Journal of Applied Fluid Mechanics, Vol. 8, No. 3, pp. 409- 417, 2015.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.18869/acadpub.jafm.67.222.22472

Hall Effects on Unsteady Hydromagnetic Flow Past an Accelerated Porous Plate in a Rotating System

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(Received November 27, 2013; accepted June 25, 2014)

ABSTRACT

An unsteady hydromagnetic flow of a viscous incompressible electrically conducting fluid past an accelerated porous flat plate in the presence of a uniform transverse magnetic field in a rotating system taking the Hall effects into account have been presented. An analytical solution describing the flow at large and small times after the start is obtained by the use of Laplace transform technique. The influences of the physical parameters acting on the flow are discussed in detail with the help of several graphs. It is found that interplay of Coriolis force and hydromagnetic force in the presence of Hall currents plays an important role in characterizing the flow behavior.

Keywords: Hall currents; Hydromagnetic flow; Rotation; Accelerated porous plate; Suction/blowing.

NOMENCLATURE

B_0	strength of applied magnetic field	u, v, w_0	velocity components along coordinates axes
$\vec{B}$	magnetic field vector	u_1, v_1	dimensionless velocity components
$\vec{E}$	electric field vector	(x, y, z)	Cartesian co-ordinates
E_x, E_y, E_z	components of electric field		
F	complex fluid velocity	α, β	defined by (24)
$i = \sqrt{-1}$	complex quantity	η	non-dimensional variable
$\vec{j}$	current density vector	σ	electric conductivity
j_x, j_y, j_z	components of current density	λ	defined by (22)
$\vec{k}$	unity vector	μ_e	magnetic permeability
K^2	rotation parameter	ν	kinematic viscosity
M^2	magnetic parameter	ρ	fluid density
m	Hall parameter	τ	non-dimensional time
p_0	fluid pressure	τ_x, τ_y	shear stresses at the plate $\eta = 0$
p	Laplace transform variable	τ_e	collision time of electron
$\vec{q}$	fluid velocity vector	ω_e	cyclotron frequency
S	suction parameter	Ω	angular velocity
t	time		

1. INTRODUCTION

In recent years, considerable interest has been given to the theory of rotating fluids due to its application in cosmic and geophysical sciences. The rotating flow of an electrically conducting fluid in the presence of a magnetic field is encountered in cosmical and geophysical fluid dynamics. It is also important in the solar physics involved in the sunspot development, the solar cycle and the structure of rotating magnetic stars. It is well known that a number of astronomical bodies posses fluid interiors and magnetic fields. Changes in the rotation rate of such objects suggest the possible importance of hydromagnetic spin-up. The hydromagnetic flow of a viscous incompressible electrically conducting fluid induced by a porous plate in the presence of rotating system is of considerable interest in the technical field due to its frequent occurrence in industrial and technological applications. The mechanism of conduction in ionized gases in the presence of strong magnetic field is different from that in metallic substance. The electric current in ionized gases is generally carried by electrons, which undergo successive collisions with other charged or neutral particles. In the ionized gases the current is not proportional to the applied potential except when the field is very weak in an ionized gas where the density is low and the magnetic field is very strong, the conductivity normal to the magnetic field is reduced due to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions and a current is induced in a direction normal to both electric and magnetic fields. This phenomenon, well known in the literature, is called the Hall effects. Hall effects are commonly used in distributors for ignition timing (and in some types of crank and camshaft position sensors for injection pulse timing, speed sensing, etc.). Hall effects are used as a direct replacement for the mechanical breaker points used in earlier automotive applications in Automotive ignition and fuel injection. Hall effects devices when appropriately packaged are immune to dust, dirt, mud and water. These characteristics make Hall effects devices better for position sensing than alternative means such as optical and electromechanical sensing. Hall effects sensors may be used in various sensors such as rotating speed sensors (bicycle wheels, gear-teeth, automotive speedometers, electronic ignition systems), fluid flow sensors, current sensors, and pressure sensors. Common applications are often found where a robust and contactless switch or potentiometer is required. These include: electric airsoft guns, triggers of electropneumatic paintball guns, gocart speed controls, smart phones and some global positioning systems.

The study of hydromagnetic viscous flows with Hall currents has important engineering applications in problems of magnetohydrodynamic generators and of Hall accelerators as well as in flight magnetohydrodynamics. The unsteady hydromagnetic flow of an incompressible electrically conducting viscous fluid induced by a porous plate is of considerable interest in the technical field due to its frequent occurrence in industrial and technological applications. It is well known that a number of astronomical bodies posses fluid interiors and magnetic fields. It is also important in the solar physics involved in the sunspot development, the solar cycle and the structure of magnetic stars. The hydromagnetic flow near an accelerated plate in the presence of a magnetic field was examined Soundalgekar (1965). Katagiri (1969) discussed the effect of Hall currents on the boundary layer flow past a semi-infinite flat plate. Hall effects on hydromagnetic flow near a porous plate was studied by Pop (1971). Pop and Soundalgekar (1974) investigated the effects of Hall currents on hydromagnetic flow near a porous plate. Gupta (1975) investigated the effect of Hall currents on the steady magnetohydrodynamic flow of an electrically conducting fluid past an infinite porous flat plate. The oscillatory magnetohydrodynamic flow past a flat plate with Hall effects was described by Datta and Jana (1976). Debnath *et al*. (1979) examined the effects of Hall current on unsteady hydromagnetic flow past a porous plate in a rotating fluid system. Raptis and Ram (1984) presented the effects of Hall current and rotation. The effect of Hall currents on hydromagnetic free convection flow near an accelerated porous plate was investigated by Hossain and Mohammad (1988). Pop and Watanabe (1994) studied the Hall effects on magnetohydrodynamic free convection about a semi-infinite vertical flat plate. Takhar (2002) discussed the MHD flow over a moving plate in a rotating fluid with magnetic field, Hall currents and free stream velocity. Hayat and Abbas (2004) studied the fluctuating rotating flow of second grade fluid past a porous heated plate with variable suction and Hall current. Hayat and Abbas (2007) analyzed the effects of Hall Current and heat transfer on the flow in a porous medium with slip condition. Deka (2008) studied the Hall effects on MHD flow past an accelerated plate. The Hall effects on hydromagnetic flow on an oscillatory porous plate was described by Maji *et al*. (2009). Gupta *et al*. (2011) examined the Hall effects on MHD shear flow past an infinite porous flat plate with suction and blowing at the plate. Resently, Deka and Das (2013) have presented the Hall effects on radiating MHD flow past an accelerated plate in a rotating fluid. Sandeep and Sugunamma (2014) have examined the radiation and inclined magnetic field effects on unsteady hydromagnetic free convection flow past an impulsively moving vertical plate in a porous medium.

