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Radiation, Heat Generation and Viscous Dissipation Effects on MHD Boundary Layer Flow for the Blasius and Sakiadis Flows with a Convective Surface Boundary Condition

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ABSTRACT

This study is devoted to investigate the radiation, heat generation viscous dissipation and magnetohydrodynamic effects on the laminar boundary layer about a flat-plate in a uniform stream of fluid (Blasius flow), and about a moving plate in a quiescent ambient fluid (Sakiadis flow) both under a convective surface boundary condition. Using a similarity variable, the governing nonlinear partial differential equations have been transformed into a set of coupled nonlinear ordinary differential equations, which are solved numerically by using shooting technique alongside with the forth order of Runge-Kutta method and the variations of dimensionless surface temperature and fluid-solid interface characteristics for different values of Magnetic field parameter M, Grashof number Gr, Prandtl number Pr, radiation parameter N_R, Heat generation parameter Q, Convective parameter γ and the Eckert number Ec, which characterizes our convection processes are graphed and tabulated. Quite different and interesting behaviors were encountered for Blasius flow compared with a Sakiadis flow. A comparison with previously published results on special cases of the problem shows excellent agreement.

Keywords: Heat transfer; MHD; Blasius/Sakiadis flows; Heat generation; Thermal radiation; Eckert number; Convective surface boundary condition; Similarity solution.

1. INTRODUCTION

Investigations of magnetohydrodynamic (MHD) boundary layer flow and heat transfer of viscous fluids over a flat sheet are important in many manufacturing processes, such as polymer extrusion, drawing of copper wires, continuous stretching of plastic films and artificial fibers, hot rolling, wire drawing, glass-fiber, metal extrusion, and metal spinning. Among these studies, Sakiadis (1961), initiated the study of the boundary layer flow over a stretched surface moving with a constant velocity and formulated a boundary-layer equation for two-dimensional and axisymmetric flows. Tsou *et al*. (1967) analyzed the effect of heat transfer in the boundary layer on a continuous moving surface with a constant velocity and experimentally confirmed the numerical results of Sakiadis (1961). The similarity solution for Magnetohydromagnetic mixed convection of heat and mass transfer for hiemenz flow through a porous media as explained by Chamkha and Khaled (2000).

Erickson *et al*. (1966) extended Sakiadas problem to include blowing or suction at the moving surface and investigated its effects on the heat and mass transfer in the boundary layer. Kao (1976) investigated free convection from vertical plates with sinusoidal temperature variation and constant transpiration. The effect of radiation on free convection flow of fluid with variable viscosity from a porous plate is discussed Anwar and Hossain *et al.* (2001). Danberg and Fansber (1976) investigated the non-similar solution for the flow in the boundary layer past a wall i.e. stretched with a velocity proportional to distant along the wall. Gupta and Gupta (1977) studied the heat and mass transfer corresponding to similarity solution for the boundary layer over an isothermal stretching sheet subject to blowing or suction. Chen and Char (1998) investigated the effects of variable surface temperature and variable surface heat flux on the heat transfer characteristics of a linearly stretching sheet subject blowing or suction. Ali (1995) has reported flow and heat characteristics on a stretched surface subject to power-law velocity and temperature distributions. Aziz (2009) have discussed a similarity solution for laminar boundary layer over a plate with a convective boundary condition. Vajravelu and Hadjinicolaou (1997) studied the convective heat transfer in an electrically conducting fluid near an isothermal stretching sheet and they studied the effect of internal heat generation or absorption. Rahman *et al.* (2012) have analyzed the Local similarity solutions for unsteady two-dimensional forced convective heat and mass transfer flow along a wedge with thermophoresis.

Recently, Makinde and Olanrewaju (2010) studied the effects of thermal buoyancy on the laminar boundary layer about a vertical plate in a uniform stream of fluid under a convective surface boundary condition. Very more recently, Olanrewaju *et al*. (2011) studied the Radiation and viscous dissipation effects for the Blasius and Sakiadis flows with a convective surface boundary condition.

Hence, the purpose of the present work is to extend the work of Olanrewaju *et al*. (2011) to include the effects of Buoyancy and Magnetohydrodynamic on radiation and viscous dissipation effects for the Blasius and Sakiadis flows with a convective surface boundary condition. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables, and these have been solved numerically. The effects of magnetic field, Radiation parameter, Eckert number, Prandtl number, Grashof number, and convective surface boundary condition parameter on fluid velocity and temperature have been shown graphically. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

2. MATHEMATICAL ANALYSIS

Taking into account the viscous dissipation and the thermal radiation terms in the energy equation, the governing equations of motion and heat transfer for the classical Blasius flat-plate flow problem can be summarized by the following boundary value problem (Makinde and Olanrewaju 2010; Aziz 2009; Bataller 2008).

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u+g\beta\,(T-T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{M}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{q}{\rho c_p}(T-T_\infty) \tag{3}$$

where ν is the fluid kinematics viscosity, ρ - the density, σ - the electric conductivity of the fluid, β the coefficient of thermal expansions, k - the thermal conductivity, C_∞ - the free stream concentration, B_0 - the magnetic induction, U_∞ - the free stream velocity and g is the gravitational acceleration .
The boundary conditions for the velocity field are:

$$u=v=0 \text{ at y=0; } u=U_\infty \text{ at x=0}$$

$$u\to U_\infty \text{ as } y\to\infty \tag{4}$$

For the Blasius flat plate flow problem and

$$u=U_w\text{; v=0 at y=0}$$
$$u\to 0 \text{ as } y\to 0 \tag{5}$$

For the classical Sakiadis flat plate flow problem respectively.

The boundary conditions at the plate surface and far into the cold fluid may be written as

$$-K\frac{\partial T}{\partial y}(x,0)=h_f[T_f-T\,(x,0)],$$
$$T(x,\infty)=T_\infty \tag{6}$$

Here u and v are the velocity components along the flow direction (x-direction) and normal to flow direction (y-direction), υ is the kinematic viscosity, k is the thermal conductivity, c is the specific heat of the fluid at constant pressure, β - the coefficient of thermal expansion, ρ is the density, g is the acceleration due to gravity, μ is the dynamic viscosity, q_r is the radiative heat flux in the y-direction, T is the temperature of the fluid inside the thermal boundary layer, B_0 - the magnetic induction, The cold fluid on the right surface of the plate generates heat internally at the volumetric rate q ,U_∞ is a constant free stream velocity and U_w is the plate velocity. It is assumed that the physical properties of the fluid are constant, and the Boussinesq and boundary layer approximation may be adopted for steady laminar flow. The fluid is considered to be gray; absorbing- emitting radiation but non-scattering medium.

