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MHD Flow of Nanofluids over an Exponentially Stretching Sheet Embedded in a Stratified Medium with Suction and Radiation Effects

P. Loganthan [1] and C.Vimala [2†]

[1, 2] *Department of Mathematics, Anna University, Chennai- 600 025, India*

†*Corresponding Author Email: vimala_100@rediffmail.com*

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ABSTRACT

An analysis has been carried out to investigate the influence of the combined effects of MHD, suction and radiation on the forced convection boundary layer flow of a nanofluid over an exponentially stretching sheet, embedded in a thermally stratified medium. The governing boundary layer equations of the problem are formulated and transformed into ordinary differential equations, using a similarity transformation. The resulting ordinary differential equations are solved numerically, by the shooting method. The effects of the governing parameters on the flow and heat transfer characteristics are studied and discussed in detail. Different types of nanoparticles, namely, Cu, Ag, Al_2O_3 and TiO_2, with water as the base fluid, are studied. It is found that the effects of the radiation parameter, volume fraction and suction are the same on the temperature profiles, in contrast to the effects of thermal stratification. Comparisons with previously published works are performed in some special cases, and found to be in good agreement.

Keywords: Exponentially stretching sheet, Suction, Nanofluid, MHD, Thermal stratification, Thermal radiation.

NOMENCLATURE

b, c	constants
B	magnetic field strength
C_p	specific heat
C_f	local skin friction coefficient
f	dimensionless stream function
f_w	suction parameter
k	thermal conductivity
L	reference length
M	magnetic parameter
N_R	radiation parameter
Nu_x	local Nusselt number
Pr	Prandtl number
qr	radiation heat flux
Re_x	local Reynolds number
St	stratification parameter
T	temperature of the fluid
U_o	reference velocity
U_w	velocity of stretching surface
u, v	velocity components in the x, y directions
x, y	dimensionless Cartesian co-ordinates

Greek symbols

α_f	thermal diffusivity of the nanofluid
ρ	density
ψ	stream function
θ	dimensionless thermal function
η	similarity variable
μ	dynamic viscosity
ν	kinematic viscosity
τ_w	skin friction or shear stress

Subscripts

f	fluid
s	solid
nf	nanofluid
w	stretching surface conditions
0	fluid conditions adjacent to the stretching surface
∞	fluid conditions far away from the stretching surface

Superscripts

$'$	differentiation with respect to η

1. INTRODUCTION

The study of the boundary layer flow and heat transfer over a stretching surface, has grown considerably during the last few decades, due to its enormous applications in several industries and environments, such as the extrusion of plastic sheets, thinning and annealing of copper wires, making electronic chips, cooling of metallic sheets, hot rolling, metal extrusion, glass fiber production, wire drawing, paper production, etc. The problem of the flow along a moving plate in a quiescent ambient fluid was first studied by Sakiadis (1961). Later, Crane (1970) examined the problem of an incompressible boundary layer flow over a stretching sheet, which moves with a velocity varying linearly with the distance from a fixed point. After this pioneering work, rapidly increasing number of papers have reported various aspects of this problem. Most of the available studies are restricted to the linear stretching of the sheet. However, it is known that realistically, the stretching of the sheet need not necessarily be linear (Gupta and Gupta, 1977). For example, in polymer processing and metal spinning processing, the quality of the final product depends on the rate of heat transfer at the stretching continuous surface with exponential variations of the stretching velocity and temperature distribution. Magyari and Keller (1999) first considered the boundary layer flow due to an exponentially stretching sheet, and analyzed the heat and mass transfer in the boundary layers with an exponentially varying wall temperature. Partha *et al.* (2005) obtained a similarity solution for a mixed convection flow past an exponentially stretching surface, by taking into account the influence of the viscous dissipation on the convective transport. Sajid and Hayat (2008) showed the influence of thermal radiation on the flow over an exponentially stretching sheet, and solved the problem analytically, using the homotopy analysis method. Later, Bidin and Nazar (2009) presented a numerical solution to the same problem. El-Aziz (2009) studied the viscous dissipation effect on the mixed convection flow of a micropolar fluid over an exponentially stretching sheet. Loganathan and Stepha (2013) analyzed the problem of chemical reaction and mass transfer effects on the flow of a micropolar fluid past a continuously moving porous plate with variable viscosity.