In a recent paper, Deka (2008) has made an exact solution of the Hall effects on an MHD flow past an accelerated plate in a rotating system. On a keen perusal into Deka's work, we have observed that his solution is incorrect due to wrongly written the equations of motion (1) and (2). He has shown that for a given value of Hall parameter m, the transverse velocity v_1 vanishes when $\Omega = mM^2/(1+m^2)$, which does not actually happen where Ω is the rotation parameter, S the magnetic parameter and m the Hall parameter. He got this result due to error in Eqs. (1) and (2). In this paper, we have examined the effects of Hall currents and rotation on a hydromagnetic flow of a viscous incompressible electrically conducting fluid past an accelerated porous flat plate in the presence of a uniform transverse magnetic field. It is assumed that the magnetic Reynolds number is small enough to neglect induced hydromagnetic effects. Effects of governing parameters on the fluid velocity components, and the shear stresses at the plate are presented graphically and tabulated.

2. MATHEMATICAL FORMULATION AND ITS SOLUTION

Consider the unsteady hydromagnetic flow of a viscous incompressible electrically conducting fluid past an accelerated porous flat plate in the presence of a uniform transverse magnetic field in a rotating system. Choose a Cartesian co-ordinates system with x - axis along the plate in the direction of the flow, the z-axis is normal to the plate and the y-axis perpendicular to xy-plane (see in Fig. 1). Initially, at time $t \le 0$, both the plate and the fluid are assumed to be at rest. At time $t > 0$, the plate at $z = 0$ starts to move in its own plane with the velocity at, where t is the time and a being a constant. A uniform magnetic field of strength B_0 is imposed perpendicular to the plate. The plate is electrically non-conducting. The effects of Hall currents and rotation give rise to a force in y-direction, which induces a cross flow in that direction. Since plate is of infinite extent in x and y - directions and is electrically non-conducting, all physical quantities depend on z and t only. Also no applied or polarized voltages exist so the effect of polarization of fluid is negligible. This corresponds to the case where no energy is added or extracted from the fluid by electrical means (Cowling 1957). It is assumed that the induced magnetic field generated by fluid motion is negligible in comparison to the applied one. This assumption is justified because magnetic Reynolds number is very small for liquid metals and partially ionized fluids which are commonly used in industrial applications. The equation of continuity $\nabla \cdot \vec{q} = 0$ gives $w = -w_0$ where $\vec{q} \equiv (u, v, -w_0)$, u, v and w_0 being the velocity components along the coordinates axes.

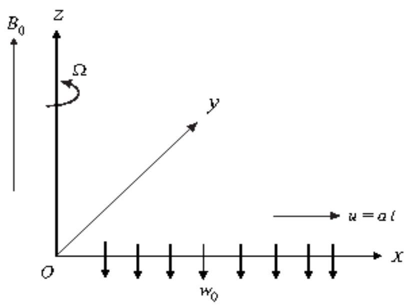

Fig. 1. Geometry of the problem

The equation of momentum in a rotating frame of reference is

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \vec{\nabla})\vec{q} + 2\vec{\Omega} \times \vec{k}$$

$$= -\frac{1}{\rho}\nabla p_0 + \nu \nabla^2 \vec{q} + \frac{1}{\rho}(\vec{j} \times \vec{B}), \tag{1}$$

where p_0 is fluid pressure including centrifugal force.

The initial and boundary conditions are

$$\begin{aligned} &t \le 0 : u = v = 0 \quad \text{for all} \quad z \ge 0, \\ &t > 0 : u = at, \ v = 0 \quad \text{at} \quad z = 0, \\ &t > 0 : u \to 0, \ v \to 0 \quad \text{as} \quad z \to \infty, \end{aligned} \tag{2}$$

where a is a constant.

The generalized Ohm's law, on taking Hall currents into account and neglecting ion-slip and thermo-electric effect, is (see Cowling 1957)

$$\vec{j} + \frac{\omega_e \tau_e}{B_0}\left(\vec{j} \times \vec{B}\right) = \sigma\left(\vec{E} + \mu_e \vec{q} \times \vec{B}\right), \tag{3}$$

Where $\vec{j}$ is the current density vector, $\vec{B}$ the magnetic field vector, $\vec{E}$ the electric field vector, ω_e the cyclotron frequency, σ the electrical conductivity of the fluid and τ_e the collision time of electron and μ_e the magnetic permeability.