The radiative heat flux q_r is described by Roseland approximation such that,

$$q_r=-\frac{4\sigma^*}{3K'}\frac{\partial T^4}{\partial y} \tag{7}$$

Where $\sigma*$ and K' are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. Following Bataller (2008), we assume that the temperature differences within the flow are sufficiently small so that the T^4 can be expressed as a linear function after using Taylor series to expand T^4 about the free stream temperature T_∞ and neglecting higher-order terms. This result is the following approximation:

$$T^4\approx 4T_\infty^3 T-3T_\infty^4 \tag{8}$$

Using (7) and (8) in (3), we obtain

$$\frac{\partial q_r}{\partial y}=-\frac{16\sigma^*}{3K'}\frac{\partial T^4}{\partial y} \tag{9}$$

In view of Eq. (9) and Eq. (8), Eq. (3) reduces to:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\alpha+\frac{16\sigma^* T_\infty^3}{3\rho c_p K'}\right)\frac{\partial^2 T}{\partial y^2}+\frac{\alpha}{k}\left(\frac{\partial u}{\partial y}\right)^2+\frac{q}{\rho c_p}(T-T_\infty) \tag{10}$$

where $\alpha = \dfrac{k}{\rho c_p}$ is the thermal diffusivity.

From the equation above, it is clearly seen that the influence of radiation is to enhance the thermal diffusivity. If we take $N_R = \dfrac{kK'}{4\sigma^* T_\infty^3}$ as the radiation parameter, Eq. (10) becomes:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\alpha}{k_0}\frac{\partial^2 T}{\partial y^2} + \frac{\alpha\mu}{k}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{q}{\rho c_p}(T - T_\infty) \quad (11)$$

where $k_0 = \dfrac{3N_R}{3N_R + 4}$. It is worth citing here that the classical solution for energy equation, Eq. (11), without thermal radiation and viscous dissipation influences can be obtained from the above equation which reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \text{ as } N_R \to \infty \ \ (ie \ \ k_0 \to 1)$$

We introduce a similarity variable η and a dimensionless stream function f(η) as

$$\eta = y\sqrt{\frac{U}{\upsilon x}} = \frac{y}{x}\sqrt{\mathrm{Re}_x},\ \frac{u}{U} = f',\ v = \frac{1}{2}\sqrt{\frac{U\upsilon}{x}}(\eta f' - f) \quad (12)$$

where prime denotes differentiation with respect to η and Re_x is the local Reynolds number ($= \dfrac{Ux}{\upsilon}$), we obtain by deriving Eq. (12):

$$\frac{\partial u}{\partial x} = -\frac{U}{2}\frac{\eta}{x}f'',\ \frac{\partial v}{\partial y} = \frac{U}{2}\frac{\eta}{x}f'' \quad (13)$$

And the equation of continuity is satisfied identically

$$\frac{\partial u}{\partial y} = Uf''\sqrt{\frac{U}{\upsilon x}},\ \frac{\partial^2 u}{\partial y^2} = \frac{U^2}{\upsilon x}f''' \quad (14)$$

Nothing that in Eq. (12) and Eq. (14), $U = U_\infty$ represents Blasius flow, whereas $U = U_w$ indicates Sakiadis flow, respectively. We also assume the bottom surface of the plate is heated by convection from a hot fluid at uniform temperature T_f which provides a heat transfer coefficient h_f.

Defining the non-dimensional temperature $\theta(\eta)$, The Prandtl number *Pr*, The Eckert number *Ec*, Magnetic field parameter *M*, Grashof number *Gr* and Heat generation parameter as:

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \mathrm{Pr} = \frac{\upsilon}{\alpha},\ Ec = \frac{U^2}{k(T_w - T_\infty)} \quad (15)$$

$$M = \frac{\sigma B_0^2 x}{\rho U},\ Gr = \frac{g\beta(T_w - T_\infty)x}{U^2},\ Q = \frac{q}{xU}$$

By substituting Eq. (12) and Eq. (14) into Eq. (2) and Eq. (11) we have:

$$f''' + \frac{1}{2}ff'' - Mf' + Gr\theta = 0 \quad (16)$$

$$\theta'' + \frac{\mathrm{Pr}\,k_0}{2}f\theta' + Ec\,\mathrm{Pr}\,f''^2 + prQ\theta = 0 \quad (17)$$

where *Ec* is the Eckert number, when k_0=1 and *Ec=0*, the thermal radiation and the viscous dissipation effects are not considered.

The transformed boundary conditions are:

$$f = 0,\ f' = 0,\ \theta' = -\gamma[1 - \theta(0)] \ \ at \ \ \eta = 0 \quad (18)$$

$$f' = 1,\ \theta = 0 \ \ as \ \ \eta \to \infty$$

for the Blasius flow, and

$$f = 0,\ f' = 1,\ \theta' = -\gamma[1 - \theta(0)] \ \ at \ \ \eta = 0 \quad (19)$$

$$f' = 0,\ \theta = 0 \ \ as \ \ \eta \to \infty$$

for the Sakiadis case, respectively.

where

$$\gamma = \frac{h_f}{k}\sqrt{\frac{\upsilon x}{U_\infty}} \quad (20)$$

For the momentum and energy equations to have a similarity solution, the parameters γ must be constants and not functions of x as in Eq. (20). This condition can be met if the heat transfer coefficient h_f is proportional to $x^{-1/2}$.

We therefore assume

$$h_f = cx^{-\frac{1}{2}} \quad (21)$$

where c is constant.

Putting Eq. (21) into Eq. (20), we have:

$$\gamma = \frac{c}{k}\sqrt{\frac{\upsilon}{U_\infty}} \quad (22)$$

Here, γ is defined by Eq. (22), the solutions of Eq. (16) and Eq. (19) yield the similarity solutions, however, the solutions generated are the local similarity solutions whenever γ is defined as in Eq. (20).

3. SOLUTION OF THE PROBLEM

The equations (16) and (17) are coupled and non-linear ordinary differential equations and hence analytical solution is not possible. Hence the dimensionless governing Eq. (16) and Eq. (17) together with the boundary conditions (Eq. (18) and Eq.(19)) are solved numerically by using Runge-Kutta fourth order technique along with shooting technique (Jain *et al.* (1985)). The step size $\Delta\eta$ =0.05 is used to obtain the numerical solution with decimal place accuracy as the criterion of convergence. The shooting method for linear equations is based on replacing the boundary value problem by two initial value problems, and solution of the boundary value problem is a linear combination between the solutions of the two initial value problems. The shooting method for the nonlinear boundary value problem is similar to the linear case, except that the solution of the nonlinear problem cannot be simply expressed as a linear combination between the solutions of the two initial value problems. The numerical computations have been done by the symbolic computation software Mathematica. Solving Eq. (17) by the nonlinear shooting method we obtain θ. Hence, Eq. (16) and Eq. (17) reduce to a system of linear equations with variable coefficients which could be solved by the linear shooting method to obtain *f*. The functions f', and θ are shown in figures. From the process of numerical computation, the skin-friction coefficient and Nusselt number, which are respectively proportional to $f''(0),\ and\ -\theta'(0)$ are also sorted out and their numerical values are presented in a tabular form.

