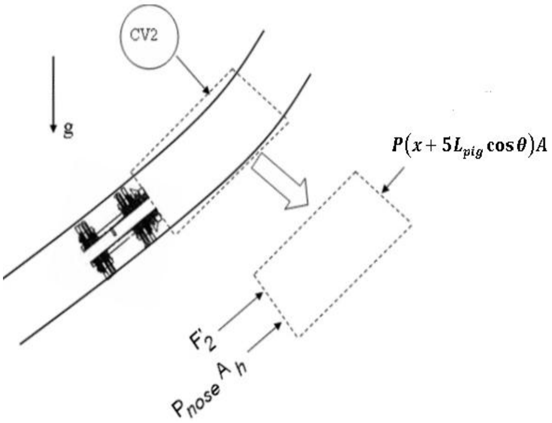


Fig. 3. The control volume No. 2

where

$$K_{total} = K_{SC} + K_V + K_{SE}$$

$$K_{SC} = 0.42(1 - \frac{d^2_{valve}}{d^2})$$

$$K_V = f(\frac{h}{d_{valve}}) \quad , \qquad (13)$$

$$K_{SE} = (1 - \frac{d^2_{valve}}{D^2})$$

$$\min(K_{total}) = (1 - \frac{d^2_{valve}}{D^2})(1.42 - \frac{d^2_{valve}}{D^2})$$

The normal force N is obtained if one substitutes Eqs. (3) and (5) in Eq. (1)

$$N = m[\frac{f''(x)}{\sqrt{1+f'(x)^2}}\dot{x}^2 + \frac{1}{\sqrt{1+f'(x)^2}}g] \qquad (14)$$

The final equation of the pig can be derived by substituting all the terms in Eq. (2). This equation is a second order ordinary differential equation that can be solved numerically.

2.1.2 Gas Flow Model in 2-D Pipeline

To determine $V(x)$, $\rho(x)$ and $P(x)$, the velocity, density and pressure of the fluid in position x respectively, the following assumptions are employed:

1. The fluid flow is steady-state.

2. The fluid flow is isothermal along the pipeline.

3. The coefficient of friction losses along the path is constant.

Gas flow in long constant-area ducts, such as natural gas pipelines, is essentially isothermal. Mach numbers in such flows are generally low, but significant pressure changes can occur as a result of frictional effects acting over long duct lengths. Hence, such flows cannot be treated as incompressible. The assumption of isothermal flow is much more appropriate Fox et. al. (2004).

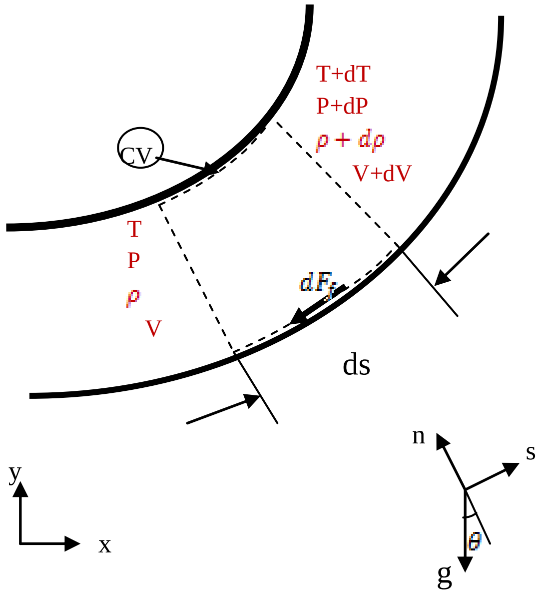


Fig. 4. The differential control volume in 2-D pipeline

The flow dynamics can be modeled based on three fundamental fluid equations: Continuity equation, Momentum equation and State equation. To determine the fluid equation, a differential control volume is employed as shown in Fig. 4.

Continuity and momentum equations can be written respectively as:

$$\rho dV + V d\rho = 0 \qquad (15)$$

$$-dF_f - (\rho + d\rho/2)gA\,ds\sin\theta + PA - (P+dP)A = \dot{m}(V+dV) - \dot{m}V \qquad (16)$$

If s is measured along the pipeline then ds can be written as follows:

$$ds = \frac{dx}{\cos\theta} = \sqrt{1+f'(x)^2}\,dx \qquad (17)$$

So Eq. (16) can be simplified to:

$$-\frac{dF_f}{A} - \rho g f'(x)dx - dP = \rho V dV \qquad (18)$$

Friction force F_f is known as:

$$dF_f = \tau_w dA_w = \tau_w p ds \qquad (19)$$

Shear stress τ_w can be written as:

$$\tau_w = \frac{dF_f}{2}\frac{dP}{dx} = \frac{f\rho V^2}{8} \qquad (20)$$

where f is the coefficient of friction loss in pipeline. Using hydraulic diameter D_h ,p can be described as:

$$p = 4A/D_h \qquad (21)$$

Combining Eqs (17), (19)-(21) we have:

$$dF_f = \frac{fA}{D_h}\frac{\rho V^2}{2}\sqrt{1+f'(x)^2}\,dx \qquad (22)$$

Substituting Eq.(22) in Eq.(18) and dividing by P , we get:

$$\frac{dP}{P} = -\frac{f}{D_h}\frac{\rho V^2}{2P}\sqrt{1+f'(x)^2}dx - \frac{\rho g}{P}f'(x)dx - \frac{\rho V dV}{P} \qquad (23)$$

