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Effects of Stress Work on MHD Natural Convection Flow along a Vertical Wavy Surface with Joule Heating

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Abstract

An analysis is presented to investigate the influences of viscous and pressure stress work on MHD natural convection flow along a uniformly heated vertical wavy surface. The governing equations are first modified and then transformed into dimensionless non-similar equations by using set of suitable transformations. The transformed boundary layer equations are solved numerically using the implicit finite difference method, known as Keller-box scheme. Numerical results for the velocity profiles, temperature profiles, skin friction coefficient, the rate of heat transfers, streamlines and isotherms are shown graphically. Some results of skin friction, rate of heat transfer are presented in tabular form for selected values of physical parameters.

Keywords: Natural convection, uniform surface temperature, wavy surface, magnetohydrodynamics, Joule heating and Prandtl number.

Nomenclature

B_0	applied magnetic field strength
C_{fx}	local skin friction coefficient
C_p	specific heat at constant pressure [J.kg-1.K-1]
f	dimensionless stream function
g	acceleration due to gravity [ms-2]
Gr	grashof number
k	thermal conductivity [Wm-1K-1]
k_∞	thermal conductivity of the ambient fluid [Wm-1K-1]
L	characteristic length associated with the wavy surface [m]
$\overline{n}$	unit normal to the surface
Nu_x	local Nusselt number
P	pressure of the fluid [Nm-2]
Pr	Prandtl number
Q	heat generation parameter
Q_0	heat generation constant
q_w	heat flux at the surface [Wm-2]
T	Temperature of the fluid in the boundary layer [K]
T_w	Temperature at the surface [K]
T_∞	Temperature of the ambient fluid [K]
(u, v)	Dimensionless velocity components along the (x, y) axes [ms-1]
(x, y)	Axis in the direction along and normal to the tangent of the surface
α	Amplitude of the surface waves
β	Volumetric coefficient of thermal expansion [K-1]
η	Dimensionless variable
θ	Dimensionless temperature function
ψ	Stream function [m2s-1]
μ	Viscosity of the fluid [kgm-1s-1]
μ_∞	Viscosity of the ambient fluid
ν	Kinematic viscosity [m2s-1]
ρ	Density of the fluid [kgm-3]
σ_0	Electrical conductivity
τ_w	Shearing stress

1. INTRODUCTION

The viscous dissipation and pressure work effect play an important role in natural convection in various devices which are subjected to large deceleration or which operate at high rotational speeds and also in strong gravitational field processes on large scales (on large planets) and in geological processes. Ackroyd (1974) first investigated stress work effects in laminar flat-plate natural convection. Joshi and Gebhart (1981) investigated the effect of pressure stress work and viscous dissipation in some natural convection flows. The natural convection along a vertical wavy surface was first studied by Yao (1983) and using an extended Prantdl's transposition theorem and a finite-difference scheme. He proposed a simple transformation to study the natural convection heat transfer from isothermal vertical wavy surfaces, such as sinusoidal surface. Moulic and Yao (1989) also investigated mixed convection along a vertical wavy surface. Alam et al. (1997) have also studied the problem of free convection from a wavy vertical surface. Hossain et al. (2002) have studied the problem of natural convection of fluid with temperature dependent viscosity along a heated vertical wavy surface. Natural convection heat and mass transfer along a vertical wavy surface have been investigated by Jang et al. (2003). Molla et al. (2004) have studied natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption. Tashtoush and Al-Odat (2004) investigated magnetic field effect on heat and fluid flow over a wavy surface with a variable heat flux. Alam et al (2006) investigated the effect of pressure stress work and viscous dissipation in natural convection flow along a vertical flat plate with heat conduction. Devi and Ganga (2010) studied the dissipation effects on MHD nonlinear flow and heat transfer past a porous surface with prescribed heat flux. They have analyzed viscous and Joule dissipation effects after finding analytical solutions of highly nonlinear momentum equation and confluent hypergeometric similarity solution of heat transfer equations. Recently Parveen and Alim (2011ab) investigated Joule heating effect on MHD natural convection flow along a vertical wavy surface with viscosity dependent on temperature and studied effect of temperature dependent thermal conductivity on magnetohydrodynamic natural convection flow along a vertical wavy surface. Alim et al. (2011) considered the effects of temperature dependent thermal conductivity on natural convection flow along a vertical wavy surface with heat generation. Miraj et al. (2011) investigated effects of pressure work and radiation on natural convection flow around a sphere with heat generation. The thermal conductivity of the fluid had been assumed to be constant in all the above studies. However, it is known that this physical property may be change significantly with temperature.

The present study aims to incorporate the idea of the conjugate effects of viscous dissipation and pressure work on MHD natural convection flow of viscous incompressible fluid with Joule heating along a uniformly heated vertical wavy surface. Numerical results for velocity, temperature, skin friction, the rate of heat transfer, the streamlines and the isotherms are obtained for different values of the selected parameters, such as joule heating parameter J, the viscous dissipation parameter which is characterized by Eckert number Ec, pressure work parameter Ge, magnetic parameter M and presented graphically and discussed. Some selected results of skin friction coefficient and rate of heat transfer for different values of Joule heating parameter J have been shown tabular form and then discussed.