The solenoidal relation $\nabla \cdot \vec{B} = 0$ for the magnetic field gives $B_z = B_0 =$ constant everywhere in the fluid where $\vec{B} \equiv (0, 0, B_0)$. Further, if (j_x, j_y, j_z) be the components of the current density $\vec{j}$, then the equation of conservation of the charge $\nabla \cdot \vec{j} = 0$ gives $j_z =$ constant. This constant is zero since $j_z = 0$ at the plate which is electrically non-conducting. Thus $j_z = 0$ everywhere in the flow. Since the induced magnetic field is neglected, the Maxwell's equation $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ becomes $\nabla \times \vec{E} = 0$ which gives $\frac{\partial E_x}{\partial z} = 0$ and $\frac{\partial E_y}{\partial z} = 0$. This implies that $E_x =$ constant and $E_y =$ constant everywhere in the flow.

In view of the above assumption, Eq. (3) gives

$$j_x + m j_y = \sigma(E_x + v B_0), \tag{4}$$

$$j_y - m j_x = \sigma(E_y - u B_0), \tag{5}$$

where $m = \omega_e \tau_e$ is the Hall parameter. For positive values of m, B_0 is upwards and the electrons of the conducting fluid gyrate in the same sense as the rotating system. For negative values of m, B_0 is downwards and the electrons gyrate in an opposite sense to the rotating system.

At infinity, the magnetic field is uniform so that there is no current and hence, we have

$$j_x \to 0, \ j_y \to 0 \ \text{as} \ z \to \infty. \tag{6}$$

On the use of (6), Eqs. (4) and (5) yield

$$E_x = 0, \ E_y = 0, \tag{7}$$

everywhere in the flow.

Substituting the above values of E_x and E_y in

Eqs. (4) and (5) and solving for j_x and j_y, we get

$$j_x = \frac{\sigma B_0}{1+m^2}(v+mu), \tag{8}$$

$$j_y = \frac{\sigma B_0}{1+m^2}(mv-u). \tag{9}$$

Using Eqs. (8) and (9), equations of momentum (1) along x - and y -directions are

$$\frac{\partial u}{\partial t} - w_0\frac{\partial u}{\partial z} - 2\Omega v = \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho(1+m^2)}(u-mv), \tag{10}$$

$$\frac{\partial v}{\partial t} - w_0\frac{\partial v}{\partial z} + 2\Omega u = \nu\frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2}{\rho(1+m^2)}(v+mu), \tag{11}$$

where ρ is the fluid density and ν the kinematic viscosity, w_0 is the normal velocity of suction or injection at the plate according as $w_0 > 0$ or $w_0 < 0$, respectively and $w_0 = 0$ represents the case of non-permeable plate.

Introducing non-dimensional variables

$$(u_1, v_1) = \frac{(u,v)}{(a\nu)^{\frac{1}{3}}},\ \eta = z\left(\frac{a}{\nu}\right)^{\frac{1}{3}}, \tau = t\left(\frac{a^2}{\nu}\right)^{\frac{1}{3}},$$

Equations (10) and (11) become

$$\frac{\partial u_1}{\partial \tau} - S\frac{\partial u_1}{\partial \eta} - 2K^2 v_1 = \frac{\partial^2 u_1}{\partial \eta^2} - \frac{M^2}{1+m^2}(u_1 - mv_1) \tag{12}$$

$$\frac{\partial v_1}{\partial \tau} - S\frac{\partial v_1}{\partial \eta} + 2K^2 u_1 = \frac{\partial^2 v_1}{\partial \eta^2} - \frac{M^2}{1+m^2}(v_1 + mu_1), \tag{13}$$

where $M^2 = \frac{\sigma {B_0}^2}{\rho}\left(\nu/a^2\right)^{\frac{1}{3}}$ is the magnetic parameter and $K^2 = \Omega\left(\frac{\nu}{a^2}\right)^{\frac{1}{3}}$ the rotation parameter and $S = w_0/(a\nu)^{\frac{1}{3}}$ the suction parameter.

The initial and boundary conditions (2) become

$$\tau \le 0 : u_1 = v_1 = 0 \quad \text{for all} \quad \eta \ge 0,$$

$$\tau > 0 : u_1 = \tau, v_1 = 0 \quad \text{at} \quad \eta = 0,$$

$$\tau > 0 : u_1 \to 0, v_1 \to 0 \quad \text{as} \quad \eta \to \infty. \tag{14}$$

Combining Eqs. (12) and (13), we get

$$\frac{\partial F}{\partial \tau} - S\frac{\partial F}{\partial \eta} = \frac{\partial^2 F}{\partial \eta^2} - \left[2iK^2 + \frac{M^2(1+im)}{1+m^2}\right]F, \tag{15}$$

where

$$F = u_1 + iv_1 \quad \text{and} \quad i = \sqrt{-1}. \tag{16}$$

The initial and boundary conditions for $F(\eta,\tau)$ are

$$F(\eta,0) = 0,\ F(0,\tau) = \tau,\ F(\infty,\tau) = 0, \tag{17}$$

Taking the Laplace transform of Eq. (15) and on the use of initial condition, we have

$$\frac{\partial \bar{F}}{\partial \tau} + S\frac{\partial \bar{F}}{\partial \eta} - \left[p + 2iK^2 + \frac{M^2(1+im)}{1+m^2}\right]\bar{F} = 0, \tag{18}$$

where

$$\bar{F}(\eta, p) = \int_0^\infty F(\eta, p)e^{-p\tau}\,d\tau. \tag{19}$$

and p is the Laplace parameter and $p > 0$.