Noting that $P/\rho = RT = c^2/k$, $VdV = d(V^2/2)$ we can write:

$$\frac{dP}{P} = -\frac{f}{D_h}\frac{kM^2}{2}\sqrt{1+f'(x)^2}dx - \frac{kg}{c^2}f'(x)dx - \frac{kM^2}{2}d(V^2/2) \qquad (24)$$

Where $M = V/c$ is the Mach number. For deleting $d(V^2/2)$ and $\frac{dP}{P}$, equations below can be used:

$$\frac{d(V^2)}{V^2} = \frac{dT}{T} + \frac{dM}{M^2}, \frac{d\rho}{\rho} = -\frac{1}{2}\frac{d(V^2)}{V^2}, \frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T} \qquad (25)$$

Since flow is assumed to be isothermal, so $\frac{dT}{T} = 0$. Substituting Eq. (25) in Eq. (24) results in

$$\frac{d(M^2)}{dx} = \frac{2M^2}{1-kM^2}\left[\frac{f}{D_h}\frac{kM^2}{2}\sqrt{1+f'(x)^2} + \frac{kg}{c^2}f'(x)\right] \qquad (26)$$

The equation is a one-order ordinary differential equation that can be solved numerically to obtain the flow Mach number (velocity) along the pipeline. The flow density and pressure along the pipeline can be achieved from continuity and state equation respectively, having flow velocity.

Continuity equation in any section of pipeline can be written as:

$$\rho_0 V_0 = \rho(x)V(x) \qquad (27)$$

Where ρ_0 and V_0 are initial density and velocity of flow. State equation is as follows:

$$\frac{P(x)}{P_0} = \frac{\rho(x)}{\rho_0} \qquad (28)$$

Where P_0 is initial flow pressure.

2.2 Case 2: Three-Dimensional Path

2.2.1 Pig Dynamics in 3-D pipeline

Figure 5 shows the free body diagram of a typical small pig inside a three-dimensional pipeline. The weight of the pig, dry friction force, normal force of the pipe wall and driving force are ***W***, ***f***, ***N***, ***P***, respectively. These forces, as shown in the Fig. 5, are three dimensional vectors in general. The position vector of the C.G. of the pig is denoted with $r(\lambda)$ where λ is a time dependent parameter. We can write it with respect to its components, i.e.

$$r(\lambda) = x(\lambda)i + y(\lambda)j + z(\lambda)k \qquad (29)$$

It is assumed that the gravitational force acts in the y-direction, then the weighting force of the pig is represented as $W = -mgj$. A frame of mutually orthogonal unit vectors is defined that it always travels along with a body moving in space (Thomas 1996).There are three vectors in the frame. One of them is the unit tangent vector ***τ***. Another one is ***n*** vector that gives the direction of ***dτ/ds***. The third is ***b=τ*** ×***n*** . These vectors and their derivatives are called as Serret-Frenet formulas (Sokolnikoff 1964), when available, give useful information about moving vehicles and paths they trace. The magnitude of the derivative ***dτ/ds*** tells us how much a pig's path turns to the left or right as it moves along; it is called the *curvature* of the path. The number $|db/ds|$ tells us how much a pig's path rotates or twists as the pig moves along; it is called the torsion of the pig's path.

Although there are experimental data that indicate that the dynamic contact force (friction) might be a function of the pig velocity, however as Hopkins (1992), it is considered that it is independent of the pig velocity. This force acts in the direction of the pipe axis and in the opposite of the pig's motion. The direction of the normal force ***N*** is unknown; however it does not have any component in the tangential direction of the pig's path. The driving force ***P*** acts in the direction of the pipe axis. This force depends on the pressure difference between the nose and the tail of the pig and other variables in a complex problem. The dynamic equations of the pig derived from the Newton's Second Law along ***n***, ***τ*** and ***b*** directions are as follow;

$$P(\lambda) - \mathrm{sgn}(V)f + mg_\tau = m\frac{d^2s}{dt^2} \qquad (30)$$

$$N_1 + mg_n = mV^2/R \qquad (31)$$

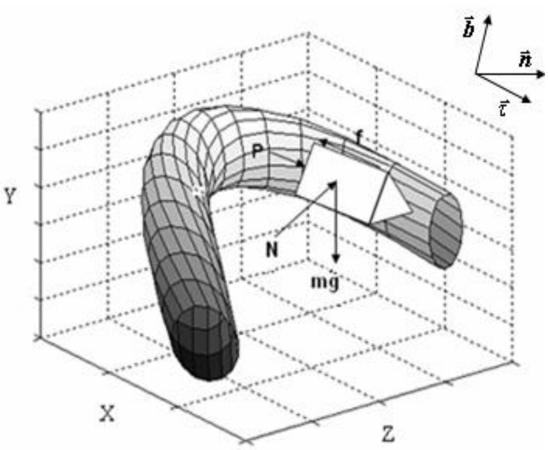


Fig. 5. Schematic view of a pig inside a 3D pipeline

$$N_2 + mg_b = 0 \tag{32}$$

where,

$$N = N_1 n + N_2 b$$

$$W = mg_\tau \tau + mg_n n + mg_b b$$

And s is measured along the pig's path. If we use the chain rule and differentiate from Eq. (29) for obtaining the ***n***, ***τ*** and ***b*** at all points and denote $d/dt = \dot{(\ )}$ and $d/d\lambda = (\ )'$, then the following relation will be obtained;