4. Results and Discussion

The governing Eq. (17) and Eq. (18) subject to the boundary conditions Eq. (19) and Eq. (20) are integrated as described in section 3. Numerical computations have been carried out for different embedded parameters coming into the flow model controlling the fluid dynamics in the flow regime. Attention is focused on positive value of the buoyancy parameters that is, Grashof number Gr>0 (which corresponds to the cooling problem). The magnetic field parameter (M) used are 0.1, 1.0, 2.0, 3.0; The Prandtl number (Pr) used are 0.72, 1, 3, 5, 7.1, 10 and 100; the convective parameters γ used are 0.1, 0.5, 1.0, 5.0, 10, and 20; the radiation parameters N_R used are 0.7, 5.0, 10, and 100; radiation parameter Q used are 0.01, 0.1, 0.2 and 0.25; and Eckert number (*Ec*) used are $Ec > 0$. Comparisons of the present results with previously works are performed and excellent agreements have been obtained. We obtained the results as shown in Tables 1 - 3 and figures 1-28 below. Table 1 shows the comparison of Olanrewaju *et al.* (2010) work for Blasius and Sakiadis flows for Prandtl numbers (Pr = 0.72, 1.0, 5.0, 10 and 100) and radiation parameter (N_R= 0.7, 5.0, 10 and 100) and it is noteworthy to mention that there is a perfect agreement in the absence of viscous dissipation parameter, buoyancy parameter, Magnetic field parameter and Heat generation parameter. Accurately, the results at γ = 0.5, Pr = 5 and N_R = 0.7 for the missed plate temperature θ (0) values were numerically obtained as θ (0) = 0.554898 for Blasius flow, and θ(0) = 0.444746 for Sakiadis flow, respectively (see table 2). In table 2 and 3, we show the influence of the embedded flow parameters on the temperature at the wall plate for the Blasius and Sakiadis flow. It is clearly seen that when Biot number (γ)and Radiation parameter (N_R) increases the wall temperature for Blasius and Sakiadis flow decreases while increase in Prandtl number Pr, and Eckert number (Ec) increases the wall temperature for both Blasius and Sakiadis flow. Table 2 shows the influence of the flow parameters on the Nusselt number and the Skin friction for Sakiadis flow. Increase in the thermal radiation parameter (N_R) and the Grashof number (Gr) bring an increase in the Nusselt number. Increase in the Convective parameter γ, Prandtl number (Pr), Magnetic parameter (M) and the Eckert number (Ec) bring a decrease in the Nusselt number. Skin friction increases with an increase in the Prandtl number (Pr), Grashof number (Gr) and the Eckert number (Ec) while increase in the Convective parameter (γ), Magnetic field parameter (M) and the radiation parameter decreases the Skin friction at the wall plate. Table 3 shows the influence of the flow parameters on the Nusselt number and the Skin friction for Blasius flow. Increase in the thermal radiation parameter (N_R) and the Magnetic field parameter (M) bring an increase in the Nusselt number. Increase in the Convective parameter (γ), Prandtl number (Pr), Grashof number (Gr) and the Eckert number Ec bring a decrease in the Nusselt number. Skin friction increases with an increase in the Prandtl number (Pr), Grashof number (Gr) and Eckert number (Ec) while increase in the Convective parameter (γ), Magnetic field parameter (M) and the Radiation parameter decreases the Skin friction at the wall plate.

4.1 Effects of Parameter Variation on Velocity Profiles

The influences of various embedded parameters on the fluid velocity are illustrated in Figs. 1 to 14. Fig. 1 depicts the effect of Grashof number on the velocity profile for Sakiadis flow and it is seen that increase in the Grashof number increases the velocity boundary layer thickness across the plate. We can see also that the same effect was seen for Blasius flow (see Fig. 8). Fig. 2 depicts the curve of velocity against span wise coordinate η for various values of Magnetic field parameter M. It is clearly seen that increases in the Magnetic field parameter decreases the velocity profile and thereby reduce the thermal boundary layer thickness. Similar effect was seen also in Fig. 9 for Blasius flow. It is interesting to note that at M = 3 the velocity remain the same meaning that it has reach a steady state. Fig. 3 also represents the curve of velocity against Span wise coordinate η for various values of Prandtl number. Increase in Prandtl number leads to a decrease in the velocity profile. It is also interesting to note that the same effect was opposite experienced in fig. 10 for Blasius flow. Fig. 4 depicts the effect of Eckert number on the velocity profile for Sakiadis flow and it is seen that increase in the Eckert number increases the velocity boundary layer thickness across the plate confirming the existing literature. The same effect was observed for in Fig. 11 for Blasius flow. Fig. 5 depicts the effect of Radiation parameter on the velocity profile for Sakiadis flow and it is seen that increase in the Radiation parameter decreases the velocity boundary layer thickness across the plate confirming the existing literature. The same effect was observed for in Fig. 12 for Blasius flow. Fig. 6 depicts the effect of Convective boundary condition parameter on the velocity profile for Sakiadis flow and it is seen that increase in the Convective parameter increases the velocity boundary layer thickness across the plate confirming the existing literature. The same effect was observed for in fig. 13 for Blasius flow. Fig. 7 depicts the effect of Heat generation parameter on the velocity profile for Sakiadis flow and it is seen that increase in the Heat generation parameter increases the velocity boundary layer thickness across the plate confirming the existing literature. The same effect was observed for in Fig. 14 for Blasius flow.

Table 1 computations showing comparison with Olangaraju et al. (2010) results of $\theta(0)_{Blasius}$ and $\theta(0)_{Sakiadis}$ for $M=0, Gr=0$, Q=0, Ec=0.

γ	Pr	N_R	$\theta(0)_{Blasius}$ Olanrewaju et al (2010)	$\theta(0)_{Sakiadis}$ Olanrewaju et al (2010)	$\theta(0)_{Blasius}$ present	$\theta(0)_{Sakiadis}$ present
0.1	5	0.7	0.19957406	0.13807609	0.199574	0.138076
0.5	5	0.7	0.55489763	0.44474556	0.554898	0.444746
1.0	5	0.7	0.71374169	0.61567320	0.713742	0.615673
10	5	0.7	0.96143981	0.94124394	0.96144	0.941244
20	5	0.7	0.98034087	0.96973278	0.980341	0.969733
1	0.72	0.7	0.83312107	0.84297896	0.833121	0.842979
1	1.0	0.7	0.81555469	0.81785952	0.815555	0.81786
1	5.0	0.7	0.71374169	0.61567320	0.713742	0.615673
1	10	0.7	0.66301284	0.51639994	0.663013	0.5154
1	100	0.7	0.47592614	0.23747971	0.475926	0.23748
5	5	0.7	0.92574298	0.88900927	0.925743	0.889009
5	5	5.0	0.90376783	0.83172654	0.903768	0.831727
5	5	10	0.90044458	0.82284675	0.900445	0.822847
5	5	100	0.89700322	0.81361511	0.897003	0.813615

Table 2 Computation of $f''(0)_{Sakiadis}, -\theta'(0)_{Sakiadis}$ and $\theta(0)_{Sakiadis}$ for several values of parameters entering the problem.

