Stagnation Point Flow of Thixotropic Fluid over a Stretching Sheet with Mass Transfer and Chemical Reaction

S. A. Shehzad[1,†], T. Hayat[2,3], S. Asghar[4] and A. Alsaedi[3]

[1]*Department of Mathematics, COMSATS Institute of Information Technology, Sahiwal 57000, Pakistan*
[2]*Department of Mathematics, Quaid-i-Azam University 45320, Islamabad 44000, Pakistan*
[3]*Department of Mathematics, Faculty of Science, King Abdulaziz University, P. O. Box 80257, Jeddah 21589, Saudi Arabia*
[4]*Department of Mathematics, COMSATS Institute of Information Technology, Islamabad 44000, Pakistan*

†*Corresponding Author Email: ali_qau70@yahoo.com*

ABSTRACT

The stagnation point flow of thixotropic fluid towards a linear stretching surface is investigated. Mass transfer with first order chemical reaction is considered. The resulting partial differential equations are reduced into the ordinary differential equations. Dimensionless velocity and concentration fields have been computed. Graphical plots are presented to illustrate the details of flow and mass transfer characteristics and their dependence upon the physical parameters. Numerical values of surface mass transfer are first computed and then analyzed.

NOMENCLATURE

a/c	stagnation point parameter	Sc	Schmidt number
C	concentration	Sh	Sherwood number
C_w	concentration at the wall	u_e	free stream velocity
C_∞	ambient fluid concentration	u_w	stretching velocity
D	mass diffusion coefficient	u,v	velocity components
f	dimensionless velocity		
j_w	mass flux	ν	kinematic viscosity
k^*	reaction rate	η	dimensionless variable
K_1, K_2	thixotropic parameters	φ	dimensionless concentration
R_1, R_2	material parameters	γ	chemical reaction parameter
Re	Reynolds number	ρ	fluid density

1. INTRODUCTION

A significant research effort has been made to explore the rheological characteristics of non-Newtonian fluids during the last few decades. In particular, such fluids are quite common in process of manufacturing coated sheets, foods, optical fibers, drilling muds, plastic polymers etc. A diverse body of conducted research has demonstrated that non-Newtonian fluids cannot be described by a single constitutive relationship. Thus a number of non-Newtonian fluid models have been proposed. In general, the modelled equations in the non-Newtonian fluids are complicated and higher order than the Navier-Stokes equations. The non-Newtonian fluids are mainly classified into three categories namely the differential, rate and integral. In existing literature, much attention has been devoted to the flows of second grade and Maxwell fluids. Although a wide range of theoretical studies have been performed for second grade and Maxwell fluids but few recent refs. in this regard may be

mentioned through the attempts by (Jamil and Fetecau (2010), Wang and Tan (2011), Jmail *et al.* (2011), Hayat *et al.* (2011), Ahmad and Asghar (2011), Rani and Reddy (2013) and Rashad *et al.* (2013)). In addition many materials including drilling muds, clays, certain oils, cosmetic products, colloids and suspension etc. become less viscous when stirred. Materials with such behavior are called thixotropic fluids. The difference between thixotropic and shear thinning fluid is that a shear thinning fluid shows a decrease in viscosity with increasing shear rate while thixotropic fluid displays a decrease in viscosity over time at constant shear rate. The details of the characteristics and restrictions on the sign and magnitude of thixotropic fluid model can be seen in the study Sadeqi *et al.* (2011). Shehzad *et al.* (2013) studied the boundary layer flow of thixotropic fluid under thermal stratified, thermal radiation and mixed convection. Hayat *et al.* (2013) invsetigated the hydromagnetic radiative flow of thixotropic fluid with variable thermal conductivity. Newtonian heating effect in boundary layer flow of thixotropic fluid has been explored by Awais *et al.* (2013).

