Comparison between Two Local Thermal Non Equilibrium Criteria in Forced Convection through a Porous Channel

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ABSTRACT

Two criteria are used and compared to investigate the local thermal equilibrium assumption in a forced convection through a porous channel. The first criterion is based on the maximum local temperature difference between the solid and fluid phases, while the second is based on the average of the local differences between the temperature of the solid phase and the fluid phase. For this purpose, the momentum and energy equations based on the Darcy-Brinkman-Forchheimer and the local thermal non equilibrium models are solved numerically using the finite volume method. The analysis focused on searching thermophysical parameters ranges which validate local thermal equilibrium hypothesis. Thus, by using the two criteria, the obtained results mainly revealed that this local thermal equilibrium assumption is verified for low thermal conductivity ratio and Reynolds number values and for high interstitial Biot number and porosity, while it is unfavorably affected by the high values of Prandtl number. However, it is also found that the parameters ranges corresponding to the local equilibrium validity depends on the selected local thermal non equilibrium criterion.

Keywords: Forced convection; Local thermal Non Equilibrium; Porous medium.

NOMENCLATURE

a_{sf}	specific surface area (m^2/m^3)		x	longitudinal axis direction (m)
C_f	inertial Forchheimer coefficient		y	transverse axis direction (m)
h_{sf}	interstitial heat transfer coefficient (W/m^2K)			
k	thermal conductivity (W/mK)		ρ	fluid density (kg/m^3)
K	permeability (m^2)		μ	dynamic viscosity (kg/ms)
L	dimensionless channel length		ν	fluid kinematics viscosity (m^2/s)
p	pressure (Pa)		θ	dimensionless temperature [-]
P	dimensionless pressure [-]		ε	porosity [-]
T	intrinsic average fluid or solid temperature (K)			

1. INTRODUCTION

Lot of research is oriented, these last decades, towards the study of heat transfer mechanisms in porous media. Most of these investigations assume local thermal equilibrium between the fluid and the solid phases at any location in the porous medium. One can cite Jana *et. al* (2014) who have analytically studied the oscillating mixed convection on the fully developed flow between two infinitely long vertical walls heated asymmetrically in a porous medium. Mixed convection heat transfer in metallic porous block with a confined slot jet was numerically studied by Marafie *et al.* (2008). They showed that there is a substantial increase in heat removal capability using porous block more particularly for high solid-to-fluid thermal conductivity ratio of the porous medium. Patil *et al.* (2013) have numerically analyzed the radiation effect on heat and mass transfer natural convection when the temperatures boundary conditions at the cold

wall of the porous annulus are varied. They particularly found that the average Nusselt and Sherwood numbers increase with the increase in Rayleigh number and the radiation parameter values at both the hot and cold walls. Ameziani *et al*. (2008) have numerically investigated the problem of unsteady natural convection which occurs in a vertical silo of granular storage, opened at both ends and filled with a saturated porous medium under local thermal equilibrium condition. They especially found that there are two types of flows, the mainly one and a counter-flow one.   An interesting numerical study of mass transfer and heat generation effects on MHD free convection flow past an inclined vertical plate in a porous medium was performed by Reddy and Reddy (2011). The results showed that the increase of the heat generation and of the thermal and solutal Grashof numbers lead to velocity and temperature augmentation in the boundary layer.

The local thermal equilibrium assumption is inappropriate for a number of cases such as heat energy storage systems and nuclear reactor modeling where the temperature difference between the fluid and the solid phase becomes crucial; therefore the local thermal non equilibrium model (LTNE) is increasingly adopted. Further of these studies are already classified, namely Badruddin *et al.* (2006), Chen *et al.* (2013), Mahmoudi and Marefat (2011), Ouyang *et al.* (2013) and Pippal and Bera (2013).. This last study takes into consideration the existence of a local thermal non-equilibrium between the fluid phase and the solid phase of the porous medium. This temperature difference, which may be significant in some applications, can lead to a poor estimation of heat exchanges (over or under-estimation). Therefore, several studies have been conducted for this purpose to establish criteria that will clearly define the conditions under which the assumption of local thermal equilibrium is valid or invalid.

