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# Scattering of Water Wave by a Surface Discontinuity over a Single Step at the Bottom

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## Abstract

The present study is concerned with the scattering of an incoming water wave over a single step below the upper surface where the height of the step may be finite or very large(infinite) in presence of a surface discontinuity. Using linear theory, the problem is formulated mathematically as a boundary value problem in two separate regions of the ocean corresponding to two different depths. By utilising the eigenfunction expansion of the velocity potentials in conjunction with the impendence conditions along the common vertical boundary of the two regions, the mathematical problem is reduced to a system of linear equations which are solved numerically to obtain the hydrodynamic coefficients. If the surface discontinuity is due to a semi-infinite floating dock over an infinite step at the bottom, use of Havelock expansion of the velocity potentials and impendence conditions, the boundary value problem leads to another system of linear equation involving integral equations. The explicit form of the reflection coefficient is computed numerically in terms of wave number of the incoming wave and a number of graphical representations is given.

**Keywords:** Water wave scattering, Surface discontinuity, Inertial surfaces, Semi-infinite dock, Step bottom, Reflection and transmission coefficient.

## 1. Introduction

The problem of water wave scattering due to the presence of floating body or by finite obstacles of a specified geometry placed on the bottom is of general interest. Earlier, a number of problem concerning surface wave propagation along with various types of floating obstacles present at the upper surface or bottom of the ocean are discussed in the treatise of Lamb (1932), Stoker (1957), Kreisel (1949) and many others. Their valuable contributions enriched the prospect of linear water wave theory in this context. Weitz and Keller (1950) considered a water wave scattering problem involving a discontinuity in the surface boundary condition due to the presence of a broken ice on a half portion of the upper surface whereas the other half being free. Evans and Linton (1994) considered the problem of scattering of incoming wave by an uneven bottom using step approximation and obtained a scattering matrix of the hydrodynamic coefficients. Ultimately they reduced this problem to a problem of scattering by a discontinuity at the upper surface in uniform finite depth water and employed residue calculus technique of complex variable theory to obtain the reflection and transmission coefficient of the incident wave. Mandal and De (2009) investigated a wave scattering problem by a discontinuity at the upper surface together with a small bottom undulation. Using perturbation technique in conjunction with Green's integral theorem they obtained hydrodynamic coefficients up to first order in terms of computable integral. Recently Basu et al. (2012) and Maiti et al. (2013) extended the problem of scattering of an incident wave by taking porous uneven or uniform bottom in presence of a surface discontinuity and they obtained the hydrodynamic coefficients in terms of wave number of the incident wave.

The problem of scattering of an incoming wave train by a semi-infinite floating dock in front of a free surface was considered by Linton (2001), Leppington (1970) and many others. Heins (1949) considered the same problem and utilised Wiener-Hopf technique for analytical solution of the mathematical problem. Chakrabarti et al. (2005)

investigated the classical dock problem by utilizing Fourier analysis and the mathematical problem was reduced to the Carleman type singular integral equation over semi-infinite range. Recently Basu et al. (2012) considered the problem of scattering of an incident wave train by a semi-infinite floating dock in presence of a small bottom undulation. They obtained the reflection and transmission coefficients by using perturbation technique and Green's integral theorem.

Scattering of an incident wave over a sudden change in depth of the ocean is another interesting aspect in the present context. The problem of diffraction of an incoming wave by a sudden change of depth has been studied earlier by Sretenskii (1950), Bartholomeusz (1958) and many others. Lee and Ayer (1981), Miles (1982) and Kirby and Dalrymple (1983) investigated the propagation of surface waves over a rectangular trench. The mathematical problem was reduced to a system of equations involving integrals which were solved numerically and they obtained the reflection and transmission coefficients for different ratios of depth. Newman (1965) considered the problem of propagation of incoming water wave from a deep sea to a region of finite depth. Butakov and Zharkov (1998) studied the problem of scattering of an incoming wave from an infinitely deep region to a region of finite depth in presence of a drifting broken ice sheet at the upper surface of the ocean. In their formulation, the problem was formulated considering two different regions and by utilising the matching condition at the vertical boundaries of the fluid domain incorporating with the eigenfunction expansion of the velocity potentials a system of linear equation was formulated and the reflection and transmission coefficients were obtained along with numerical results.