In recent years, the study of heat transfer in the presence of nanofluids has attracted enormous interest from researchers, due to its potential for a high rate of heat transfer enhancement properties. A nanofluid refers to the suspension of nanometer sized particles in a base fluid. These fluids have very high thermal conductivity and the competence to meet challenges, such as stability, sedimentation, erosion and additional pressure drop. The term nanofluid was introduced by Choi (1995). Buongiorno (2006) noted that the nanoparticles' absolute velocity can be viewed as the sum of the base fluid's velocity and a relative velocity. Karthikeyan et al. (2008) found that the effective thermal conductivity of a nanofluid increases with increasing volume fraction. The present paper has considered the problem, using the nanofluid model proposed by Tiwari and Das (2007), which was successfully utilized by many authors, such as Abu-Nada (2008), Muthamilselvan (2010), Oztop_and Abu-Nada (2008), Yacob *et al.* (2011) and Loganathan *et al.* (2013). An excellent collection of papers on nanofluids can be found in the book by Das *et al.* (2007), and the review papers by Trisaksri and Wongwises (2007), Wang and Mujumdar (2007), and Kakac and Pramuanjaroennkij (2009).

The study of magnetohydrodynamic flow and heat transfer has received considerable attention in recent years due to its wide variety of applications in engineering and technology, such as MHD generators, plasma studies, nuclear reactors and geothermal energy extractions. Bala Anki Reddy (2012) investigated the MHD and radiation effects on the unsteady flow along an exponentially accelerated isothermal vertical plate, with uniform mass diffusion in the presence of a heat source. The magnetic nanofluid aids in improving the extremely good long term stability for applications, such as mechanical seals, sensors, biomedicine, and many others. The nanofluid containing magnetic nanoparticles also acts as a super-paramagnetic fluid, which, in an alternating electromagnetic field, absorbs energy and produces a controllable hyperthermia, to be used in cancer treatment. Radiation can be quite significant at high operating temperatures. Radiative heat transfer becomes more important for the design of the pertinent equipment. Due to the significance of magnetic and radiation effects, Loganathan and Vimala (2013) analyzed the influence of magnetic and radiation effects on forced convection boundary layer flow of a nanofluid past an exponentially stretching sheet.

Futhermore, the effect of stratification is also one of the important aspects that has to be considered in the study of heat transfer. Stratification of fluids occurs due to the temperature variations or concentration differences, or the presence of different fluids of different densities. This has great importance in many engineering processes, for applications such as solar collectors, polymer production, drying processes, reactors and metallurgy, by hydromagnetic methods. Rosmila *et al.* (2012) used Lie group transformation, to study the problem of the magnetic effect on the natural convective flow of a nanofluid, over a linearly porous stretching sheet in the presence of thermal stratification. The process of suction also has its importance in many engineering activities, such as in the design of thrust bearing and radial diffusers, and thermal oil recovery.

Due to numerous applications and the importance of the factors discussed above, our objective is to study, the influence of combined effects of magnetic, suction and radiation on the boundary layer flow of nanofluids past an exponentially stretching sheet embedded in a thermally stratified medium. The present analysis finds its applications in the rotating seals, aerodynamic sensors, nano -structured magneto rheological fluids for semi active vibration dampers and biomedical applications. In the present study, the influence of the solid volume fraction, thermal radiation, suction and thermal stratification, and magnetic effects, is investigated and presented in detail. Four different types of nanofluids, namely, Cu-water, Ag-water, Al_2O_3-water and TiO_2-water are studied. The obtained results are compared with the relevant results in the existing literature, and are found to be in good agreement.

2. MATHEMATICAL FORMULATION

Consider the steady laminar two-dimensional flow of an electrically conducting nanofluid over an exponentially stretching sheet in the presence of