γ	Pr	N_R	Ec	M	Gr	Q	$f''(0)_{Sakiadis}$	$-\theta'(0)_{Sakiadis}$	$\theta(0)_{Sakiadis}$
0.1	5	0.7	2	0.1	0.1	0.01	-0.264936	-0.134	2.34
0.5	5	0.7	2	0.1	0.1	0.01	-0.294676	-0.453799	1.9076
1.0	5	0.7	2	0.1	0.1	0.01	-0.312691	-0.647418	1.64742
10	5	0.7	2	0.1	0.1	0.01	-0.350522	-1.05382	1.10538
20	5	0.7	2	0.1	0.1	0.01	-0.354088	-1.09211	1.05461
5	0.72	0.7	2	0.1	0.1	0.01	-0.381829	-0.0593541	1.01187
5	1	0.7	2	0.1	0.1	0.01	-0.376083	-135646	1.02713
5	5	0.7	2	0.1	0.1	0.01	-0.3441	-0.984851	1.19697
5	10	0.7	2	0.1	0.1	0.01	-0.328248	-0.78831	1.35766
5	5	5	2	0.1	0.1	0.01	-0.418711	-0.618085	1.12362
5	5	10	2	0.1	0.1	0.01	-0.426919	-0.548505	1.1097
5	5	100	2	0.1	0.1	0.01	-0.434733	-0.474231	1.09485
5	5	0.7	5	0.1	0.1	0.01	-0.184914	-2.15093	1.43019
5	5	0.7	7	0.1	0.1	0.01	-0.0660823	-2.71187	1.54237
5	5	0.7	8	0.1	0.1	0.01	-0.0197729	-3.21079	1.64216
5	5	0.7	2	0.5	0.1	0.01	-0.637747	-1.92724	1.38545
5	5	0.7	2	1.0	0.1	0.01	-0.903691	-2.97897	1.59579
5	5	0.7	2	0.1	0.2	0.01	-0.175952	-0.586984	1.1174
5	5	0.7	2	0.1	0.3	0.01	0.00493527	-0.417037	1.08341
5	5	0.7	2	0.1	0.1	0.1	-0.208372	-2.15195	1.43034
5	5	0.7	2	0.1	0.1	0.2	0.298626	-8.61115	2.72223
5	5	0.7	2	0.1	0.1	0.25	0.772563	-17.4461	4.48921

Table 3 Computation of $f''(0)_{Blasius}$, $-\theta'(0)_{Blasius}$ and $\theta(0)_{Blasius}$ for several values of parameters entering the problem

γ	Pr	N_R	Ec	M	Gr	Q		$-\theta'(0)_{Blasius}$	$\theta(0)_{Blasius}$
0.1	5	0.7	2	0.1	0.1	0.01	1.06612	-0.714241	8.14241
0.5	5	0.7	2	0.1	0.1	0.01	0.642248	-1.09398	3.18797
1.0	5	0.7	2	0.1	0.1	0.01	0.546451	-1.16921	2.16921
10	5	0.7	2	0.1	0.1	0.01	0.443699	-1.24493	1.12449
20	5	0.7	2	0.1	0.1	0.01	0.437431	-1.24938	1.06247
5	0.72	0.7	2	0.1	0.1	0.01	0.391918	-0.0490898	1.00982
5	1	0.7	2	0.1	0.1	0.01	0.398925	-0.130407	1.02608
5	5	0.7	2	0.1	0.1	0.01	0.456042	-1.23612	1.24722
5	10	0.7	2	0.1	0.1	0.01	0.505411	-2.64958	1.52992
5	100	0.7	2	0.1	0.1	0.01	0.929389	-33.7274	7.74548
5	5	5	2	0.1	0.1	0.01	0.376847	-0.600844	1.12017
5	5	10	2	0.1	0.1	0.01	0.368227	-0.531757	1.10635
5	5	100	2	0.1	0.1	0.01	0.35994	-0.465031	1.09301
5	5	0.7	5	0.1	0.1	0.01	0.748891	-6.81688	2.36338
5	5	0.7	7	0.1	0.1	0.01	1.13608	-17.4808	4.49615
5	5	0.7	8	0.1	0.1	0.01	1.81598	-41.0993	9.21984
5	5	0.7	2	0.5	0.1	0.01	0.196885	-0.339683	1.06794
5	5	0.7	2	1.0	0.1	0.01	0.133393	-0.329498	1.0659
5	5	0.7	2	0.1	0.2	0.01	0.736656	-2.351	1.4702
5	5	0.7	2	0.1	0.3	0.01	1.1684	-4.73695	1.94739
5	5	0.7	2	0.1	0.1	0.1	0.663865	-3.86494	1.77299
5	5	0.7	2	0.	0.1	0.2	1.33903	-15.4145	4.08291

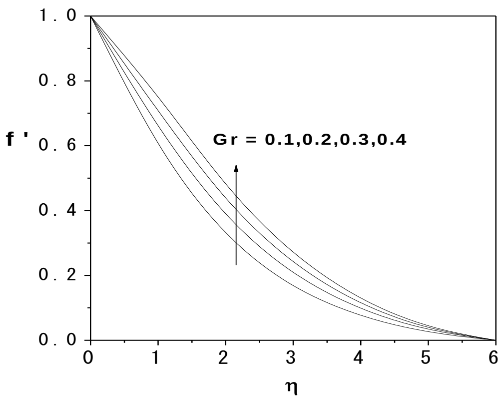

Fig. 1. Variation of the velocity component f' with *Gr* for Pr=0.72, M = γ =0.1, N_R=0.7, Q=0.01, Ec=1. (Sakiadis flow)

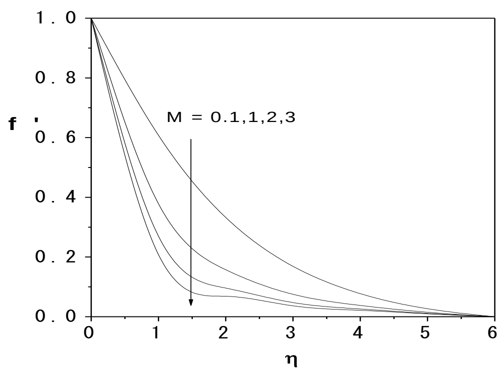

Fig. 2. Variation of the velocity component f' with *M* for Pr=0.72, Gr = γ =0.1, N_R=0.7, Q=0.01, Ec=1. (Sakiadis flow)

