

# Nonlinear Radiation Effects on Hydromagnetic Boundary Layer Flow and Heat Transfer over a Shrinking Surface

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## ABSTRACT

The effects of nonlinear radiation on hydromagnetic boundary layer flow and heat transfer over a shrinking surface is investigated in the present work. Using suitable similarity transformations, the governing nonlinear partial differential equations are transformed into nonlinear ordinary differential equations. The resultant equations which are highly nonlinear are solved numerically using Nachtsheim Swigert shooting iteration scheme together with Fourth Order Runge Kutta method. Numerical solutions for velocity, skin friction coefficient and temperature are obtained for various values of physical parameters involved in the study namely Suction parameter, Magnetic parameter, Prandtl number, Radiation parameter and Temperature ratio parameter. Numerical values for dimensionless rate of heat transfer are also obtained for various physical parameters and are shown through tables. The analytical solution of the energy equation when the radiation term is taken in linear form is obtained using Confluent hypergeometric function.

**Keywords:** Magnetic field; Heat transfer; Thermal radiation; Shrinking surface; Confluent hypergeometric function.

## NOMENCLATURE

$a$	shrinking rate	$T_\infty$	temperature of the free stream fluid
$B_0$	magnetic field strength	$u$	velocity in the $x$ direction
$C_f$	skin friction coefficient	$v$	velocity in the $y$ direction
$C_p$	specific heat at constant pressure	$v_0$	constant suction velocity
$F$	dimensionless velocity	$x$	dimensional distance along the sheet
$k$	thermal conductivity of the fluid	$y$	dimensional distance normal to the sheet
$k^*$	Rosseland mean absorption coefficient	$\eta$	dimensionless coordinate
$M^2$	magnetic parameter	$\theta$	dimensionless temperature
$Nu_x$	local Nusselt number	$\theta_w$	temperature ratio parameter
$Pr$	Prandtl number	$\mu$	dynamic viscosity of the fluid
$q_w$	heat flux at the surface	$\nu$	kinematic viscosity of the fluid
$q_r$	radiative heat flux	$\rho$	density of the fluid
$Rd$	radiation parameter	$\sigma$	electrical conductivity of the fluid
$Re_x$	local Reynolds number	$\sigma^*$	Stefan Boltzmann constant
$S$	Suction parameter	$\tau_w$	shear stress at the wall
$T$	temperature of the fluid	$\psi$	stream function
$T_w$	temperature at the wall		

## 1. INTRODUCTION

Radiation heat transfer effects on forced convection flows are very important in space technology and high temperature processes and lot of studies have been reported on the effects of radiation on the boundary layer flow of an electrically conducting fluid past a body.

Soundalgekar *et al.* (1960) examined the radiation effects on free convection flow of a gas past a semi-infinite flat plate. Viskanta and Grosh (1962) investigated the boundary layer in thermal radiation absorbing and emitting media. Mosa (1979) discussed one of the models for combined radiative hydromagnetic heat transfer. The effects of radiation of an optically dense viscous incompressible fluid past a heated vertical plate with uniform free stream velocity and surface

temperature was explored by Hossain and Takhar (1996). In this investigation, consideration had been given to gray gases that emit and absorb, but do not scatter thermal radiation. The Rosseland diffusion approximation offers one of the most straightforward simplifications of the full integro-partial differential equations.

The effects of radiation on the free convection heat transfer problem in the absence of a magnetic field and viscous dissipation was studied by Hossain *et al.* (1999). Elbashareshy (2000) investigated the radiation effect on heat transfer over a stretching surface. He considered the full term expansion for the radiation term. Duwari and Damseh (2004) presented the radiation-conduction interaction in free and mixed convection fluid flow for a vertical flat plate in the presence of a magnetic field. Radiation effects on the flow near the Stagnation point of a stretching sheet were investigated by Pop *et al.* (2004). Cortell (2008) analyzed the radiation effect in Blasius flow. Aydin and Kaya (2009) studied MHD mixed convective heat transfer flow about an inclined plate.

Swati Mukhopadhy *et al.* (2011) discussed the effect of nonlinear radiation on steady boundary layer flow and heat transfer over a porous moving plate. Anjali Devi and David Maxim Gururaj (2012) considered the effects of variable viscosity and nonlinear radiation on MHD flow with heat transfer over a surface stretching with a power law velocity. Misra and Sinha (2013) studied the effects of Hall current and thermal radiation on flow of an electrically conducting and radiating fluid through a porous medium. The effect of thermal radiation on unsteady flow and heat transfer of MHD micropolar fluid over a stretching sheet subjected to suction was investigated by Shit *et al.* (2013).

Recently, the boundary layer flow due to shrinking sheet has attracted considerable interest. In particular in the past few years much attention has been focused for the study of different types of flow and heat transfer over a shrinking sheet for various fluids due to its numerous applications. The velocity on the boundary is towards a fixed point and so the flow over a shrinking sheet is physically different from that of the stretching sheet. It is also shown that the mass suction is required generally to maintain the flow over the shrinking sheet.

Micklavcic and Wang (2006) were the first to investigate the flow over a shrinking sheet. Wang (2008) investigated the stagnation flow towards a shrinking sheet. Yao and Chen (2009) examined analytical solution branch for the Blasius equation with a shrinking sheet.

Nadeem *et al.* (2009) obtained the series solution for the stagnation flow of a second grade fluid over a shrinking sheet using homotopy analysis method. Cortell (2010) discussed on certain boundary value problem arising in shrinking sheet flows. Thermal boundary layer over a shrinking sheet is investigated by Fang and Zhang (2010) for prescribed power law wall temperature and power law wall heat flux case.