$$\tau = \frac{x'i + y'j + z'k}{\sqrt{x'^2 + y'^2 + z'^2}} \tag{33}$$

Based on the first Serret-Frenet formula, we have, $d\tau/ds = \kappa n$ where κ is the curvature and will be determined from;

$$\frac{d\tau}{ds} = \frac{d\tau}{dt}.\frac{dt}{ds} = a_1 i + a_2 j + a_3 k$$

$$a_1 = (x''y'^2 + z'^2 x'' - x'y'y'' - x'z'z'')/\Delta^2$$

$$a_2 = (x'^2 y'' + y''z'^2 - y'x'x'' - y'z'z'')/\Delta^2$$

$$a_3 = (y'^2 z'' + x'^2 z'' - z'x'x'' - z'y'y'')/\Delta^2$$

$$\Delta^2 = x'^2 + y'^2 + z'^2 \tag{34}$$

$$\kappa = \frac{1}{R} = \left|\frac{d\tau}{ds}\right| = \sqrt{a_1^2 + a_2^2 + a_3^2} \tag{35}$$

Also the normal unit vector ***n*** can be obtained from Eq. (34), i.e.

$$n = (a_1 i + a_2 j + a_3 k)/\Delta \tag{36}$$

The second normal unit vector is derived from the outer cross product of ***τ*** and ***n***, i.e.

$$b = \tau \times n = \frac{\dot\lambda}{|\dot\lambda|\sqrt{\Delta(a_1^2 + a_2^2 + a_3^2)}}$$
$$[(a_3 y' - a_2 z')i - (a_3 x' - a_1 z')j + (a_2 x' - a_1 z')k] \tag{37}$$

Since the projection of the vector ***V***1 on ***V2*** is equal to ***V***1.***V***2 /***V***2 then g_n, g_b and g_τ can be obtained as:

$$g_n = -\frac{ga_2}{\sqrt{a_1^2 + a_2^2 + a_3^2}} \tag{38}$$

$$g_b = \frac{g(a_3 x' - a_1 z')\lambda}{|\lambda|\sqrt{\Delta(a_1^2 + a_2^2 + a_3^2)}} \tag{39}$$

$$g_n = -\frac{gy'\lambda}{|\lambda|\sqrt{x'^2 + y'^2 + z'^2}} \tag{40}$$

Now, one needs to calculate the remaining unknown terms in Eqs. (30) to (32) i.e.,

$$V = \dot r(\lambda) = (x'i + y'j + z'k)\dot\lambda \tag{41}$$

If one substitutes Eqs. (37), (38) and the magnitude of Eq.(41) in Eqs. (31) and (32), the components of normal forces are determined as:

$$N_1 = m[\dot\lambda^2 \Delta\sqrt{a_1^2 + a_2^2 + a_3^2}$$
$$+ \frac{ga_2}{\sqrt{a_1^2 + a_2^2 + a_3^2}}] \tag{42}$$

$$N_2 = -\frac{mg(a_3 x' - a_1 z')\lambda}{|\lambda|\sqrt{\Delta(a_1^2 + a_2^2 + a_3^2)}} \tag{43}$$

$$|N| = \sqrt{N_1^2 + N_2^2} \quad , \quad f = \mu|N| \tag{44}$$

Finally, substituting Eqs. (44) and (41) in Eq. (30) results in:

$$p(\lambda) - \mathrm{sgn}(V)\mu|N| - m\frac{gy'\dot\lambda}{|\dot\lambda|\sqrt{x'^2 + y'^2 + z'^2}} =$$
$$m\frac{\dot\lambda}{|\dot\lambda|}[\frac{\dot\lambda^2(x'x'' + y'y'' + z'z'')}{\sqrt{x'^2 + y'^2 + z'^2}} + \frac{(x'^2 + y'^2 + z'^2)}{\sqrt{x'^2 + y'^2 + z'^2}}\ddot\lambda] \tag{45}$$

The term $p(\lambda)$ can be obtained from momentum and energy equations in pipeline direction as follows

$$
\begin{aligned}
&p(\lambda)=[P(\lambda+5L_{pig}\frac{1}{\sqrt{x'^2+y'^2+z'^2}})-\\
&P(\lambda)]A+[(\frac{k_{total}}{2}+1)\frac{\rho^2(\lambda)A^2}{\rho_h A_h}-\rho(\lambda)A]\\
&(V(\lambda)-V_{pig})^2+2[\rho(\lambda+5L_{pig}\frac{1}{\sqrt{x'^2+y'^2+z'^2}})\\
&+\rho(\lambda)]gL_{pig}A\frac{y'\dot{\lambda}}{|\dot{\lambda}|\sqrt{x'^2+y'^2+z'^2}}+\\
&+[-\frac{\rho^2(\lambda+5L_{pig}\frac{1}{\sqrt{x'^2+y'^2+z'^2}})A^2}{\rho_h A_h}\\
&+\rho(\lambda+5L_{pig}\frac{1}{\sqrt{x'^2+y'^2+z'^2}})A]\\
&(V(\lambda+5L_{pig}\frac{1}{\sqrt{x'^2+y'^2+z'^2}})-V_{pig})^2
\end{aligned}
\qquad (46)
$$