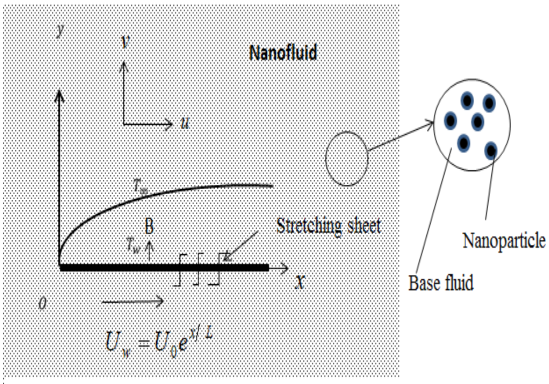

Fig.1. Physical model and coordinate system

radiation. The x axis is taken along the stretching sheet in the direction of the motion, and the y axis is normal to it. There are no pressure gradients in the x and y directions. $\frac{\partial p}{\partial y} = 0$ as the momentum boundary layer is very thin. $\frac{\partial p}{\partial x} = 0$ as U_∞ is constant for the flow over a flat surface at zero incidence. (The application of Bernoulli's equation reveals that pressure is constant in the x direction in the free stream). The sheet stretches at a velocity of U_w and a given temperature distribution T_w, and is embedded in a thermally stratified medium, through a quiescent ambient fluid of variable temperature T_∞ where $T_w > T_\infty$. A uniform magnetic field B is applied normal to the sheet as shown in Fig. 1. The induced magnetic field is assumed to be small compared to the applied magnetic field, and is neglected (the magnetic Reynolds number is small). The fluid is a water based nanofluid containing four different types of nanoparticles, namely, Cu, Ag, Al_2O_3 and TiO_2. It is also assumed that the base fluid and the suspended nanoparticles are in thermal equilibrium, and no slip occurs between them. Under the above assumptions, and the following model equations proposed by Tiwari and Das (2007), the basic continuity, momentum and energy equation for this problem can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu_{nf}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2 u}{\rho_{nf}} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{(\rho C_p)_{nf}}\left(k_{nf}\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}\right). \tag{3}$$

The boundary conditions of Eqs. (1)-(3) are taken to be

$$\begin{cases} u = U_w = U_o e^{x/L}, v = -V_w(x), T = T_w(x) \text{ at } y = 0 \\ u = v = 0, T = T_\infty \text{ as } y \to \infty \end{cases} \tag{4}$$

where u and v are the velocity components in the x and y directions, T is the local temperature of the nanofluid, L is the characteristic length of the plate. U_0 is the velocity parameter of the stretching surface, q_r is the radiation heat flux and $V_w(x) = V_o e^{x/2L} = f_w$ is the constant parameter with $f_w > 0$ for suction and $f_w < 0$ for injection.

The magnetic field $B(x)$ is of the form $B = B_0 e^{x/(2L)}$ where B_0 is the constant magnetic field. It is assumed that $T_w = T_0 + be^{x/2L}$, $T_\infty = T_0 + ce^{x/2L}$ where T_0 is the parameter of the temperature distribution in the stretching surface, $b > 0$ and $c \geq 0$ are constants.

The thermo physical properties of the nanofluids, namely, viscosity μ_{nf}, density ρ_{nf}, thermal diffusivity α_{nf}, and heat capacitance $(\rho Cp)_{nf}$ are given by (Brinkman, 1952; Xuan *et al.*, 2000).

$$\begin{cases} \mu_{nf} = \dfrac{\mu_f}{(1-\varphi)^{2.5}}, \\ \rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s, \\ \alpha_{nf} = \dfrac{k_{nf}}{(\rho C_p)_{nf}}, \\ (\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s. \end{cases} \tag{5}$$

Here, k_{nf} is the thermal conductivity of the nanofluid, ϕ is the solid volume fraction of the nanofluid, ρ_f is the density of the fluid fraction, ρ_s is the density of the solid fraction, μ_f is the viscosity of the fluid fraction, k_f is the thermal conductivity of the fluid fraction, k_s is the thermal conductivity of the solid volume fraction. $(\rho C_p)_f$ is the specific heat parameter of the fluid fraction and $(\rho C_p)_s$ is the specific heat parameter of the solid fraction.

The effective thermal conductivity of the nanofluid is approximated by the Maxwell-Garnetts model

$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)} \tag{6}$$

Using the Rosseland approximation (Brewster, 1992) for the thermal radiation, the radiative heat flux is simplified as:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{7}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient.

