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MHD Boundary Layer Flow near Stagnation Point of Linear Stretching Sheet with Variable Thermal Conductivity via He's Homotopy Perturbation Method

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ABSTRACT

MHD boundary layer flow near stagnation point of linear stretching sheet with variable thermal conductivity are solved using He's Homotopy Perturbation Method (HPM), which is one of the semi-exact method. Similarity transformation has been used to reduce the governing differential equations into an ordinary non-linear differential equation. The main advantage of HPM is that it does not require the small parameter in the equations and hence the limitations of traditional perturbations can be eliminated. In this paper firstly, the basic idea of the HPM for solving nonlinear differential equations is briefly introduced and then it is employed to derive solution of nonlinear governing equations of MHD boundary layer flow with nonlinear term. The influence of various relevant physical characteristics are presented and discussed.

Keywords: MHD; Homotopy Perturbation Method (HPM); Stretching sheet; Thermal conductivity.

NOMECLATURE

B_0	constant applied magnetic field
b	free stream velocity parameter
C_p	specific heat of the fluid at constant pressure
c	stretching parameter
f	dimensionless stream function
K	permeability of porous media
M	dimensionless magnetic field parameter
Pr	prandtl number
p	pressure
Q	volumetric rate of heat generation/absorption
q_r	radiation heat flux
R	radiation parameter
S	heat source/sink parameter
T	absolute temperature
T_w	temperature at stretching sheet
T_∞	free stream temperature
U	free stream velocity
u	velocity component in x direction
u_w	velocity of stretching sheet
v	velocity component in y direction
x	coordinate in direction of stretching sheet
y	coordinate in direction normal to stretching sheet
ϵ	perturbation parameter
η	dimensionless similarity variable
θ	dimensionless temperature
κ	thermal conductivity
κ^*	variable thermal conductivity
λ	ratio of free stream parameter
μ	viscosity of fluid
ν	kinematic viscosity
ρ	density of fluid
σ	electrical conductivity
τ_w	shear stress
Ψ	stream function
Ω	permeability parameter
Superscript	
$'$	derivative with respect to η
Subscripts	
w	properties at the plate
∞	free stream condition

1. INTRODUCTION

The fluid dynamics due to a stretching sheet is important in extrusion processes. The production of sheeting material arises in a number of industrial manufacturing processes and includes both metal and polymer sheets like the cooling of an infinite metallic plate in a cooling bath, paper production, wire drawing, drawing of plastic films, glass blowing, etc. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products. In view of these applications, Sakiadis (1961) first investigated the boundary layer flow on a continuous solid surface moving at constant speed. MHD flow in electrically conducting fluid can control the rate of cooling therefore; desired quality of product can be achieved. Flow in the neighborhood of a stagnation point in a plane was initiated by Hiemenz (1911). Stagnation point flows have been discussed by Pai (1956), Schlichting (1968) and Bansal (1977) and many other researchers. Kay (1966) reported that thermal conductivity of liquids with low Prandtl number varies linearly with temperature in range of 0°F to 400°F. Crane (1970) presented the flow over a stretching sheet and obtained similarity solution in closed analytical form. Arunachalam and Rajappa (1978) considered forced convection in liquid metals (fluid with low Prandtl number) with variable thermal conductivity and capacity in potential flow and derived explicit closed form of analytical solution. Fluid flow and heat transfer characteristics on stretching sheet with variable temperature condition have been investigated by Grubka and Bobba (1985). Watanabe (1988) discussed stability of boundary layer and effect of suction/injection in MHD flow under pressure gradient. Convective heat transfer at a stretching sheet has been presented by Vajravelu and Nayfeh (1978). Sharma and Jat (1994) analyzed flow and heat transfer between two vertical plates under viscous fluid injection through porous plate, the other being a stretching sheet. Chiam (1997) discussed the heat transfer in fluid flow on stretching sheet at stagnation point in presence of internal dissipation, heat source/sink and stress with constant fluid properties. Chen (1998) considered laminar mixed convection adjacent to vertical, continuously stretching sheet. Chaim (1998) studied heat transfer in fluid flow of low Prandtl number with variable thermal conductivity induced due to stretching sheet and compared the numerical results with perturbation solution. Chamka and Khaled (2000) considered Hiemenz flow in the presence of magnetic field through porous media. Sharma and Mishra (2001) investigated steady MHD flow through horizontal channel: lower being a stretching sheet and upper being a permeable plate bounded by porous medium. Sriramalu *etal.* (2001) studied steady flow and heat transfer of a viscous incompressible fluid flow through porous medium over a stretching sheet. Mahapatra and Gupta (2001) investigated the magnetohydrodynamic stagnation-point flow towards isothermal stretching sheet and pointed that velocity decreases/increases with the increase in magnetic field intensity when free stream velocity is smaller/greater, respectively than the stretching velocity. Mahapatra and Gupta (2002) studied heat transfer in stagnation-point flow towards stretching sheet with viscous dissipation effect. Khan et. al. (2003) presented viscoelastic MHD flow, heat and mass transfer over a porous stretching sheet with dissipation energy and stress work. Pop *etal*. (2004) discussed the flow over stretching sheet near a stagnation point taking radiation effect. Seddeek and Salem (2005) investigated the heat and mass transfer distributions on stretching surface with variable viscosity and thermal diffusivity. Vyas and Srivastava (2010) present a numerical study for the steady two-dimensional radiative MHD boundary−layer flow of an incompressible, viscous, electrically conducting fluid caused by a non-isothermal linearly stretching sheet placed at the bottom of fluid saturated porous medium.