2. FORMULATION OF THE PROBLEM

Steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid along a vertical wavy surface in presence of uniform transverse magnetic field of strength B_0 with temperature dependent physical properties like viscosity and thermal conductivity is considered. It is assumed that the wavy surface is electrically insulated and is maintained at a uniform temperature T_w. Far above the wavy plate, the fluid is stationary and is kept at a temperature T_∞. The surface temperature T_w is greater than the ambient temperature T_∞ that is $T_w > T_\infty$. The flow configuration of the wavy surface and the two-dimensional Cartesian coordinate system are shown in figure 1.

The boundary layer analysis outlined below allows $\bar{\sigma}(X)$ being arbitrary, but our detailed numerical work assumed that the surface exhibits sinusoidal deformations. The wavy surface may be defined by

$$Y_w = \bar{\sigma}(X) = \alpha \sin\left(\frac{n\pi X}{L}\right) \tag{1}$$

Where, α is the amplitude and L is the wave length associated with the wavy surface.

The governing equations of such flow of magnetic field in presence of heat generation/absorption with viscosity variation along a vertical wavy surface under the usual Boussinesq approximations can be written in a dimensional form as:

Continuity Equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

X-Momentum Equation

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{1}{\rho}\frac{\partial P}{\partial X} + \frac{1}{\rho}\nabla.\left(\mu\nabla U\right) + g\beta(T - T_\infty) - \frac{\sigma_0 B_0^2}{\rho}U \tag{3}$$

Y-Momentum Equation

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{1}{\rho}\frac{\partial P}{\partial Y} + \frac{1}{\rho}\nabla.\left(\mu\nabla V\right) \tag{4}$$

Energy Equation

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{k}{\rho C_P}\nabla^2 T + \frac{T\beta}{\rho C_P}U\frac{\partial P}{\partial X} + \frac{\mu}{\rho C_P}\left(\frac{\partial U}{\partial Y}\right)^2 + \frac{\sigma_0 {B_0}^2}{\rho c_p}U^2 \tag{5}$$

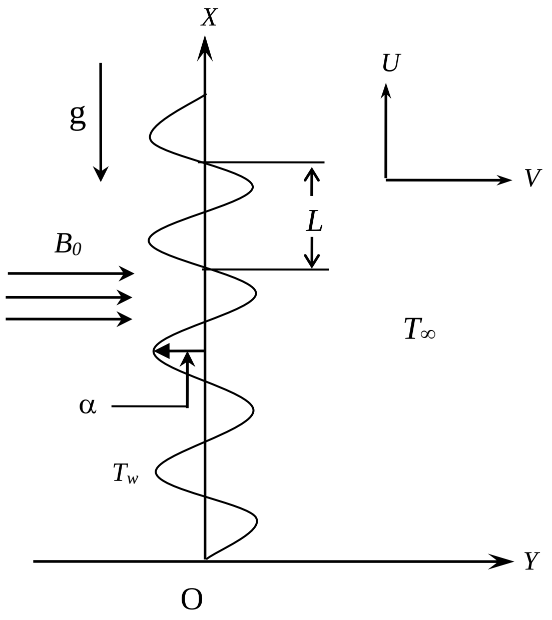

Fig. 1. Physical model and coordinate system

where (X, Y) are the dimensional coordinates along and normal to the tangent of the surface and (U, V) are the velocity components parallel to (X, Y), $\nabla^2(=\partial^2/\partial x^2+\partial^2/\partial y^2)$ is the Laplacian operator, g is the acceleration due to earth gravity, P is the dimensional pressure of the fluid, ρ is the density, T is the temperature of the fluid in the boundary layer, C_P is the specific heat at constant pressure and $\nu(=\mu/\rho)$ is the kinematic viscosity and $\mu(T)$ is the dynamic viscosity of the fluid in the boundary layer region depending on the fluid temperature, k is the thermal conductivity of the fluid, σ_0 is the electrical conductivity of the fluid, B_0 is the strength of magnetic field and β is the volumetric coefficient of thermal expansion.

The boundary conditions for the present problem are

$$U=0, V=0, T=T_w \text{ at } Y=y_w=\bar{\sigma}(X); \quad U=0, T=T_\infty, P=p_\infty \text{ as } Y\to\infty \tag{6}$$

Where, T_w is the surface temperature, T_∞ is the ambient temperature of the fluid and P_∞ is the pressure of fluid outside the boundary layer.

Using Prandtl's transposition theorem to transform the irregular wavy surface into a flat surface as extended by Yao (1983) and boundary layer approximation, the following dimensionless variables are introduced for non-dimensional governing equations.

$$\begin{aligned} &x=\frac{X}{L},\ y=\frac{Y-\bar{\sigma}}{L}Gr^{\frac{1}{4}},\\ &p=\frac{L^2}{\rho\nu^2}Gr^{-1}P, u=\frac{U}{u_0}=\frac{\rho L}{\mu}Gr^{-1/2}U,\\ &v=\frac{\rho L}{\mu_\infty}Gr^{-1/4}\left(V-\sigma_x U\right), \theta=\frac{T-T_\infty}{T_w-T_\infty},\\ &\sigma_x=\frac{d\bar{\sigma}}{dX}=\frac{d\sigma}{dx},\ Gr=\frac{g\beta(T_w-T_\infty)}{\nu^2}L^3 \end{aligned} \tag{7}$$

Where, $u_0=\frac{\mu}{\rho L}Gr^{1/2}$ is characteristic velocity, θ is the dimensionless temperature function and (u, v) are the dimensionless velocity components parallel to (x, y). Here (x, y) are not orthogonal but a regular rectangular computational grid can be easily fitted in the transformed coordinates. It is also worthwhile to point out that (u, v) are the velocity components parallel to (x, y) which are not parallel to the wavy surface, p is the dimensionless pressure of the fluid, L is the wave length associated with the wavy surface and Gr is the Grashof number. Introducing the above dimensionless dependent and independent variables into equations (2)–(5), the following dimensionless form of the governing equations are obtained after ignoring terms of smaller orders of magnitude in Gr, the Grashof number defined in (7).