In the present paper, the problem of scattering of an incoming water wave in presence of a discontinuity at the upper surface is considered when the fluid region is of finite depth and the bottom has a sudden change in its depth from $h_1$ to $h_2$, $(h_1 > h_2)$ i.e there is a single step at the bottom. Assuming linear theory, the problem is mathematically formulated as a two dimensional boundary value problem. At first the discontinuity at the upper surface arises due to a change in wave number of the incoming wave when the upper surface of the ocean is covered by two different floating inertial surfaces in two halves. The discontinuity at the upper surface may also be presented in the form of a semi-infinite floating dock of negligible thickness occupying one half of the ocean in front of the free surface and the bottom geometry remains same as before. When a train of progressive wave arrives and interacts with the discontinuity then some part of it is reflected and transmitted through the ocean whereas there will be no propagation of the incident wave as well as no transmission in presence of a semi-infinite floating dock in front of the free surface. In the mathematical formulation, we find that the velocity potential satisfies two different boundary value problems in two region of the ocean of different depths. These boundary value problems are being solved and the potentials are expressed in terms of depth eigenfunctions in their respective regions. By utilising the impendence conditions at the vertical boundaries of the fluid domain along with the eigenfunction expansion of the velocity potentials, a system of linear equations is generated involving computable integrals. The resulting matrix equations involving the hydrodynamic coefficients are evaluated numerically. Finally, we consider that the free surface region is of infinitely deep and right-half of the upper surface is covered by a semi-infinite floating dock over finite depth water so that the bottom has an infinite step. In this case, the solution of the velocity potentials for the free surface region is obtained by using Havelock expansion (Havelock (1929)) of the potential. The remaining problem is solved by combining the matching conditions at the vertical fluid boundary leading to another system of linear equations involving integral equations. The explicit analytical form of the reflection coefficient is obtained by using the far field behavior of the potential functions in the free surface region which is computed numerically after solving the matrix equations. In each case, the hydrodynamic coefficients are plotted in figures in terms of wave number of the incident wave for both finite and infinite step respectively.

## 2. Mathematical formulation of the problem

For the mathematical formulation of the problem, a right handed cartesian co-ordinate system $(x,y)$ is chosen in which $y = 0$ represents the undisturbed upper surface and the direction of $y$-axis is vertically downward into the fluid region. The fluid is considered to be inviscid, incompressible and a train of progressive wave arrives from $x \to -\infty$ where the direction of the positive $x$-axis being opposite to the direction of the incoming wave field. Assuming that the motion in water is two-dimensional, irrotational and time-harmonic with angular frequency $\sigma$, it can be described by the velocity potential $\psi(x,y,t) = Re[\phi(x,y)e^{-i\sigma t}]$ where $\phi(x,y)$ satisfies the Laplace equation:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0, \quad \text{in the fluid region.} \tag{1}$$

### 2.1 Case-A: Two floating inertial surfaces over a finite step:

We consider that the discontinuity at the upper surface occurs due to the presence of two floating inertial surfaces of uniform area densities $\varepsilon_1\rho$ and $\varepsilon_2\rho$ ($\rho$ being the density of water), occupying the regions $y=0, x<0$ and $y=0, x>0$ respectively. The bottom of the ocean has a finite step due to a sudden change of depth from $h_1$ to $h_2$, $(h_1 > h_2)$. The boundary conditions at the upper surface are given by

$$K_1\phi + \frac{\partial\phi}{\partial y} = 0 \quad \text{on} \quad y=0, x<0, \tag{2}$$

$$K_2\phi + \frac{\partial\phi}{\partial y} = 0 \quad \text{on} \quad y=0, x>0. \tag{3}$$

This produces a discontinuity in the upper surface of the ocean at the point (0,0), where $K_1 = \frac{K}{1-\varepsilon_1 K}, K_2 = \frac{K}{1-\varepsilon_2 K}$ and $\varepsilon_1,\varepsilon_2 < \frac{g}{\sigma^2}$ $(K = \frac{\sigma^2}{g}$, g is the acceleration due to gravity) so that the time-harmonic progressive wave can propagate with angular frequency $\sigma$ along the inertial surfaces. The boundary conditions at the bottom are given by

$$\frac{\partial\phi}{\partial y} = 0 \quad \text{on} \quad y = h_i, \quad (i=1,2). \tag{4}$$

The boundary condition along the vertical wall $h_2 < y < h_1, x=0$ is given by

$$\frac{\partial\phi}{\partial x} = 0 \quad \text{on} \quad x=0, h_2<y<h_1. \tag{5}$$

Besides, there is a singularity at the edge of the step (Mandal and Chakrabarti (2000)) which is of the form

$$r^{\frac{1}{3}}\nabla\phi(x,y) \text{ is bounded as } r\to 0, \tag{6}$$

where r is the distance from submerged edge of the step. When a train of progressive wave having potentials $e^{ik_0x}\psi_0^1(y)$ arrives from negative infinity direction, the far field behavior of $\phi(x,y)$ is specified as

$$\phi(x,y) \sim \begin{cases} Te^{is_0x}\psi_0^2(y) & \text{as } x\to\infty, \\ e^{ik_0x}\psi_0^1(y) + Re^{-ik_0x}\psi_0^1(y) & \text{as } x\to-\infty. \end{cases} \tag{7}$$