Most of the problems in MHD area are nonlinear. Except a limited number of these problems have precise analytical solution, most of them do not have exact solution, and so these nonlinear equations should be solved using other proper methods. Most scientists believe that the combination of numerical and semi-exact analytical methods can lead to applicable results. In this paper one of the semi-exact method which is called HPM has been introduced and applied in MHD boundary layer flow near stagnation point of linear stretching sheet with variable thermal conductivity. The initial work in HPM was studied by J. H. He (1998, 2000, 2001, 2005, 2009) and after that these investigations inspired a lot of researchers Ariel *et al.* (2006), Beléndez *et. al* (2008), Ganji and Rajabi (2006), Ganji and Ganji (2008), Hosein *et al.* (2008), Jhankal (2014), Ma *et. al* (2008), Siddiqui *et al*. (2008), Zhang and He (2006), Zhang *et al.* (2008) and many others to solve nonlinear equations with this method.

Motivated by the above investigations and practical applications, the present paper deals with the steady of effects of variable thermal conductivity, heat source/sink and variable free stream on flow of a viscous incompressible electrically conducting fluid and heat transfer on a non-conducting stretching sheet in the presence of transverse magnetic field near a stagnation point. The governing partial differential equations are transformed into ordinary differential equations using similarity transformation and then solved using He's Homotopy Perturbation Method (HPM), which is one of the semi-exact methods. The heat source and sink is included in the work to understand the effect of internal heat generation and absorption Chaim (1998).

1.1 Basic idea of homotopy perturbation method (HPM)

To illustrate the basic ideas of the HPM, we consider the following nonlinear differential equation

$$A(u) - f(r) = 0,\ r \in \Omega \tag{1}$$

Subject to the boundary conditions

$$B\left(u, \frac{\partial u}{\partial \eta}\right) = 0,\ r \in \Gamma \tag{2}$$

Where A is a general differential operator, B is a boundary operator, f(r) is a known analytical function and Γ is the boundary of the domain Ω. A can be divided into two parts which are L and N, where L is linear and N is nonlinear. Therefore Eq. (1) can be rewritten as follows:

$$\mathrm{L}(u) + N(u) - f(r) = 0,\ r \in \Omega \tag{3}$$

By the homotopy perturbation technique, we construct a homotopy

$v(r,p): \Omega \times [0,1] \to R$, which satisfies:

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \ \ p \in [0,1], r \in \Omega \quad (4)$$

where $p \in [0,1]$ is an embedding parameter and u_0 is an initial approximation that satisfies the boundary condition. Obviously, from these definitions we will have:

$$H(v,0) = L(v) - L(u_0) = 0$$

$$H(v,1) = A(v) - f(r) = 0$$

The changing process of p from zero to one is just that of $v(r,p)$ from $u_0(r)$ to $u(r)$. In topology, this is called deformation and $L(v) - L(u_0)$ and $A(v) - f(r)$ are called homotopy. According to the HPM, we can first use the embedding parameter p as a "small parameter" and assuming that the solution of Eq. (4) can be written as a power series in p:

$$v = v_0 + pv_1 + p^2 v_2 \quad (5)$$

Setting $p = 1$, results in the approximate solution of (1):

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots \quad (6)$$

The convergence and stability of this method was shown in Hosein *et al.* (2008).

2. MATHEMATICAL FORMULATION

Consider steady two-dimensional flow of a viscous incompressible electrically conducting fluid of variable thermal conductivity in the vicinity of a stagnation point on a non-conducting stretching sheet in the presence of transverse magnetic field and volumetric rate of heat generation/absorption. The stretching sheet has uniform temperature T_w, linear velocity $u_w(x)$. It is assumed that external field is zero, the electric field owing to polarization of charges and Hall Effect are neglected. Stretching sheet is placed in the plane $y = 0$ and x-axis is taken along the sheet. The fluid occupies the upper half of the plane i.e. $y > 0$.

The governing equations of continuity, momentum and energy under the influence of externally imposed transverse magnetic field with variable thermal conductivity in the boundary layer are (Bansal 1994):

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (7)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\frac{\partial^2 u}{\partial x^2} - \frac{\sigma B_0^2}{\rho}u - \frac{v}{K}u \quad (8)$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\kappa^*\frac{\partial T}{\partial y}\right) + Q(T - T_0) - \frac{\partial q_r}{\partial y} \quad (9)$$

In the free stream $u = U(x) = bx$, Eq. (8) reduces to:

$$U\frac{dU}{dx} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\sigma B_0^2}{\rho}U \quad (10)$$

Eliminating $\frac{\partial p}{\partial x}$ between Eq. (8) and Eq. (10), we obtain:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + v\frac{\partial^2 u}{\partial x^2} - \frac{\sigma B_0^2}{\rho}(u - U) - \frac{v}{K}u \quad (11)$$

Along with the boundary conditions

$$y = 0: \ \ u = u_w(x) = cx, \ \ v = 0, \ \ T = T_w$$

$$y \to \infty: \ \ u = U(x), \ \ T = T_w \quad (12)$$

Using the Rosseland approximation for radiation (Brewster 1972), the radiation heat flux q_r could be expressed by:

$$q_r = \frac{4\sigma^*}{3\kappa_0}\frac{\partial T^4}{\partial y} \quad (13)$$

Where σ^* the represents the Stefan-Boltzman constant and κ_0 is the Rosseland mean absorption coefficient. Assuming the temperature difference within the flow is such T^4 that may be expanded in a Taylor series about T_∞ and neglecting higher orders we get:

$$T^4 \cong 4T_\infty^3 T - T_\infty^4 \quad (14)$$

Following Arunachalam and Rajappa (1978) and Chaim (1997), the thermal conductivity κ^* is taken of form as given below

$$\kappa^* = \kappa(1 + \epsilon\theta) \quad (15)$$

The continuity Eq. (7) is satisfied by introducing a stream function Ψ such that

$$u = \frac{\partial \Psi}{\partial y} \text{ and } = -\frac{\partial \Psi}{\partial x} \quad (16)$$

The momentum and energy equations can be transformed into the corresponding ordinary nonlinear differential equations by the following transformation:

$$\eta = y\sqrt{\frac{c}{v}}, \ \Psi = \sqrt{cv}xf(\eta), \ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (17)$$

Where η is the independent similarity variable. The transformed nonlinear ordinary equations are:

$$f''' + ff'' - f'^2 - M^2(f' - \lambda) - \Omega f' + \lambda^2 = \quad (18)$$

$$(1 + R + \epsilon\theta)\theta'' + \epsilon\theta'^2 + \Pr(\theta' f + S\theta) = 0 \quad (19)$$