The stagnation point flow over a stretching sheet has great concern in extrusion process, paper production, drawing of plastic sheets, continuous casting etc. Considerable progress has been made regarding the stretching and stagnation point flows in the past. Chiam (1994) studied the two-dimensional stagnation-point flow of viscous fluid towards a linear stretching surface. In this study, the stretching velocity is taken equal to the free stream velocity and consequently no boundary layer is observed. Mahapatra *et al.* (2009) examined the stagnation point flow when stretching and free stream velocities are different. They found that boundary layers exist in this situation. The effect of thermal radiation on magnetohydrodynamic stagnation point flow in a porous space with mixed convection has been investigated by Hayat *et al.* (2010). Slip and heat transfer effects on boundary layer stagnation point flow of viscous fluid towards a shrinking surface are studied by Bhattacharyya (2011). Boundary layer stagnation point flow of viscous fluid and heat transfer with thermal radiation has been examined by Bhattacharyya and Layek (2011). Hayat *et al.* (2011) investigated the stagnation point flow of Maxwell fluid over a stretched sheet with mass transfer. Melting heat transfer in the stagnation point flow of Powell-Eyring fluid was studied by Hayat *et al.* (2013). Shateyi and Makinde (2013) considered the stagnation point flow over a convectively heated disk. Hayat *et al.* (2013) investigated the stagnation point flow of Maxwell fluid with convective boundary condition and thermal radiation. MHD axisymmetric stagnation point flow over a shrinking sheet has been explored by Mahapatra and Nandy (2013). Singh and Sharma (2014) discussed the heat and mass transfer effects in boundary layer flow near a stagnation point.

Many practical diffusive operations involve the molecular diffusion of species in the presence of chemical reaction within or at the boundary. There

are two types of chemical reaction namely homogenous and heterogeneous. Homogenous reaction exists uniformly throughout a given phase while the heterogeneous reaction occurs in a restricted region (or within the boundary of a phase). The flow analysis with heat and mass transfer in presence of chemical reaction is important in chemical and hydrometallurgical industries. For instance, smog formation represents a first order homogenous chemical reaction. Having these facts in mind, the object of current study is to investigate the stagnation point flow of thixotropic fluid towards a stretching surface with chemical reaction. First order chemical reaction is considered. In all the above reported studies of the thixotropic fluid deal in the absence of stagnation point flow, mass transfer and chemical reaction. The present study deals with the stagnation point flow of thixotropic fluid in presence of mass transfer and chemical reaction. The relevant nonlinear problem is solved by homotopy analysis method (HAM) by Liao (2003), Yao (2009), Vosughi *et al.* (2011), Rashidi *et al.* (2011), Turkyilmazoglu (2012), Shehzad *et al.* (2013), Malvandi *et al.* (2014) and Hayat *et al.* (2014). Convergence of the obtained solutions is checked. Influence of pertinent parameters on the flow quantities is addressed.

2. DEFINITION OF FLOW PROBLEM

We consider the stagnation point flow of an incompressible thixotropic fluid towards a stretching sheet with chemical reactive species A first order chemical reaction is considered. The sheet is stretched with a velocity $u_w(x) = cx$ (where c is a constant). We choose the Cartesian coordinate system in such a way that $x-$axis is along the stretching surface and $y-$axis perpendicular to it. The equations governing the boundary layer flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + v\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{6R_1}{\rho}\left(\frac{\partial u}{\partial y}\right)^2\left(\frac{\partial^2 u}{\partial y^2}\right)$$

$$+\frac{4R_2}{\rho}\left(\begin{array}{c}\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial^2 u}{\partial y^2}\right)\left(u\frac{\partial^2 u}{\partial x\partial y} + v\frac{\partial^2 u}{\partial y^2}\right)\\ +\left(\frac{\partial u}{\partial y}\right)^2\left(\begin{array}{c}u\frac{\partial^3 u}{\partial x\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\\ +\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y} + \frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y^2}\end{array}\right)\end{array}\right), \tag{2}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k^*(C - C_\infty), \tag{3}$$

Where u is $x-$component of the velocity, v is $y-$component of the velocity, R_1 and R_2 are the material constants, v is the kinematic viscosity of fluid, ρ is the density of fluid, C is the concentration field, D is the mass diffusion and k^* is the reaction rate.

In addition to the above three equations, the boundary conditions for this problem are

$$u = u_w = cx, \ v = 0, \ C = C_w \ \text{at} \ y = 0,$$
$$u = u_e = ax, \ C \to C_\infty \ \text{as} \ y \to \infty, \tag{4}$$

where C is the stretching rate.