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{8}$$

$$\begin{aligned} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}&=-\frac{\partial p}{\partial x}+Gr^{1/4}\sigma_x\frac{\partial p}{\partial y}\\ &+\left(1+\sigma_x^2\right)\frac{\partial^2 u}{\partial y^2}-Mu+\theta \end{aligned} \tag{9}$$

$$\begin{aligned} \sigma_x\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)&=-Gr^{1/4}\frac{\partial p}{\partial y}\\ &+\sigma_x\left(1+\sigma_x^2\right)\frac{\partial^2 u}{\partial y^2}-\sigma_{xx}u^2 \end{aligned} \tag{10}$$

$$\begin{aligned} &u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{Pr}\left(1+\sigma_x^2\right)\frac{\partial^2\theta}{\partial y^2}\\ &-Ge\left(\frac{T_\infty}{T_w-T_\infty}+\theta\right)u+Ec\left(\frac{\partial u}{\partial y}\right)^2+Ju^2 \end{aligned} \tag{11}$$

It is worth noting that the σ_x and σ_{xx} indicate the first and second differentiations of σ with respect to x, therefore, $\sigma_x=d\bar{\sigma}/dX=d\sigma/dx$ and $\sigma_{xx}=d\sigma_x/dx$.

In the above equations *Pr*, *M*, *Ge*, *Ec* and *J* are respectively known as the Prandtl number, the magnetic parameter, pressure work parameter, Eckert number and joule heating parameter, which are defined as

$$\begin{aligned} &Pr=\frac{C_p\mu_\infty}{k}, M=\frac{\sigma_0 B_0^2 L^2}{\mu Gr^{1/2}}, Ge=\frac{g\beta L}{c_p}\\ &Ec=\frac{u_0^2}{C_p(T_w-T_\infty)}, J=\frac{\sigma_0 B_0^2\nu Gr^{1/2}}{\rho C_p\left(T_w-T_\infty\right)} \end{aligned} \tag{12}$$

It can easily be seen that the convection induced by the wavy surface is described by equations (8)–(11). For the present problem this pressure gradient ($\partial p/\partial x=0$) is zero. Thus, the elimination of $\partial p/\partial y$ from equations (9) and (10) leads to

$$\begin{aligned} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}&=\left(1+\sigma_x^2\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_x\sigma_{xx}}{1+\sigma_x^2}u^2\\ &-\frac{M}{1+\sigma_x^2}u+\frac{1}{1+\sigma_x^2}\theta \end{aligned} \tag{13}$$

The corresponding boundary conditions for the present problem then turn into

$$\left.\begin{array}{l} u = v = 0,\ \theta = 1\ \ at\ \ y = 0 \\ u = \theta = 0,\ as\ \ y \to \infty \end{array}\right\} \qquad (14)$$

Now we introduce the following transformations to reduce the governing equations to a convenient form:

$$\psi = x^{3/4} f(x,\eta),\ \eta = yx^{-1/4},\ \theta = \theta(x,\eta) \qquad (15)$$

Where, $f(\eta)$ is the dimensionless stream function, η is the dimensionless similarity variable and ψ is the stream function that satisfies the continuity equation (8) and is related to the velocity components in the usual way as

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x} \qquad (16)$$

Introducing the transformations given in equation (15) and using (16) into equations (13) and (11) are transformed into the new co-ordinate system. Thus the resulting equations are

$$(1+\sigma_x^2) f''' + \frac{3}{4} ff'' - \left(\frac{1}{2} + \frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^2}\right)(f')^2 + \frac{1}{1+\sigma_x{}^2}\theta - \frac{Mx^{1/2}}{1+\sigma_x{}^2} f' = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \qquad (17)$$

$$\frac{1}{Pr}(1+\sigma_x^2)\theta'' + \frac{3}{4} f\theta' + Ecx(f'')^2\{\ \} - Ge\left(\frac{T_\infty}{T_w - T_\infty} + \theta\right)xf' + Jx^{\frac{3}{2}}(f')^2 = x\left(f'\frac{\partial \theta}{\partial x} - \theta'\frac{\partial f}{\partial x}\right) \qquad (18)$$

The boundary conditions (14) now take the following form:

$$\left.\begin{array}{l} f(x,0) = f'(x,0) = 0,\ \theta(x,0) = 1 \\ f'(x,\infty) = 0,\ \theta(x,\infty) = 0 \end{array}\right\} \qquad (19)$$

Here, prime denotes the derivatives with respect to η.

However, once we know the values of the functions f and θ and their derivatives, it is important to calculate the values of the shearing stress τ_w in terms of the local skin friction coefficient C_{fx} and the rate of heat transfer in terms of local Nusselt number Nu_x from the following relations:

$$C_{fx} = \frac{2\tau_w}{\rho U^2};\ \ Nu_x = \frac{q_w X}{k(T_w - T_\infty)} \qquad (20)$$

$$where\ \ q_w = -k(\bar{n}.\nabla T)_{y=0},\quad \tau_w = (\mu\ \ \bar{n}.\nabla\bar{u})_{y=0},\ U = \mu_\infty Gr^{1/2} / \rho L \qquad (21)$$

Here $\bar{n} = \dfrac{\bar{i}f_x + \bar{j}f_y}{\sqrt{f_x^2 + f_y^2}}$ is the unit normal to the surface.