The transformed boundary conditions are:

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1,$$

$$f'(\infty) \to 0, \theta(\infty) \to 0. \quad (20)$$

Where prime denotes differentiation with respect to η, $\lambda = b/c$ is the ratio of free stream velocity parameter to stretching sheet parameter, $\Omega = v/Kc$ is the permeability parameter, $M^2 = \frac{\sigma B_0^2}{\rho c}$ is the dimensionless magnetic parameter, $R = \frac{16\sigma^* T_\infty^3}{3\kappa_0\kappa}$ is the thermal radiation parameter, $S = Q/\rho c C_p$ is the heat source/sink parameter and $Pr = {}^{\mu C_p}/_{\kappa}$ is the Prandtl number.

Solution with Homotopy perturbation method (HPM):

According to the HPM, the homotopy form of Eq. (18) and Eq. (19) are constructed as follows:

$$(1-p)\left(f''' - m_1^2 f'\right) + p\left(f''' + ff'' - f'^2 - m_1^2 f'\right) = -m_2 \quad (21)$$

$$(1-p)\left[(1+R)\theta'' PrS\theta\right] + p\left[(1+R)\theta'' + \epsilon\theta\theta'' + Prf\theta' + PrS\theta\right] = 0 \quad (22)$$

Where $m_1^2 = M^2 + \Omega$, and $m_2 = M^2\lambda + \lambda^2$.

We consider f and θ as the following:

$$f = f_0 + pf_1 + p^2 f_2$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 \quad (23)$$

By substituting Eq. (23) into Eq. (21) and Eq.(22), and then,

(I) Terms independent of p give:

$$f_0''' - m_1^2 f_0' = -m_2 \quad (24)$$

$$(1+R)\theta_0'' = 0 \quad (25)$$

The boundary conditions are:

$$f_0(0) = 0,\ f_0'(0) = 1,\ f_0'(\infty) = 0,\ \theta_0(0) = 1,\ \theta_0(\infty) = 0. \quad (26)$$

(II) Terms containing only p give:

$$f_1''' - m_1^2 f_1' = f_0'^2 - f_0 f_0'' \quad (27)$$

$$(1+R)\theta_1'' = -(\epsilon\theta_0''\theta_0 + \epsilon\theta_0'^2 + Prf_0\theta_0') \quad (28)$$

The boundary conditions are:

$$f_1(0) = 0,\ f_1'(0) = 0,\ f_1'(\infty) = 0,\ \theta_1(0) = 0,\ \theta_1(\infty) = 0 \quad (29)$$

Solving Eqs. (24)-(25) and (27)-(28) with boundary conditions (26) and (29) respectively, we have:

$$f_0 = A_3 + A_2 e^{m_1\eta} + A_1 e^{-m_1\eta} + \frac{\eta m_2}{m_1^2} \quad (30)$$

$$f_1 = A_4 + A_5 e^{m_1\eta} + A_6 e^{-m_1\eta} + \left(\frac{m_2A_2}{m_1^3} - \frac{A_1A_2}{2}\right)\eta e^{m_1\eta} + \left(-\frac{m_2A_3}{m_1^3} - \frac{A_1A_3}{2}\right)\eta e^{-m_1\eta} + \frac{3m_2A_2}{4m_1^3}\eta^2 e^{m_1\eta} - \frac{3m_2A_3}{4m_1^3}\eta^2 e^{-m_1\eta} - \frac{m_2A_2}{4m_1^2}\eta^3 e^{m_1\eta} - \frac{m_2A_3}{4m_1^2}\eta^3 e^{-m_1\eta} + \left(4A_2A_3 - \frac{m_2^2}{m_1^6}\right)\eta \quad (31)$$

$$\theta_0 = cosC\eta + B_1 sinC\eta \quad (32)$$

$$\theta_1 = B_2 sinC\eta + B_3 cosC\eta + B_5 cos2C\eta + B_6 sin2C\eta + B_7\eta cosC\eta + B_8\eta sinC\eta + B_9\eta^2 cosC\eta + B_{10}\eta^2 sinC\eta + B_{11}e^{m_1\eta}cosC\eta + B_{12}e^{m_1\eta}sinC\eta + B_{13}e^{-m_1\eta}cosC\eta + B_{14}e^{-m_1\eta}sinC\eta \quad (33)$$

The constant coefficients, can be calculated using boundary conditions, the boundary condition η=∞ were replaced by those at η=10 in accordance with standard practice in the boundary layer analysis. If p→1, we find the approximate solution of Eqs. (21) and (22).