The corresponding boundary conditions for $\bar{F}$ are

$$\bar{F}(0,p) = \frac{1}{p^2},\ \bar{F}(\infty, p) = 0. \tag{20}$$

The solution of Eq. (18) subject to the boundary conditions (20) is

$$\bar{F}(\eta, p) = \frac{1}{p^2}e^{-\frac{S}{2}\eta}\exp(-\sqrt{\lambda+p}\ \eta), \tag{21}$$

where

$$\lambda = \left[\frac{M^2(1+im)}{1+m^2} + 2iK^2\right] + \frac{S^2}{4}. \tag{22}$$

On the use of the inverse Laplace transform, Eq. (18) becomes

$$\begin{aligned} F(\eta,\tau) = \frac{1}{2}\Bigg[&\left\{\tau + \frac{\eta}{2(\alpha+i\beta)}\right\}e^{\{(\alpha+i\beta)-\frac{S}{2}\}\eta} \\ &\times \operatorname{erfc}\left\{\frac{\eta}{2\sqrt{\tau}} + (\alpha+i\beta)\sqrt{\tau}\right\} \\ &+\left\{\tau - \frac{\eta}{2(\alpha+i\beta)}\right\}e^{-\{(\alpha+i\beta)+\frac{S}{2}\}\eta} \\ &\times \operatorname{erfc}\left\{\frac{\eta}{2\sqrt{\tau}} - (\alpha+i\beta)\sqrt{\tau}\right\}\Bigg] \end{aligned} \tag{23}$$

where

$$\alpha, \beta = \frac{1}{\sqrt{2}}\left[\left\{\left(\frac{M^2}{1+m^2} + \frac{S^2}{4}\right)^2 + \left(2K^2 + \frac{M^2}{1+m^2}\right)^2\right\}^{\frac{1}{2}} \pm \left(\frac{M^2}{1+m^2} + \frac{S^2}{4}\right)\right]^{\frac{1}{2}} \tag{24}$$

Equation (23) does not coincident with the Eq. (15) of Deka (2008) when $S = 0$ due to the mathematical error in Eqs.(1) and (2) of his paper (as discussed in the introduction).

3. Small Time Solution

To get some physical insight into the flow pattern, we shall examine the solution (23) for small and large times τ. For small times τ, the method given by Carslaw and Jaeger (1959) is very useful where small time corresponds to large p. For small times, Eq. (21) can be rewritten as

$$\bar{F}(\eta,p)=e^{-\left(\frac{S}{2}\eta+\lambda\tau\right)}\sum_{n=0}^{\infty}(n+1)\lambda^n\frac{e^{-\sqrt{p}\,\eta}}{p^{n+2}}. \quad (25)$$

where λ is given by Eq. (22).

Taking the inverse Laplace transform of Eq. (21), we have

$$F(\eta,\tau)=e^{-\left\{\frac{S}{2}\eta+(\alpha+i\beta)^2\tau\right\}}$$

$$\times\sum_{n=0}^{\infty}(n+1)(\alpha+i\beta)^{2n}(4\tau)^{n+1}j^{2n+2}\operatorname{erfc}\left(\frac{\eta}{2\sqrt{\tau}}\right) \quad (26)$$

On the use of Eq. (16), Eq. (26) yields

$$u_1(\eta,\tau)=e^{-\left\{\frac{S}{2}\eta+(\alpha^2-\beta^2)\tau\right\}}$$

$$\times\Big[\cos 2\alpha\beta\tau\left\{(4\tau)T_2+2(\alpha^2-\beta^2)(4\tau)^2T_4+\cdots\right\}$$

$$+\sin 2\alpha\beta\tau\left\{4\alpha\beta(4\tau)^2T_4+12\alpha\beta(\alpha^2-\beta^2)(4\tau)^3T_6+\cdots\right\}\Big] \quad (27)$$

$$v_1(\eta,\tau)=e^{-\left\{\frac{S}{2}\eta+(\alpha^2-\beta^2)\tau\right\}}$$

$$\times\Big[\cos 2\alpha\beta\tau\left\{4\alpha\beta(4\tau)^2T_4+12\alpha\beta(\alpha^2-\beta^2)(4\tau)^3T_6+\cdots\right\}$$

$$-\sin 2\alpha\beta\tau\left\{(4\tau)T_2+2(\alpha^2-\beta^2)(4\tau)^2T_4+\cdots\right\}\Big], \quad (28)$$

where α and β is given by (24) and

$$T_{2n+2}=j^{2n+2}\operatorname{erfc}\left(\frac{\eta}{2\sqrt{\tau}}\right),$$

$$j^n\operatorname{erfc}(\eta)=j^{n-1}\operatorname{erfc}(\eta),\quad j^0\operatorname{erfc}(\eta)=\operatorname{erfc}(\eta). \quad (29)$$

Eqs. (27) and (28) show that the Hall effects become important only when terms of order τ is taken into account.

