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Magneto-thermal Stability of Rotating Walters' (model B') Visco-elastic Compressible Fluid in the Presence of Hall Currents and Suspended Particles in Porous Medium

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ABSTRACT

In the present paper, the stability of a compressible rotating Walters' B' viscoelastic fluid through a porous medium is considered in the presence of uniform vertical magnetic field, Hall current and suspended particles. It is found that the region of stability increases with the increase in magnetic field, rotation and compressibility and it decreases with the increase in suspended particles and Hall currents. Also, it has been established that the medium permeability has destabilizing effect in the absence of rotation while in the presence of rotation it may have stabilizing effect on the system. Graphs are plotted to find the region of stability in the presence of various physical parameters like Hall current parameter, suspended particles, rotation, magnetic field and medium permeability. The principle of exchange of stabilities holds good under certain conditions and the modes may be non-oscillatory or oscillatory.

Keywords: Hall current, Rotation; Porous medium**;** Suspended particles; Magnetic field.

1. INTRODUCTION

The basic concepts of fluid mechanical phenomena have been given in Bansal (2004) and Gupta and Gupta (2013) to understand various fascinating and diverse applications of fluid mechanics. A comprehensive and detailed overview of the onset of thermal stability of an incompressible Newtonian fluid has been well documented by Chandrasekhar (1981) and Drazin and Reid (1981). Rayleigh (1883) studied the character of equilibrium of an inviscid and incompressible fluid with vertical density variation. For the case of compressible fluids, the governing equations become quite tedious. To overcome this situation, Boussinesq tried to justify the approximation for compressible fluids when the density variations are mainly due to thermal effects in the equations of motion. Spiegel and Veronis (1960) have simplified the system of equations for compressible fluids by assuming that the vertical height of the fluid is much smaller than the scale height as defined by them and the perturbations in density, temperature and pressure do not exceed their total static variations. The effects of suspended particles on various stability problems have relevance importance in several scientific and engineering applications such as geophysics, chemical engineering and astrophysics. In geophysical situations, the fluid is generally not pure but contains some dust particles. In astrophysical context, a comet consists of a dusty snowball which is a mixture of frozen gases and changes from solid to gas and vice-versa. Scanlon and Segel (1973) have investigated the effect of suspended particles on Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by the particles. So the effect of suspended particles was to accelerate the onset of convection. The thermal instability problems of compressible fluids with Hall currents and suspended particles through a porous medium have been studied by Sharma and Gupta (1993). Sharma and Aggarwal (2006) have studied the effect of compressibility and suspended particles on thermal stability in a Walters' (model B') elastic-viscous fluid in hydromagnetics.

Bhatia and Steiner (1973) have studied the problem of thermal instability of a Maxwellian visco-elastic fluid in the presence of a magnetic field and found that the magnetic field has a stabilizing influence on the overstable mode of convection in a visco-elastic fluid layer. Kumar *et al*. (2010) have studied the thermal convection in a Walters' (model B') elastic-viscous dusty fluid in hydromagnetics under the influences of compressibility and rotation. There are many elastico-viscous fluids that cannot be characterized by Oldroyd's constitutive relations or Maxwell's constitutive relations. Two such classes of fluids are Walters' B' fluid and Rivlin-Ericksen fluid. Walters' (1962) reported that the mixture of polymethyl methacrylate and pyridine at 25^0C containing 30.5g of polymer with density 0.98g per litre behaves very nearly as the Walters' B' fluid. Walters' B' fluids have relevance importance in agriculture, plastic industry, chemical technology, petroleum industry, industrial technology, food processing industry and polymer industry. Polymers are used in the manufacture of contact lenses, seats, foam, plastics, space crafts, tyres, rubber, belts, load carrying ropes, communications appliances, cushions, adhesives etc.

Thermal convective instability through a porous medium has merited extensive attention over the years and is of fundamental importance in a wide range of scientific and technical purposes like solidification, chemical processing industry, geophysical fluid dynamics, petroleum industry, recovery of crude oil from earth's interior etc. A porous medium is defined as a solid with inter-connected voids. A detailed study of convection through porous medium has been covered extensively in Nield and Bejan (2006). Liquid saturated porous material are often present on and below the surface of the earth in the form of dust particles, limestone and other sediments permeated by groundwater or oil. Recently, the development of geothermal power resources has increased considerable attention in convection through porous medium. The gross effect when the fluid slowly percolates through the pores of a homogeneous and isotropic medium is governed by Darcy's law which states that the usual viscous term in the equations of motion of fluid is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\mathbf{q}\right]$, where μ, μ', k_1 and q denotes, respectively, the viscosity, visco-elasticity, medium permeability and Darcian (filter) velocity of Walters' B' fluid.