Thus, Wong and Saeid (2009) investigated numerically the thermal characteristics of the jet impingement cooling with buoyancy effect in porous media in the mixed convection regime under local thermal non-equilibrium conditions. They showed that the increase in the porosity scaled thermal conductivity ratio parameter or heat transfer coefficient parameter brings the solid and fluid towards thermal equilibrium model and the mixed convection behavior is more significant for higher Rayleigh number. Al-Sumaily *et al.* (2013) have numerically studied the validation of the thermal equilibrium assumption in forced convection steady and pulsating flows over a cylinder embedded in a porous channel. They concluded that the higher Reynolds and Prandtl numbers have the effect of limiting the LTE condition. However, high Biot number, thermal conductivities ratio and porosity have a positive impact to satisfy the LTE assumption. Khashan and AL-Nimr (2005) have performed a numerical study to examine the validity of the local thermal equilibrium LTE assumption for forced

convective heat transfer of non-Newtonian fluids in a channel confined by two horizontal wall planes. The results obtained over broad ranges of representative dimensionless parameters are used to map conditions at which the local thermal equilibrium assumption can or cannot be employed. Jaballah *et al*. (2012) have studied numerically the mixed convection in a channel with porous layers using the thermal non equilibrium model. They showed clearly the limits of the heat transfer coefficient and thermal conductivity ratio parameters over which the two models LTE and LTNE of the two phases fluid and solid will be identical. Finally, a numerical study of natural convection heat transfer in a porous cavity heated from below using a thermal non-equilibrium model was proposed by Khashan *et al*. (2006) who examined the significance of the local thermal non-equilibrium model over a broad range of dimensionless groups such as Rayleigh number, Darcy number, effective fluid to solid thermal conductivity ratio and the modified Biot number.

The main objective of this study is to examine the effect of the dimensionless control parameters such as the interstitial Biot number, the thermal conductivities ratio, the Reynolds number, the Prandtl number and the porosity, on the local thermal equilibrium assumption. A comparison will also be made between two most appropriate LTNE criteria, one based on the maximum value of the temperature difference between the solid and fluid phases (Wong and Saeid 2009; AL-Sumairy *et al*. 2013) and the other on the average value of the temperature difference between the two phases across the entire channel (Khashan and Al-Nimr 2005; Jaballah *et al*. 2012; Khashan *et al*. 2006), in order to check the degree of dependence of each criterion on the LTE assumption.

2. MATHEMATICAL FORMULATION

The system under consideration is a two dimensional parallel plate channel filled with an isotropic and homogenous porous medium saturated with a single phase fluid. At the inlet the velocity (U_{in}) and the temperature (T_{in}) are uniform and constant. The walls are maintained at a constant temperature (T_w) higher than the inlet fluid temperature as shown in Fig. 1. The flow is considered to be two-dimensional, laminar, incompressible and in steady state with neglecting viscous dissipation. The thermo-physical properties of the fluid are assumed to be constant.

The dynamic field is modeled using the Brinkman-Forchheimer extended Darcy model and the thermal field is described by the two temperatures equations model, which takes into account the local thermal non equilibrium between the two phases, Nield and Bejan (2006). Therefore, the steady state dimensionless governing equations are presented as:
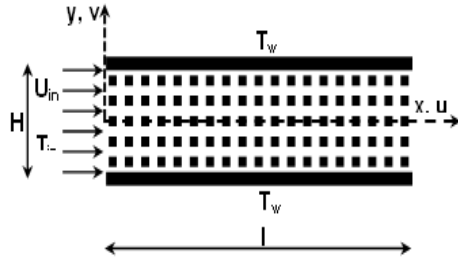
Fig. 1. Schematic representation of physical domain

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad (1)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\varepsilon^2 \frac{\partial P}{\partial X} + \frac{\varepsilon}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) - \frac{\varepsilon^2}{ReDa}U - \frac{\varepsilon^2 C_F}{\sqrt{Da}}\sqrt{U^2 + V^2}U \qquad (2)$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\varepsilon^2 \frac{\partial P}{\partial Y} + \frac{\varepsilon}{Re}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) - \frac{\varepsilon^2}{ReDa}V - \frac{\varepsilon^2 C_F}{\sqrt{Da}}\sqrt{U^2 + V^2}V \qquad (3)$$

$$U\frac{\partial \theta_f}{\partial X} + V\frac{\partial \theta_f}{\partial Y} = \frac{\varepsilon}{RePr}\left(\frac{\partial^2 \theta_f}{\partial X^2} + \frac{\partial^2 \theta_f}{\partial Y^2}\right) + \frac{Bi}{RePr}\left(\theta_s - \theta_f\right) \qquad (4)$$

$$\frac{\partial^2 \theta_s}{\partial X^2} + \frac{\partial^2 \theta_s}{\partial Y^2} + \frac{Bi}{(1-\varepsilon)R_k}\left(\theta_f - \theta_s\right) = 0 \qquad (5)$$