Where $V(\lambda)$, $\rho(\lambda)$ and $P(\lambda)$ are the velocity, density and pressure of the fluid, respectively that will be obtained in part 2.2.2. Substituting Eq. (46) in Eq. (45) and rearranging it with respect to λ results in the final differential equation of the pig. This will be a nonlinear differential equation with respect to λ, which can be solved by a numerical technique such as Runge-Kutta based on initial conditions. When parameter λ is determined in each time instant *t*, then the position and the velocity of the pig can be calculated.

2.2.1 Gas Flow Model in 3-D Pipeline

To determine the fluid equation, a differential control volume is employed as shown in Figure 6.

Continuity and momentum equations can be written respectively as:

$$\rho dV+Vd\rho=0 \qquad (47)$$

$$
\begin{aligned}
&-dF_f-(\rho+d\rho/2)gA\,ds\frac{gy'\dot{\lambda}}{|\dot{\lambda}|\sqrt{x'^2+y'^2+z'^2}}+\\
&PA-(P+dP)A=\dot{m}(V+dV)-\dot{m}V
\end{aligned}
\qquad (48)
$$

As *s* is measured along the pig's path we can write

$$ds=\sqrt{x'^2+y'^2+z'^2}\,d\lambda \qquad (49)$$

Substituting Eqs (49) and (20)-(21) in Eq. (19), one can obtain

$$dF_f=\frac{fA}{D_h}\frac{\rho V^2}{2}\sqrt{x'^2+y'^2+z'^2}\,d\lambda \qquad (50)$$

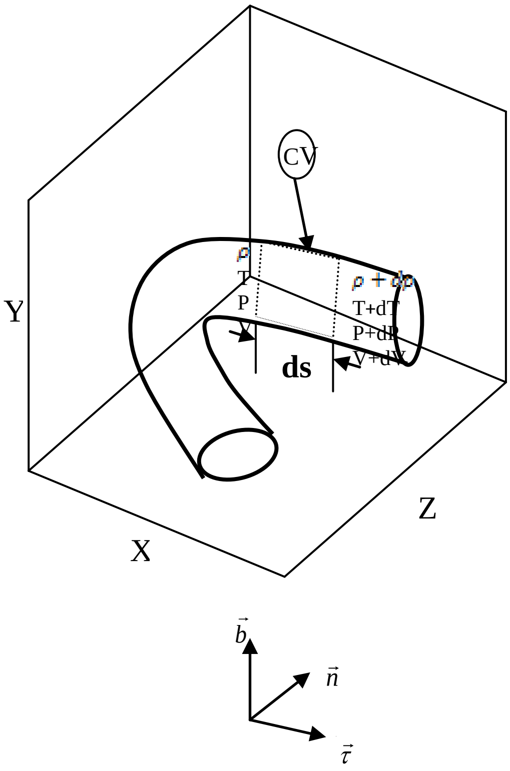


Fig. 6. The differential control volume in 3-D pipeline

Using the same sequence as in part 2.2.1 differential equation of Mach number changes can be achieved as;

$$
\begin{aligned}
&\frac{d(M^2)}{dx}=\frac{2M^2}{1-kM^2}[\frac{f}{D_h}\frac{kM^2}{2}\sqrt{x'^2+y'^2+z'^2}\\
&+\frac{kg}{c^2}y']
\end{aligned}
\qquad (51)
$$

The equation is a one-order ordinary differential equation that can be solved numerically to obtain the flow Mach number (velocity) along the pipeline. The flow density and pressure along the pipeline can be achieved from continuity and state equation i. e. equations (27) and (28), respectively.

3. NUMERICAL EXAMPLES

To show the efficiency of the developed model, some numerical studies are designed and solved for two and three dimensional pipelines. In all examples, Runge-Kutta-Fehlberg method is used to solve the differential equations by writing a program in Matlab software.

3.1 Example 1

For the first case study, let us assume a 2D pipeline with the curve equation as $y(x)=0.5(x+\sin x)$. Figure 7 shows the pipeline curve. The numerical values for the mass of the pig, length of the pig, pipe diameter and the bypass diameter are assumed to be 400kg, 1.4 m, 0.7366 m and 0.57 m, respectively.