If an electric field is imposed at right angle to the magnetic field strength then the whole current will not flow along the direction of electric field. This nature of the electric current to flow across an electric field in the presence of magnetic field is known as Hall effect. Hall current plays an important role in many geophysical and astrophysical problems as well as in flow of laboratory plasmas. Sunil *et al*. (2000) have studied the effect of Hall currents on thermal instability of Walters' (model B') fluid. Gupta and Aggarwal (2011) have investigated thermal instability of compressible Walters' (model B') fluid in the presence of Hall currents and suspended particles and found that compressibility and magnetic field have a stabilizing effect whereas Hall currents and suspended particles have destabilizing effects. Rana (2013) has discussed the problem of thermosolutal convection in Walters' (model B') rotating fluid permeated with suspended particles and variable gravity field in porous medium in hydromagnetics. The Magnetohydrodynamic thermoconvective problems of Walters' B' fluid including the effects of various parameters like rotation, variable gravity and suspended particles through a Brinkman porous medium have been studied by Kumar *et al*. (2013).

Keeping in mind, the importance and applications of non-Newtonian elastico-viscous fluids in Geology, Petroleum Industry, Chemical Technology, and various applications mentioned above, the study and investigations have become important and attractive on such fluids. Stability is discussed analytically as well as graphically.

2. Mathematical Formulation of the Problem

We consider a static state, in which a compressible Walters' B' visco-elastic fluid of depth d is arranged in horizontal strata in a porous medium of porosity $\in$ and permeability k_1. The system is assumed to be rotating with angular velocity Ω (0, 0, Ω) along the vertical axis. A uniform vertical magnetic field intensity H (0, 0, H) pervade the system. A temperature gradient $\beta=\left|\frac{dT}{dz}\right|$ is maintained by underside heating. Both the boundaries are taken to be free and perfect conductors of heat. The pressure p, density ρ, viscosity μ and viscoelasticity μ' depend upon the vertical co-ordinate z- only. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.

Let $T_{ij}, \tau_{ij}, e_{ij}, \delta_{ij}, q_i$, *and* x_i denotes the stress tensor, shear stress tensor, rate of strain tensor, Kronecker delta, velocity vector and position vector, respectively. The constitutive relations for the Walters' B' viscoelastic fluid are

$$T_{ij}=-p\delta_{ij}+\tau_{ij},\ \tau_{ij}=2\left(\mu-\mu'\partial/\partial t\right)e_{ij},\ e_{ij}=\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right) \quad (1)$$

The governing mathematical equations of motion and continuity relevant to the problem are

$$\frac{\rho}{\in}\left[\frac{\partial\mathbf{q}}{\partial t}+\frac{1}{\in}(\mathbf{q}.\nabla)\mathbf{q}\right]=-\nabla p+\rho\mathbf{X_i}-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\mathbf{q} \quad (2)$$
$$+\frac{2\rho}{\in}(\mathbf{q}\times\mathbf{\Omega})+\frac{K'N_0}{\in}(\mathbf{q_d}-\mathbf{q})+\frac{\mu_e}{4\pi}(\nabla\times\mathbf{H})\times\mathbf{H}$$

$$\in\frac{\partial\rho}{\partial t}+\nabla.\left(\rho\mathbf{q}\right)=0 \quad (3)$$

Where $\rho, \upsilon, \upsilon', \mu_e, \alpha, \mathbf{q}, \mathbf{q_d}, N_0\left(\bar{x}, t\right)$ denote, respectively, the density of compressible fluid, kinematic viscosity, kinematic visco-elasticity, magnetic permeability, co-efficient of volume expansion, velocity of pure fluid, velocity of suspendzed particles and number density of the suspended particles. The symbol $\mathbf{X_i}=-\mathbf{g}\lambda_i$ represents external force term due to gravity variation, $\bar{x}=\left(x,y,z\right)$ and $K'=6\pi\rho\upsilon\delta$ where δ being particle radius, is the Stokes' drag coefficient.

The presence of suspended particles adds an extra force term, in equation of motion, proportional to velocity difference between particles and fluid. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. Inter-particle reactions are ignored as the distances between the particles are assumed to be quite large compared with their diameters. The effects of pressure, magnetic field and gravity on the particles are very small and hence ignored.

If mN_0 is the mass of particles per unit volume, then the equations of motion and continuity for the particles are:

$$mN_0\left[\frac{\partial \mathbf{q_d}}{\partial t}+\frac{1}{\in}(\mathbf{q_d}.\nabla)\mathbf{q_d}\right]=K'N_0(\mathbf{q}-\mathbf{q_d}) \tag{4}$$

$$\in\frac{\partial N_0}{\partial t}+\nabla.(N_0\mathbf{q_d})=0 \tag{5}$$

Let $\rho_s, c_s, c_v, c_{pt}, T$ and k denote, respectively, the density of solid material, heat capacity of solid material, the specific heat at constant volume, heat capacity of suspended particles, the temperature and the effective thermal conductivity of the pure fluid. Assuming that the suspended particles and the fluid are in thermal equilibrium, the equations of heat conduction gives:

$$\begin{aligned}&\left[\in\rho c_v+\rho_s c_s(1-\in)\right]\frac{\partial T}{\partial t}+\rho c_v(\mathbf{q}.\nabla)T\\&+mN_0 c_{pt}\left(\in\frac{\partial}{\partial t}+\mathbf{q_d}.\nabla\right)T=k\nabla^2 T\end{aligned} \tag{6}$$

The Maxwell's equation yields

$$\begin{aligned}&\in\frac{\partial \mathbf{H}}{\partial t}=\nabla\times(\mathbf{q}\times\mathbf{H})+\in\eta\nabla^2\mathbf{H}\\&-\frac{\in}{4\pi N'e}\nabla\times\left[(\nabla\times\mathbf{H})\times\mathbf{H}\right]\end{aligned} \tag{7}$$

$$\nabla.\mathbf{H}=0 \tag{8}$$

Where η, N' and e denote the electrical resistivity, electron number density and the charge of an electron, respectively.