Here, $\psi_0^1(y) = N_0^1\cosh k_0(h_1 - y), \psi_0^2(y) = N_0^2\cosh s_0(h_2 - y)$ and $N_0^1 = (\frac{2k_0h_1}{2k_0h_1+\sinh 2k_0h_1})^{1/2}, N_0^2 = (\frac{2s_0h_2}{2s_0h_2+\sinh 2s_0h_2})^{1/2}$. $k_0$ and $s_0$ are given by the unique positive real roots of the following two transcendental equations (Das and Mandal (2005)) $k\tanh kh_1 = K_1$ and $s\tanh sh_2 = K_2$ respectively. Here $R$ and $T$ are the unknown reflection and transmission coefficients (complex) which are to be determined.

### 2.2 Case-B: Semi-infinite floating dock over a finite step:

Discontinuity at the upper surface may occurs due to the presence of a semi-infinite floating dock of negligible thickness occupying the region $x>0$, $y=0$ in front of the free surface $x<0$, $y=0$ over the same bottom geometry as before. Instead of the conditions Eq. (2) and Eq. (3), the boundary conditions at the upper surface are given by

$$K\phi + \frac{\partial\phi}{\partial y} = 0 \quad \text{on} \quad y=0, x<0, \tag{8}$$

$$\frac{\partial\phi}{\partial y} = 0 \quad \text{on} \quad y=0, x>0. \tag{9}$$

As there is no propagation of incoming wave in the dock region, the incident wave field $e^{ik_0x}\psi_0^1(y)$ from negative infinity direction is reflected by the dock and the far field conditions for $\phi(x,y)$ are

$$\phi(x,y) \sim \begin{cases} 0 & \text{as } x\to\infty, \\ e^{ik_0x}\psi_0^1(y) + Re^{-ik_0x}\psi_0^1(y) & \text{as } x\to-\infty. \end{cases} \tag{10}$$

### 2.3 Case-C: Semi-infinite floating dock over an infinite step:

In this case, the free surface region is of infinite depth and the dock region remains of finite depth H so that there is an infinite step at the bottom (Newman (1965)). Along with Eq. (6), Eq. (8) and Eq. (9), the boundary conditions given by the Eq. (4),Eq. (5) andEq. (7) are

$$\nabla\phi \longrightarrow 0 \text{ as } y\to\infty, \tag{11}$$

$$\frac{\partial\phi}{\partial x} = 0 \text{ on } x=0, H<y<\infty \tag{12}$$

and

$$\phi(x,y) \sim \begin{cases} 0 & \text{as } x\to\infty, \\ \phi_I e^{-Ky+iKx} + \phi_R e^{-Ky-iKx} & \text{as } x\to-\infty. \end{cases} \tag{13}$$

$\phi_I$ and $\phi_R$ are the incident and reflected wave potentials respectively and $K$ is the root of the dispersion relation $k = \frac{\sigma^2}{g}$. In each case, determination of the reflection coefficients $R$ is of our prime concern.

## 3. Method of Mathematical Derivation and Solution

We divide the fluid domain into two regions accordingly to two different depths namely the region-1 for $-\infty < x < 0, 0 < y < h_1$ and the region-2 for $0 < x < \infty, 0 < y < h_2$ respectively. The velocity potential $\phi(x,y)$ in these two regions is defined as

$$\phi(x,y) = \begin{cases} \phi_1(x,y) & \text{in the region-1}, \\ \phi_2(x,y) & \text{in the region-2}. \end{cases} \tag{14}$$

The matching conditions due to the continuity of pressure and the horizontal velocity of flow across the common vertical boundary of the two regions are given by

$$\phi_1(0,y)=\phi_2(0,y) \qquad \text{for } 0<y<h_2, \tag{15}$$

$$\phi_{1x}(0,y)=\phi_{2x}(0,y) \qquad \text{for } 0<y<h_2. \tag{16}$$

First we consider the Case-A where the mathematical boundary value problem is to solve the basic equationEq. (1) along with the boundary conditions given by Eq. (2)-(7). Now according to Eq. (14), the velocity potentials $\phi_1(x,y)$ and $\phi_2(x,y)$ are given by

$$\phi_1(x,y)=e^{ik_0x}\psi_0^1+\sum_{n=0}^{\infty}A_ne^{k_nx}\psi_n^1(y) \quad \text{in the region-1}, \tag{17}$$

$$\phi_2(x,y)=\sum_{n=0}^{\infty}B_ne^{-s_nx}\psi_n^2(y) \quad \text{in the region-2} \tag{18}$$