For large times, Eq. (23) can be written in the following form

$$u_1+iv_1=e^{-\frac{S}{2}\eta}\left[2\left\{\frac{\tau}{2}-\frac{\eta}{4(\alpha+i\beta)}\right\}e^{-(\alpha+i\beta)\eta}\right.$$

$$+\left\{\frac{\tau}{2}+\frac{\eta}{4(\alpha+i\beta)}\right\}e^{(\alpha+i\beta)\eta}$$

$$\times\operatorname{erfc}\left\{(\alpha+i\beta)\sqrt{\tau}+\frac{\eta}{2\sqrt{\tau}}\right\}$$

$$-\left\{\frac{\tau}{2}-\frac{\eta}{4(\alpha+i\beta)}\right\}e^{-(\alpha+i\beta)\eta}$$

$$\left.\times\operatorname{erfc}\left\{(\alpha+i\beta)\sqrt{\tau}-\frac{\eta}{2\sqrt{\tau}}\right\}\right] \quad (30)$$

For $\eta\ll 2\sqrt{\tau}$ and $\tau\gg 1$, Eq. (30) approximates to

$$u_1(\eta,\tau)=\tau e^{-\left(\frac{S}{2}+\alpha\right)\eta}\cos\beta\eta$$

$$+\sqrt{\frac{\tau}{\pi}}\frac{e^{-\left\{\frac{S}{2}\eta+(\alpha^2-\beta^2)\tau\right\}}}{\alpha^2+\beta^2}$$

$$\times\big[\alpha\big(\cos 2\alpha\beta\tau\sinh\alpha\eta\cos\beta\eta$$

$$+\sin 2\alpha\beta\tau\cosh\alpha\eta\sin\beta\eta\big)$$

$$+\beta\big(\cos 2\alpha\beta\tau\cosh\alpha\eta\sin\beta\eta$$

$$-\sin 2\alpha\beta\tau\sinh\alpha\eta\cos\beta\eta\big)\big] \quad (31)$$

$$v_1(\eta,\tau)=-\tau e^{-\left(\frac{S}{2}+\alpha\right)\eta}\sin\beta\eta$$

$$+\sqrt{\frac{\tau}{\pi}}\frac{e^{-\left\{\frac{S}{2}\eta+(\alpha^2-\beta^2)\tau\right\}}}{\alpha^2+\beta^2}$$

$$\times\big[\alpha\big(\cos 2\alpha\beta\tau\cosh\alpha\eta\sin\beta\eta$$

$$-\sin 2\alpha\beta\tau\sinh\alpha\eta\cos\beta\eta\big)$$

$$-\beta\big(\cos 2\alpha\beta\tau\sinh\alpha\eta\cos\beta\eta$$

$$+\sin 2\alpha\beta\tau\cosh\alpha\eta\sin\beta\eta\big)\big] \quad (32)$$

4. Result and Discussion

We have presented the non-dimensional velocity components for several values of magnetic parameter M^2, Hall parameter m, rotation parameter K^2, suction parameter S and time τ in Figs. 2 to 6. It is seen from Fig. 2 that both the primary velocity u_1 and the magnitude of the secondary velocity v_1 decrease with an increase in magnetic parameter M^2. The imposition of the transverse magnetic field tends to retard the fluid flow. This phenomenon has an excellent agreement with the physical fact that the Lorentz force generated in present flow model due to interaction of the transverse magnetic field and the fluid velocity acts as a resistive force to the fluid flow which serves to decelerate the flow. The reduction of the boundary layer velocity due to the imposition of the transverse magnetic field causes the pressure gradient to drop and as a consequence the boundary layer separation is prevented to some extent. It also resists the transition from laminar to turbulent flow which causes the viscous drag to increase and as a result the flow is stabilized. As such the magnetic field is an effective regulatory mechanism for the flow regime. Hall currents tend to accelerate secondary fluid velocity which is consistent with the fact that Hall currents induce secondary flow in the flow field. This is a new phenomenon, which appears as a result of including the Hall term. The case $m = 0$ corresponds to the neglect of the Hall effects. It is found from Fig. 4 that the primary velocity u_1 decreases while the magnitude of the secondary velocity v_1 increases with an increase in rotation parameter K^2. This implies that rotation tends to retard primary fluid velocity. Although rotation induces the secondary fluid velocity in the flow field by suppressing the primary fluid velocity, its accelerating effect is prevalent only in the region near to the plate. This is due to the reason that Coriolis force is dominant in the region near to the axis of rotation. An increase in suction parameter S leads to decrease both the primary velocity u_1 and the magnitude of the secondary velocity v_1 as shown in Fig. 5 It is observed that the suction/blowing exerts a strong influence on the velocity profiles. It is observed from Fig. 6 that both the primary velocity u_1 and the magnitude of the secondary velocity v_1 increase with an increase in time τ. This implies that primary and secondary fluid velocities are getting accelerated with the progress of time.

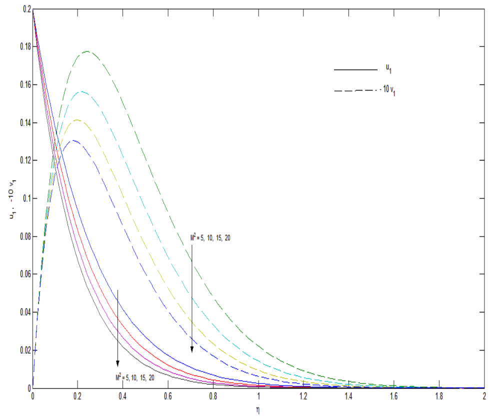

Fig. 2. Velocity profiles for M^2 when $K^2 = 3$, $S = 0.5$, $m = 0.2$ and $\tau = 0.2$