The steady state of the system is

$$\begin{aligned}&\mathbf{q}=(0,0,0),\ \mathbf{q_d}=(0,0,0), p=p(z), \rho=\rho(z),\\&T=T(z), \mathbf{H}=[\mathbf{0},\mathbf{0},\mathbf{H}],\ N_0=constant.\end{aligned} \tag{9}$$

Following any state variable X can be expressed in the form

$$f(x,y,z,t)=f_m+f_0(z)+f'(x,y,z,t) \tag{10}$$

Where f_m stands for the constant space distribution of f, f_0 is the variation in the absence of motion and $f'(x,y,z,t)$ stands for the fluctuations in f resulting from the motion of fluid. According to Spiegel and Veronis (1960), it has been shown that the equations governing for compressible fluids are equivalent to those for incompressible fluids if the static temperature gradient β is replaced by its excess over the adiabatic gradient $\left(\beta-\frac{g}{c_p}\right)$.

Where

$$\begin{aligned}&T=T_0-\beta z,\ \rho=\rho_m\begin{bmatrix}1-\alpha_m(T-T_m)\\+K_m(p-p_m)\end{bmatrix},\\&\alpha_m=-\left(\frac{1}{\rho}\frac{\partial\rho}{\partial T}\right)_m, K_m=\left(\frac{1}{\rho}\frac{\partial\rho}{\partial p}\right)_m,\\&p(z)=p_m-g\int_0^z(\rho_0+\rho_m)dz.\end{aligned} \tag{11}$$

Where ρ_0 and T_0 stands for the density and temperature of the fluid at the lower boundary whereas p_m and ρ_m stands for a constant space distribution of pressure p and density ρ respectively.

3. MATHEMATICAL ANALYSIS AND DISPERSION RELATION

Here, we shall analyze the stability of the basic state defined by Eq. (9) by applying usual perturbation technique. Let $\mathbf{q}$ (u,v,w), $\mathbf{q_d}$ (l,r,s), N, θ, δp, $\delta\rho$ and $\mathbf{h}(\mathbf{h_x},\mathbf{h_y},\mathbf{h_z})$ denote, respectively, the perturbation in fluid velocity $\mathbf{q}$(0,0,0), the perturbation in particle velocity $\mathbf{q_d}$ (0,0,0), perturbation in particle number density N_0, temperature T, pressure p, density ρ and magnetic field **H**. The density variation $\delta\rho$ due to temperature perturbation θ is given by

$$\delta\rho=-\alpha\rho_m\theta. \tag{12}$$

Then, the linearized perturbation equations governing the motion of fluid are

$$\begin{aligned}&\frac{1}{\in}\frac{\partial\mathbf{q}}{\partial t}=-\frac{1}{\rho_m}\nabla(\delta p)-\mathbf{g}\left(\frac{\delta\rho}{\rho_m}\right)\lambda_i-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\mathbf{q}\\&+\frac{2}{\in}(\mathbf{q}\times\mathbf{\Omega})+\frac{K'N}{\rho_m\in}(\mathbf{q_d}-\mathbf{q})+\frac{\mu_e}{4\pi\rho_m}(\nabla\times\mathbf{h})\times\mathbf{H}\end{aligned} \tag{13}$$

$$\nabla.\mathbf{q}=0 \tag{14}$$

$$\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\mathbf{q_d}=\mathbf{q} \tag{15}$$

$$\left(E+b\in\right)\frac{\partial\theta}{\partial t}=\left(\beta-\frac{\mathbf{g}}{c_p}\right)\left(\mathbf{w}+b\mathbf{s}\right)+\kappa\nabla^2\theta \tag{16}$$

$$\in\frac{\partial N}{\partial t}+\nabla.\left(N\mathbf{q_d}\right)=0 \tag{17}$$

$$\nabla.\mathbf{h}=0 \tag{18}$$

$$\in\frac{\partial\mathbf{h}}{\partial t}=\left(\nabla\mathbf{H}\right)\mathbf{q}+\in\eta\nabla^2\mathbf{h}-\frac{\in}{4\pi N'e}\nabla\times\left[\left(\nabla\times\mathbf{h}\right)\times\mathbf{H}\right] \tag{19}$$

Where

$$E=\in+\left(1-\in\right)\left(\frac{\rho_s C_s}{\rho_m C_v}\right), b=\frac{mN_0C_{pt}}{\rho_m C_v}, \kappa=\frac{k}{\rho_m C_v}, \lambda_i=\left(0,0,1\right)$$

And w, s, be the vertical fluid and particle velocity, respectively.