Using the transformation (15) and (21) into equation (20) the local skin friction coefficient C_{fx} and the rate of heat transfer in terms of the local Nusselt number Nu_x take the following forms:

$$\frac{1}{2}(Gr / x)^{1/4} C_{fx} = \sqrt{1+\sigma_x^2}\ \ f''(x,0) \qquad (22)$$

$$Gr^{-1/4} x^{-3/4} Nu_x = -\sqrt{1+\sigma_x^2}\,\theta'(x,0) \qquad (23)$$

For the computational purpose the period of oscillations in the waviness of this surface has been considered to be π.

3. Method of Solution

The governing partial differential equations are reduced to dimensionless local non-similar equations by adopting appropriate transformations. The transformed boundary layer equations are solved numerically using Keller box method described by Keller (1978) and Cebeci and Bradshaw (1984) and used by many other authors.

4. Results And Discussions

The effects of stress work on MHD natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface with joule heating has been investigated. Although there are six parameters of interest in the present problem, the effects of magnetic parameter M, viscous dissipation parameter Ec, pressure work parameter Ge, and the joule heating parameter J on the surface shear stress in terms of local skin friction coefficient, the rate of heat transfer in terms of the local Nusselt number, the velocity and temperature profiles, the streamlines and the isotherms are focused. Numerical values of skin friction coefficient C_{fx} and rate of heat transfer Nu_x are calculated from equations (22) and (23) for the wavy surface from lower stagnation point at x = 0.0 to x = 2.0 presented in tabular form in the Table 1 and comparison of the present numerical results of the values of skin friction coefficient C_{fx}, and the heat transfer coefficient Nu_x, with Parveen and Alim (2011b) have been shown in Table 2.

Table 1 Skin friction coefficient and rate of heat transfer against *x* for different values of joule heating parameter *J* with other controlling parameters *Pr* = 0.72, α = 0.1, *M* = 0.2, *Ec* = 10.0 and *Ge* = 0.01.

x	***J* = 0.0**		***J* = 0.4**	
	C_{fx}	Nu_x	C_{fx}	Nu_x
0.0000	0.7437	0.3283	0.7437	0.3283
0.5050	1.0595	-0.8423	1.0711	-0.8916
1.0050	0.8787	-1.0399	0.9073	-1.1913
1.5050	1.3317	-4.6785	1.4180	-5.4669
2.0000	1.1292	-3.9747	1.2493	-5.1074

Table 2 Comparison of the values of skin friction coefficient C_{fx}, and the heat transfer coefficient Nu_x with Parveen and Alim (2011b) and present work for the variation of Prandtl number Pr while Ec = 0.0, Ge = 0.0, J = 0.0 and M = 0.8 with α = 0.2.

Pr	C_{fx}		Nu_x	
	Parveen and Alim (2011b)	Present work	Parveen and Alim (2011b)	Present work
0.73	0.81128	0.81098	0.32999	0.33004
1.73	0.72029	0.72032	0.44434	0.44441
4.24	0.62331	0.62329	0.59015	0.59021
7.00	0.57038	0.57043	0.68546	0.68554

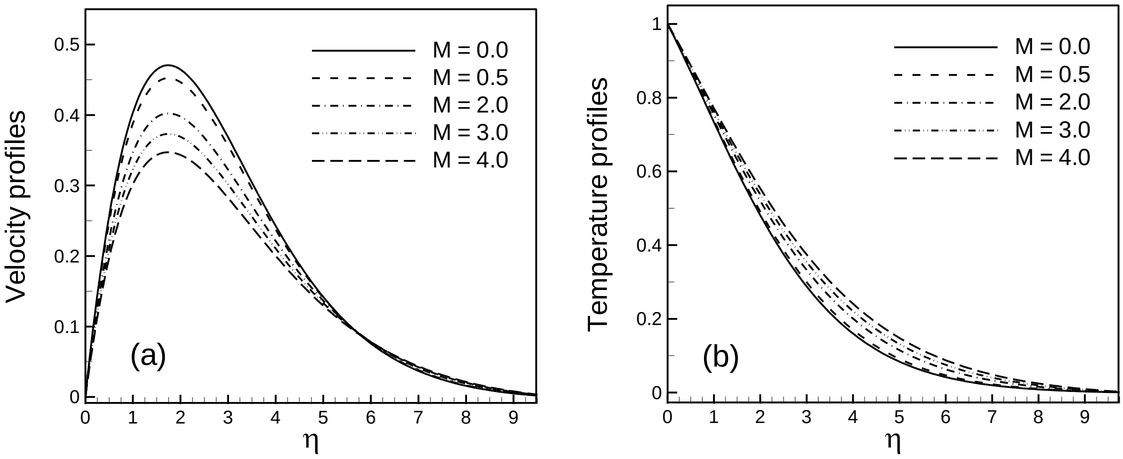

Fig. 2. (a) Velocity and (b) Temperature profiles against η for different values of *M* with α = 0.2, *Pr* = 0.72, *Ec* = 10.0, *Ge* = 0.01 and *J* = 0.01.

Here, the stress work and the joule heating parameter *J* are ignored to make the numerical data comparable with Parveen and Alim (2011b) for different values of Prandtl number Pr. It is obvious from the comparison table that the present results agreed well with the results of Parveen and Alim (2011b).