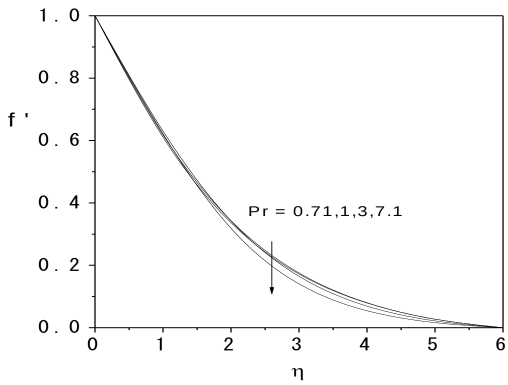

Fig. 3. Variation of the velocity component f' with *Pr* for Gr=M = γ =0.1, N_R=0.7, Q=0.01, Ec=1. (Sakiadis flow)

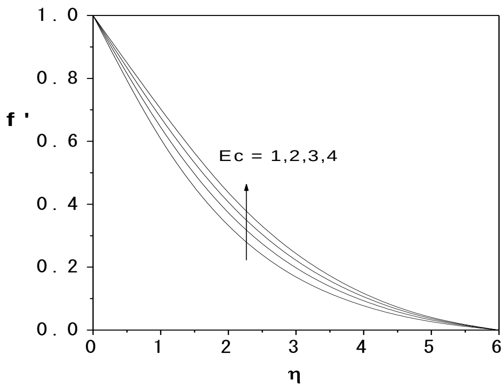

Fig. 4. Variation of the velocity component f' with *Ec* for Pr=0.72, Gr=M = γ =0.1, Q=0.01, N_R=0.7. (Sakiadis flow)

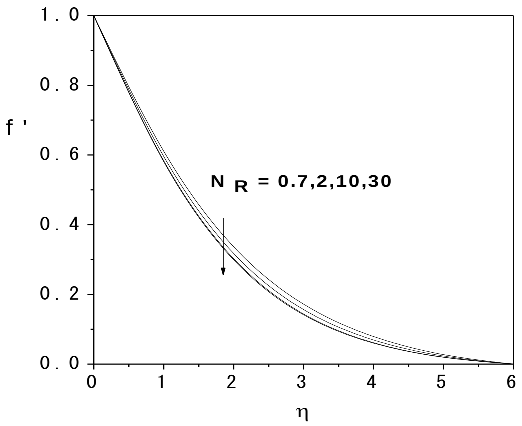

Fig. 5. Variation of the velocity component f' with N_R for Pr=0.72, Gr=M = γ =0.1, Q=0.01, Ec=1. (Sakiadis flow)

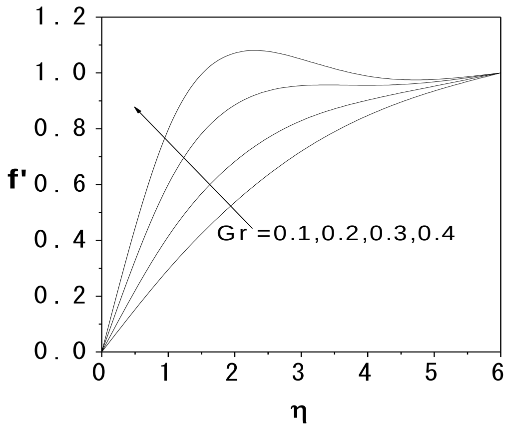

Fig. 8. Variation of the velocity component f' with *Gr* for Pr=0.72, M = γ =0.1, N_R=0.7, Q=0.01, Ec=1.(Blasius flow)

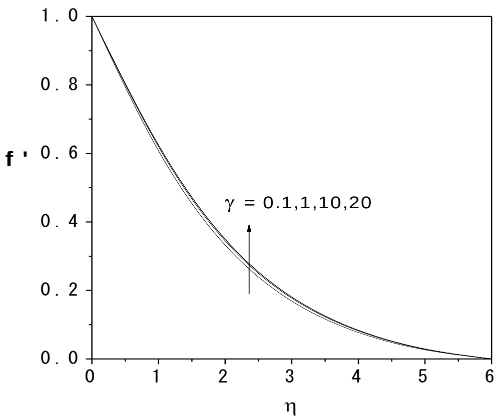

Fig. 6. Variation of the velocity component f' with γ for Pr=0.72, Gr=M =0.1, N_R=0.7, Q=0.01, Ec=1. (Sakiadis flow)

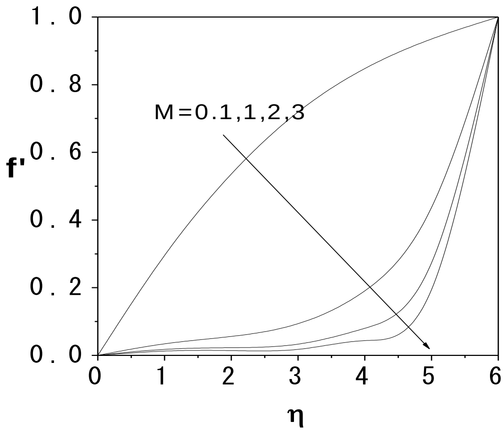

Fig. 9. Variation of the velocity component f' with *M* for Pr=0.72, Gr = γ =0.1, N_R=0.7, Q=0.01, Ec=1. (Blasius flow)

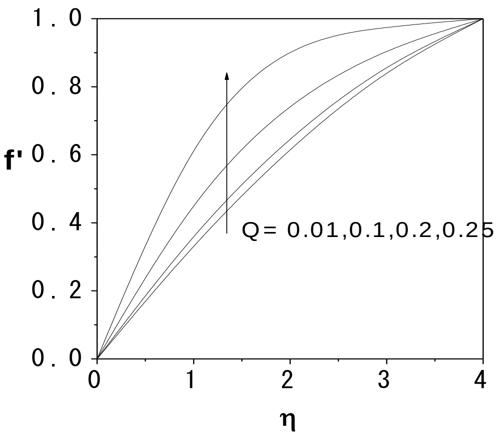

Fig. 7. Variation of the velocity component f' with Q for Pr=0.72, Gr= γ =M =0.1, N_R=0.7, Ec=1. (Sakiadis flow)