To seek the solution, the following transformations are defined:

$$u = cxf'(\eta), \ v = -\sqrt{cv}f(\eta), \ \eta = y\sqrt{\frac{c}{v}}. \tag{5}$$

Now incompressibility condition is automatically satisfied and the problem is reduced as follows:

$$f''' + ff'' - f'^2 + K_1(x)f''^2 f''' + K_2(x) \tag{6}$$

$$(ff''^2 f''' + f''^4 - ff'f'''^2 - ff''^2 f^{iv}) + a^2/c^2 = 0,$$

$$\varphi'' + Scf\varphi' - Sc\gamma\varphi = 0, \tag{7}$$

$$f = 0, \ f' = 1, \ \varphi = 1 \ \text{at} \ \eta = 0,$$
$$f' = a/c, \ \varphi \to 0 \ \text{as} \ \eta \to \infty \tag{8}$$

in which $K_1(x) = -\dfrac{6R_t c^3 x^2}{\rho v^2}$ and $K_2(x) = \dfrac{4R_2 c^4 x^2}{\rho v^2}$ are the dimensionless non-Newtonian parameters of the thixotropic fluid, $Sc = v/D$ is the Schmidt number and $\gamma = k^*/c$ is the dimensionless chemical reaction parameter.

The local Sherwood number is

$$Sh = \frac{xj_w}{D(C_w - C_\infty)} \tag{9}$$

in which the mass flux j_w is given by

$$j_w = -D\left(\frac{\partial C}{\partial y}\right)_{y=0}.$$

Dimensionless form of Eq. (9) is

$$Sh/\text{Re}_x^{1/2} = -\varphi'(0). \tag{10}$$

3. HOMOTROPY ANALYSIS SOLUTIONS

We express f and θ by a set of base functions

$$\{\eta^k \exp(-n\eta), \ k \geq 0, n \geq 0\} \tag{11}$$

as follows

$$f_m(\eta) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a_{m,n}^k \eta^k \exp(-n\eta), \tag{12}$$

$$\varphi_m(\eta) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} b_{m,n}^k \eta^k \exp(-n\eta). \tag{13}$$

In above equations $a_{m,n}^k$ and $b_{m,n}^k$ are the coefficients. Further the initial approximations and

auxiliary linear operators are selected as

$$f_0(\eta) = (a/c)\eta + (1 - a/c)[1 - \exp(-\eta)], \tag{14}$$
$$\varphi_0(\eta) = \exp(-\eta),$$
$$L_f = f''' - f', \ L_\varphi = \varphi'' - \varphi \tag{15}$$

With

$$L_f(C_1 + C_2 e^\eta + C_3 e^{-\eta}) = 0, \tag{16}$$
$$L_\varphi(C_4 e^\eta + C_5 e^{-\eta}) = 0,$$

Where C_i $(i = 1 - 5)$ represent the arbitrary constants. The problems at zeroth order deformation are

$$(1-q)L_f\left[\hat{f}(\eta;q) - f_0(\eta)\right] = qh_f\mathbf{N}_f\left[\hat{f}(\eta;q)\right], \tag{17}$$

$$(1-q)L_\varphi\left[\hat{\varphi}(\eta;q) - \varphi_0(\eta)\right]$$
$$= qh_\varphi\mathbf{N}_\varphi\left[\hat{\varphi}(\eta;q), \hat{f}(\eta;q)\right], \tag{18}$$

$$\hat{f}(0;q) = 0, \hat{f}'(0;q) = 1, \hat{f}'(\infty;q) = a/c,$$
$$\hat{\varphi}(0;q) = 1 \ \text{and} \ \hat{\varphi}(\infty;q) = 0, \tag{19}$$