The governing parameters appearing in (2)-(5) are defined as:

$$Re = \frac{U_{in}H}{\nu}; Da = \frac{K}{H^2}; Pr = \frac{\nu}{\alpha};$$

$$Bi = \frac{h_{sf}a_{sf}H^2}{k_f}; R_k = \frac{k_s}{k_f}.$$

The above dimensionless parameters are obtained by introducing the following dimensionless variables:

$$X = \frac{x}{H}; Y = \frac{y}{H}; U = \frac{u}{U_{in}}; V = \frac{v}{U_{in}};$$

$$P = \frac{p}{\rho U_{in}^2}; \theta = \frac{T - T_{in}}{T_w - T_{in}}$$

In carrying out this investigation, no slip velocity condition is imposed at the solid walls. The fluid and solid phases are assumed to be in local thermal equilibrium at the inlet and at the channel walls. What is more, the Neumann boundary condition is used here to solve the equations for the upper half of the channel only in order to reduce the computational execution time and storage requirements. Accordingly, the dimensionless boundary conditions can be mathematically expressed as follows:

$$U(0,Y) = 1, \ V(0,Y) = \theta_s(0,Y) = \theta_f(0,Y) = 0 \quad (6)$$

$$U(X,0.5) = V(X,0.5) = 0, \ \theta_s(X,0.5) = \theta_f(X,0.5) = 1 \qquad (7)$$

$$\frac{\partial U(X,0)}{\partial Y} = \frac{\partial V(X,0)}{\partial Y} = \frac{\partial \theta_f(X,0)}{\partial Y} = \frac{\partial \theta_s(X,0)}{\partial Y} = 0 \qquad (8)$$

$$\frac{\partial U(L,Y)}{\partial X} = \frac{\partial V(L,Y)}{\partial X} = \frac{\partial \theta_f(L,Y)}{\partial X} = \frac{\partial \theta_s(L,Y)}{\partial X} = 0 \qquad (9)$$

The objective of this work, as previously mentioned, is to make a comparative study between two most common LTNE criteria used in the literature, which are mathematically expressed as:

Model A:

$$LTNE = \frac{\sum_N |\theta_s - \theta_f|}{N} \qquad (10)$$

Model B:

$$LTNE = Max_N |\theta_s - \theta_f| \qquad (11)$$

Where N is the total number of nodes in the computational domain.

It has been well admitted that the LTNE parameter takes 0.05 as a reference value to delineate local thermal equilibrium validity zone condition. Thus, the LTE assumption is verified if LTNE ≤ 0.05 and inversely, the local thermal non equilibrium is so pronounced if LTNE > 0.05.

3. NUMERICAL PROCEDURE

The numerical computation was carried out with the control volume approach (Patankar 1980) using rectangular cells with constant mesh spacing in both directions (axial and transverse). The first order upwind scheme is applied for the convection–diffusion formulation in (2)-(5). The central differencing scheme is used for the energy equations diffusion terms as well as in the momentum equation. Staggered grids and SIMPLE algorithm as suggested by Patankar (1980) are adopted to treat the coupling between velocity and pressure fields. The set of obtained algebraic equations are solved using a numerical algorithm based on an iterative Strongly Implicit Procedure developed by Stone (1968). The iteration process is terminated if the following condition is satisfied:

$$max\left|\frac{\Phi_{i,j}^{m+1} - \Phi_{i,j}^m}{\Phi_{i,j}^m}\right| \leq 10^{-7} \qquad (12)$$

Where Φ represents a dependent variable U, V, P, θ_f and θ_s. The indices i, j indicate the grid point and m denotes the iteration number. Relaxation factors are employed on the above dependent variables to avoid divergence during the iteration.

Concerning the mesh refinement, to analyze the effect of the grid size on the numerical solution, various grid systems from 200 x 100 to 350 x 140 (in X- and Y-

directions, respectively) are tested to compare the LTNE criterion values for each model (A and B). The results are presented in Table 1 for various values of Re, Bi and R_k at fixed Da = 10^{-4} and Pr= 0.71. The maximum discrepancy in the values of LTNE criterion for the both models between the grid 250 x120 and the finest one (350 x140) for all values of Re, Bi and R_k does not exceed 4.5 % in maximum. Thus, the mesh size of 250 x 120 nodes, which is uniform in the both directions (axial and transverse), is considered good enough to generate grid independence results.