Numerical values of local shearing stress and the rate of heat transfer are calculated from equations (22) and (23) in terms of the skin-friction coefficients C_{fx} and Nusselt number Nu_x respectively for a wide range of the axial distance variable *x* starting from the leading edge for different values of the parameters *Pr, Ec, M, Ge, J* and α.

The velocity and temperature of the flow field is found to change more or less with the variation of the flow parameters. The effect of the flow parameters on the velocity and temperature fields, the skin friction coefficients, the rate of heat transfer, streamlines and the isotherms are analyzed with the help of graphs.

The effects for different values of magnetic parameter M on the velocity and temperature have been presented graphically in figures 2(a) and 2(b). It has been seen from figure 2(a) that for the higher values of magnetic parameter M the velocity decreases to the position of η = 5.5 and from that position of η velocity increases with magnetic parameter M that is, velocity profiles meet together at the position of η = 5.5 and cross the side and increasing with magnetic parameter M. The maximum values of velocities are recorded as 0.47071, 0.45227, 0.40234, 0.37326 and 0.34727 for magnetic parameter M = 0.0, 0.5, 2.0, 3.0, 4.0 respectively which occur at the same position η = 1.73814. Here, it is observed that at η = 1.73814, the maximum velocity decreases by 26.22 % as the magnetic parameter M change from 0.0 to 4.0. The values of temperature are recorded as 0.66955, 0.67611, 0.69481, 0.70637 and 0.71717 for magnetic parameter M = 0.0, 0.5, 2.0, 3.0, 4.0 at the same position of η = 1.23788 and the temperature increases by 7.10 %.

In figures 3(a) and 3(b) the effects for different values of the Eckert number *Ec* on the velocity and temperature profiles have been shown graphically. It has been seen from figure 3(a) that as the Eckert number *Ec* increases, the velocities rising up to the position of η = 1.73814 for the Eckert number *Ec* = 0.0, 5.0, 10.0, 20.0, 25.0 and from that position of η velocities fall down slowly and finally approaches to zero. It is also observed from figure 3(b) that as the Eckert number *Ec* increases, the temperature profiles increases. The maximum values of velocities are recorded as 0.46690, 0.46880, 0.47071, 0.47454 and 0.47647 for the Eckert number *Ec* = 0.0, 5.0, 10.0, 20.0, 25.0 respectively which occur at the same position η = 1.73814 and the maximum velocity increases by 2.04 %. Temperatures are recorded as 0.66008, 0.66479, 0.66955, 0.67924 and

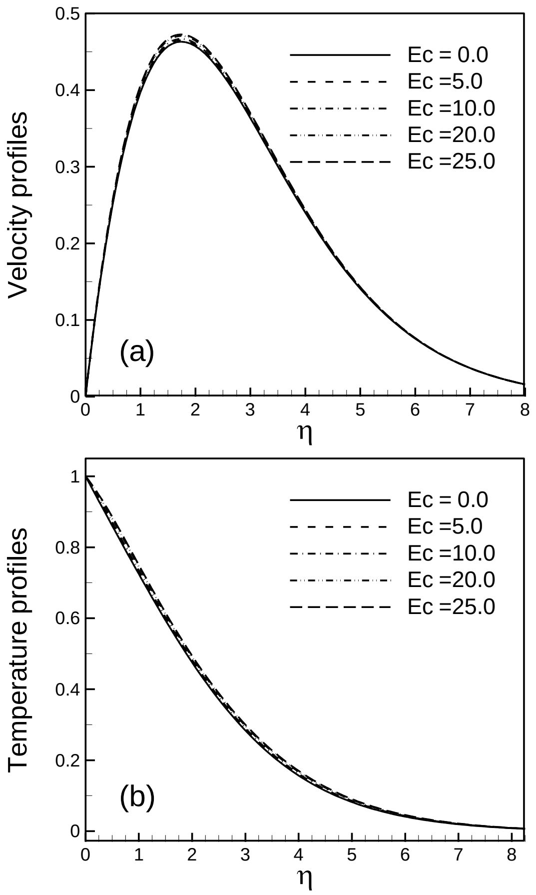

Fig. 3. (a) Velocity and (b) Temperature profiles against η for different values of *Ec* with α = 0.2, *Pr* = 0.72, *M* = 0.1, *Ge* = 0.01 and *J* = 0.01.

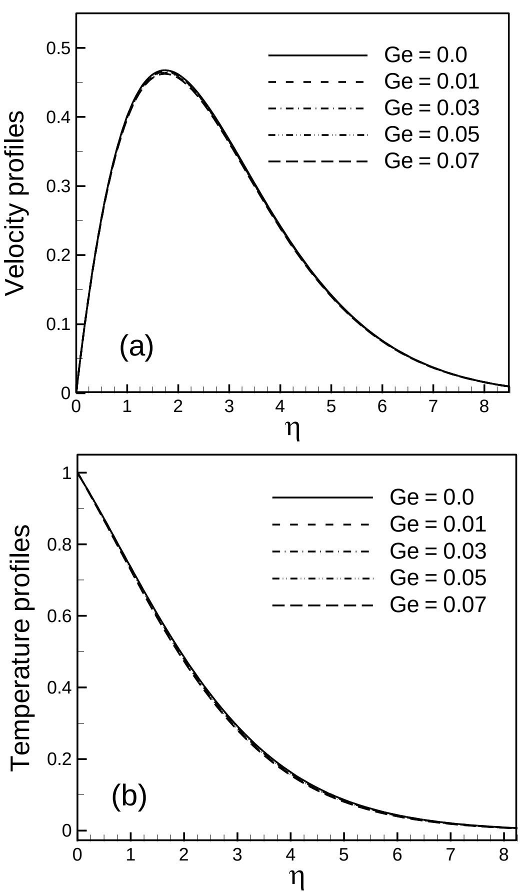

Fig. 4. (a) Velocity and (b) Temperature profiles against η for different values of *Ge* with α = 0.2, *Pr* = 0.72, *M* = 0.1, *Ec* = 10.0 and *J* = 0.01.