Writing Eq. (13) in component form and eliminating δp between them on using Eqs. (14) and (15), we obtain

$$\begin{gathered}\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}\left(\nabla^2\mathbf{w}\right)=-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\\ \nabla^2\mathbf{w}-\frac{mN}{\in\rho_m}\frac{\partial}{\partial t}\left(\nabla^2\mathbf{w}\right)+\mathbf{g}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\alpha\theta\\ -\frac{2\mathbf{\Omega}}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial\zeta}{\partial z}+\frac{\mu_e\mathbf{H}}{4\pi\rho_m}\left(\frac{m}{k'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial z}\left(\nabla^2\mathbf{h_z}\right)\end{gathered} \tag{20}$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is the three dimensional Laplacian operator.

$$\begin{gathered}\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial\zeta}{\partial t}=-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\zeta-\\ \frac{mN}{\in\rho_m}\frac{\partial\zeta}{\partial t}+\frac{\mu_e\mathbf{H}}{4\pi\rho_m}\left(\frac{m}{k'}\frac{\partial}{\partial t}+1\right)\frac{\partial\xi}{\partial z}+\frac{2\mathbf{\Omega}}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial\mathbf{w}}{\partial z}\end{gathered} \tag{21}$$

Where $\xi=\frac{\partial\mathbf{h_y}}{\partial x}-\frac{\partial\mathbf{h_x}}{\partial y}$ is the z-component of current density and $\zeta=\frac{\partial\mathbf{v}}{\partial x}-\frac{\partial\mathbf{u}}{\partial y}$ is the z-component of vorticity.

Equation (16) with the help of Eq. (15) gives

$$\begin{gathered}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left[\left(E+b\in\right)\frac{\partial}{\partial t}-\kappa\nabla^2\right]\theta=\\ \beta\left(\frac{G-1}{G}\right)\left[\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)+b\right]\mathbf{w}\end{gathered} \tag{22}$$

Equation (19) along with Eq. (18) gives

$$\in\frac{\partial\xi}{\partial t}=\mathbf{H}\frac{\partial\zeta}{\partial z}+\in\eta\nabla^2\xi+\mathbf{M_1}\frac{\partial}{\partial z}\left(\nabla^2\mathbf{h_z}\right) \tag{23}$$

$$\in\frac{\partial\mathbf{h_z}}{\partial t}=\mathbf{H}\frac{\partial\mathbf{w}}{\partial z}+\in\eta\nabla^2\mathbf{h_z}-\mathbf{M_1}\frac{\partial\xi}{\partial z} \tag{24}$$

Where $\mathbf{M_1}=\frac{\in\mathbf{H}}{4\pi N'e}$ is the Hall current parameter and $M=\frac{\in N_0}{N}$ is the ratio of particle number densities.

Now, analyzing the perturbation quantities using normal mode method by considering solution with a dependence on x, y and t of the form:

$$\left[\mathbf{w},\mathbf{s},\theta,\zeta,\mathbf{h_z},\xi\right]=\left[\begin{matrix}\mathbf{W}(z),\mathbf{S}(z),\Theta(z),\\ \mathbf{Z}(z),\mathbf{K}(z),\mathbf{X}(z)\end{matrix}\right]\exp\left(ik_x x+ik_y y+nt\right) \tag{25}$$

Where $k^2=\left(k_x{}^2+k_y{}^2\right)$, is the resultant wave number of the disturbances and n be the frequency of the harmonic disturbance, which is, in general, a complex constant.

Now, the dimensionless and linearized perturbation equations on using Eq. (25) in Eqs. (20) to (24), become

$$\begin{gathered}\left[\frac{\sigma}{\in}\left(1+\frac{f}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right]\left(D^2-a^2\right)\mathbf{W}(z)+\\ \frac{\mathbf{g}\alpha a^2d^2\Theta}{\upsilon}+\frac{2\mathbf{\Omega}d^3}{\in\upsilon}D\mathbf{Z}-\frac{\mu_e\mathbf{H}d}{4\pi\rho_m\upsilon}\left(D^2-a^2\right)D\mathbf{K}=0\end{gathered} \tag{26}$$

$$\left[\frac{\sigma}{\in}\left(1+\frac{f}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right]\mathbf{Z}=\frac{\mu_e\mathbf{H}d}{4\pi\rho_m\upsilon}D\mathbf{X}+\left(\frac{2\mathbf{\Omega}d}{\in\upsilon}\right)D\mathbf{W} \tag{27}$$

$$\left[\left(\left(D^2-a^2\right)-E_1p_1\sigma\right)\right]\Theta=-\left(\frac{\beta d^2}{\kappa}\right)\left(\frac{G-1}{G}\right)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\mathbf{W} \tag{28}$$

$$\left[\sigma\in-\frac{\left(D^2-a^2\right)\in}{p_2}\right]\mathbf{K}=\left(\frac{\mathbf{H}d}{\upsilon}\right)D\mathbf{W}-\frac{\mathbf{M_1}d}{\upsilon}D\mathbf{X} \tag{29}$$

$$\left[\sigma\in-\frac{\left(D^2-a^2\right)\in}{p_2}\right]\mathbf{X}=\left(\frac{\mathbf{H}d}{\upsilon}\right)D\mathbf{Z}+\frac{\mathbf{M_1}}{\upsilon d}D\left(D^2-a^2\right)\mathbf{K} \tag{30}$$

where the following non-dimensional quantities and parameters are introduced:

$z = z^{*}d$, $a = kd$, $\sigma = \frac{nd^{2}}{\upsilon}, \tau = \frac{m}{K'}, \tau_1 = \frac{\tau\upsilon}{d^{2}}$,

$F = \frac{\upsilon'}{d^{2}}, N_0 = \frac{\rho_m f}{m}, E_1 = E + b \in$, $B = b+1$.

$P_l = \frac{k_1}{d^{2}}$, is the dimensionless medium permeability, $p_1 = \frac{\upsilon}{\kappa}$, is the thermal Prandtl number, $p_2 = \frac{\upsilon}{\eta}$, is the magnetic Prandtl number.

The exact solution for the lowest mode subject to the boundary conditions (for the case of two free boundaries),

$W = D^2W = 0$, $DZ = DK = 0$

and $\Theta = 0$ at z = 0 and 1. (31)

is defined as

$$W = W_0 \sin \pi z \tag{32}$$

where W_0 is a constant.

Eliminating K, Θ, X and Z between Eqs. (26) to (30) and using Eqs. (31) and (32), we obtain the dispersion relation

$$\begin{aligned} R_1 = {} & \left(\frac{G}{G-1}\right)\left\{\frac{\left(1+x+E_1 p_1 i\sigma_1\right)}{x\left(B+\tau_1\pi^2 i\sigma_1\right)}\right\} \\ & \left\{ \left[\frac{i\sigma_1}{\in}\left(1+\frac{f}{1+\tau_1\pi^2 i\sigma_1}\right)+\frac{1-F\pi^2 i\sigma_1}{P}\right](1+x) \right. \\ & \left(1+x+E_1 p_1 i\sigma_1\right)+\frac{Q_1}{\in} \\ & \left[\frac{(1+x)\left(1+x+E_1 p_1 i\sigma_1\right)\left(1+x+p_2 i\sigma_1\pi^2\right)+ Q_1\left(1+x+p_2 i\sigma_1\pi^2\right)}{\left[M_1(1+x)+\left(1+x+p_2 i\sigma_1\pi^2\right)^2\right]\left(1+x+p_2 i\sigma_1\right) +Q_1\left(1+x+p_2 i\sigma_1\right)}\right] \\ & \left. +\frac{\frac{T_{A_1}}{\in^2}}{\left[\frac{i\sigma_1}{\in}\left(1+\frac{f}{1+\tau_1\pi^2 i\sigma_1}\right)+\frac{1-F\pi^2 i\sigma_1}{P}\right]+\frac{Q_1}{\left(1+x+p_2 i\sigma_1\pi^2\right)}} \right\} \end{aligned} \tag{33}$$

where

$R_1 = \frac{R}{\pi^4}, T_{A_1} = \frac{T_A}{\pi^4}, x = \frac{a^2}{\pi^2}, i\sigma_1 = \frac{\sigma}{\pi^4}, P = \pi^2 P_l, Q_1 = \frac{Q}{\pi^4}$.

$R = \frac{g\alpha\beta d^{4}}{\upsilon\kappa}$, is the thermal Rayleigh number,

$Q = \frac{\mu_e H^{2} d^{2}}{4\pi\rho_m \upsilon\eta}$, is the Chandrasekhar number and

$T_A = \left(\frac{2\Omega d^{2}}{\upsilon}\right)^2$, is the Taylor number.

Equation (33) is required dispersion relation accounting the effects of rotation, Hall currents, compressibility, magnetic field and medium permeability on thermal instability of Walters' (model B') elastico-viscous fluid permeated with suspended particles through a porous medium.

4. RESULTS AND DISCUSSION

4.1 Case of Stationary Convection

When the instability sets in as stationary convection, the marginal state will be characterized by putting $\sigma = 0$ in Eq. (33) then the Rayleigh number reduces to

$$R_1 = \left(\frac{G}{G-1}\right)\frac{(1+x)}{xB}\left[\frac{Q_1\left[(1+x)^2+Q_1\right]}{\in\left[(1+x)\left\{M_1+(1+x)\right\}+Q_1\right]} + \frac{T_{A_1}(1+x)P}{\left\{(1+x)+Q_1 P\right\}\in^2}+\frac{(1+x)^2}{P}\right] \tag{34}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters T_{A_1}, B, P, Q_1, M_1. Hence, it is clear that for stationary convection Walters' (Model B') elastico-viscous fluid behave like an ordinary Newtonian fluid since elastico-viscous parameter F vanishes with σ .