Further, the present numerical code is also validated with published numerical results obtained by Wong and Sayed (2009) in the case of the mixed convection regime in the jet impingement cooling from an isothermal heated surface immersed in a confined porous channel under local thermal non-equilibrium conditions. Fig. 2.b shows the LTNE parameter variation versus interstitial Biot number for different values of solid to fluid thermal conductivity ratio compared with those obtained by Wong and Saeid (2009) in the same operating conditions (Fig 2.a). As seen in these figures, the agreement between the results obtained by the present numerical code and the results from the literature is satisfactory.

Table 1 Comparison of the results with different mesh sizes for various Re, R_k and Bi values with Da=10^{-4} and Pr=0.71

LTNE Criterion values (Model A)

R_k	Bi	200 x 100	250 x 120	350 x 140
		Re=100		
10	1	0.2108	0.1954	0.1876
10	10	0.1347	0.1220	0.1165
10	10	2.57×10^{-2}	2.34×10^{-2}	2.24×10^{-2}
		Re=500		
100	1	0.6589	0.6268	0.6045
100	10	0.5562	0.5316	0.5141
100	10	0.2071	0.2001	0.1955

LTNE Criterion values (Model B)

Rk	Bi	200 x 100	250 x 120	350 x 140
		Re=100		
10	1	0.6139	0.6017	0.5966
10	10	0.4742	0.4722	0.4728
10	10	0.2661	0.2728	0.2774
		Re=500		
100	1	0.9026	0.8756	0.8592
100	10	0.8396	0.8232	0.8132
100	10	0.6391	0.6421	0.6458

4. RESULTS

The comparison between two LTNE criteria (Model A and Model B) defined by (10) and (11),

respectively, assessing the local thermal equilibrium validity assumption was performed under various flow conditions and thermo physical properties of the porous medium which are expressed in a various range of dimensionless controlling parameter such as Biot number (0.1 ≤ Bi ≤ 1000), solid-to-fluid thermal conductivity ratio (0.1 ≤ R_k ≤ 1000), Reynolds number (0.1 ≤ Re ≤ 1000), Prandtl number (0.7 ≤ Pr ≤ 100), porosity (0.5 ≤ ε ≤ 0.98) for inertial Forchheimer coefficient (C_F = 0.1).

The results are presented and discussed in terms of the LTNE parameter profiles for the two models A and B as function of the governing parameters. The LNTE reference value parameter (LTNE=0.05), representing the LTE validity assumption limit, is represented by the horizontal dashed line on all the following figures.
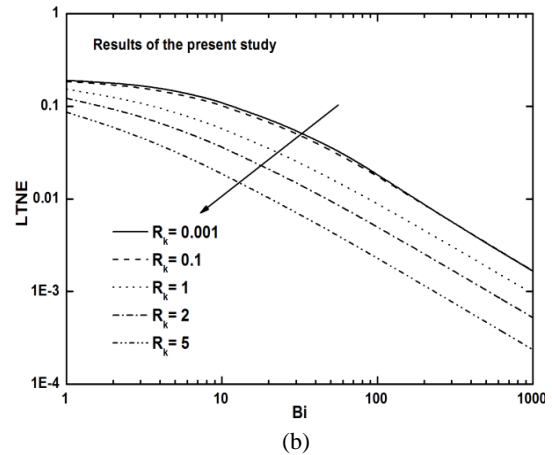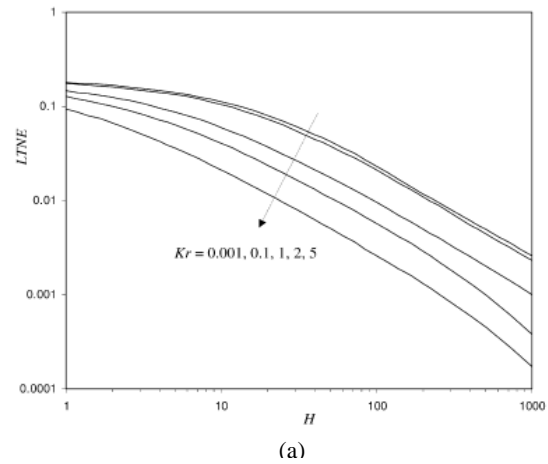