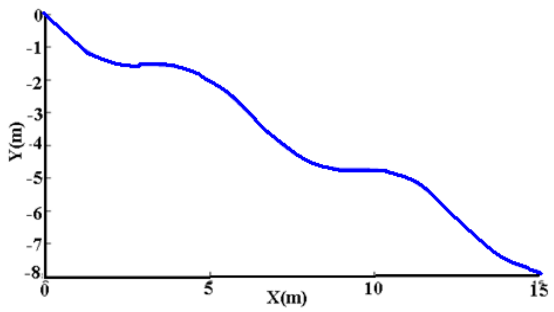


Fig. 7. The pipeline curve in 2D for example 1

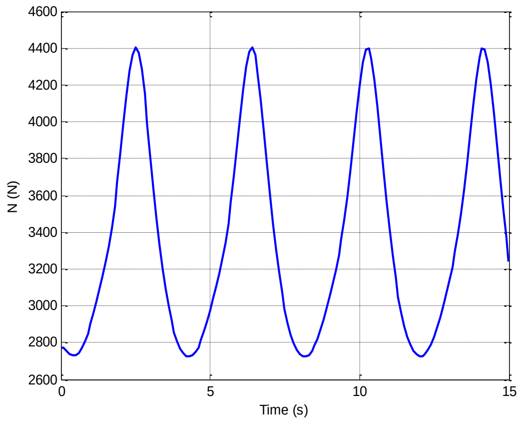


Fig. 9. Normal force exerted on the pig for example 1.

The numerical values for fluid that used in all numerical examples are given in table 1.The dynamic and static coefficients of friction are assumed to be 0.2 and 0.3, respectively. The pig position at initial time is at the point $x(0)=y(0)=0$ and its velocity is zero. The aim of this example is to test the validation of the formulations for the simpler case, i.e. two-dimensional path. The geometric curvilinear periodic nature of this pipeline can help this examination. Figure 8 shows the velocity of the pig with respect to time. This figure shows the periodic variations of the pig's velocity. Figure 9 also shows the periodic nature of the normal force acting on the pig.

Table 1 The Numerical values for simulations

Temperature (T)	278 K
Gas constant (R)	287 J/kg K
k	1.4
Inlet pressure of gas(P_0)	59×10^5 Pa
Inlet flow of gas (Q_0)	1.37 m^3/s
Inlet density of gas (ρ_0)	105.44 kg/m^3
coefficient of friction loss (f)	0.038

3.2 Example 2

The geometry of the selected pipeline for the third test case is shown in Fig. 10. It is similar to a well- known

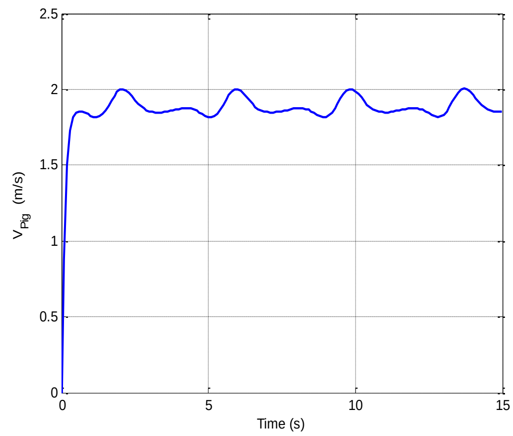


Fig. 8. Velocity of the pig for example 1.

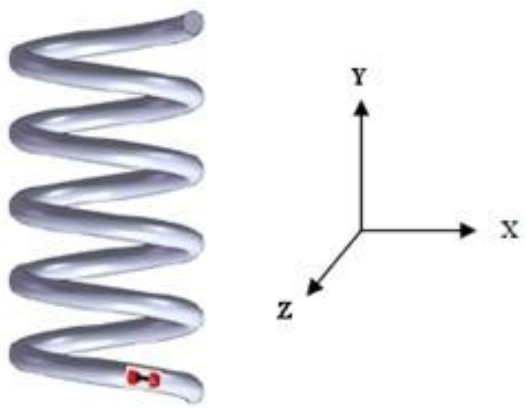


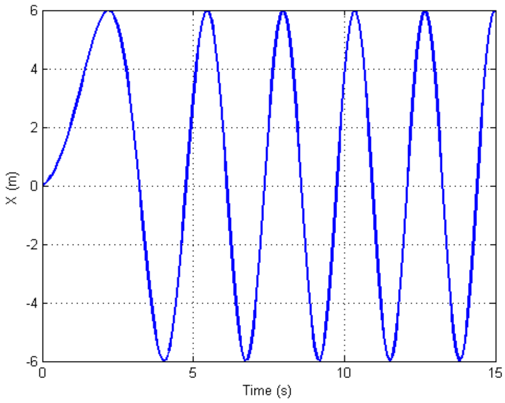
Fig. 10. Selected geometry of pipeline for example 2.

space curve such as helix with the following parametric equations

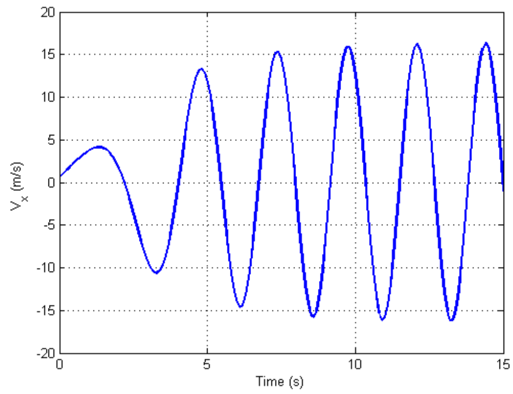
$$x(\lambda)=6\sin\lambda,\ y(\lambda)=-6\lambda,\ z(\lambda)=6\cos\lambda$$

The numerical values for the mass of the pig, length of the pig, pipe diameter and the bypass diameter are assumed to be 400 kg , 1.4 m, 0.7366 m and 0.67 m , respectively. Initial conditions for solving dynamic equation are $\lambda=0, \dot{\lambda}=0.1$ thus $x(0)=0, y(0)=0, z(0)=6$ m and $\dot{x}(0)=0.6, \dot{y}(0)=-0.6, \dot{z}(0)=0$ m/s. The numerical values for fluid that used in all numerical examples are given in table 1.