In the absence of rotation and porous medium $\left(i.e. T_{A_1} = 0, \in = 1, P = 1\right)$, Eq. (34) reduces to:

$$R_1 = \left(\frac{G}{G-1}\right)\frac{(1+x)}{xB}\left[(1+x)^2+\frac{Q_1\left[(1+x)^2+Q_1\right]}{\left[(1+x)\left\{M_1+(1+x)\right\}+Q_1\right]}\right] \tag{35}$$

Which is identical with the expression derived by Gupta and Aggarwal (2011).

In the absence of Hall currents, Eq. (35) yields

$$R_1 = \left(\frac{G}{G-1}\right)\frac{(1+x)}{xB}\left[(1+x)^2+Q_1\right] \tag{36}$$

Which is identical with the expression derived by Sharma and Aggarwal (2006).

From Eq. (34), we obtain

$$R_1 = R_1^{*}\left(\frac{G}{G-1}\right) \tag{37}$$

Where R_1 and R_1^* denote, respectively, the critical Rayleigh numbers in the presence and absence of compressibility. The negative and infinite values of critical Rayleigh number for G<1 and G=1 are not relevant for the problem under consideration. Hence the effect of compressibility is to postpone the onset of thermal convection.

To study the effects of suspended particles, Hall current parameter, rotation, magnetic field and medium permeability, we will examine the behaviour of $\frac{dR_1}{dB}, \frac{dR_1}{dM_1}, \frac{dR_1}{dT_{A_1}}, \frac{dR_1}{dQ_1}$ *and* $\frac{dR_1}{dP}$ analytically.

Equation (34) gives

$$\frac{dR_1}{dB} = -\left(\frac{G}{G-1}\right)\frac{(1+x)}{xB^2}\left[\frac{(1+x)^2}{P} + \frac{T_A(1+x)P}{\{(1+x)+Q_1P\}\in^2} + \frac{Q_1\left[(1+x)^2+Q_1\right]}{\in\left[(1+x)\{M_1+(1+x)\}+Q_1\right]}\right] \quad (38)$$

$$\frac{dR_1}{dM_1} = -\left(\frac{G}{G-1}\right)\frac{(1+x)^2}{xB}\left[\frac{Q_1\left[(1+x)^2+Q_1\right]}{\in\left[(1+x)\{M_1+(1+x)\}+Q_1\right]^2}\right] \quad (39)$$

Equations (38) and (39) show that suspended particles and Hall currents have a destabilizing effect on the system. These destabilizing influences of suspended particles and Hall currents agree with the previous work reported by Gupta & Aggarwal (2011).

Equation (34) also yields

$$\frac{dR_1}{dT_{A_1}} = \left(\frac{G}{G-1}\right)\frac{(1+x)}{xB}\left[\frac{(1+x)P}{\{(1+x)+Q_1P\}\in^2}\right] \quad (40)$$

$$\frac{dR_1}{dQ_1} = \left(\frac{G}{G-1}\right)\frac{(1+x)}{xB}\left[\frac{\left[(1+x)\{M_1+(1+x)\}\right]\left[(1+x)^2+2Q_1\right]+Q_1^2}{\in\left[(1+x)\{M_1+(1+x)\}+Q_1\right]^2}\right] \quad (41)$$

Equations. (40) and (41) show that rotation and magnetic field have stabilizing effects on the system. These stabilizing effects are an agreement of the earlier works of Sharma and Aggarwal (2006) and Gupta and Aggarwal (2011).

From Eq. (34), we also get

$$\frac{dR_1}{dP} = -\left(\frac{G}{G-1}\right)\frac{(1+x)}{xB}\left[\frac{(1+x)^2}{P^2} - \frac{T_{A_1}(1+x)^2}{\{(1+x)+QP\}^2\in^2}\right] \quad (42)$$

From Eq. (42), we observe that in the absence of rotation, medium permeability has destabilizing effect whereas in the presence of rotation, it has destabilizing effect when $\{\in(1+x)+Q_1P\}^2 > T_AP_l^2$ and stabilizing effect when $\{\in(1+x)+QP\} < TP$.

The dispersion *relation (34)* is analyzed numerically. Graphs have been plotted for various values of the parameters for determining the region of stability.

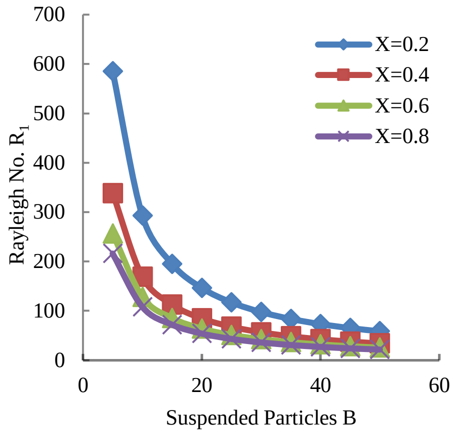

Fig. 1. Variation of Rayleigh number R_1 with suspended particles B for G=10, T_{A_1} = 10 , M_1 =10, Q_1 = 50, $\in$= 0.1 , P = 0.1 for fixed wave numbers x = 0.2, x = 0.4, x=0.6 and x = 0.8.