Hence, we express T^4 as a linear function of temperature T using the Taylor series about a free stream temperature T_∞ and neglecting the higher order terms, we get:

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \tag{8}$$

By using Eqs. (7) and (8), the energy equation becomes

$$\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3\left(\rho C_p\right)_{nf} k^*}\frac{\partial^2 T}{\partial y^2} \tag{9}$$

Introducing the following transformations (Sajid and Hayat, 2008)

$$\begin{cases} \eta = \left(\dfrac{U_0}{2\nu_f L}\right)^{1/2} e^{x/2L} y, \\ \psi = \left(2U_0 \nu_f L\right)^{1/2} e^{x/2L} f(\eta), \\ \theta(\eta) = \dfrac{T - T_\infty}{T_w - T_\infty}. \end{cases} \tag{10}$$

The stream function ψ satisfies Eq. (1) automatically with

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}. \tag{11}$$

On substituting the transformations (10), Eqs. (2) and (3) are reduced to the following set of ordinary differential equations:

$$f''' + (1-\varphi)^{2.5}\begin{Bmatrix}(1-\varphi+(\varphi\rho_s/\rho_f))(ff''-2f'^2)\\ -Mf'\end{Bmatrix} = 0 \tag{12}$$

$$\begin{aligned}&(1+\frac{4}{3}N_R)\frac{1}{\Pr}\frac{k_{nf}}{k_f}\theta'' + (1-\varphi \\ &+\varphi((\rho C_p)_s/(\rho C_p)_f) \\ &\{(f\theta' - f'\theta) - f'St\} = 0\end{aligned} \tag{13}$$

Subject to the transformed boundary conditions are:

$$f = f_w, \quad f' = 1, \quad \theta = 1 - St \quad \text{at} \quad \eta=0, \quad f' \to 0, \quad \theta \to 0 \quad \text{as} \quad \eta \to \infty. \tag{14}$$

where the primes denote differentiation, with respect to the similarity variable η, $M = \dfrac{2\sigma B_o^2 L}{\rho_f U_o}$ is the magnetic parameter, $\Pr = \dfrac{\nu_f}{\alpha_f}$, is the Prandtl number, $N_R = \dfrac{4\sigma^* T_\infty^3}{k_{nf} k^*}$ is the radiation parameter, and $St = c/b$ is the stratification parameter. $St > 0$ implies a stably stratified environment, while $St = 0$ corresponds to an unstratified environment.

The skin friction coefficient C_f and local Nusselt number Nu_x, are defined as

$$C_f = \frac{\tau_w}{\rho_f U^2} \text{ and } Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)} \tag{15}$$

where τ_w is the skin friction or the shear stress and q_w is the heat flux from the plate; these are given by

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} \text{ and } q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{16}$$

Substituting the transformations (10) into (15) and (16), we obtain

$$\begin{aligned}&\sqrt{2\mathrm{Re}}C_f = \frac{1}{(1-\varphi)^{2.5}}f''(0), \\ &\mathrm{Re}_x^{-1/2} Nu_x = -\frac{k_{nf}}{k_f}\sqrt{\frac{x}{2L}}\theta'(0)\end{aligned} \tag{17}$$

where $\mathrm{Re}_x = \dfrac{xU_0 e^{x/L}}{\nu_f}$ is the local Reynolds number.

2. NUMERICAL METHOD

The system of non-linear ordinary differential Eqs. (12) and (13) along with boundary conditions (14) forms a two-point boundary value problem. A total of five initial conditions are required to solve these equations. Hence, two initial conditions $f''(0)$ and $\theta'(0)$ which are not prescribed are to be obtained, using the numerical shooting technique by utilizing the Natchtsheim-Swiget iteration technique (Alan Adams and Rogers, 1973) and then solved numerically, using the fourth order Runge-Kutta integration scheme. Here, various groups of parameters M, St, f_w, ϕ, N_R and Pr are considered in different phases. We have chosen a step size $\Delta\eta = 0.01$, to satisfy the convergence criterion of 10^{-4} in all of different phases mentioned above.

3. RESULTS AND DISCUSSION

In order to obtain a clear insight into the physical problem, the numerical results are displayed with the help of graphical and tabular illustrations. In the current investigation, different types of nanofluids, namely, Cu-water, Ag-water Al_2O_3-water and TiO_2-water have been studied with the Prandtl number Pr = 6.2 and the range of particles fraction ϕ is

Table 1 Thermophysical properties of the fluid and nanoparticles (Oztop and Abu-Nada, 2008).