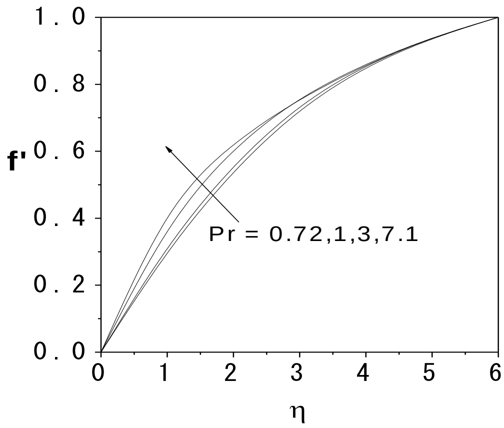

Fig. 10. Variation of the velocity component f' with *Pr* for Gr = M = γ =0.1, N_R=0.7, Q=0.01,Ec=1. (Blasius flow)

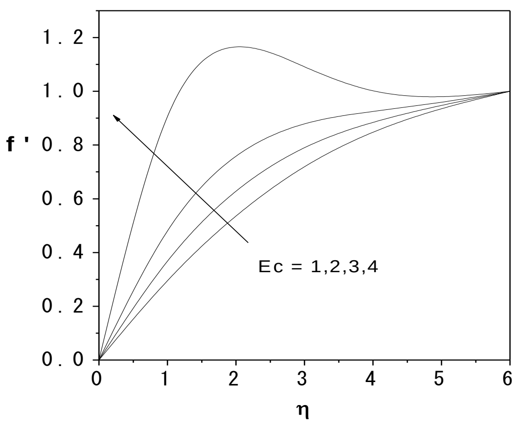

Fig. 11 Variation of the velocity component f' with *Ec* for Pr=0.72, Gr =M = γ =0.1, Q=0.01, N_R=0.7. (Blasius flow)

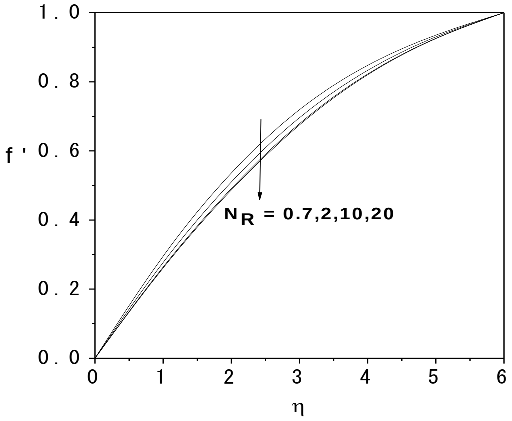

Fig. 12. Variation of the velocity component f' with N_R for Pr=0.72, Gr=M = γ =0.1, Q=0.01, Ec=1. (Blasius flow)

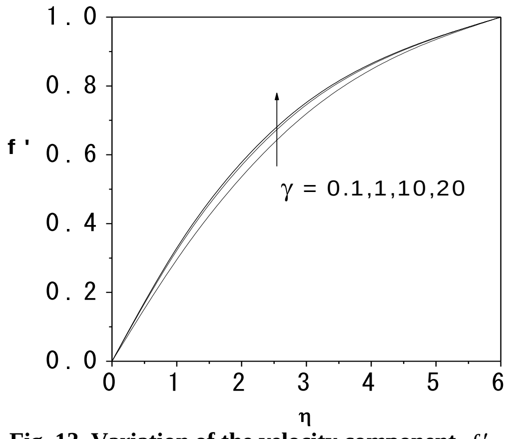

Fig. 13. Variation of the velocity component f' with γ for Pr=0.72, Gr = M =0.1, N_R=0.7, Q=0.01,Ec=1. (Blasius flow)

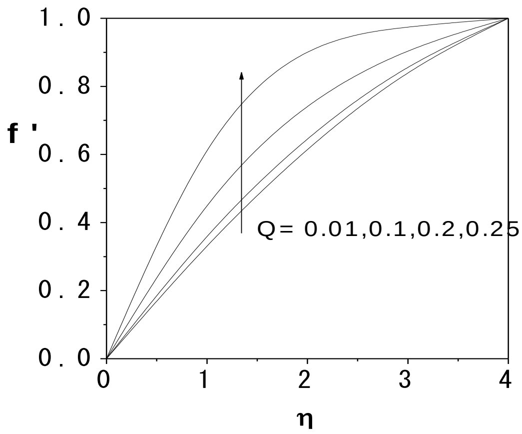

Fig. 14. Variation of the velocity component f' with for Pr=0.72, Gr = γ = M =0.1, N_R=0.7, Ec=1. (Blasius flow)

4.2 Effects of Parameter Variation on Temperature Profiles

The influences of various embedded parameters on the fluid temperature are illustrated in Figs. 15 to 28. Fig. 15 depicts the effect of Grashof number on the temperature profile for Sakiadis flow and it is seen that increase in the Grashof number decreases the temperature boundary layer thickness across the plate. But an increase in the Grashof number increases the temperature boundary layer thickness across the plate for Blasius flow (see Fig. 22). Fig. 16 depicts the curve of temperature against span wise coordinate η for various values of Magnetic field parameter M. It is clearly seen that increases in the Magnetic field parameter increases the temperature profile and thereby reduce the thermal boundary layer thickness. Opposite results was seen also in Fig. 23 for Blasius flow. Fig. 17 also represents the curve of velocity against Span wise coordinate η for various values of Prandtl number. Increase in Prandtl number leads to an increase in the velocity profile. It is also interesting to note that the same effect was experienced in fig. 24. Fig. 18 depicts the effect of Eckert number on the temperature profile for Sakiadis flow and it is seen that increase in the Eckert number increases the temperature boundary layer thickness across the plate confirming the existing literature. The same effect was observed for in Fig. 25 for Blasius flow. Fig. 19 depicts the effect of Radiation parameter on the temperature profile for Sakiadis flow and it is seen that increase in the Radiation parameter decreases the temperature boundary layer thickness across the plate confirming the existing literature. The same effect was observed for in Fig. 26 for Blasius flow. Fig. 20 depicts the effect of Convective boundary condition parameter on the temperature profile for Sakiadis flow and it is seen that increase in the Convective parameter increases the temperature boundary layer thickness across the plate confirming the existing literature. The same effect was observed for in Fig. 27 for Blasius flow. Fig. 21 depicts the effect of Heat generation parameter on the temperature profile for Sakiadis flow and it is seen that increase in the Heat generation parameter increases the temperature boundary layer thickness across the plate confirming the existing literature. The same effect was observed for in Fig. 28 for Blasius flow.