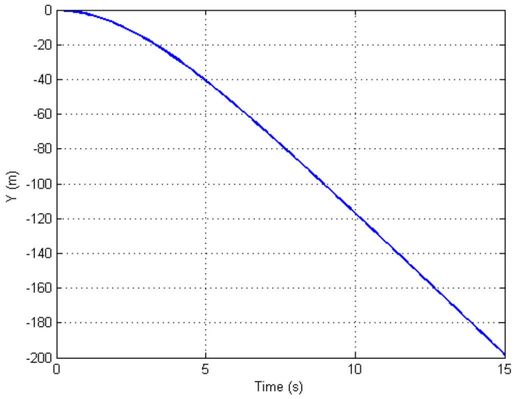
This example shows the validation of the above formulation for three dimensional path, where its geometric is curvilinear periodic nature. Figures 11a to 11f show the position and velocities of the pig in the x, y and z-directions with respect to time. The positions of the pig in the x and y-directions have a periodic nature. The velocities in these directions have the same sorts of trends. However, the velocity of the pig increases up to a certain level and then remains constant. This is true because of the special geometry of the pipeline. Since there is no difference between any two points of the



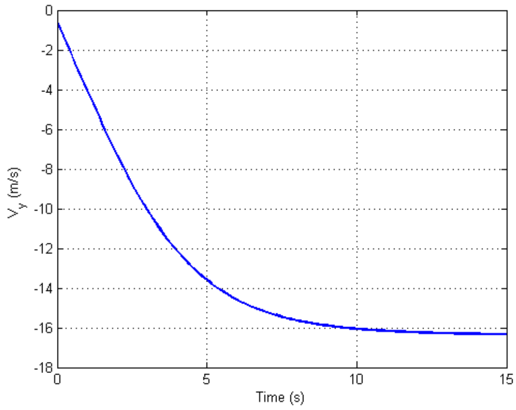
a) Position of the pig in x direction.



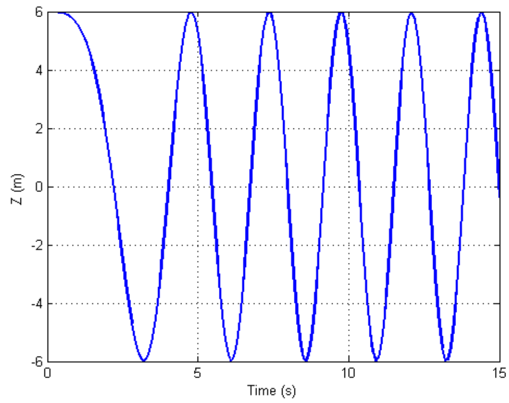
b) Velocity of the pig in x direction.



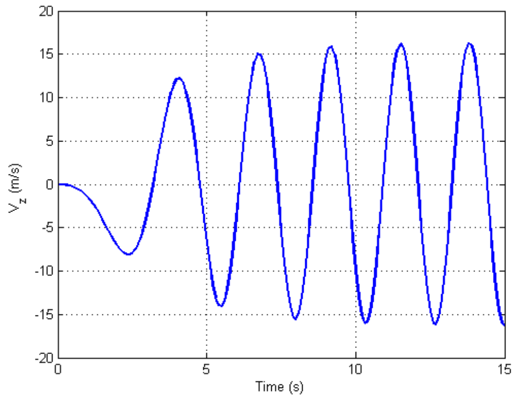
c) Position of the pig in y direction.



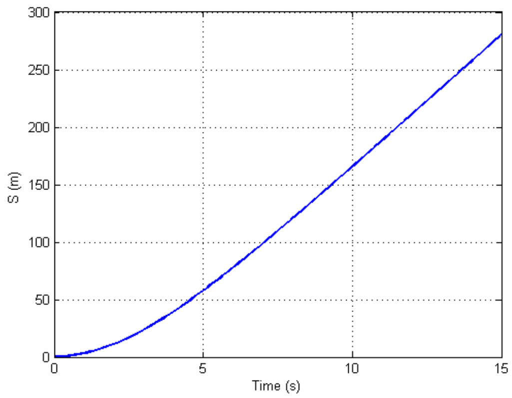
d) Velocity of the pig in y direction.