	C_P (J/kg K)	K (W/m K)	ρ (kg/m^3)
Water	4179	0.613	997.1
Copper (Cu)	385	401	8933
Silver (Ag)	235	429	10500
Alumina (Al_2O_3)	765	40	3970
Titanium oxide (TiO_2)	686.2	8.9538	4250

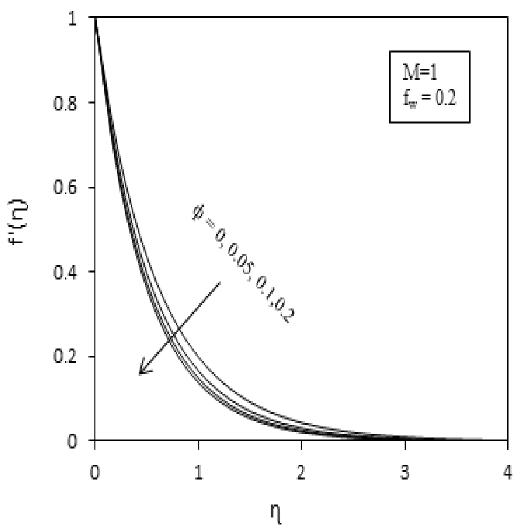

Fig. 2. Velocity profiles of Cu-water nanofluid for different values of volume fraction parameter ϕ.

analyzed, for the values between 0 and 0.2. The effect of the thermal stratification level is considered for a range 0≤ St ≤ 1. The thermophysical properties of the base fluid (water) and the nanoparticles chosen, are listed in Table 1.

The influence of the physical parameters, namely, volume fraction ϕ, suction parameter f_w, magnetic parameter M, stratification parameter St and the radiation parameter N_R in the flow and heat transfer analysis, are depicted in Figs. 2-12. In order to test the accuracy of the present results, the values of the temperature gradient $-\theta'(0)$ for $M = St = f_w = \phi = N_R = 0$ are compared with those of the previously published results, as shown in Table 2, and are found to be in good agreement.

Figures 2 and 3 display the effect of the volume fraction on the velocity and temperature profiles, for the Cu-water nanofluid. In the presence of a uniform strength of the magnetic field, it is observed that the velocity of the fluid decreases, whereas the temperature of the fluid increases with the increase in the volume fraction of the nanoparticles. This agrees with the physical behavior, in that, when the addition of copper increases the thermal

Table 2 Comparison of the results of the temperature gradient -θ′(0) when *St* = 0, *M* = 0, f_w = 0, N_R = 0 and ϕ = 0

Pr	Magyari and Keller (1999)	El.Aziz (2009)	Bidin and Nazar (2009)	Present results
1	0.954782	0.954785	0.9548	0.954955
3	1.869075	1.869074	1.8691	1.869074
5	2.500135	2.500132		2.500184
10	3.660379	3.660372		3.660379

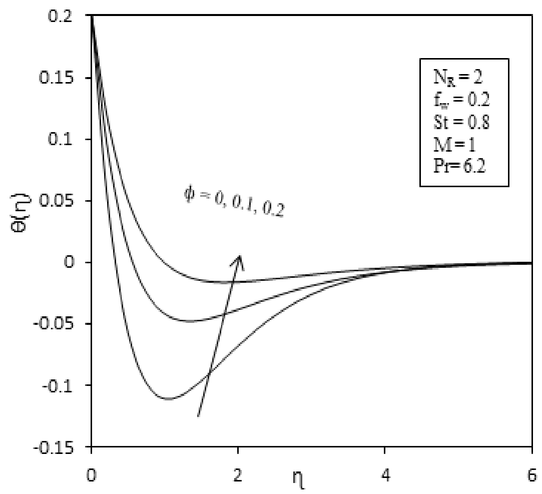

Fig. 3. Temperature profiles of Cu-water nanofluid with different values of ϕ.

conductivity increases, and then the thermal boundary layer thickness increases.

The values of the magnetic parameter are considered as 0.5, 1 and 1.5 in the present analysis. Figures 4 and 5 exhibit that an increase in the magnetic parameter decreases the velocity profiles, while increasing the temperature profiles. This clearly indicates that the transverse magnetic field opposes the flow. This is because, as the magnetic field increases the Lorentz force also increases, which then produces more resistance to the motion of the flow. On the other hand, as the Lorentz force increases, the fluid exhibits aresistance to this force, by increasing the friction between its layers, thereby resulting in an increase in the temperature.