The constant coefficient,

$A_i\ (i = 1,2,3,\ldots.8)$, $B_j\ (j = 1,2,3,\ldots.14)$, m_1, m_2 and C are defined as in the Appendix.

3. RESULTS AND DISCUSSION

In order to get a physical insight into the problem, numerical computations are performed for various values of the physical parameters involved in the equations viz. the magnetic parameter M, ratio of free stream parameter λ, permeability parameter Ω, perturbation parameter ϵ, thermal radiation parameter R and fixed value of Prandtl number (Pr) 1.0. To ensure the occurrence of steady flow near the sheet by confining the generated vorticity inside the boundary layer, the magnetic field is taken quite strong by assigning large values of M. Calculated results are presented in Figs 1 to 6 to understand the effect of parameters on flow and temperature field.

On the other hand, the skin friction coefficient $f''(0)$ and heat transfer coefficient $-\theta'(0)$ against magnetic parameter (M), for various values λ, and given values of Ω, Pr, ϵ, R and S are presented in Figs. 7 and 8 respectively.

Figure 1 shows the effect of magnetic parameter M on velocity profile for λ=0.5 and Ω=0.25. From this figure it is observed that the dimensionless velocity $f'(\eta)$ decreases with increasing values of M. This happens due to Lorentz force arising from the interaction of magnetic and electric fields during the motion of the electrically conducting fluid. This force has the tendency to slow down the motion of fluid in the boundary layer.

Figure 2 shows the effect of λ on the velocity profile. From this plot it is observed that the velocity increases with increasing values of λ. Accordingly, the thickness of momentum boundary layer decreases.

Figure 3 shows the effect of Ω on the velocity profile. We infer from this figure that the velocity decreases with increasing values of Ω. This happens due to the increased restriction resulting from decreasing the porosity of porous medium.

Figure 4 is plotted for temperature profiles for different values of M. It is observed that the temperature decreases with magnetic parameter M.

Figure 5 shows the effect of Ω on temperature. We infer from this figure that the temperature decreases with increasing values of Ω.

Figure 6 illustrates the effect of λ on the temperature profile. We infer from this figure that the temperature decreases for η ≤ 3.5 and increases for η ≥ 3.5 with increasing values of λ.

Figure 7 represents the skin friction parameter against magnetic parameter M for various values of λ. It is noted that for increasing values of λ, the skin friction increases.

Figure 8 represents the variation of the temperature gradient which is significant in evaluating the rate of heat transfer. The rate of heat transfer increases with increasing values of λ.

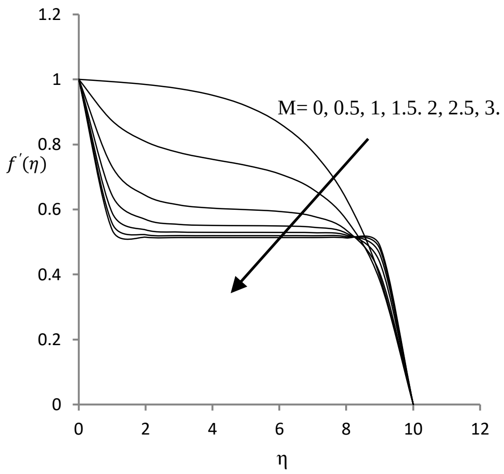

Fig. 1. Velocity profile for various values of magnetic parameter M, when λ=0.5 and Ω=0.25.