with $A_0=|R|, B_0=|T|$ and $A_n, B_n$ $(n=1,2,.....)$ are the unknown constants to be determined. Here $\psi_n^1(y)$, $\psi_n^2(y)$ are the orthogonal depth eigenfunctions for the two regions (region-1 and region-2) respectively which are given by $\psi_n^1(y)=N_n^1\cos k_n(h_1-y)$, $\psi_n^2(y)=N_n^2\cos s_n(h_2-y)$ and $N_n^1=\left(\frac{2k_nh_1}{(2k_nh_1+\sin 2k_nh_1)}\right)^{1/2}, N_n^2=\left(\frac{2s_nh_2}{(2s_nh_2+\sin 2s_nh_2)}\right)^{1/2}$ $(n=1,2,......)$. $k_n$ and $s_n$ $(n=1,2,3,.....)$ are given by the positive real roots of the following two transcendental equations $k_n\tan k_nh_1+K_1=0$ and $s_n\tan s_nh_2+K_2=0$ respectively. Now multiplying $\psi_n^2(y)$, $(n=0,1,2,.......)$ to the matching condition given by Eq.(15) and integrating between the limits 0 to $h_2$, we obtain,

$$\int_0^{h_2}\phi_1(0,y)\psi_n^2(y)dy=\int_0^{h_2}\phi_2(0,y)\psi_n^2(y)dy. \tag{19}$$

Again, multiplying $\psi_n^1(y)$, $(n=0,1,2,.......)$ to Eq. (16) and integrating between 0 to $h_2$ we obtain,

$$\int_0^{h_2}\phi_{1x}(0,y)\psi_n^1(y)dy=\int_0^{h_2}\phi_{2x}(0,y)\psi_n^1(y)dy \tag{20}$$

i.e using Eq. (5), Eq. (20) can be written as

$$\int_0^{h_1}\phi_{1x}(0,y)\psi_n^1(y)dy=\int_0^{h_2}\phi_{2x}(0,y)\psi_n^1(y)dy \tag{21}$$

Now we consider the Case-B where the boundary value problem can be formulated by Eq. (1) together with boundary conditions given by Eq. (4)-(6) and Eq. (8)-(10). Now following Eq. (14), the eigenfunction expansion of the potentials $\phi_1(x,y)$ and $\phi_2(x,y)$ in this case are respectively given by

$$\phi_1(x,y)=e^{ik_0x}\psi_0^1(y)+\sum_{n=0}^{\infty}C_ne^{k_nx}\psi_n^1(y) \quad \text{in the region-1}, \tag{22}$$

$$\phi_2(x,y)=\sum_{n=1}^{\infty}D_ne^{-s_nx}\psi_n^2(y) \quad \text{in the region-2} \tag{23}$$

with $C_0=|R|$ and $C_n, D_n$ $(n=1,2,.....)$ are the unknown constants to be determined. Here $\psi_n^1(y)=N_n^1\cos k_n(h_1-y)$, $\psi_n^2(y)=N_n^2\cos s_n(h_2-y)$ and $k_n$, $s_n$ $(n=1,2,3,.....)$ are given by $k_n\tan k_nh_1+K_1=0$ and $s_n=\frac{n\pi}{h_2}$ respectively. By a similar analysis, we multiply $\psi_n^2(y)$ and $\psi_n^1(y)$ to the matching condition given by Eq. (15) and Eq. (16) successively and integrating between the limit 0 to $h_2$, we obtain,

$$\int_0^{h_2}\phi_1(0,y)\psi_n^2(y)dy=\int_0^{h_2}\phi_2(0,y)\psi_n^2(y)dy \tag{24}$$

and

$$\int_0^{h_2}\phi_{1x}(0,y)\psi_n^1(y)dy=\int_0^{h_2}\phi_{2x}(0,y)\psi_n^1(y)dy \tag{25}$$

i.e using Eq.(5), Eq.(25) can be written as

$$\int_0^{h_1}\phi_{1x}(0,y)\psi_n^1(y)dy=\int_0^{h_2}\phi_{2x}(0,y)\psi_n^1(y)dy. \tag{26}$$

Next we consider the Case-C where the boundary value problem is to solve Eq. (1) together with boundary conditions given by Eq. (6), Eq. (8)-(9) and Eq. (11)-(13). The unknown reflection coefficient in this case is given by

$$R=\frac{\phi_R}{\phi_I}. \tag{27}$$

As before, we specify the potential function $\phi(x,y)=\phi_1(x,y)$ for the free surface region $0<y<\infty, -\infty<x<0$ and $\phi(x,y)=\phi_2(x,y)$ for the semi-infinite dock occupying the region $0<y<H, 0<x<\infty$. Considering the continuity of the horizontal velocity of flow across the $x$-direction along $x=0, 0<y<H$, we get

$$\phi_{1x}(0,y)=\phi_{2x}(0,y)=u(y), \quad \text{say, in } 0<y<H, \tag{28}$$

where $u(y)$ is the linear velocity across the common vertical boundary of the two region. The potential function $\phi_1(x,y)$ for the free surface region $(-\infty<x<0, 0<y<\infty)$ can be written as

$$\phi_1(x,y)=\phi_Ie^{-Ky+iKx}+\phi_0(x,y), \tag{29}$$

where the additional potential function $\phi_0(x,y)$ characterises the reflected wave far from the discontinuity and satisfies the following boundary value problem