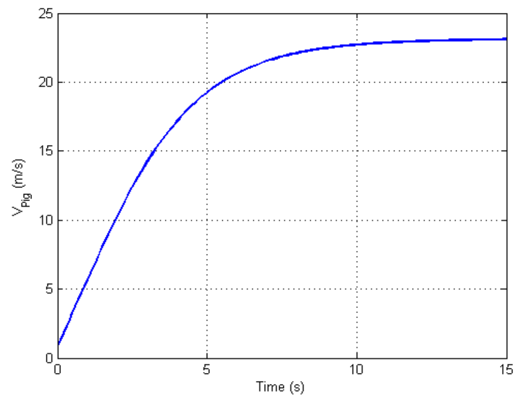
e) Position of the pig in z direction.



f) Velocity of the pig in z direction.



g) Distance from inlet of the pipe.



h) Velocity of the pig in tangential direction.

Fig. 11. Simulation results of pipeline of the example 3

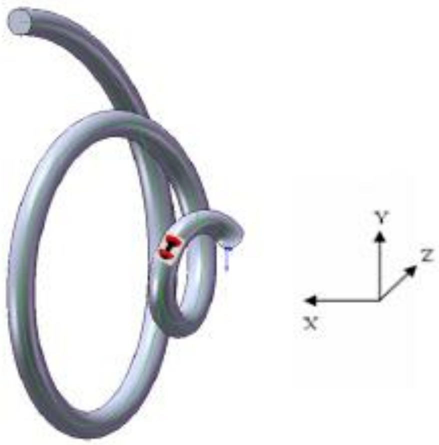


Fig. 12. Selected geometry of pipeline for example 3.

path and also the area of hole is constant, the pig's velocity reaches a constant value. These results validate the modeling and formulation of the present research. Also, these results are very similar to the case 2 of Lesani et. al. (2012) qualitatively.

3.3 Example 3

The geometry of the selected pipeline for the third test case is shown in Fig. 12. The parametric equations of this path are

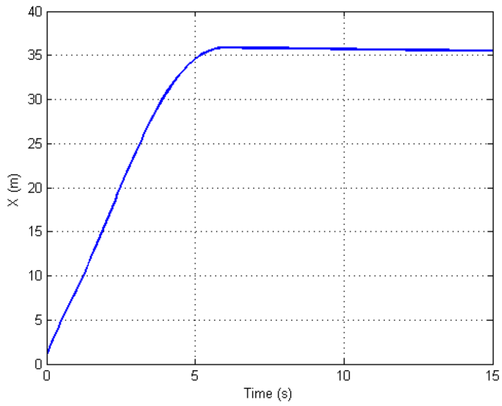
$$x(\lambda)=\lambda^2,\ y(\lambda)=\lambda^2\sin\lambda,\ z(\lambda)=\lambda^2\cos\lambda$$

The numerical values for the mass of the pig, length of the pig, pipe diameter and the bypass diameter are assumed to be 400 kg, 1.4 m, 0.7366 m and 0.67 m, respectively. Initial conditions for solving dynamic equation are $\lambda=1,\dot{\lambda}=5$ thus $x(1)=1, y(1)=\sin 1, z(1)=\cos 1$ m and $\dot{x}(1)=10, \dot{y}(1)=11.12, \dot{z}(1)=1.2$ m/s. The pig does not move from origin at *t=0* and its motion is recorded up to 15 seconds. The numerical values for fluid that used in all numerical examples are given in table 1.

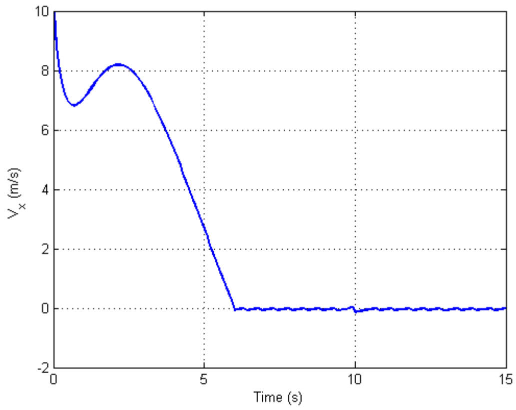
The results are shown in Fig. 13. The geometry of the pipeline for this test case is arbitrary. All figures show that the pig stops after 6 seconds, where the equilibrium conditions establish for pig in the pipeline at this moment. In this case, the direction of the friction force and also its type (static or dynamic) varies in some times of motion. Also, the gravity force of the pig and the fluid driving force frequently vary. This example shows the ability of the formulation for pigging estimation. The best pig can be select for a special pipeline based on the formulation in this research. Also, sticking of a pig in a special pipeline can be estimated by this formulation.