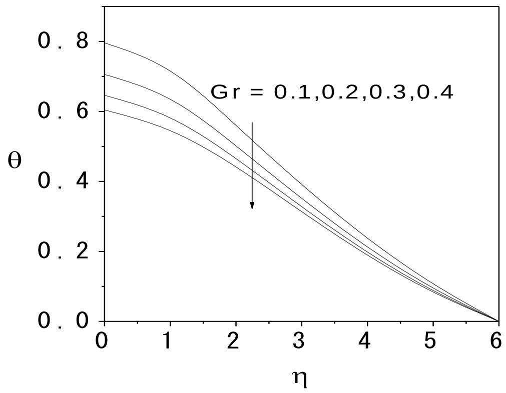

Fig. 15. Variation of the temperature θ with *Gr* for Pr = 0.72, M = γ =0.1, N_R = 0.7, Q = 0.01, Ec=1. (Sakiadis flow)

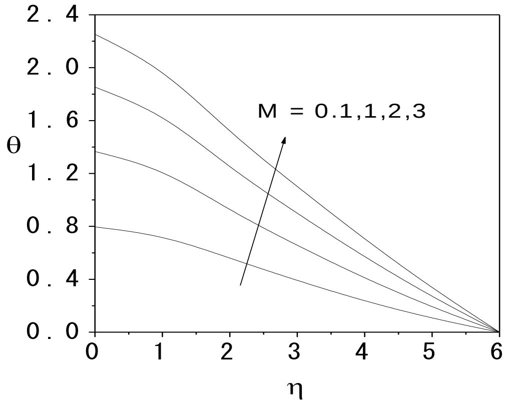

Fig. 16. Variation of the temperature θ with *M* for Pr = 0.72, Gr = γ =0.1, N_R=0.7, Q =0.01,Ec=1. (Sakiadis flow)

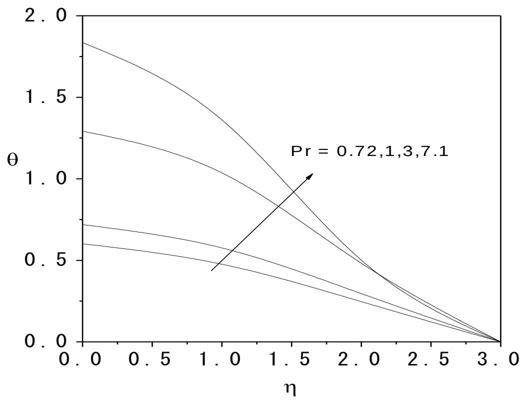

Fig. 17. Variation of the temperature θ with *Pr* for Gr=M = γ =0.1, N_R=0.7, Q=0.01,Ec=1. (Sakiadis flow)

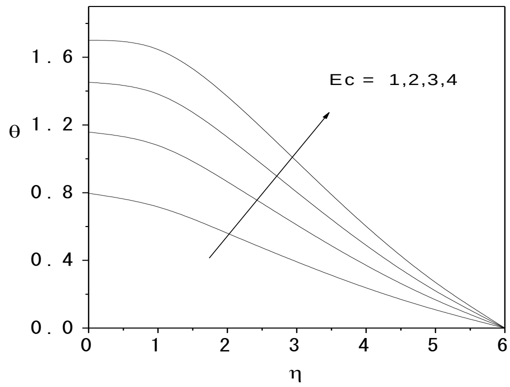

Fig. 18. Variation of the temperature θ with *Ec* for Pr=0.72, Gr=M = γ =0.1,Q=0.01, N_R=0.7. (Sakiadis flow)

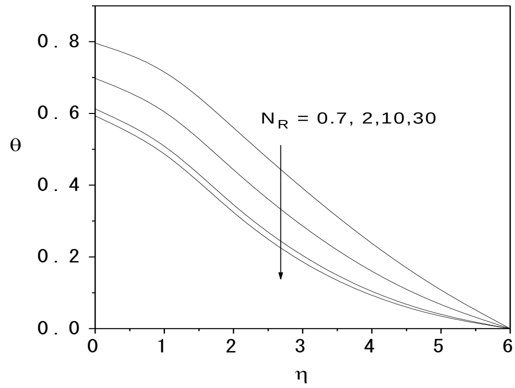

Fig. 19. Variation of the temperature θ with N_R for Pr=0.72, Gr=M = γ =0.1, Q=0.01,Ec=1. (Sakiadis flow)

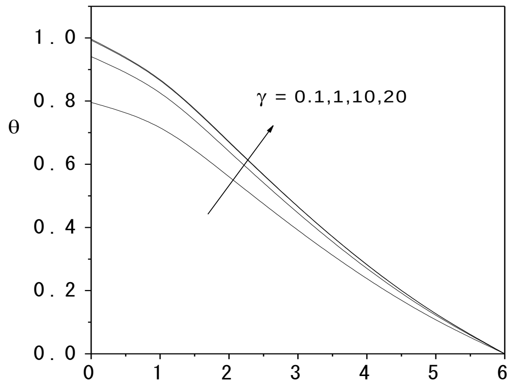

Fig. 20. Variation of the temperature θ with γ for Pr = 0.72, Gr=M =0.1, N_R=0.7, Q =0.01, Ec=1. (Sakiadis flow)

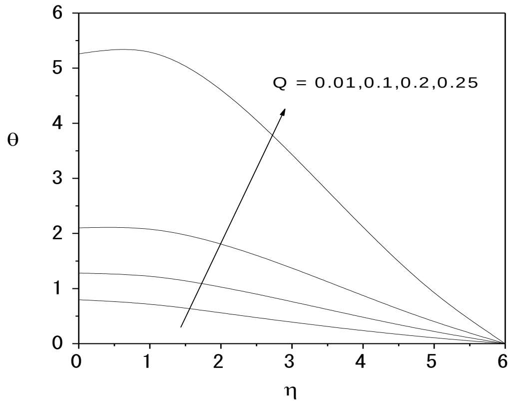

Fig. 21. Variation of the temperature θ with Q for Pr=0.72, Gr= γ =M =0.1, N_R=0.7, Ec=1. (Sakiadis flow)

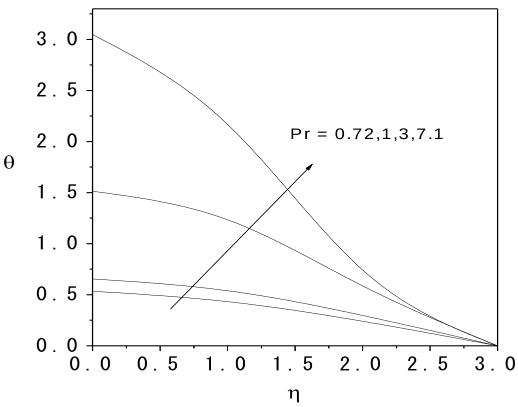

Fig. 24. Variation of the temperature θ with *Pr* for Gr=M = γ =0.1, N_R=0.7, Q=0.01, Ec=1. (Blasius flow)