In which q is an embedding parameter and h_f and h_φ are the non-zero auxiliary parameters. The nonlinear operators $\mathbf{N}_f$ and $\mathbf{N}_\varphi$ are

$$\mathbf{N}_f[\hat{f}(\eta,q)] = \frac{\partial^3\hat{f}(\eta,q)}{\partial\eta^3} + \hat{f}(\eta,q)\frac{\partial^2\hat{f}(\eta,q)}{\partial\eta^2}$$

$$-\left(\frac{\partial\hat{f}(\eta,q)}{\partial\eta}\right)^2 + K_1(x)\left(\frac{\partial^2\hat{f}(\eta,q)}{\partial\eta^2}\right)^2$$

$$\frac{\partial^3\hat{f}(\eta,q)}{\partial\eta^3} + a^2/c^2$$

$$+K_2(x)\begin{bmatrix} \frac{\partial\hat{f}(\eta,q)}{\partial\eta}\left(\frac{\partial^2\hat{f}(\eta,q)}{\partial\eta^2}\right)^2\frac{\partial^3\hat{f}(\eta,q)}{\partial\eta^3} + \\ \left(\frac{\partial^2\hat{f}(\eta,q)}{\partial\eta^2}\right)^4 - \hat{f}(\eta,q) \\ \frac{\partial^2\hat{f}(\eta,q)}{\partial\eta^2}\left(\frac{\partial^3\hat{f}(\eta,q)}{\partial\eta^3}\right)^2 \\ -\hat{f}(\eta,q)\left(\frac{\partial^2\hat{f}(\eta,q)}{\partial\eta^2}\right)^2\frac{\partial^4\hat{f}(\eta,q)}{\partial\eta^4} \end{bmatrix}, \tag{20}$$

$$\mathbf{N}_\varphi\left[\hat{\varphi}(\eta;q), \hat{f}(\eta;q)\right] = \frac{\partial^2\hat{\varphi}(\eta;q)}{\partial\eta^2}$$
$$+ Sc\hat{f}(\eta;q)\frac{\partial\hat{\varphi}(\eta;q)}{\partial\eta} - Sc\gamma\hat{\varphi}(\eta;q). \tag{21}$$

Taking $q = 0$ and $q = 1$, one can write

$$\hat{f}(\eta;0) = f_0(\eta) \ \text{and} \ \hat{f}(\eta;1) = f(\eta), \tag{22}$$

$$\hat{\varphi}(\eta;0) = \varphi_0(\eta) \ \text{and} \ \hat{\varphi}(\eta;1) = \varphi(\eta), \tag{23}$$

and when q increases from 0 to 0 then $f(\eta,q)$,

$\varphi(\eta,q)$ vary from $f_0(\eta)$ to $f(\eta)$ and $\varphi_0(\eta)$ to $\varphi(\eta)$. Using Taylor's series we have

$$f(\eta,q) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)q^m,$$

$$f_m(\eta) = \frac{1}{m!}\frac{\partial^m f(\eta;q)}{\partial\eta^m}\bigg|_{q=0}, \tag{24}$$

$$\varphi(\eta,q) = \varphi_0(\eta) + \sum_{m=1}^{\infty} \varphi_m(\eta)q^m,$$

$$\varphi_m(\eta) = \frac{1}{m!}\frac{\partial^m \varphi(\eta;q)}{\partial\eta^m}\bigg|_{q=0}. \tag{25}$$

The convergence of series (24) and (25) strongly depends upon h_f and h_φ. We select h_f and h_φ in such a way that the series (24) and (25) converge at $q=1$. Hence we can write

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \tag{26}$$

$$\varphi(\eta) = \varphi_0(\eta) + \sum_{m=1}^{\infty} \varphi_m(\eta), \tag{27}$$

Where the special solutions f_m and φ_m are given below

$$f_m(\eta) = f_m^*(\eta) + C_1 + C_2 e^\eta + C_3 e^{-\eta}, \tag{28}$$

$$\varphi_m(\eta) = \varphi_m^*(\eta) + C_4 e^\eta + C_5 e^{-\eta}. \tag{29}$$

4. CONVERGENCE ANALYSIS AND DISCUSSION

The auxiliary parameter h_f and h_φ have the key role in controlling and adjusting the convergence of the series solutions. For permissible values of h_f and h_φ, the $h-$ curves are plotted at 14th -order of approximations.

Figure 1 demonstrates that the admissible values of h_f and h_φ are $-0.75 \le h_f \le -0.2$ and $-1.0 \le h_\varphi \le -0.3$. Further, the presented series solutions converge in the whole region of η when $h_f = -0.6$ and $h_\varphi = -0.7$. It is further found from Table 1 that 12th and 16th order deformations are sufficient for f and φ, respectively.