(a)

(b)

Fig. 2. Comparison of the LTNE parameter versus interstitial Biot number, for different R_k values, (a) Wong and Saeid (2009), (b) Present numerical results

Figure 3 shows the results concerning the solid to fluid thermal conductivity ratio effect on the LTNE parameter for the two models A and B for different fixed values of the Biot number values. It is clear from this figure that the increase in solid-to-fluid thermal conductivity ratio causes the LTNE increase. Indeed,

when the conductivity ratio increases it follows at the interface a temperature gradient increase in the fluid comparatively to the gradient in the solid. This creates a relative drop in the fluid temperature near the interface, leading to a decrease in convective heat exchange with the interstitial fluid. It follows a slow exchange between the two phases, causing LTNE rise.

For low Biot numbers, the LTE assumption is not verified over all values of thermal conductivity ratio for the both models. As Bi increases (Bi=10 & 100), the LTNE parameter takes lower values than the reference one for the model A. Otherwise, the LTE assumption is verified at weak R_k values (until $R_k\approx$ 2) for Bi=10 and over all R_k values for Bi=100. For the model B, the LTNE parameter profile takes much higher values than the reference one (dashed line) for all fixed Bi values. It can be noted that the increase in R_k has an unfavorable effect on the LTE assumption and the gap between the two models is also important over all values of R_k and Bi.
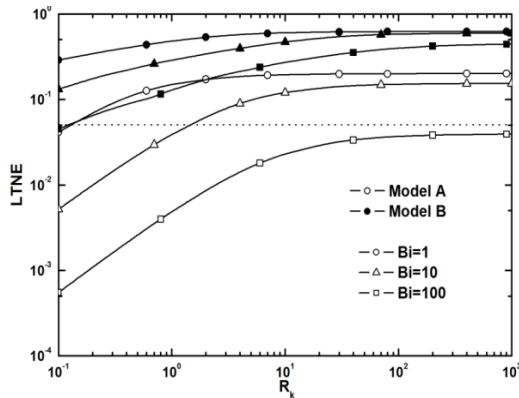

Fig. 3. Variation of the LTNE parameter versus thermal conductivity ratio, for different Bi values. Re=100, Da=10^{-4}, Pr= 0.7 and ε =0.9

Figure 4 displays the interstitial Biot number effect on the LTNE parameter for the two models (A &B) for different fixed thermal conductivity ratio values. The curves show that increasing the interstitial Biot number led to a reduction in the magnitude of the LTNE parameter values for the both cases (models A&B). Thus, augmenting Bi value improves considerably the interstitial thermal communication between the fluid and the solid phases. This leads the two phases temperatures to be closer witch satisfy the thermal equilibrium condition. The degree of validity of LTE assumption depends thereby on the value of Bi: when Bi increases the thermal equilibrium is strongly verified for the model A. Moreover, as it is shown by the above figure, the model B does not satisfy the LTE assumption for all the range of Bi numbers and for almost all the R_k values. The intersection point between the LTNE parameter profile and the horizontal dashed line represents the minimum value of the Biot number required to satisfy the thermal equilibrium condition between the two

phases, and it is clear that this minimum depends strongly on the thermal conductivity ratio. Thus, as the ratio R_k increases, the required Biot number value from which the LTE assumption is verified also rises.
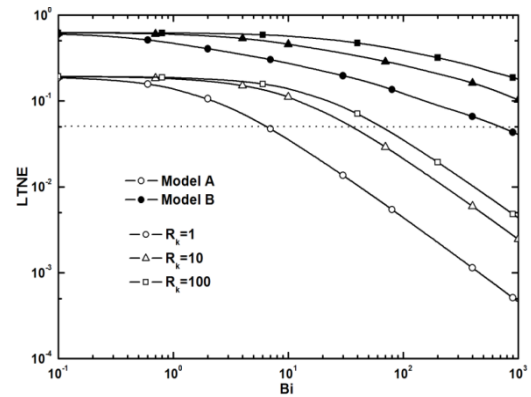