$i)$ $\dfrac{\partial^2\phi_0}{\partial x^2}+\dfrac{\partial^2\phi_0}{\partial y^2}=0$ in the fluid region,

$ii)$ $\phi_0(x,y)\to\phi_Re^{-Ky-iKx}$ as $x\to-\infty$, *iii)* $\frac{\partial\phi_0}{\partial x} = v(y)$ on $x=0, 0<y<\infty$,

where $v(y)$ is specified by combining Eq. (28),Eq. (29) together with condition (iii) of the above boundary value problem

$$v(y) = \begin{cases} u(y) - iK\phi_I e^{-Ky} & \text{for } 0<y<H, \\ -iK\phi_I e^{-Ky} & \text{for } H<y<\infty. \end{cases} \tag{30}$$

Using Havelock expansion (Havelock (1929)), the potential $\phi_0(x,y)$ for the above boundary value problem (i)-(iii) can be specified as

$$\phi_0(x,y) = 2ie^{-Ky-iKx}\int_0^\infty v(\xi)e^{-K\xi}d\xi + \frac{2}{\pi}\int_0^\infty v(\xi)\chi(\xi,x,y)d\xi \tag{31}$$

$$\chi(\xi,x,y) = \int_0^\infty e^{-k|x|}\frac{(kcosky - Ksinky)(kcosk\xi - Ksink\xi)}{k(k^2+K^2)}dk.$$

Substituting the expression of $v(y)$ from Eq. (30) in $\phi_0(x,y)$, we obtain

$$\phi_0(x,y) = \phi_I e^{-Ky-iKx} + 2ie^{-Ky-iKx}\int_0^H u(\xi)e^{-K\xi}d\xi + \frac{2}{\pi}\int_0^H u(\xi)\chi(\xi,x,y)d\xi. \tag{32}$$

After substitution of Eq. (32) to Eq. (29) for $\phi_1(x,y)$, we obtain,

$$\phi_1(x,y) = 2\phi_I e^{-Ky}cosKx + 2ie^{-Ky-iKx}\int_0^H u(\xi)e^{-K\xi}d\xi + \frac{2}{\pi}\int_0^H u(\xi)\chi(\xi,x,y)d\xi. \tag{33}$$

Now the potential function $\phi_2(x,y)$ is of the form

$$\phi_2(x,y) = \sum_{n=1}^{\infty} E_n e^{-s_n x}\psi_n(y)$$

$$\text{for } 0<x<\infty, 0<y<H, \tag{34}$$

where $E_n$ are the unknown constants and $s_n = \frac{n\pi}{H}$ $(n=1,2,3,...)$. Here $\psi_n(y) = \cos s_n(H-y)$, $(n=1,2,3,......)$ forms a complete set of orthogonal eigenfunction for the semi-infinite dock region. Now by using Eq. (28), we obtain,

$$u(y) = -\sum_{n=1}^{\infty} u_n\psi_n(y), \tag{35}$$

where $u_n = s_n E_n$. Using the orthogonality of $\psi_n(y)(n=1,2,3,...)$ in Eq. (34) and Eq. (35), we obtain,

$$\phi_2(x,y) = \sum_{n=1}^{\infty}\frac{e^{-s_n x}}{s_n}\left(\int_0^H u(\xi)\psi_n(\xi)d\xi\right)\psi_n(y). \tag{36}$$

Since $\phi_1(0,y) = \phi_2(0,y)$ across $0<y<H$, from Eq. (33), (35) and Eq. (36), we obtain

$$\sum_{n=1}^{\infty} u_n\int_0^H Q(\xi,y)\psi_n(\xi)d\xi = \phi_I e^{-Ky}, \tag{37}$$

where

$$Q(\xi,y) = ie^{-K(y+\xi)} + \sum_{n=1}^{\infty}\frac{\cos s_n(H-\xi)\cos s_n(H-y)}{2s_nH + sin2s_nH} + \frac{1}{\pi}\chi(\xi,0,y).$$