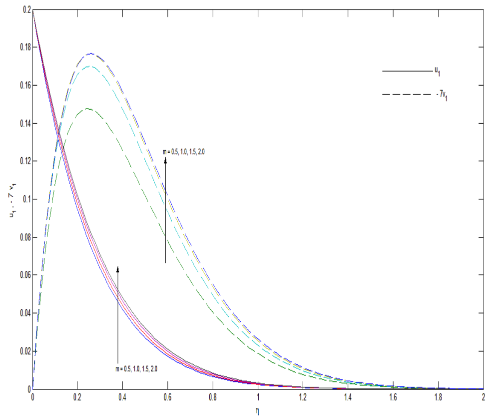

Fig. 3. Velocity profiles for m when $M^2 = 5$, $K^2 = 3$, $S = 0.5$ and $\tau = 0.2$

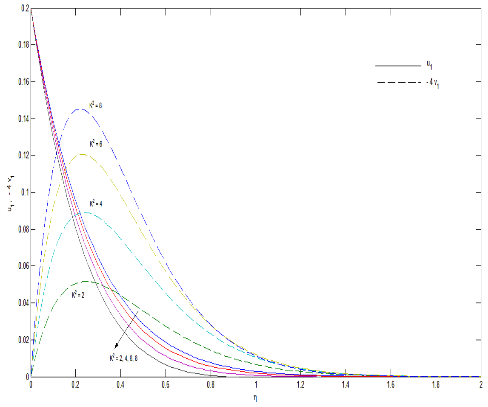

Fig. 4. Velocity profiles for K^2 when $M^2 = 5$, $S = 0.5$, $m = 0.2$ and $\tau = 0.2$.

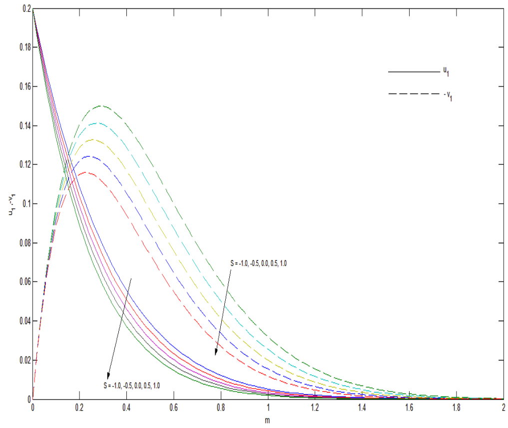

Fig. 5. Velocity profiles for S when $M^2 = 5$, $K^2 = 3$, $m = 0.2$ and $\tau = 0.2$.

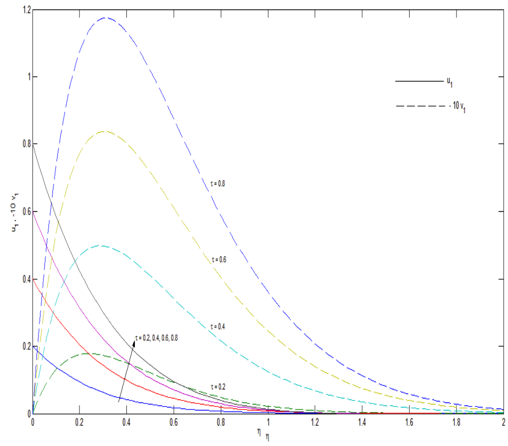

Fig. 6. Velocity profiles for τ when $M^2 = 5$, $S = 0.5$, $m = 0.2$ and $K^2 = 3$

For small values of time, we have drawn the primary velocity u_1 and the secondary velocity v_1 on using the exact solution given by the Eq. (23) and the series solution given by Eqs. (27) and (28) in Figs.7 and 8 respectively. It is seen from Figs.7 and 8 that the series solution given by Eqs. (27) and (28) converges more quickly than the exact solution given by Eq. (23) for small times. Hence, we conclude that for small times, the numerical values of the velocity components u_1 and v_1 can be computed from Eqs. (27) and (28) instead of Eq. (23).

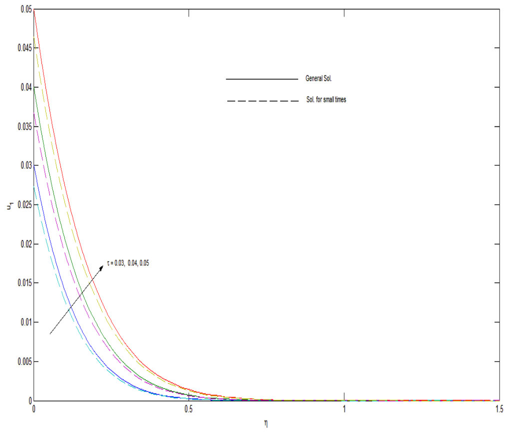

Fig. 7. u_1 for general and small time solutions when $M^2 = 5$, $K^2 = 3$, $m = 0.2$ and $S = 0.5$.

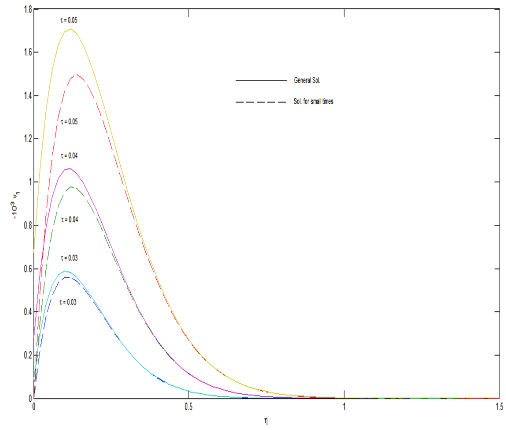

Fig. 8. v_1 for general and small time solutions when $M^2 = 5$, $K^2 = 3$, $m = 0.2$ and $S = 0.5$.