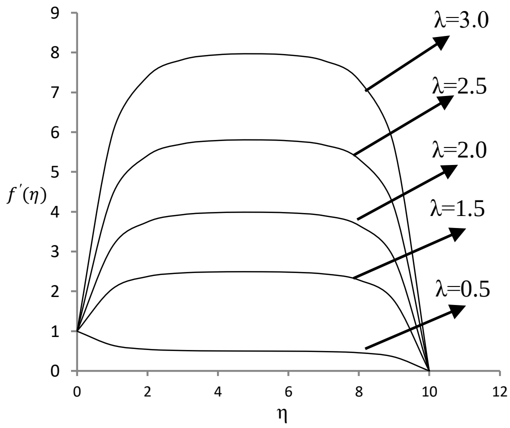

Fig. 2. Velocity profile for various values of λ, when M=1.0 and Ω=0.5.

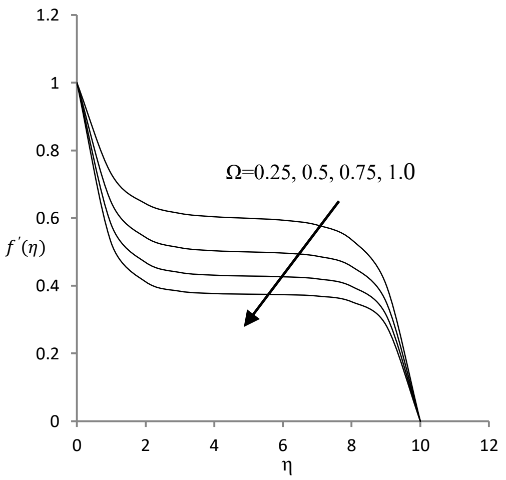

Fig. 3. Velocity profile for various values of Ω, when M=1.0 and λ =0.5.

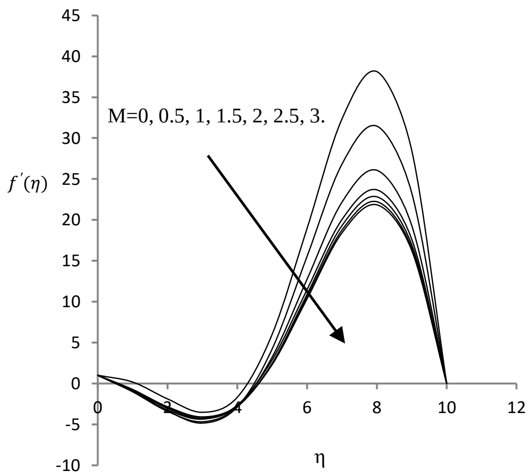

Fig. 4. Temperature profile for various values of magnetic parameter M, when Pr=1.0, λ=0.5, Ω=0.25, R=0.25, S=0.4 and ϵ=0.1.

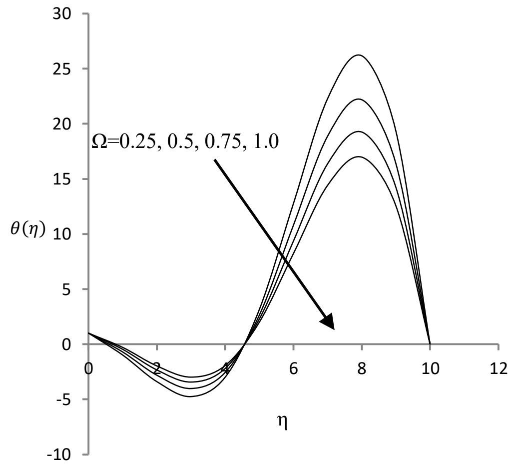

Fig. 5. Temperature profile for various values of Ω, when Pr=1.0, λ=0.5, M=1.0, R=0.25, S=0.4 and ϵ=0.1.