## 4. Determination of the Hydrodynamic Coefficients

In the Case-A, using the expressions of $\phi_1(x,y)$ and $\phi_2(x,y)$ given by Eq. (17) and Eq. (18) to Eq. (19) and Eq. (21), we get a system of linear equation involving the unknown constants $A_n, B_n$ $(n=0,1,2,...)$ in which $A_0 = |R|$ and $B_0 = |T|$ respectively. For the Case-B, we get $C_0 = |R|$, the unknown reflection coefficient, which can be determined after eliminating the unknown constants $C_n, D_n$ $(n=1,2,...)$ from the system of linear equations given by Eq. (24) and Eq. (26). In all cases, the resulting system of linear equations can be solved numerically for the unknown constants after truncating the infinite sums up to desired accuracy. Now, multiplying both sides ofEq. (37) by $\psi_l(y) = \cos s_l(H-y)$, $(l=1,2,3......)$ and integrating between the limits 0 to H, we obtain,

$$\sum_{n=1}^{\infty} u_n\int_0^H\int_0^H Q(\xi,y)\cos s_n(H-\xi)\cos s_l(H-y)dyd\xi = \phi_I\int_0^H e^{-Ky}\cos s_n(H-y)dy. \tag{38}$$

The reflection coefficient can be obtained from the far field behavior of the additional potential $\phi_0(x,y)$ satisfying the boundary value problem (i)-(iii) in section-3 for the Case-C. As $R = \frac{\phi_R}{\phi_I}$ and $\phi_0(x,y) \to \phi_R e^{-Ky-iKx}$ as $x\to-\infty$ and therefore from Eq. (31), we have

$$R = 1 + 2i\int_0^H e^{-K\xi}u(\xi)d\xi. \tag{39}$$

Using $u(y) = -\sum_{n=1}^{\infty} u_n\psi_n(y)$,Eq. (39) reduces to,

$$R = 1 + 2iKe^{-KH}\sum_{n=1}^{\infty}\frac{U_n}{K^2+s_n^2}, \tag{40}$$

where the unknowns $U_n = \frac{u_n}{\phi_I}$ $(n=1,2,3,....)$. can be eliminated from the system of linear equations given by Eq. (38).

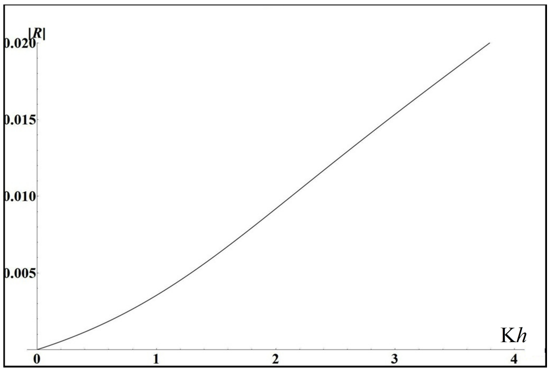


**Fig. 1. Case-A: $|R|$ for $h_1 = h_2 = h$.**

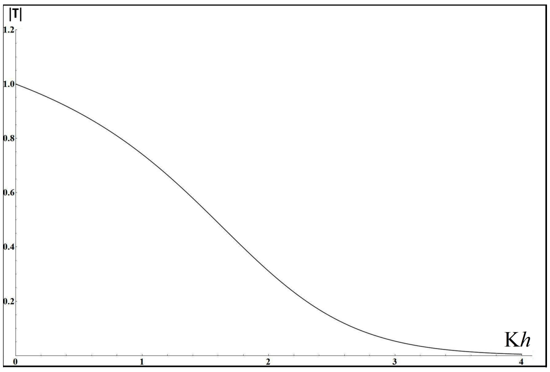


**Fig. 2. Case-A: $|T|$ for $h_1 = h_2 = h$.**

## 5. Numerical computation and graphical results

In this section, hydrodynamic coefficients are evaluated numerically by solving the system of linear equations in each case as discussed in section-3 and section-4. For computational purpose, the infinite series are being truncated up to a finite number of term so that a modest degree of accuracy is maintained. The computed values of the hydrodynamic coefficients are plotted graphically against the wave number of the incident wave in each case.
First we consider that the depth of the ocean is uniform and of finite depth h in presence of a discontinuity for two floating inertial surfaces at the upper boundary. Now following Eq. (19) and Eq. (21), the computed values of reflection and transmission coefficient are plotted in Fig. 1 and Fig. 2 respectively against $Kh$.