0.68417 for the Eckert number *Ec* = 0.0, 5.0, 10.0, 20.0, 25.0 respectively at the same position of η = 1.23788 and the temperature profiles increases by 3.60 %. The velocity boundary layer thickness and thermal boundary layer thickness are unchanged. The effects for different values of the pressure work parameter *Ge* on the velocity and temperature profiles have been presented graphically in figures 4(a) and 4(b) respectively. For the higher values of the pressure work parameter *Ge* both the velocity and the temperature decrease slightly.

The different values of joule heating parameter *J* on the velocity and temperature profiles have been presented graphically in figures 5(a) and 5(b) respectively. For the higher values of the joule heating parameter *J* both the velocity and the temperature rise up slightly.

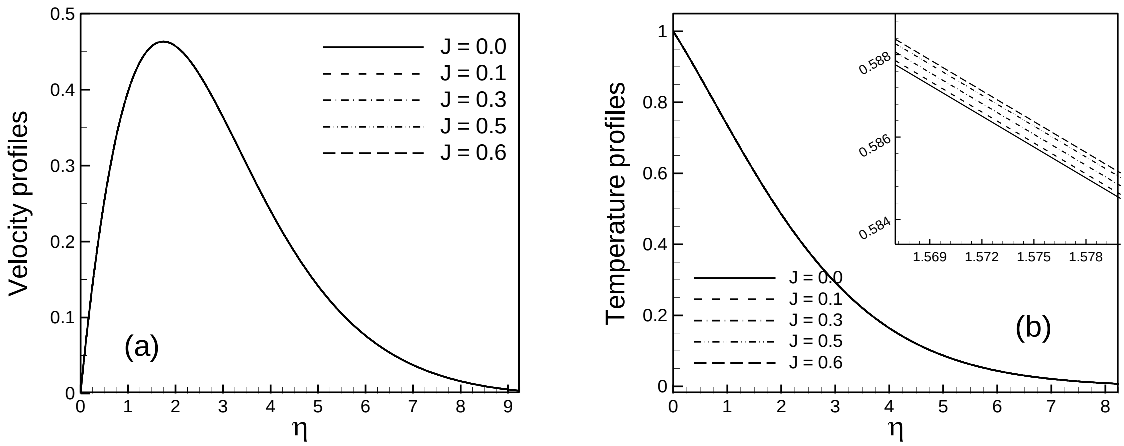

Fig. 5. (a) Velocity and (b) Temperature profiles against η for different values of *J* with α = 0.2, Pr = 0.72, M = 0.2, Ec = 10.0 and Ge = 0.01.

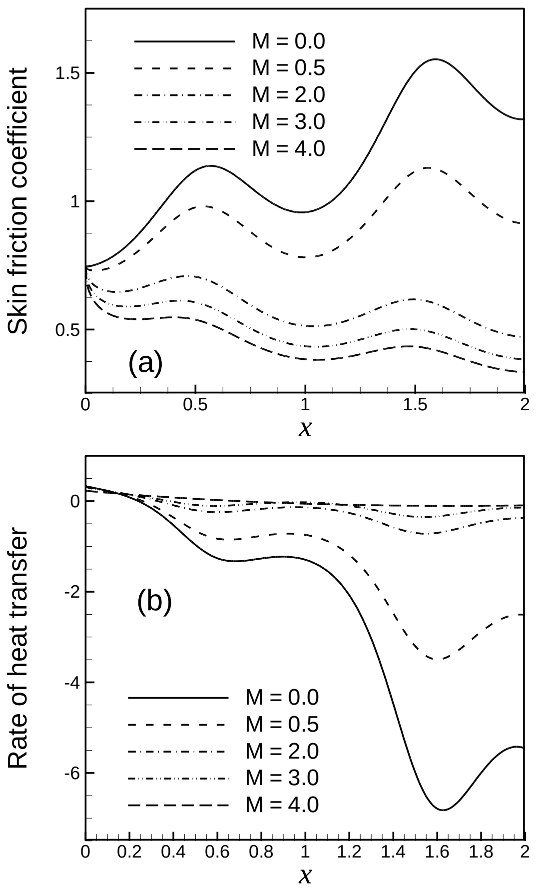

Fig. 6. (a) Skin friction coefficient and (b) Rate of heat transfer against x for different values of M with α = 0.2, Pr = 0.72, Ec = 10.0, Ge = 0.01 and J = 0.01.