In Figs. 2 to 8, the influence of emerging parameters on the velocity and concentration fields is studied. In particular, the Figs. 2 to 5 represent the variations of thixotropic parameters K_1, K_2, stagnation point parameter a/c, Schmidt number Sc and chemical reaction parameter γ. Figures 2 to 4 illustrate the effects of thixotropic parameters K_1, K_2 and stagnation point parameter a/c on the velocity profile f'. From Figs. 2 and 3 it can be seen that velocity field and boundary layer thickness are increasing functions of K_1 and K_2.

Fig. 3 depicts that an increase in the velocity field is more pronounced in case of K_2. when compared with K_1 Fig. 4 elucidates that the velocity profile f'. is an increasing function of stagnation point parameter a/c Here the velocity field is increased but the momentum boundary layer thickness is reduced. The effects of a/c Sc and γ on the concentration profile are examined in the Figs. 5 to 8. Figure 5 provides the variation of a/c on φ in destructive $(\gamma > 0)$ chemical reaction. Increase in value of a/c decreases φ The variation of Schmidt number Sc on φ is observed in Fig. 6 It is noticed that the concentration field φ decreases when Sc increases. Here the Schmidt number is dependent on the mass diffusion coefficient. Larger Schmidt number corresponds to weaker mass diffusion coefficient. Such weaker diffusion coefficient is responsible for the redaction in concentration field. As expected the fluid concentration increases due to an increase in generative chemical reaction parameter $(\gamma < 0)$ (see Fig. 8).The fluid concentration φ has the opposite behavior for destructive chemical reaction parameter $(\gamma > 0)$ when compared with that of generative chemical reaction (Fig. 7)
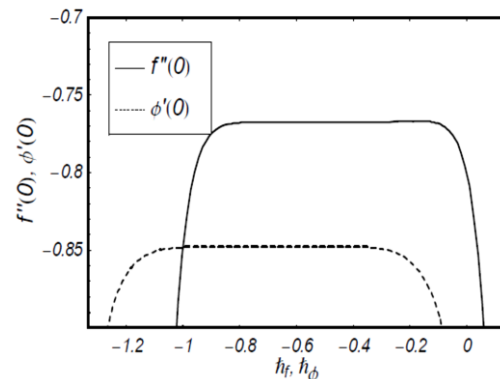

Fig. 1. $h-$ curves for the functions f and φ when $K_1 = 0.4$, $K_2 = 0.5$, $a/c = 0.2$, $Sc = 0.8$ and $\gamma = 0.5$.
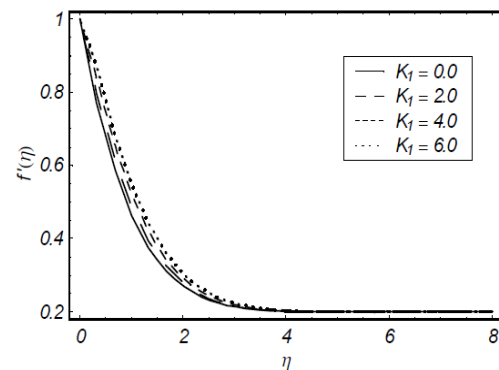

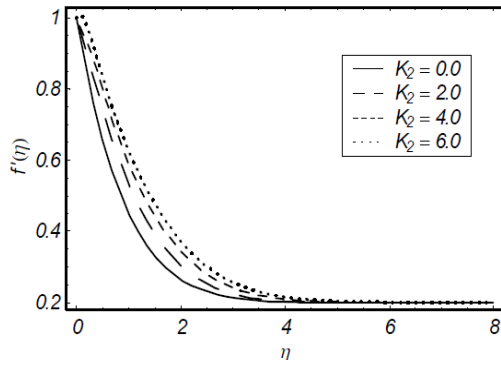**Fig. 2. Influence of K_1 on velocity profile $f'(\eta)$ when $K_2 = 0.5$, and $a/c = 0.2$,**