The non-dimensional shear stresses τ_x and τ_y due to the primary and secondary flows at the plate $\eta = 0$ repectively obtained from Eq. (23) are

$$\tau_x + i\tau_y = -\frac{S\tau}{2} - \frac{1}{2(\alpha + i\beta)} \times \left\{1 + 2(\alpha + i\beta)^2 \tau\right\} \mathrm{erf}\left\{\sqrt{(\alpha + i\beta)\tau}\right\} - \sqrt{\frac{\tau}{\pi}}\, e^{-(\alpha + i\beta)^2 \tau}, \tag{33}$$

where α and β are given by Eq. (24).

The numerical results of the non-dimensional shear stresses τ_x and τ_y at the plate $\eta = 0$ for several values of rotation parameter K^2, magnetic parameter M^2, suction parameter S and time τ against the Hall parameter m are presented in Figs. 9 to 12. Fig. 9 shows that the absolute values of the shear stresses τ_x and τ_y increase with an increase in rotation parameter K^2. Rotation tends to enhance both the shear stresses at the plate. On the other hand, the absolute value of the shear stress τ_x decreases whereas the absolute value of the shear stress τ_y increases with an increase in Hall parameter m. This implies that, the Hall currents have tendency to reduce the shear stress due to the primary flow whereas these physical quantities have reverse effect on the shear stress due to secondary flow. It is seen from Fig. 10 that the absolute value of the shear stress τ_x increases while the absolute value of the shear stress τ_y decreases for $m \le 0.2$ and it increases for $m > 0.2$ for increasing magnetic parameter M^2. Fig.11 displays that the absolute value of the shear stresses τ_x increases whereas the absolute value of the shear stress τ_y decreases with an increase in suction parameter S. It is found from Fig.12 that the absolute values of the shear stresses τ_x and τ_y increase with an increase in time τ. As time progresses, shear stresses are getting enhanced.

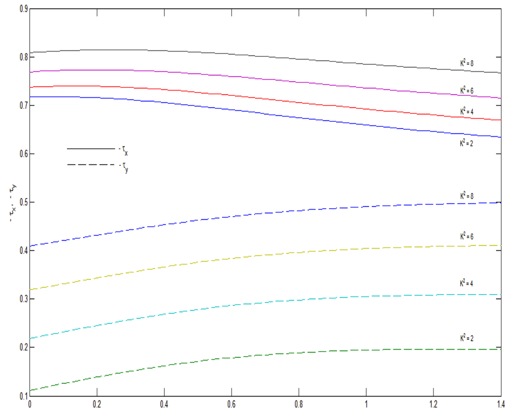

Fig. 9. Shear stresses τ_x and τ_y for K^2 when $M^2 = 5$, $S = 0.5$ and $\tau = 0.2$

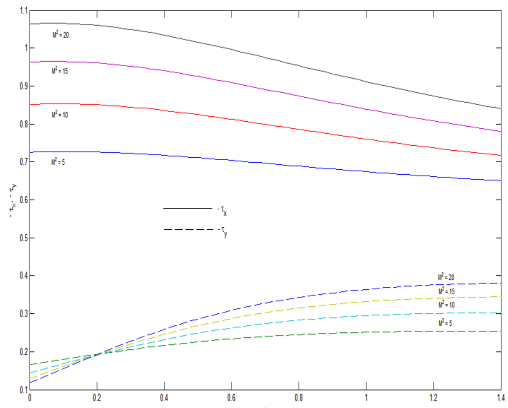

Fig. 10. Shear stresses τ_x and τ_y for M^2 when $\tau = 0.2$, $S = 0.5$ and $K^2 = 3$

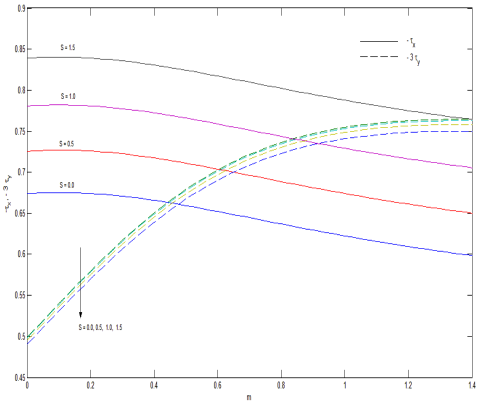

Fig. 11. Shear stresses τ_x and τ_y for S when $K^2 = 3$, $M^2 = 5$ and $\tau = 0.2$

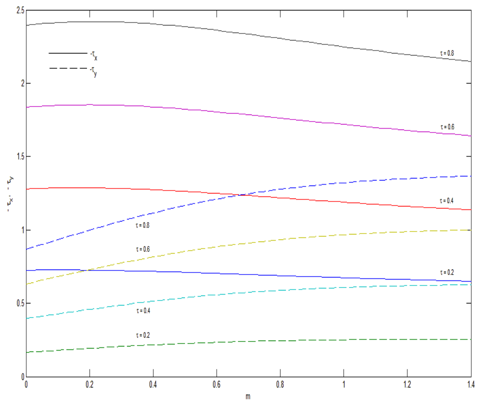

Fig. 12. Shear stresses τ_x and τ_y for τ when $M^2 = 5$, $K^2 = 3$ and $S = 0.5$.