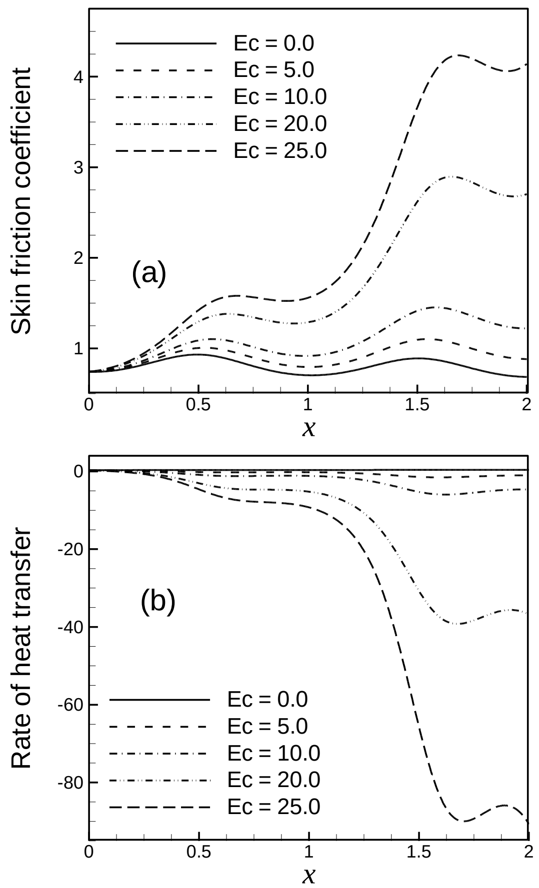

Fig. 7. (a) Skin friction coefficient and (b) Rate of heat transfer against *x* for different values of *Ec* with α = 0.2, *Pr* = 0.72, *M* = 0.1, *Ge* = 0.01 and *J* = 0.01.

In figures 6(a) and 6(b) effects of magnetic parameter *M* on skin friction and the rate of heat transfer have been presented. From figure 6(a) it is found that skin friction decreases significantly for greater magnetic field strength. This is physically realizable as the magnetic field retards the velocity field and consequently reduces the frictional force at the wall. However rate of heat transfer opposite pattern of skin friction due to the higher values of magnetic parameter *M* which are presented in figure 6(b).

The different values of the Eckert number *Ec* of the skin friction coefficients and the rate of heat transfer are shown graphically in figures 7(a) and 7(b) respectively. In this case the values of local skin friction coefficient C_{fx} are recorded to be 0.92433, 1.15578, 1.51578, 2.46985 and 4.03501 for *Ec* = 0.0, 5.0, 10.0, 20.0, 25.0 which occur at same point *x* = 1.51. From the figure 7(a), it is observed that at *x* = 1.51, the skin friction coefficient increases by 336.53 % due to the higher value of viscous dissipation parameter *Ec.* However, the values of rate of heat transfer are found to be .40024, -1.63443, -6.11295, -36.29311 and -77.95609 for *Ec* = 0.0, 5.0, 10.0, 20.0, 25.0 which occur at same point *x* = 1.51. It is seen from the figure 7(b) that for higher values of the Eckert number the rate of heat transfer decreases that is heat transfer slows down for higher viscous dissipation parameter *Ec.*

In figures 8(a) and 8(b) the skin friction coefficient C_{fx} and local rate of heat transfer Nu_x for different values of pressure work parameter *Ge* have been displayed. It is observed from the figure 8(a) that the skin friction coefficient decreasing down for higher values of pressure work parameter. It is seen from the figure 8(b) that the local rate of heat transfer increasing up with higher values of pressure work parameter.

In figures 9(a) and 9(b) the skin friction coefficient C_{fx} and local rate of heat transfer Nu_x for different values of joule heating parameter *J* have been displayed. It is observed from the figure 9(a) that for higher values of joule heating parameter *J*, skin friction increasing up to the axial position of x. It is seen from the figure 9(b) that for higher values of joule heating parameter *J* the rate of heat transfer decreasing down due to reduction of temperature difference between solid wall and the fluid.

In figure 10(a) and 10(b) show that streamlines and isotherms for selected values of the joule heating parameter *J* =.0.0 and 0.5 respectively. In figure 10 (a) have been shown the value of stream function ψ is 0.0 near the wall and then ψ increases gradually in the downstream within the boundary layer and away from the wall. In this case the maximum values of stream function ψ_{max} are found as 3.7

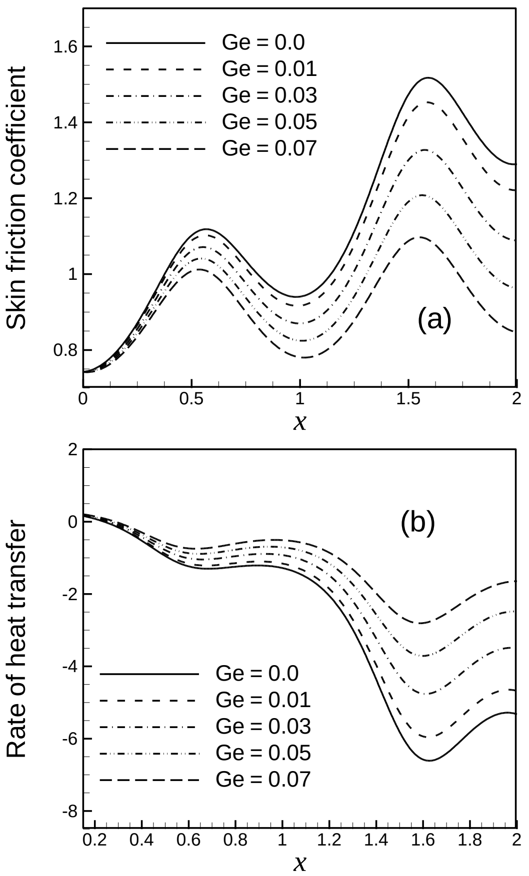

Fig. 8. (a) Skin friction coefficient and (b) Rate of heat transfer against *x* for different values of *Ge* with α = 0.2, *Pr* = 0.72, *M* = 0.1, *Ec* = 10.0 and *J* = 0.01