Fig. 3. Influence of K_2 on velocity profile $f'(\eta)$ when $K_1 = 0.4$ and $a/c = 0.2$,
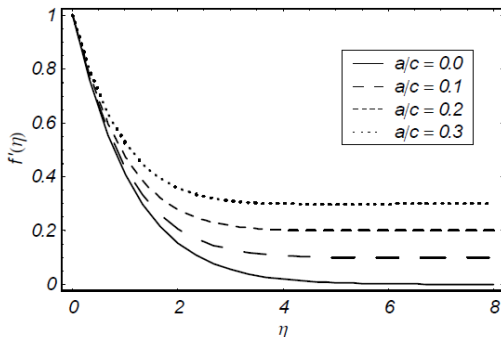

Fig. 4. Influence of stagnation point parameter a/c on velocity profile $f'(\eta)$ when $K_1 = 0.4$ and $K_2 = 0.5$,
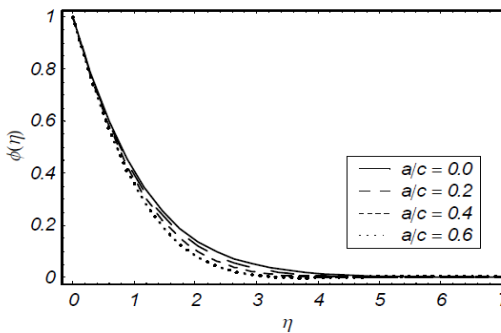

Fig. 5. Influence of stagnation point parameter a/c on concentration profile $\varphi(\eta)$ when $K_1 = 0.4$ $K_2 = 0.5$, $Sc = 0.8$ and $\gamma = 0.4$.
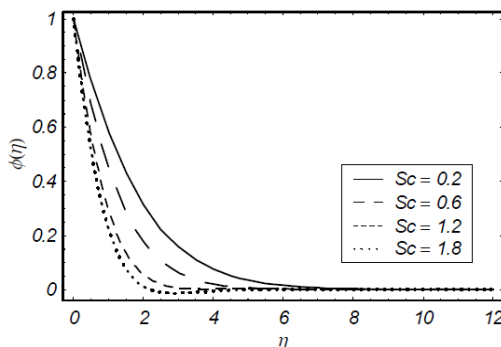

Fig. 6. Influence of Schmidt number Sc on concentration profile $\varphi(\eta)$ when $K_1 = 0.4$ $K_2 = 0.5$, $a/c = 0.8$ and $\gamma = 0.4$.

Fig. 7. Influence of reaction parameter $\gamma > 0$ on concentration profile $\varphi(\eta)$ when $K_1 = 0.4$ $K_2 = 0.5$, $a/c = 0.2$, and $Sc = 0.8$
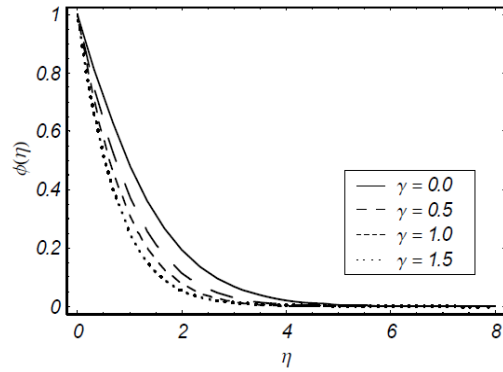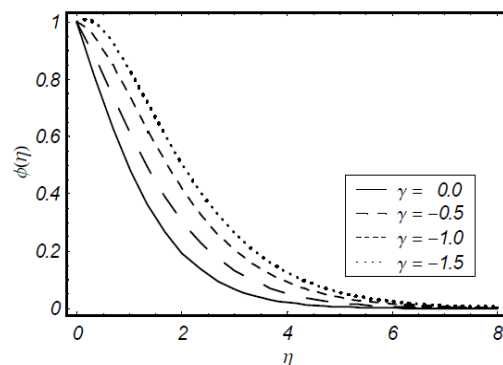

Fig. 8. Influence of reaction parameter $\gamma < 0$ on concentration profile $\varphi(\eta)$ when $K_1 = 0.4$ $K_2 = 0.5$, $a/c = 0.2$, and $Sc = 0.8$

The values of surface mass transfer rate $-\varphi'(0)$ are presented in the Tables 2 and 3. The surface mass transfer rate $-\varphi'(0)$ increases by increasing K_1 and K_2. However it decreases for large values of a/c. The surface mass transfer rate $-\varphi'(0)$ increases by increasing both Schmidt number and chemical reaction parameter.