It is observed that the values of $|R|$ increases with $Kh$ but the values of $|T|$ decreases simultaneously which can be explained by the energy identity relation $|R^2| + \frac{1}{\alpha}|T^2| = 1, \alpha = \frac{s_0}{k_0}$ (Evans and Linton, 1994). In Case-A, when there is a finite step at the bottom of height $l = h_1 - h_2, (h_1 > h_2)$, the reflection and transmission coefficients are computed numerically for different step heights by using Eq. (19) and Eq. (21). Fig. 3 and Fig. 4 depicts these values of $|R|$ and $|T|$ respectively against $Kl$ by taking $h_1/l = 3,5,\ h_2/l = 1,\ \varepsilon_1/l = 0.04$ and $\varepsilon_2/l = 0.08$. Comparing with the Fig. 1 and Fig. 2, it is noted that as the step height $l$ at the bottom increases, the values of $|R|$ increases but $|T|$ decreases against $Kl$. The above results can be explained as there is a change of wave number of the incident wave at the upper surface of the ocean due to the presence of two different floating inertial surfaces and the presence of a finite step of height $l$ at the bottom. However, to observe effect of area densities of the inertial surfaces on the reflection and transmission coefficients, the Fig. 5 and Fig. 6 are plotted by taking $h_1/l = 5,\ h_2/l = 1, \varepsilon_1/l = 0.01, 0.06$ and $\varepsilon_2/l = 0.08$ respectively.

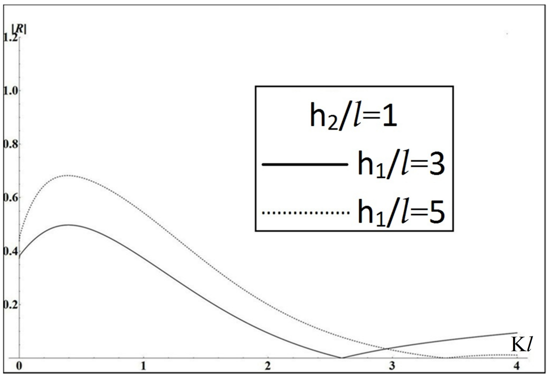


**Fig. 3. Case-A: $|R|$ for $h_1/l = 3,5; h_2/l = 1$.**

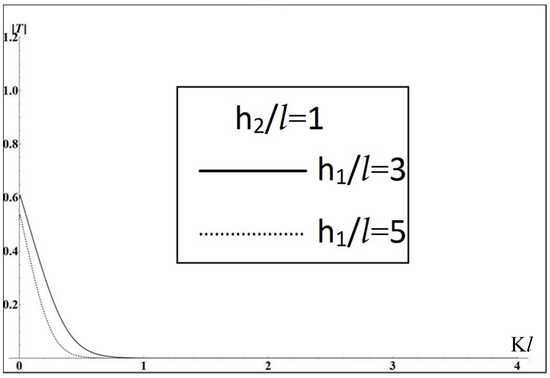


**Fig. 4. Case-A: $|T|$ for $h_1/l = 3,5; h_2/l = 1$.**

The above results can be explained as there is a change of wave number of the incident wave at the upper surface of the ocean due to the presence of two different floating inertial surfaces and the presence of a finite step of height $l$ at the bottom. However, to observe effect of area densities of the inertial surfaces on the reflection and transmission coefficients, the Fig. 5 and Fig. 6 are plotted by taking $h_1/l = 5,\ h_2/l = 1, \varepsilon_1/l = 0.01, 0.06$ and $\varepsilon_2/l = 0.08$ respectively.

For the Case-B, where the right half of the upper surface of the ocean is covered by a semi-infinite floating dock over a finite step of height