For small time, the non-dimensional shear stresses τ_x due to the primary flow and τ_y due to the secondary flow at the plate $\eta = 0$ are given by

$$\tau_x = -e^{(\alpha^2-\beta^2)\tau}\left[\frac{S}{2}A_1(0,\tau) + \frac{1}{2\sqrt{\tau}}A_2(0,\tau)\right], \quad (34)$$

$$\tau_y = -e^{(\alpha^2-\beta^2)\tau}\left[\frac{S}{2}B_1(0,\tau) + \frac{1}{2\sqrt{\tau}}B_2(0,\tau)\right], \quad (35)$$

where

$$A_1(\eta,\tau) = \cos 2\alpha\beta\tau \times\left[(4\tau)T_2 + 2(\alpha^2-\beta^2)(4\tau)^2T_4 + \cdots\right]$$
$$+\sin 2\alpha\beta\tau\left[4\alpha\beta(4\tau)^2T_4 + 12\alpha\beta(\alpha^2-\beta^2)(4\tau)^3T_6 + \cdots\right]$$

$$B_1(\eta,\tau) = \cos 2\alpha\beta\tau\left[4\alpha\beta(4\tau)^2T_4 + \right.$$
$$\left. +12\alpha\beta(\alpha^2-\beta^2)(4\tau)^3T_6 + \cdots\right]$$
$$-\sin 2\alpha\beta\tau\left\{(4\tau)T_2 + 2(\alpha^2-\beta^2)(4\tau)^2T_4 + \cdots\right\}$$

$$A_2(\eta,\tau) = \cos 2\alpha\beta\tau \times\left[(4\tau)Y_1 + 2(\alpha^2-\beta^2)(4\tau)^2Y_3 + \cdots\right]$$
$$+\sin 2\alpha\beta\tau\left[4\alpha\beta(4\tau)^2Y_3 + 12\alpha\beta(\alpha^2-\beta^2)(4\tau)^3Y_5 + \cdots\right]$$

$$B_2(\eta,\tau) = \cos 2\alpha\beta\tau$$
$$\times\left[4\alpha\beta(4\tau)^2Y_3 + 12\alpha\beta(\alpha^2-\beta^2)(4\tau)^3Y_5 + \cdots\right]$$
$$-\sin 2\alpha\beta\tau\left\{(4\tau)Y_1 + 2(\alpha^2-\beta^2)(4\tau)^2Y_3 + \cdots\right\}$$

$$Y_{2n+1} = j^{2n+1}\operatorname{erfc}\left(\eta/2\sqrt{\tau}\right),$$
$$j^{-1}\operatorname{erfc}\left(\eta/2\sqrt{\tau}\right) = \frac{2}{\sqrt{\pi}}e^{-z^2}. \quad (36)$$

The numerical results of the non-dimensional shear stresses τ_x due to the primary flow and the shear stress τ_y due to the secondary flow at the plate $\eta = 0$ for the general solution and the solution for small time calculated from Eqs. (33), (34) and (35) respectively are given in Tables 1 and 2 for several values of Hall parameter m and time τ. It is observed from Tables 1 and 2 that for small time solution, the shear stresses calculated from Eqs. (34) and (35) give better result than that calculated from Eq. (33).

Table 1 Shear stress τ_x at the plate $\eta = 0$ when $M^2 = 5$, $K^2 = 3$ and $S = 0.5$

	$-10\tau_x$ (For general solution)			$-10\tau_x$ (Solution for small times)		
$m \backslash \tau$	0.001	0.002	0.003	0.001	0.002	0.003
0.2	0.35991	0.51127	0.62856	0.35992	0.51131	0.62867
0.4	0.35985	0.51111	0.62825	0.35986	0.51117	0.62844
0.6	0.35977	0.51089	0.62786	0.35979	0.51098	0.62811
0.8	0.35970	0.51068	0.62748	0.35972	0.51078	0.62776

Table 2 Shear stress τ_y at the plate $\eta = 0$ when $M^2 = 5$, $K^2 = 3$ and $S = 0.5$

	$-10^3\tau_y$ (For general solution)			$-10^3\tau_y$ (Solution for small times)		
$m \backslash \tau$	0.001	0.002	0.003	0.001	0.002	0.003
0.2	0.08258	0.23365	0.42893	0.08272	0.23367	0.42869
0.4	0.09165	0.25929	0.47606	0.09178	0.25930	0.47572
0.6	0.09737	0.27553	0.50594	0.09752	0.27553	0.50554
0.8	0.10015	0.28342	0.52051	0.10030	0.28343	0.52008

5. CONCLUSION

An investigation of the effects of Hall currents and rotation on unsteady hydromagnetic flow of a viscous incompressible electrically conducting fluid past an accelerated vertical porous plate in a rotating system has been carried out. Hall current tends to accelerate the primary and secondary fluid velocities. Rotation has tendency to retard primary fluid velocity and tends to accelerate secondary fluid velocity. The primary and secondary fluid velocities are getting accelerated with the progress of time. Hall currents have tendency to reduce the shear stress due to the primary flow whereas these physical quantities have reverse effect on the shear stress due to the secondary flow. Rotation tends to enhance both the shear stresses at the plate. It is interesting to note that for small times, the series solution converges more rapidly than the exact solution. This study of Hall currents in rotating environment will be useful in dealing with real engineering problems.

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