Figures 6 and 7 depict the effects of the suction parameter f_w on the velocity and temperature profiles, respectively. It is observed that on increasing value of f_w, the velocity of the fluid decreases, and the temperature of the fluid increases. This is due to the fact that as f_w increases, the resistance of the fluid increases, and has a tendency to slow down the velocity of the fluid, and to increase the temperature profiles.

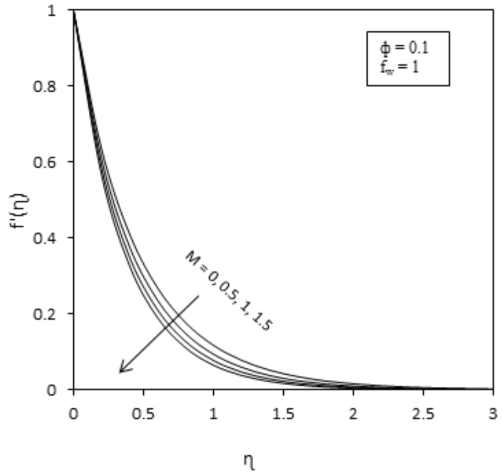

Fig. 4. Velocity profiles of Cu-water nanofluid for different values of magnetic parameter M

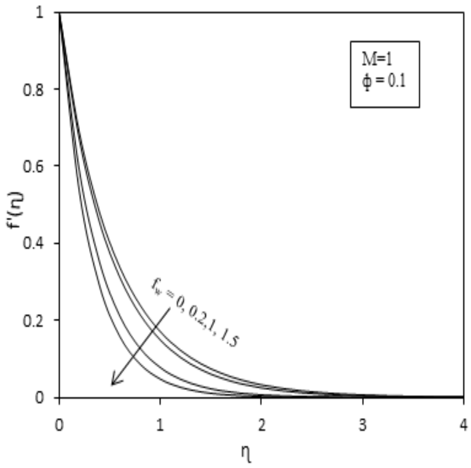

Fig. 6. Velocity profiles of Cu-water nanofluid for different values of suction parameter f_w

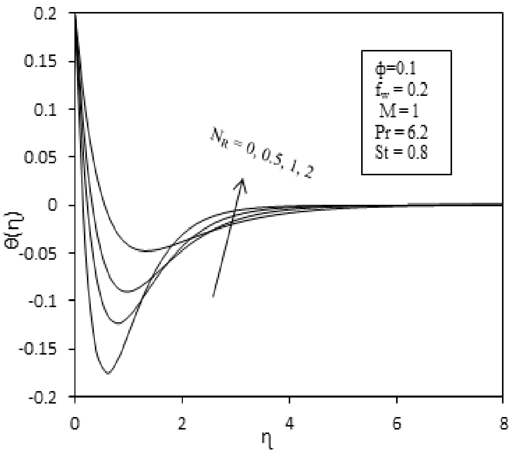

Fig. 8. Temperature profiles of Cu-water nanofluid with different values of N_R

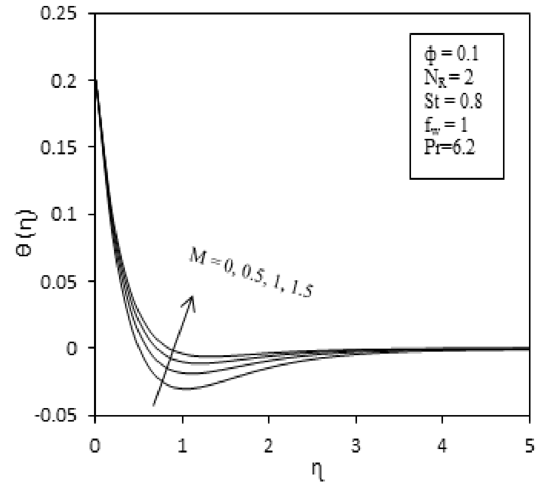

Fig. 5. Temperature profiles of Cu-water nanofluid with different values of M.