4. Discussion

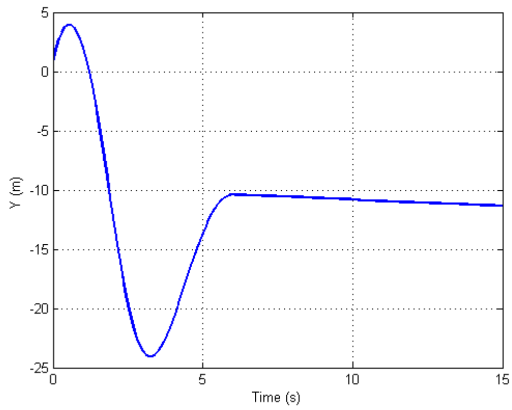
Since this is a theoretical research and there is no experimental data for comparison, some numerical



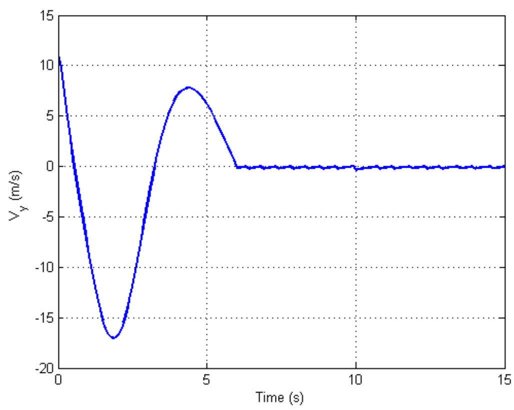
a) Position of the pig in x direction.



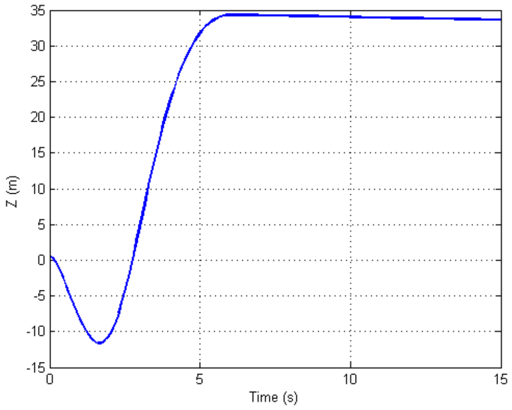
b) Velocity of the pig in x direction.



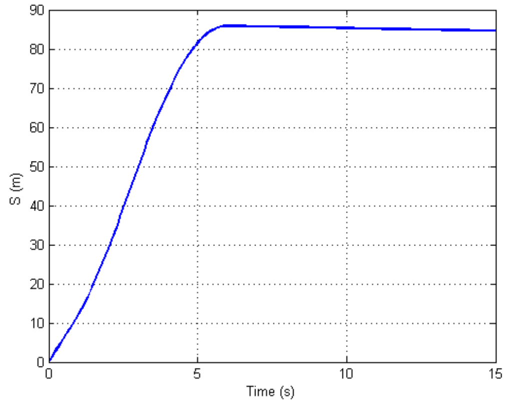
c) Position of the pig in y direction.



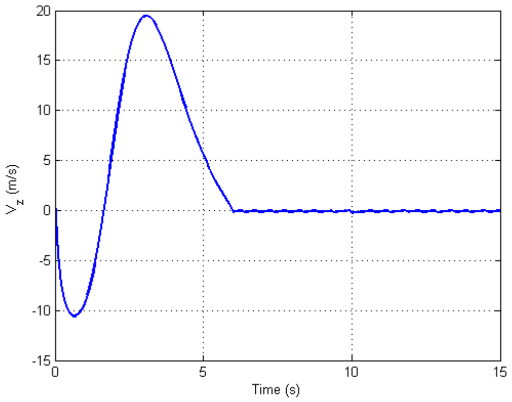
d) Velocity of the pig in y direction.



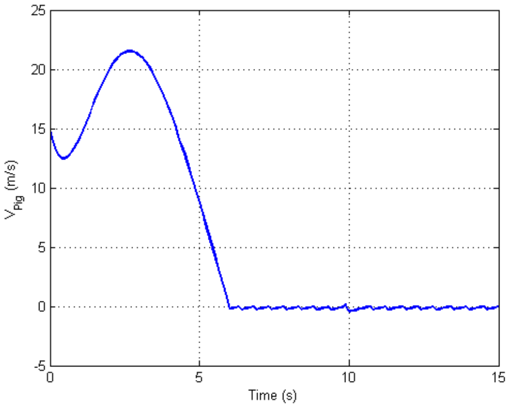
e) Position of the pig in z direction.



f) Distance from inlet of the pipe.



g) Velocity of the pig in z direction.



h) Velocity of the pig in tangential direction.

Fig. 13. Simulation results of pipeline of the example 3.

examples with special geometry were selected to approve the derived model. The results can easily show the ability of the present model to estimate the position and velocity of the pig in any time and any pipeline. This model can predict the sticking of a pig in a pipeline as in example 3. This is an important problem in pigging operation. Using this model can help the selection of pigging operation such as required upstream pressure and coefficient of friction. Time duration of pipeline pigging and length of the pipeline can be determined using the present equations, for instance in example 2, it takes 15 seconds for pig to move 280 m in the pipeline. This model can be extended to transient pig motion, large pigs and non-continues pipelines.

5. CONCLUSION

This research extends the dynamic analysis of pipeline pigs which run in the gas pipelines. In other words, in this paper, fluid in the pipeline is assumed to be compressible. This assumption is new comparing to previous work of the authors. Three numerical case studies are selected to validate the derived equations. The results from these three examples are in agreement with the physical nature of the problem. The third example clearly shows that these equations can estimate the stop time of the pig. This is an important prediction, because knowing this will be helpful in pigging operations. The derived equations of this research can be applied to speed control of the pigs in gas pipelines. It is expected that the present equations also are validated experimentally in future studies.

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