Fig. 4. Variation of the LTNE parameter versus interfacial Biot number, for different R_k values. Re=100, Da=10^{-4}, Pr= 0.7 and ε =0.9
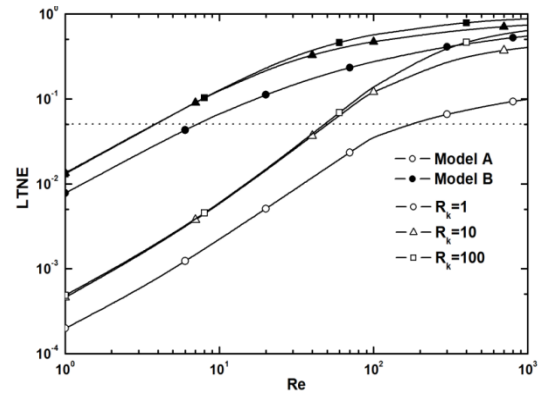

Fig. 5. Variation of the LTNE parameter versus Reynolds number for different R_k values with fixed Bi=10, Da=10^{-4}, Pr= 0.7 and ε =0.9

The Reynolds number effect on the LTNE parameter for the two criteria and for various solid-to-fluid thermal conductivity ratio values is presented in Fig. 5. The local thermal equilibrium condition is widely verified at low Reynolds number values for the both models. The increase in Reynolds number leads to the reduction of the conduction effects and to increase the convection effect, causing the augmentation of the temperature difference between the two phases. The high flow intensity doesn't allow a sufficient interstitial thermal communication. For a given R_k value, the intersection point between the LTNE parameter profile and the reference dashed line represents the maximum value of the Reynolds number (Re_{max}) by which the LTE assumption is verified. It is found that by choosing the model B, this maximum value is about $Re_{max}\approx7$ and the equivalent one for the model A is about $Re_{max}\approx180$ (for a given R_k=1). Thus, the validity range of the LTE condition

for the Model A is more important than for the model B.

Figures 6 and 7 show the porous medium porosity effect on the LTNE parameter for the two models for different values of thermal conductivity ratio and interstitial Biot number, respectively. It is clear from these figures that rising the porous medium porosity leads to the decrease in the LTNE parameter value approaching towards local thermal equilibrium between the two phases. For a given high Bi number, the LTNE parameter values for the model A are less than the reference value (LTNE=0.05), which means that the LTE assumption is widely verified for the whole range of the porosity values, comparatively to the model B whose the LTNE profile is so far to the reference LTE value. Physically, It means, that by increasing the porosity, the fluid amount inside the porous medium increases which allows a better mixing between the two phases and hence a reduction in the temperature difference between the fluid and the solid phases. It is worth to notice that, under the same operating conditions, the validity of the LTE assumption is considerably affected by the selected model and the conclusions can be expressed in the opposite way, ie the LTE for the model A and the LTNE for the model B.

The LTNE parameter variation with Prandtl number for different values of solid-to-fluid thermal conductivity ratio (R_k=10, 100, 1000) and for two values of Bi (10 and 100), is presented in Fig. 8 and Fig. 9. The results show that the evolution is overall in the same way as with the Reynolds number. When the Prandtl number augments, the LTNE parameter increases more significantly for high R_k numbers. For moderate Bi value (Bi=10), the LTE assumption in not verified for the two models over the whole range of Pr values. Indeed, this can be explained by the fact that the interstitial heat transfer coefficient weakness effect outweighs the conductive diffusion effects. When Biot numbers are high (Bi=100), the local thermal equilibrium condition is satisfied with the model A for low Pr numbers until a critical value which is strongly affected by the R_k value. From this critical point the LTNE is again pronounced indicating that the fluid thermal diffusion effect begins to outweigh the interstitial convective exchange influence. However, the LTNE parameter values of the model B are higher than the reference LTE value. The deviation between the two models is important for low values of Pr and diminishes by increasing Pr. Thus It appears from these results that a high Prandtl number does not allow a good thermal communication between the two phases, which can be physically explained by the relatively low fluid thermal diffusivity value corresponding to a low fluid conductivity.
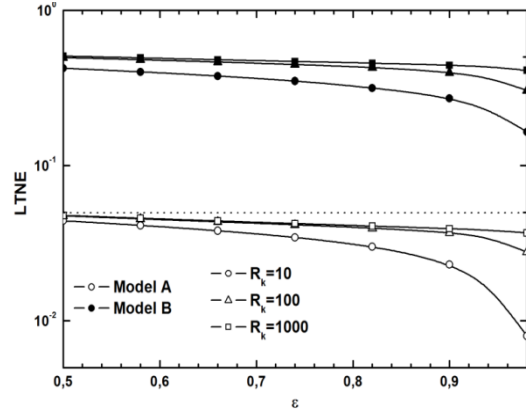