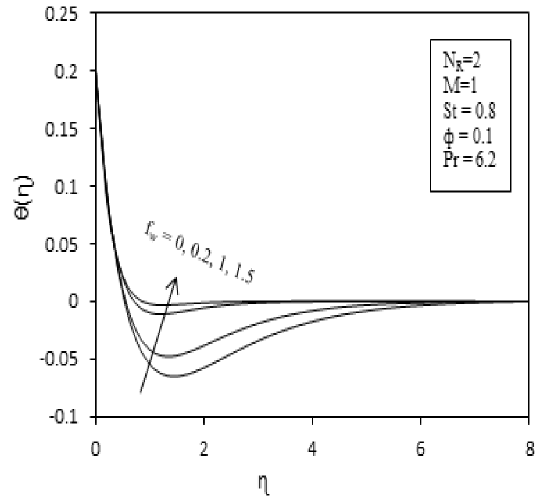

Fig. 7. Temperature profiles of Cu-water nanofluid with different values of f_w

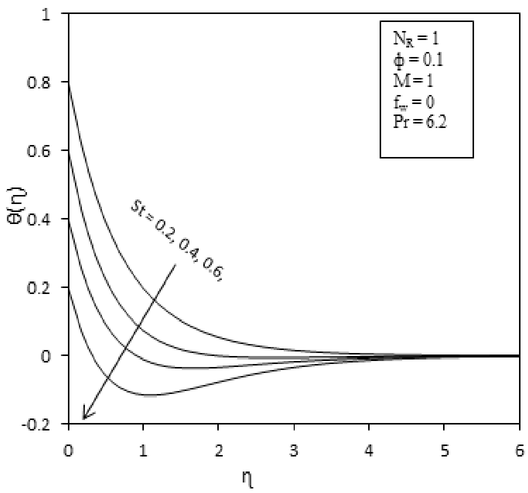

Fig. 9. Temperature profiles of Cu-water nanofluid with different values of stratification parameter.

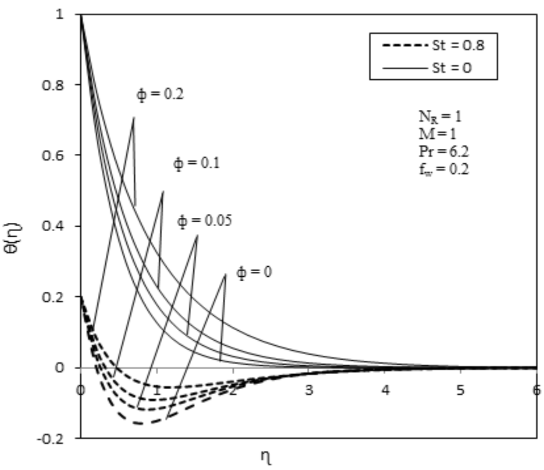

Fig. 10. Volume fraction effects of Cu-water nanofluid on temperature in stratified and unstratified medium.

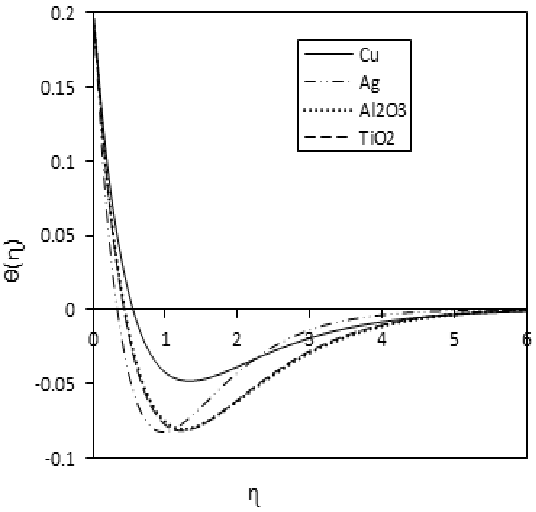

Fig. 12. Temperature profiles of different types of nanoparticles when NR = 2, f_w = 0.2, M = 1, St= 0.8 and ϕ = 0.1.

This is because of the combined effects of the strength of thermal stratification, and the presence of the nanoparticle volume fraction.

Figure 8 shows the influence of the radiation parameter on the temperature profile. When the radiation parameter increases from 0 to 2, the thermal boundary layer thickness is increased. This agrees with the physical behavior that, when the radiation parameter N_R increases, the mean absorption coefficient k will decrease, which in turn, increases the divergence of the radiative heat flux. Hence, the rate of radiative heat transferred to the nanofluid will be increased, so that the nanofluid temperature will be increased. Since the Eq. (12) is uncoupled from the temperature equation, the values of N_R will cause no change in the velocity profile.