Table 1 Convergence of homotopy solutions for different order of approximations when $K_1 = 0.4$ $K_2 = 0.5$, $a/c = 0.2$, $Sc = 0.8$ $\gamma = 0.5$, $h_f = -0.6$ and $h_\varphi = -0.7$.

Order of approximations	$-f''(0)$	$-\varphi'(0)$
1	0.759317	0.877333
5	0.767386	0.847801
12	0.767444	0.847690
16	0.767444	0.847691
25	0.767444	0.847691
35	0.767444	0.847691

Table 2 Numerical values of local Sherwood number $-\varphi'(0)$ for different values of K_1, K_2, Sc and γ when $a/c = 0.2$,

K_1	K_2	Sc	γ	$-\varphi'(0)$
0.0	0.0	0.8	0.5	0.839923
0.5				0.842755
1.0				0.844963
2.0				0.848376
0.0	0.3			0.844042
	0.8			0.849083
	1.0			0.850737
0.4	0.5	0.6		0.525735
		1.0		0.955795
		1.5		1.17993
			0.3	0.745382
			0.8	0.981174
			1.2	1.13462

Table 3. Numerical values of local Sherwood number for different values of Sc, γ and a/c when $K_1 = K_2 = 0$.

Sc	γ	a/c	$-\varphi'(0)$
0.5	0.5	0.2	0.651519
1.0			0.947175
1.5			1.17733
0.8	0.0		0.544838
	0.6		0.887229
	1.7		1.29661
		0.0	0.824839
		0.3	0.850034
		0.5	0.872783

REFERENCES

Ahmad, A. and S. Asghar, (2011). Flow of a second grade fluid over a sheet stretching with arbitrary velocities subject to a transverse magnetic field, *Appl. Math. Letters,* 24, 1905-1909.

Awais, M., T. Hayat, A. Qayyum, and A. Alsaedi (2013). Newtonian heating in a flow of thixotropic fluid, *Europ. Phys. J. Plus*, 128, 114.

Bhattacharyya, K. (2011). Dual solutions in unsteady stagnation-point flow over a shrinking sheet, *Chin. Phys. Lett., 28,* 084702.

Bhattacharyya, K. and G. C. Layek (2011). Effects of suction/blowing on steady boundary layer stagnation-point flow and heat transfer towards a shrinking sheet with thermal radiation, *Int. J. Heat Mass Transfer,* 54, 302-307.

Chiam, T. C. (1994). Stagnation point flow towards a stretching plate, *J. Phys. Soc. Jpn.,* 63, 2443-2444.

Hayat, T., Z. Abbas, I. Pop, and S. Asghar (2010). Effects of radiation and magnetic field on the mixed convection stagnation-point flow over a vertical stretching sheet in a porous medium, *Int. J. Heat Mass Transfer,* 53, 466-474.

Hayat, T., M. Awais, M. Qasim, and A. A. Hendi (2011). Effects of mass transfer on the stagnation point flow of an upper-convected Maxwell (UCM) fluid, *Int. J. Heat Mass Transfer,* 54, 3777-3782.

Hayat, T., M. Farooq, A. Alsaedi, and Z. Iqbal, (2013). Melting heat transfer in the stagnation point flow of Powell-Eyring fluid, *J. Thermophys. Heat Transfer,* 27, 761-766.

Hayat, T., S. A. Shehzad, M. Qasim, and S. Obaidat (2011). Steady flow of Maxwell fluid with convective boundary conditions, *Z. Naturforsch.* 66a, 417-422.

Hayat, T., S.A. Shehzad, and S. Asghar (2013). MHD flow of thixotropic fluid with variable thermal conductivity and thermal radiation, *Walailak J. Sci. Tech.,* 10, 29-42.

Hayat, T., S. A. Shehzad, S. Al-Mezel, and A. Alsaedi (2014). Three-dimensional flow of an Oldroyd-B fluid over a bidirectional stretching surface with prescribed surface temperature and prescribed surface heat flux, *J. Hydrol. Hydromech.* 62, 117-125.