Fig. 6. Variation of the LTNE parameter versus porosity for different Rk values with fixed Re=100, Bi=100, Da=10-4 and Pr= 0.7

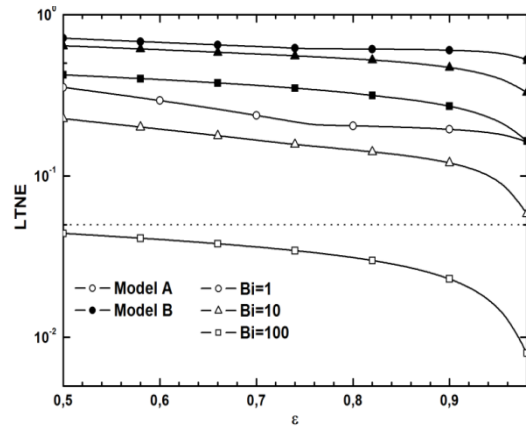**Fig. 7. Variation of the LTNE parameter versus porosity for different Bi values with fixed Re=100, Rk=10, Da=10-4 and Pr= 0.7**
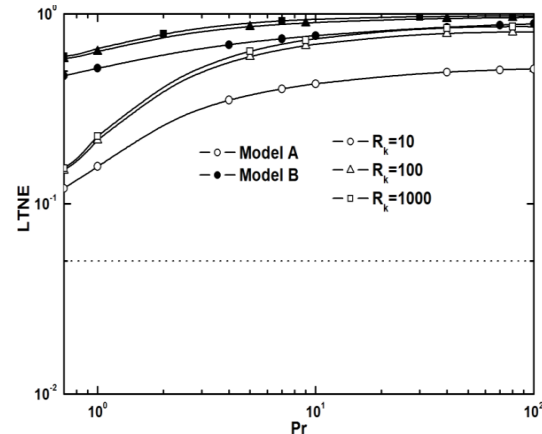

Fig. 8. Variation of the LTNE parameter versus Prandtl number for different R_k values with fixed Re=100, Bi=10, Da=10-4 and ε =0.9
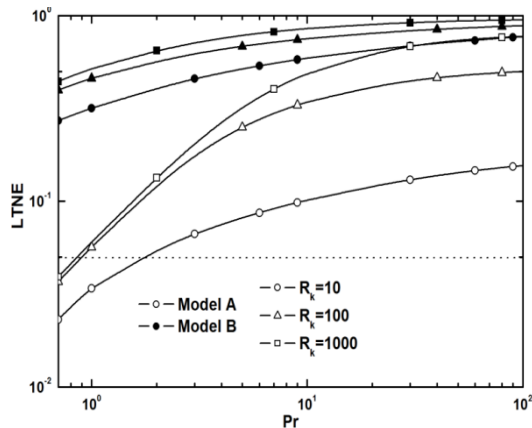
Fig. 9. Variation of the LTNE parameter versus Prandtl number for different R_k with fixed Re=100, Bi=100, Da=10^{-4} and ε =0.9

5. CONCLUSION

Forced convection in a channel filled with a saturated porous medium is numerically investigated using the finite volume method. The Brinkman-Forchheimer extended Darcy model is used to describe the fluid movement while the thermal field is modeled by the two energy equations model taking into account the local thermal non equilibrium between the two phases. The study was conducted in the perspective to determine under what circumstances the LTE assumption is valid, and to compare between two most LTNE criteria used in the literature. It can be concluded from this numerical study that the LTNE parameter values augment with the increase of the solid-to-fluid thermal conductivity ratio and with the Reynolds and Prandtl numbers. Under these conditions, the local thermal equilibrium condition cannot be satisfied. Inversely, the LTNE parameter values diminish when the interstitial Biot number and the porosity of the porous medium increase. Thus, the circumstances of low values of R_k, Re and Pr and high values of Bi and ε are all found to have favorable effects to satisfy the local thermal equilibrium assumption.

It is worth to notice from this study that the LTNE parameter values of the model B are often higher than for the model A. Indeed, as mentioned earlier, the local thermal equilibrium assumption is verified, for many cases, by using the model A (based on the average value of the temperature difference between the two phases) and not verified when using the model B (based on the maximum value of the local temperature difference between the solid and fluid phases), under the same operating conditions. Thus, it is recommended to be careful when exploiting the conclusions defining the LTE assumption validity, which can be strongly affected by the selected LTNE criterion.

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