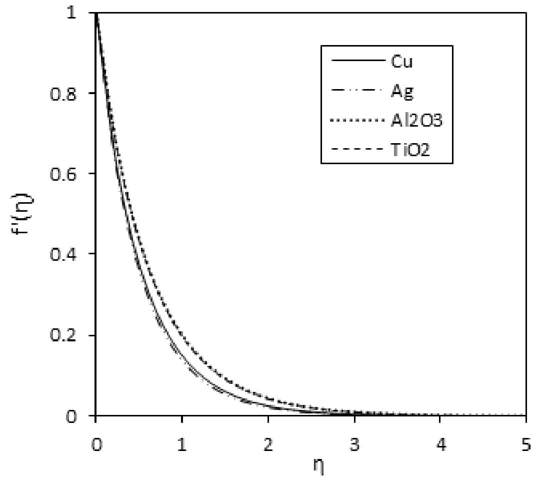

Fig. 11. Velocity profiles of different types of nanoparticles when N_R = 2, f_w = 0.2, M = 1, St = 0.8 and ϕ = 0.1.

Table 3 Values of $-f''(0)$ and $-\theta'(0)$ for various parameters ϕ, *St* when *M* = N_R = 1 and f_w =0.2

Pr	ϕ	f_w	$-f''(0)$	$St=0.8$	$St=0$
				$-\theta'(0)$	$-\theta'(0)$
6.2	0	0.2	1.7280486	1.9752295	1.2651814
6.2	0.05	0.2	1.9285381	1.7318596	1.0674257
6.2	0.1	0.2	2.0510345	1.5305518	0.9195955
6.2	0.2	0.2	2.1320622	1.2204669	0.7124357

Figure 9 shows the influence of the stratification parameter on the temperature profile. It is observed that the temperature of the Cu-water nanofluid is decreased significantly, with an increase of the thermal stratification parameter. From Fig.10 it is noted, that the volume fraction is more influential in the stratified medium, than in the unstratified medium. Besides, a negative temperature profile has been observed; this is because for a high thermal stratification, the temperature at the wall is reduced to a value lower than that of the free stream.

It is found that the effects of the radiation parameter, volume fraction, and suction are the same on the temperature profiles, in contrast to the effects of the thermal stratification parameter.

Figures 11 and 12 illustrate the velocity and temperature profiles of different nanoparticles, namely, Cu, Ag, Al_2O_3 and TiO_2. It can be seen, that on using different kinds of nanofluids, the momentum and the thermal boundary layer thickness change, which means that the additions of nanoparticles in the base fluid, are capable of changing the velocity and temperature profiles in the boundary layer.

Table 3 displays the numerical results of $-f''(0)$ and $-\theta'(0)$ for various values of ϕ and St when M = N_R = 1, f_w = 0.2 and Pr = 6.2. From Table 3, it is seen that, with the increase in ϕ the magnitudes of $|f''(0)|$ increases, and $|\theta'(0)|$ decreases for different kinds of nanofluids, which in turn, increases the skin friction; whereas it decreases the heat transfer rate with the volume fraction ϕ, but increases with the stratification parameter St .

5. CONCLUSIONS

The present study gives the numerical solutions for the effects of MHD, suction and thermal radiation in the two-dimensional boundary layer flow of nanofluids past an exponentially stretching sheet, embedded in a thermally stratified medium. The shooting technique, along with the fourth order Rung-Kutta integration scheme is employed, to obtain the solution of the governing equations. The obtained results are compared with the known results available in the literature, and the comparison shows good agreement.

The results from the present study can be summarized as follows:

- An increase in the magnetic parameter M, volume fraction parameter ϕ, and the suction parameter f_w decreases the thickness of the velocity boundary layers.
- An increase in the stratification parameter, leads to a decrease in the nanofluid's temperature, whereas the opposite is true with increasing ϕ, M, f_w and N_R. Besides, a negative temperature is observed due to the influence of thermal stratification.
- The addition of nanoparticles in the base fluid is capable of changing the velocity and temperature profiles in the boundary layer. Cu-water nanofluid shows a thicker thermal boundary layer than the other nanofluids.
- On using different kinds of nanofluids, the momentum and the thermal boundary layer thickness change, which means that the addition of nanoparticles in the base fluid are important in the cooling and heating processes.
- The magnitude of the skin friction coefficient increases, with increasing values of the volume fraction parameter ϕ.
- The heat transfer rate decreases with increasing values of ϕ, while an opposite effect is observed for the stratification parameter St.

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