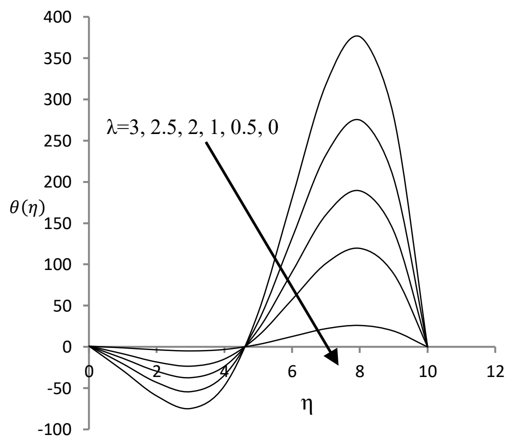

Fig. 6. Temperature profile for various values of λ, when Pr=1.0, Ω =0.25, M=1.0, R=0.25, S=0.4 and ϵ=0.1.

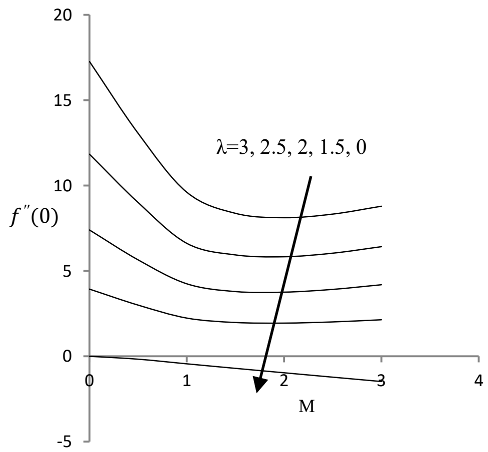

Fig. 7. Skin friction against magnetic parameter M, for various values of λ (when Ω=0.25).

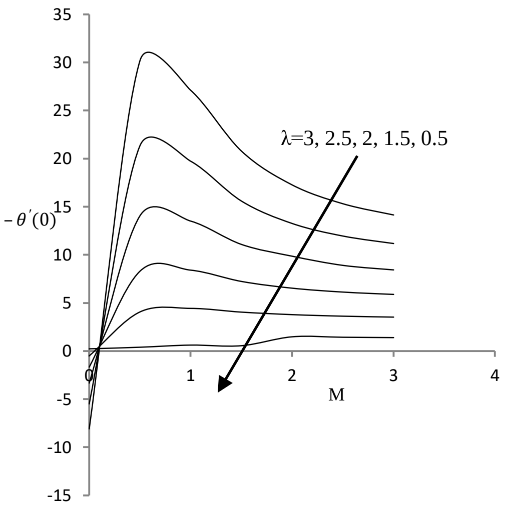

Fig. 8. Nusselt number against magnetic parameter M for various values of λ, when Pr=1.0, Ω =0.25, R=0.25, S=0.4 and ϵ=0.1.

4. CONCLUSION

In this study, the momentum and energy equations are solved with homotopy perturbation method (HPM) of MHD boundary layer flow near stagnation point of linear stretching sheet with variable thermal conductivity. Result shows that velocity and temperature decreases with increasing values of M and Ω and velocity increases with the increasing values of λ, but temperature decreases for η≤3.5 and increases when η>3.5 with increasing values of λ.

It is also noted that for increasing value of λ, the skin friction increases, the similar results can be drawn for rate of heat transfer.

The homotopy perturbation method (HPM) suggested in this article is an efficient method for obtaining the solution of the nonlinear partial differential equations. Therefore, this method is a powerful mathematical tool to solve any system of partial differential equations linear and nonlinear.

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APPENDIX

$m_1=\sqrt{M^2+\Omega},\ m_2=M^2\lambda+\lambda^2$,

$$A_1=\frac{-1}{m_1(e^{10m_1}-e^{-10m_1})}\left[e^{10m_1}+\frac{m_2}{m_1^2}(1-e^{10m_1})\right]$$

$A_2=\frac{1}{m_1}+A_1-\frac{m_2}{m_1^3}$,

$A_3=-(A_1+A_2)$,

$A_4=-(A_5+A_6)$,

$A_5=A_6-\frac{A_7}{m_1}$,

$A_6=\frac{A_7e^{10m_1}-A_8}{m_1(e^{10m_1}-e^{-10m_1})}$,

$A_7=\frac{A_2m_2}{m_1^3}-\frac{A_2A_3}{2}-\frac{A_1m_2}{m_1^3}-\frac{A_1A_3}{2}+4A_1A_2-\frac{m_2^2}{m_1^6}$,