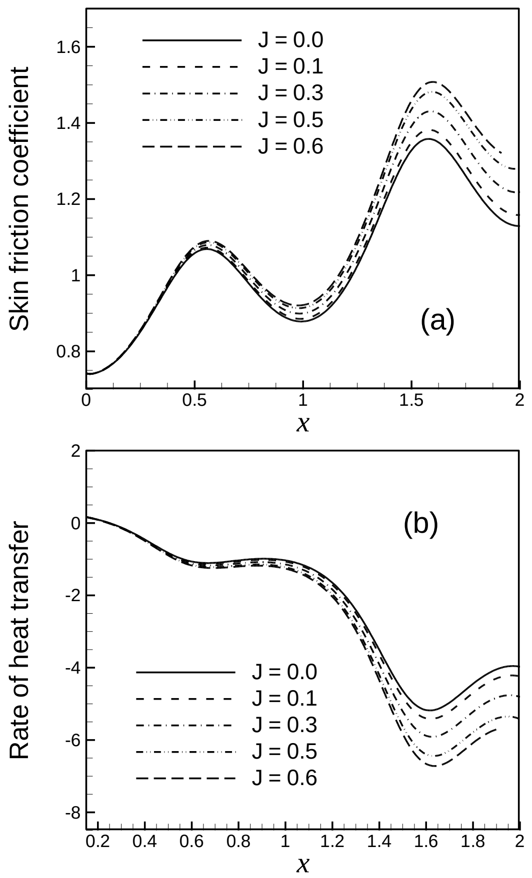

Fig. 9. (a) Skin friction coefficient and (b) Rate of heat transfer against *x* for different values of *J* with α = 0.2, *Pr* = 0.72, *M* = 0.2, *Ec* = 10.0 and *Ge* = 0.01.

and 3.9 for the values of the joule heating parameter *J* equal to 0.0 and 0.5 respectively. The isolines of temperature (isotherms) distribution show that temperature decreases significantly as the values of the joule heating parameter *J* increases which have been presented in figure 10(b). The value of isotherm is 1.0 at the wall and isotherms decreases slowly along the y-direction and finally approach to zero. The maximum values of isotherms are recorded as 1.7 and 2.1 for the values of joule heating parameter *J* equal to 0.0 and 0.5 respectively.

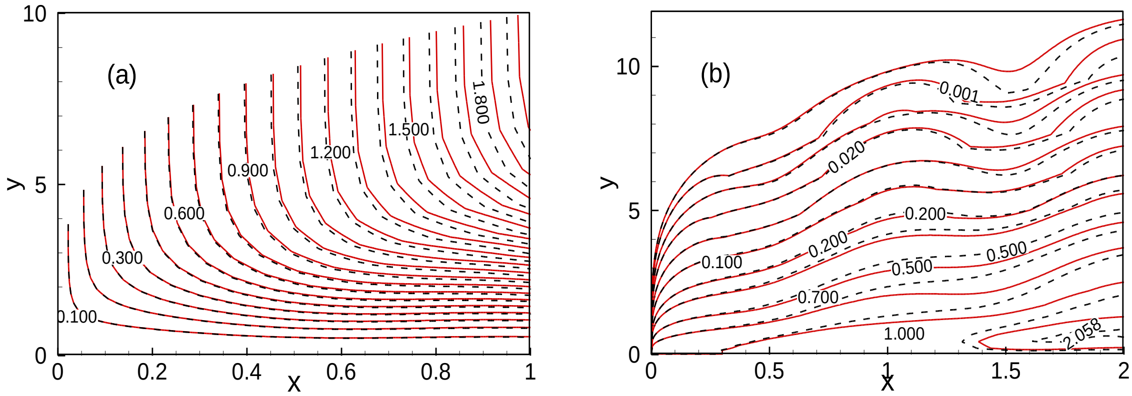

Fig. 10. (a) Streamlines and (b) Isotherms for *J* = 0.0 (Red solid lines), *J* = 0.5 (Black dashed lines), with α = 0.2, *Pr* = 0.72, *M* = 0.2, *Ec* = 10.0 and *Ge* = 0.01

5. CONCLUSION

The effects of the Prandtl number *Pr,* the magnetic parameter *M,* the viscous dissipation parameter *Ec,* the pressure work parameter *Ge,* the joule heating parameter *J* and the amplitude of the waviness of surface α on MHD natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface have been studied in detail. From the present investigations the following conclusions may be drawn:

Magnetic field strength enhancement causes the temperature and the rate of heat transfer rise and the velocity reduction within the boundary layer. At the position of η = 5.5 the velocity becomes constant and then cross the side and increasing with magnetic parameter. The local skin friction coefficient decreases due to the greater magnetic field strength.

The velocity profiles, the temperature profiles and the frictional force enhances for the higher values of the Eckert number *Ec*, the joule heating parameter *J* but the rate of heat transfer reduces significantly for all those cases.
The velocity, the temperature, the skin friction reduce and the rate of heat transfer rise up for higher values of the pressure work parameter *Ge*.

The value of stream function ψ is 0.0 near the wall and then ψ increases gradually in the downstream within the boundary layer and away from the wall and isolines of temperature (isotherms) show that temperature is 1.0 at the wall and decreases slowly away from the wall and finally approach to zero for the selected values of the joule heating parameter*J*.

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