Hayat, T., M. Waqas, S. A. Shehzad, and A. Alsaedi (2013). Mixed convection radiative flow of Maxwell fluid near a stagnation point with convective condition. *J. Mech.*, 29, 403-409.

Jamil, M. and C. Fetecau, (2010). Helical flows of Maxwell fluid between coaxial cylinders with given shear stresses on the boundary. *Nonlinear Analysis: Real World Appl.,* 11, 4302-4311.

Jamil, M., A. Rauf, C. Fetecau, and N. A. Khan, (2011). Helical flows of second grade fluid due to constantly accelerated shear stresses. *Commun. Nonlinear Sci. Numer. Simulat.*, 16, 1959-1969.

Liao, S. J. (2003). *Beyond perturbation: Introduction to homotopy analysis method.* Chapman and Hall, CRC Press, Boca Raton.

Mahapatra, T. R., S. K. Nandy, and A. S. Gupta (2009). Magnetohydrodynamic stagnation-point flow of a power-law fluid towards a stretching surface. *Int. J. Non-Linear Mech.,* 44, 123-128.

Mahapatra, T. R. and S. K. Nandy (2013)

Momentum and heat transfer in MHD axisymmetric stagnation-point flow over a shrinking sheet. *J. Appl. Fluid Mech.* 6, 121-129.

Malvandi, A., F. Hedayati, and M. R. H. Nobari (2014). An HAM analysis of stagnation-point flow of a nanofluid over a porous stretching sheet with heat generation. *J. Appl. Fluid Mech.*, 7 135-145.

Rani, H. P. and G. J. Reddy (2013). Soret and Dufour effects on transient double diffusive free convection of couple-stress fluid past a vertical cylinder, *J. Appl. Fluid Mech.*, 6, 545-554.

Rashad, A. M., A. J. Chamkha, and M. M. M. Abdou (2013). Mixed convection flow of non-Newtonian fluid from vertical surface saturated in a porous medium filled with a nanofluid. *J. Appl. Fluid Mech.,* 6, 301-309.

Rashidi, M. M., S. A. M. Pour, and S. Abbasbandy (2011). Analytic approximate solutions for heat transfer of a micropolar fluid through a porous medium with radiation. *Commun. Nonlinear Sci. Numer. Simulat.,* 16, 1874-1889.

Sadeqi, S., N. Khabazi, and K. Sadeghy (2011). Blasius flow of thixotropic fluids: A numerical study. *Commun. Nonlinear Sci. Numer. Simulat.,* 16, 711-721.

Shateyi, S. and O. D. Makinde (2013). Hydromagnetic stagnation-point flow towards a radially stretching convectively heated disk, *Math. Problems Eng.*, 2013, 616947.

Shehzad, S. A., A. Alsaedi, and T. Hayat (2013). Hydromagnetic steady flow of Maxwell fluid over a bidirectional stretching surface with prescribed surface temperature and prescribed surface heat flux, *Plos One*, 8, 68139.

Shehzad, S. A., M. Qasim, A. Alsaedi, T. Hayat, and M. S. Alhuthali (2013) Combined effects of thermal stratification and thermal radiation in mixed convection flow of thixotropic fluid. *Europ. Phys. J. Plus,* 128, 7.

Singh, G. and P. R. Sharma, (2014). Heat and mass transfer in the boundary layer flow along a vertical isothermal reactive plate near stagnation point: Existence of dual solution. *J. Appl. Fluid Mech.*, 7, 25-33.

Turkyilmazoglu, M. (2012). Solution of the Thomas-Fermi equation with a convergent approach, *Commun. Nonlinear Sci. Numer. Simulat.* 17, 4097-4103.

Vosughi, H., E. Shivanian, and S. Abbasbandy (2011).A new analytical technique to solve Volterra's integral equations. *Math. Methods Appl. Sci.,* 34, 1243-1253.

Wang, S. and W. C. Tan (2011). Stability analysis of soret-driven double-diffusive convection of Maxwell fluid in a porous medium. *Int. J. Heat Fluid Flow,* 32, 88-94.

Yao, B. (2009). Approximate analytical solution to the Falkner-Skan wedge flow with the permeable wall of uniform suction. *Commun. Nonlinear Sci. Numer. Simulat.,* 14, 3320-3326.