$A_8=\left(\frac{A_2m_2}{m_1^3}-\frac{A_2A_3}{2}\right)(1+10m_1)e^{10m_1}+\left(-\frac{A_1m_2}{m_1^3}-\frac{A_1A_3}{2}\right)(1-10m_1)e^{-10m_1}\frac{A_1m_2}{m_1^3}-\frac{A_2m_2}{4m_1^2}(300+1000m_1)e^{10m_1}+\frac{3A_2m_2}{4m_1^3}(20+100m_1)e^{10m_1}-\frac{A_1m_2}{4m_1^2}(300-1000m_1)e^{-10m_1}-\frac{3A_2m_2}{4m_1^3}(20-100m_1)e^{-10m_1}+4A_1A_2-\frac{m_2^2}{m_1^6}$,

$C=\sqrt{\frac{PrS}{1+R}}$,

$B_1=-cot10C$,

$B_2=\frac{\epsilon(1-B_1^2)}{3(1+R)}+\frac{PrCA_2}{(1+R).(4C^2+m_1^2)}\left\{\frac{2C}{m_1}+B_1\right\}-\frac{PrCA_3}{(1+R).(4C^2+m_1^2)}\left\{\frac{2C}{m_1}-B_1\right\}$,

$B_3=-\frac{\epsilon(1-B_1^2)}{3(1+R)}cos20C-\frac{2\epsilon B_1}{3(1+R)}sin20C-\frac{5Pr}{(1+R)}\left\{A_1+\frac{B_1m_2}{2m_1^2}\right\}cos10C-\frac{5PrB_1}{(1+R)}\left\{A_1-\frac{m_2}{2m_1^2}\right\}sin10C-\frac{50Prm_2}{m_1^2(1+R)}cos10C-\frac{50PrB_1m_2}{m_1^2(1+R)}sin10C-\frac{PrCA_2}{(1+R).(4C^2+m_1^2)}\left\{\frac{2C}{m_1}+B_1\right\}e^{10m_1}cos10C+\frac{PrCA_2}{(1+R).(4C^2+m_1^2)}\left\{\frac{2CB_1}{m_1}+1\right\}e^{10m_1}sin10C+\frac{PrCA_3}{(1+R).(4C^2+m_1^2)}\left\{\frac{2C}{m_1}-B_1\right\}e^{-10m_1}cos10C+\frac{PrCA_3}{(1+R).(4C^2+m_1^2)}\left\{\frac{2C}{m_1}+1\right\}e^{-10m_1}sin10C$,

$B_4=-cosec10C(B_2cos10C+B_3)$,

$B_5=-\frac{\epsilon(1-B_1^2)}{3(1+R)}$,

$B_6=-\frac{2\epsilon B_1}{3(1+R)}$,

$B_7=-\frac{Pr}{2(1+R)}\left\{A_1+\frac{B_1m_2}{2m_1^2}\right\}$,

$B_8=-\frac{PrB_1}{2(1+R)}\left\{A_1-\frac{m_2}{2m_1^2}\right\}$,

$B_9=-\frac{Prm_2}{2m_1^2(1+R)}$,

$B_{10}=-\frac{PrB_1m_2}{2m_1^2(1+R)}$,

$B_{11}=-\frac{PrCA_2}{(1+R).(4C^2+m_1^2)}\left\{\frac{2C}{m_1}+B_1\right\}$,

$B_{12}=-\frac{PrCA_2}{(1+R).(4C^2+m_1^2)}\left\{\frac{2CB_1}{m_1}+1\right\}$,

$B_{13}=-\frac{PrCA_3}{(1+R).(4C^2+m_1^2)}\left\{\frac{2C}{m_1}-B_1\right\}$,

$B_{14}=-\frac{PrCA_3}{(1+R).(4C^2+m_1^2)}\left\{\frac{2C}{m_1}+1\right\}$.