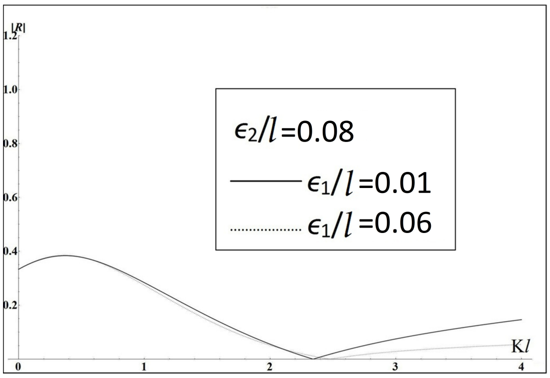


**Fig. 5. Case-A:** $|R|$ **for** $\varepsilon_1/l = 0.01, 0.06; \varepsilon_2/l = 0.08$**.**

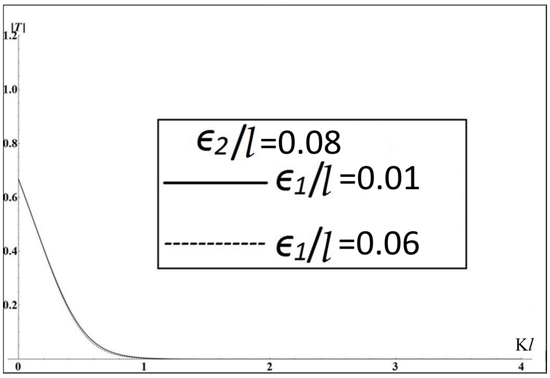


**Fig. 6. Case-A:** $|T|$ **for** $\varepsilon_1/l = 0.01, 0.06; \varepsilon_2/l = 0.08$**.**

$l = h_1 - h_2, (h_1 > h_2)$ at the bottom, there will be no propagation as well as transmission of the incoming wave field in the dock region. The reflection coefficient can be obtained by solving the system of equations given by Eq. (24) and Eq. (26) after truncation of the infinite sums as before and the computed values of $|R|$ are plotted in Fig. 7 and Fig. 8 respectively. Figure 7 depicts $|R| = 1$ for the case of uniform depth of the ocean by taking $h_1 = h_2 = h$. However, for different step heights $l = h_1 - h_2, (h_1 > h_2)$, following Eq. (24) and Eq. (26) the computed values of $|R|$ are plotted in Fig. 8 by taking $h_1/l = 5, 7$ and $h_2/l = 1$. In Fig. 3, it is observed that over the same bottom topography, the values of $|R|$ first increases with $Kl$ and then decreases but in Fig. 8, it gradually increases with $Kl$. Comparing these two figures, the graph of reflection coefficient is different due to the presence of two different types of discontinuity at the upper surface.

Finally for the case of an infinite step at the bottom (Case-C), the reflection coefficient $|R|$ is evaluated numerically by utilising the expression given by Eq. (40). These computed values of $|R|$ are depicted in Fig. 9 against $KH$ while the dock region remains of finite depth H.

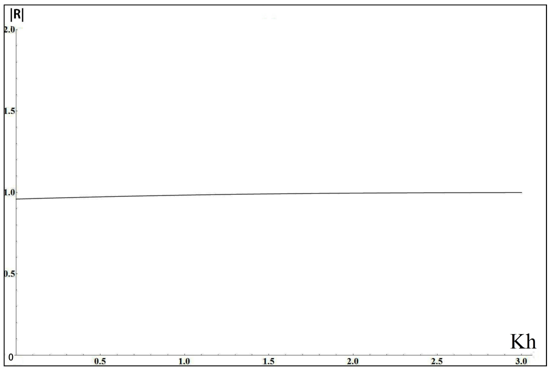


**Fig. 7. Case-B:** $|R|$ **for** $h_1 = h_2 = h$**.**

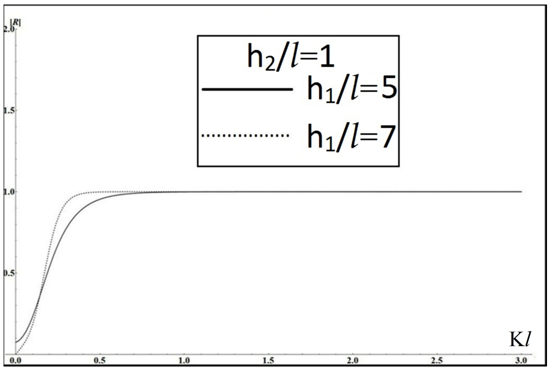


**Fig. 8.** Case-B: $|R|$ for $h_1/l = 5, 7; h_2/l = 1$.

## 6. Conclusions

The discontinuity at the upper surface is being presented either by two floating inertial surfaces or by a semi-infinite floating dock in front of a free surface. Here boundary value problems for the two regions of different finite depths are solved by using eigenfunction expansion of the velocity potentials in conjunction with the matching conditions at the vertical boundary between two regions. A system of linear equations involving unknown constants are then obtained and solved numerically. Numerical values of these hydrodynamic coefficients are plotted graphically against wave number for two different depths corresponding to two regions. Comparing all graphs with the case of uniform fi-

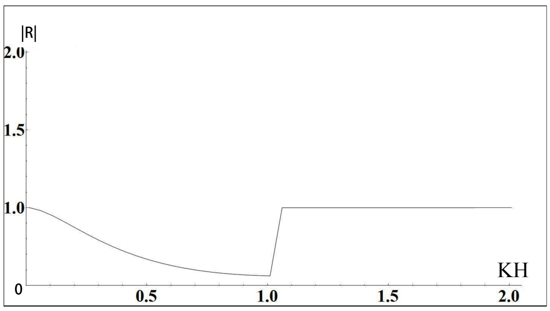


**Fig. 9.** Case-C: $|R|$ against $KH$.

nite depth it is observed that the change of depth at the bottom has an effect on the values of reflection or transmission coefficients. For the presence of a semi-infinite floating dock in front of the free surface over an infinite step, the boundary value problem is being solved by using Havelock expansion. Finally, by using the continuity of the horizontal velocity of flow across the common vertical boundary of the two regions along with the eigenfunction expansion of the velocity potentials of the finite depth region give rises to another system of equations involving integral equations. The explicit form of the reflection coefficient is thus obtained from the far field behavior of the potential function and is computed numerically for graphical representation.

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