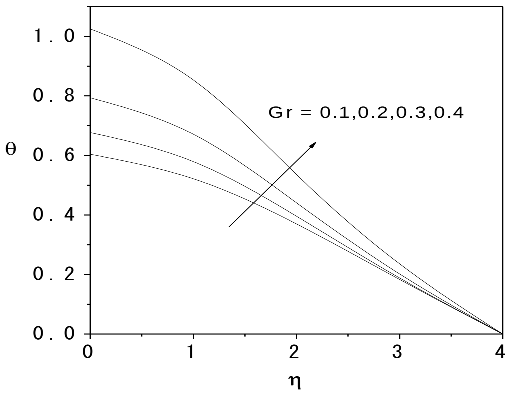

Fig. 22. Variation of the temperature θ with *Gr* for Pr=0.72, M = γ =0.1, N_R=0.7, Q=0.01, Ec=1. (Blasius flow)

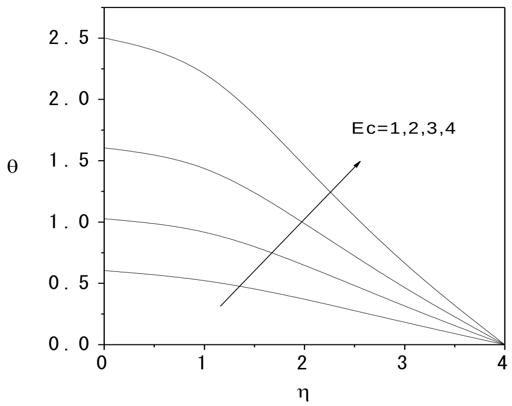

Fig. 25. Variation of the temperature θ with *Ec* for Pr=0.72, Gr=M = γ =0.1, Q=0.01, N_R=0.7. (Blasius flow)

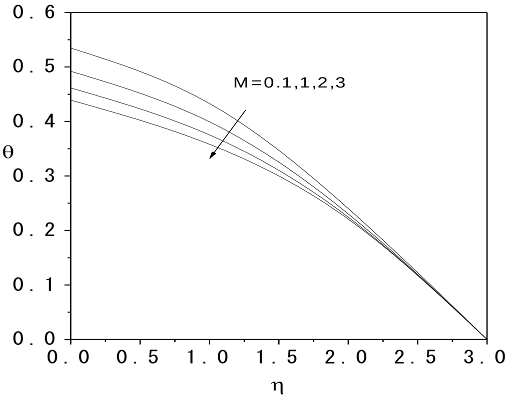

Fig. 23. Variation of the temperature θ with *M* for Pr = 0.72, Gr = γ = 0.1, N_R=0.7, Q=0.01, Ec=1. (Blasius flow)

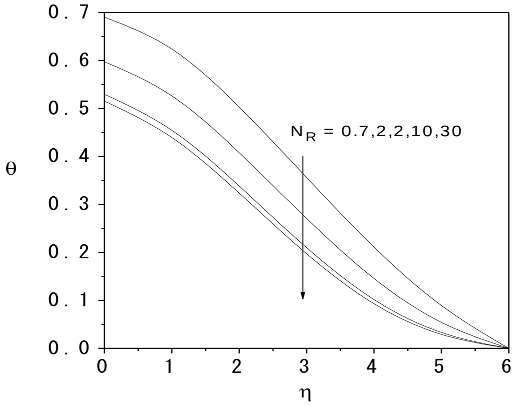

Fig. 26. Variation of the temperature θ with N_R for Pr=0.72, Gr=M = γ =0.1, Q=0.01, Ec=1. (Blasius flow)

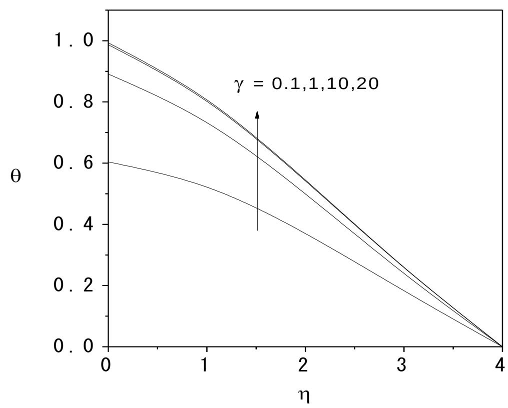

Fig. 27. Variation of the temperature θ with γ for Pr = 0.72, Gr =M =0.1, N_R=0.7, Q=0.01, Ec=1. (Blasius flow)

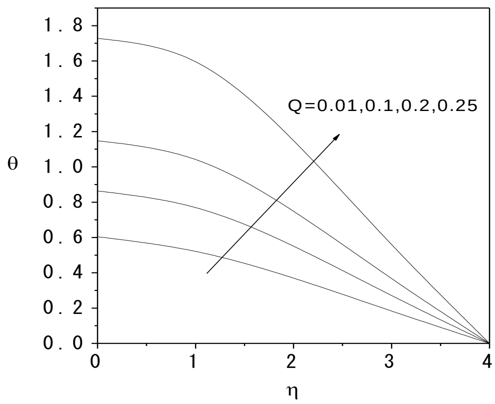

Fig. 28. Variation of the temperature θ with Q for Pr=0.72, Gr= γ =M =0.1, N_R=0.7, Ec=1. (Blasius flow)

5. CONCLUSION

In this article studied the numerical solutions of the Blasius and Sakiadis momentum, thermal boundary layer over a horizontal flat plate and heat transfer in the presence of Magnetic parameter, thermal radiation and the viscous dissipation parameters under a convective surface boundary condition. The lower boundary of the plate is at a constant temperature T_f whereas the upper boundary of the surface is maintained at a constant temperature T_w. It is also noted that the temperature of the free stream is assumed as and also we have $T_f > T_w > T_\infty$. where T_w is the temperature at the wall surface. The transformed partial differential equations together with the boundary conditions are solved numerically by a shooting technique along with the forth order Runge-Kutta method for better accuracy. Comparisons have been analyzed and the numerical results are listed and graphed. The combined effects of increasing the Grashof number, the Eckert number, the Prandtl number and the convective parameter tend to reduce the velocity boundary layer thickness along the plate except for the magnetic field parameter, Heat generation parameter and radiation parameter. The combined effects of increasing the Eckert number, the Prandtl number and the convective parameter tend to reduce the thermal boundary layer thickness along the plate which as a result yields a reduction in the fluid temperature. On the contrary, the values of $\theta(0)_{Blasius}$ *and* $\theta(0)_{Sakiadis}$ increase with increasing Pr, Ec and decreases with increasing γ. In general, the Blasius flow gives a thicker thermal boundary layer compared with the Sakiadis flow, but this trend can be reversed at low values of embedded parameters Sakiadis controlling the flow model. Finally, in the limiting cases, $N_R \to \infty (i.e., k_0 \to 1)$ the thermal radiation influence can be neglected.

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