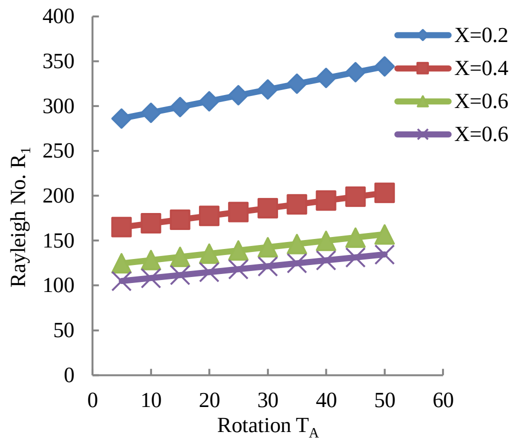

Fig. 2. Variation of Rayleigh number R_1 with rotation T_{A_1} for G=10, B = 10 , M_1 =10, Q_1 = 50, $\in$= 0.1 , P = 0.1 for fixed wave numbers x = 0.2, x = 0.4, x=0.6 and x = 0.8.

Figures 1 and 5 show the decrease in the values of Rayleigh number R_1 with the increase in suspended particles B, Hall current parameter M_1, respectively. This shows that the effect of suspended particles and Hall currents is to accelerate the onset of convection. Figures 2 and 3 show that the values of Rayleigh number R_1 increases with the increase in rotation T_A and magnetic field Q_1, respectively, thereby depicting the stabilizing effects of rotation T_A and magnetic field Q_1.

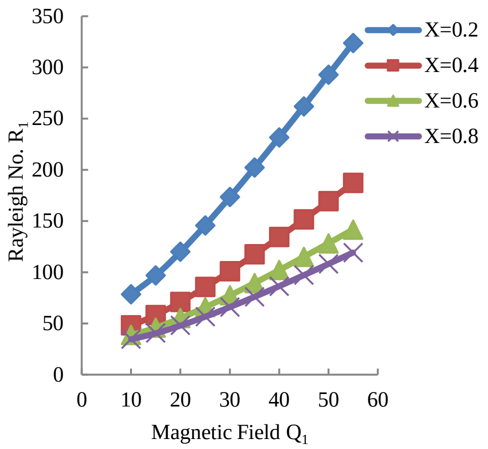

Fig. 3. Variation of Rayleigh number R_1 with magnetic field Q_1 for G=10, B=10, $T_{A_1}=10$, M_1 =10, $P=0.1$, $\in=0.1$ for fixed wave numbers x = 0.2, x = 0.4, x=0.6 and x = 0.8.

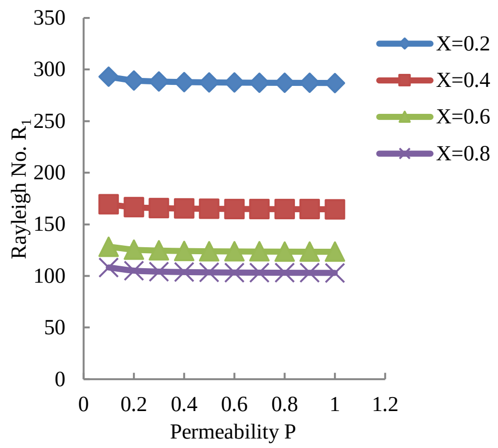

Fig. 4. Variation of Rayleigh number R_1 with medium permeability P for G=10, B=10, $T_{A_1}=10$, M_1 =10, $Q_1=50$, $\in=0.1$ for fixed wave numbers x = 0.2, x = 0.4, x=0.6 and x = 0.8.

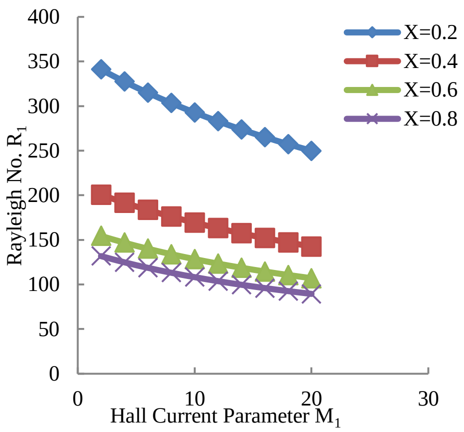

Fig. 5. Variation of Rayleigh number R_1 with Hall current parameter M_1 for G=10, B=10, $T_{A_1}=10$, $Q_1=50$, $P=0.1$, $\in=0.1$ for fixed wave numbers x = 0.2, x = 0.4, x=0.6 and x = 0.8.

Figure 4 shows that the medium permeability has a destabilizing effect for P=0.1 to P=0.2 and stabilizing effect for P=0.2 to P=1.2. Therefore, medium permeability has both stabilizing and destabilizing influences on the system.

5. Principle of Exchange of Stabilities and Oscillatory Modes

To determine the possibility of oscillatory modes, if any for the problem on hand, we multiply Eq. (26) by W^* (complex conjugate of W) and using Eqs. (27) to (30) together with the boundary condition (31), we obtain

$$\left[\frac{\sigma}{\in}\left(1+\frac{f}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right]I_1-\frac{g\alpha\kappa a^2}{\beta\upsilon}\left(\frac{G}{G-1}\right)\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)\left(I_2+E_1p_1\sigma^*I_3\right)+\frac{\mu_e\varepsilon\eta}{4\pi\rho_m\upsilon}\left(p_2\sigma^*I_7+I_8\right)+d^2\left[\left\{\frac{\sigma^*}{\in}\left(1+\frac{f}{1+\tau_1\sigma^*}\right)+\frac{1-F\sigma^*}{P_l}\right\}I_4+\frac{\mu_e\varepsilon\eta d^2}{4\pi\rho_m\upsilon}\left(p_2\sigma^*I_5+I_6\right)\right]=0 \quad (43)$$

where

$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz$,

$I_2=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz$, $I_3=\int_0^1\left(|\Theta|^2\right)dz$,

$I_4=\int_0^1\left(|Z|^2\right)dz$, $I_5=\int_0^1\left(|X|^2\right)dz$,

$I_6=\int_0^1\left(|DX|^2+a^2|X|^2\right)dz$,

$I_7=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz$,

$I_8=\int_0^1\left(\left|D^2K\right|^2+a^4|K|^2+2a^2|DK|^2\right)dz$.

In the above integrals σ^* denotes the complex conjugate of σ. The integrals I_1-I_8 are all positive definite.

Putting $\sigma=i\sigma_i$ in Eq. (43), where σ_i is real and equating the imaginary part of Eq. (43), we obtain

$$\sigma_i\left[\left\{\frac{1}{\in}\left(1+\frac{f}{1+\tau_1^2\sigma_i^2}\right)-\frac{F}{P_l}\right\}I_1+\frac{g\alpha\kappa a^2}{\beta\upsilon}\left(\frac{G}{G-1}\right)\left\{\frac{\tau_1(B-1)I_2+\left(B+\tau_1^2\sigma_i^2\right)E_1p_1I_3}{B+\tau_1^2\sigma_i^2}\right\}-\frac{\mu_e\varepsilon}{4\pi\rho_m}(I_7)+d^2\left[\left\{\frac{F}{P_l}-\frac{1}{\in}\left(1+\frac{f}{1+\tau_1^2\sigma_i^2}\right)\right\}I_4-\frac{\mu_e\varepsilon d^2}{4\pi\rho_m}(I_5)\right]\right]=0 \quad (44)$$

Equation (44) yields that $\sigma_i = 0$ or $\sigma_i \neq 0$ which mean that modes may be non oscillatory or oscillatory. The oscillatory modes introduced due to presence of rotation, magnetic field (or Hall currents) and visco-elasticity. In the absence of magnetic field, rotation and visco-elastic parameter, Eq. (44) gives

$$\sigma_i \left[\frac{1}{\in}\left(1+\frac{f}{1+\tau_1^2\sigma_i^2}\right)I_1 + \frac{g\alpha\kappa a^2}{\beta\upsilon}\left(\frac{G}{G-1}\right) \left\{ \frac{\tau_1(B-1)I_2 + \left(B+\tau_1^2\sigma_i^2\right)E_1 p_1 I_3}{B+\tau_1^2\sigma_i^2} \right\} \right] = 0 \tag{45}$$

It is evident from Eq. (45) that if $B > 1$ and $G > 1$ then the term inside the bracket is positive which implies that $\sigma_i = 0$, and only non-oscillatory modes are prevail which implies that the principle of exchange of stabilities will hold good for the problem under consideration.

Thus $B > 1$ and $G > 1$ are the necessary conditions for the validity of principle of exchange of stabilities for the problem of thermal convection in Walters' (model B') visco-elastic compressible fluid permeated with suspended particles in the presence of rotating, magnetic field and Hall currents in a porous medium.

6. CONCLUSION

In the present note, the problem of thermal convection in Walters' (model B') visco-elastic compressible fluid permeated with suspended particles in the presence of rotating, magnetic field and Hall current in a porous medium is studied using usual perturbation technique. It is found that for the case of stationary convection, Walters' (B') elastico-viscous fluid behaves like an ordinary Newtonian fluid. We have also investigated the effect of various parameters of physical importance like suspended particles, Hall current parameter, rotation, compressibility, magnetic field and medium permeability. The effect of suspended particles and Hall current parameter is to destabilize the system whereas magnetic field, rotation and compressibility are found to have stabilizing effect on the system. The medium permeability has both stabilizing and destabilizing effect on the system under certain conditions. The oscillatory modes prevail in the system due to the presence of presence of rotation, magnetic field (or Hall currents) and visco-elastic parameter. The necessary conditions for the validity of principle of exchange of stabilities in the absence of magnetic field (or Hall currents), rotation and visco-elastic parameter are also obtained.

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