Shrinking sheet problems are also extended for electrically conducting fluids. Hayat *et al.* (2007) reported the analytical solution of magnetohydrodynamic flow of second grade fluid over a shrinking sheet. Sajid *et al.* (2008) analyzed rotating flow of a viscous fluid over a shrinking sheet for electrically conducting fluids.

Noor *et al.* (2009) investigated MHD flow adjacent to permeable shrinking sheet in a porous medium. Ali *et al.* (2010) observed the MHD viscous flow and heat transfer due to permeable shrinking sheet with constant surface heat flux. Ali *et al.* (2010) analysed the effect of radiation on unsteady three dimensional flows and heat transfer past a permeable shrinking sheet. Javed *et al.* (2011) analyzed the viscous dissipation effect on steady hydromagnetic viscous fluid for nonlinear shrinking sheet. The effects of suction/blowing on steady boundary layer stagnation point flow and heat transfer towards a shrinking sheet with thermal radiation was carried out by Bhattacharyya and Layek (2011). Jafar *et al.* (2012) investigated MHD flow and heat transfer over stretching/shrinking sheets with external magnetic field, viscous dissipation and Joule effects without the radiation effect. Recently, Mahapatra and Nandy (2013) studied on momentum and heat transfer in MHD axisymmetric stagnation point flow over a shrinking sheet.

Thermal radiation effects might play a significant role in controlling heat transfer processes in polymer processing industry. The radiative flows of an electrically conducting fluid with high temperature in the presence of magnetic field are encountered in electrical power generation, space vehicle re-entry, nuclear engineering applications and other industrial areas. Owing to these applications, the present work mainly deals with a problem of such kind. The paper discusses the effect of magnetic field on the forced convection flow of a viscous, incompressible, electrically conducting and radiating fluid from a shrinking sheet with uniform rate of suction and uniform magnetic field. Many outstanding theoretical models have been developed for radiative convection flows and radiation conductive transport. As a result, in most of the investigations, the radiation term appears in linear form. A new dimension is added to hydromagnetic boundary layer flow over a shrinking surface by the consideration of radiation term in full form. Rosseland approximation is utilized in the energy equation which gives the value for  $q_r$ . When the nonlinear radiation is considered, the assumption that the temperature difference with in the flow is sufficiently small is not made and hence the effect of radiation is taken in real sense. Based on these, the current work investigates the nonlinear radiation effects on hydromagnetic boundary layer flow and heat transfer over a shrinking surface.

## 2. PHYSICAL MODEL

### 2.1 Governing Equations

The steady, two-dimensional, hydromagnetic boundary layer flow of a viscous, incompressible, electrically conducting and radiating fluid over a shrinking surface with nonlinear radiation effects is considered. It is assumed that the components of the velocity be  $(u, v)$ . The applied magnetic field is assumed to be in the  $y$  direction and is taken as  $B_0 = B_0 \hat{j}$ . The flow configuration and the coordinate system are shown in Fig. 1.

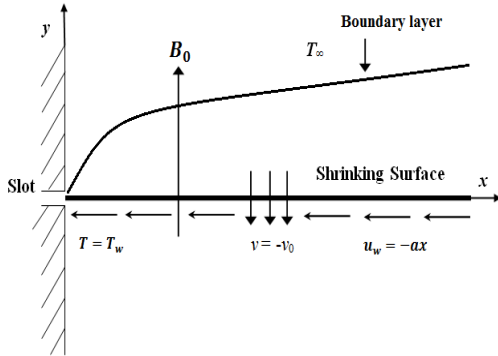


Fig. 1. Flow model and coordinate system

The fluid has constant physical properties. The fluid is assumed to be gray, emitting and absorbing, but non-scattering medium. The  $x$  axis extends parallel to the shrinking surface, while  $y$  axis extends upwards normal to the surface. The radiative heat flux in the  $x$  direction is considered to be negligible in comparison to that in the  $y$  - direction. The effects of viscous and Ohmic dissipation are neglected in the energy equation.

Under the usual boundary layer assumptions, the conservation equations of mass, momentum and energy can be described by the following equations:

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum Equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \quad (2)$$

Energy Equation:

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial}{\partial y} (q_r) \quad (3)$$

where  $u$  and  $v$  are the velocity components in the  $x$  and  $y$  directions respectively,  $T$  is the temperature of the fluid,  $\nu$  is the kinematic viscosity,  $\sigma$  is the electrical conductivity of the fluid,  $B_0$  is the magnetic field,  $\rho$  is the density of the fluid,  $C_p$  is the specific heat at constant pressure,  $k$  is the thermal

conductivity of the fluid and  $q_r$  is the radiative heat flux.

The term of Lorentz force in Eq. (2) is derived under the assumption that the induced magnetic field is assumed to be negligible in comparison to that of the applied magnetic field. This is valid when the magnetic Reynolds number is considered as very small. Since the flow is steady,  $\text{curl } E = 0$ . Also  $\text{div } E = 0$  in the absence of surface charge density. Hence  $E = 0$  is assumed.

The radiative heat flux in the energy equation is simplified utilising the Rosseland approximation for radiation [Brewster (1992)] for an optically thick layer and  $q_r$  is written as

$$q_r = -\frac{4\sigma^* T^3}{3k^*} \frac{\partial T}{\partial y} \quad (4)$$

where  $\sigma^*$  is the Stefan-Boltzmann constant,  $k^*$  is the Rosseland mean absorption coefficient. Using Eq. (4), the energy equation becomes

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \frac{\partial}{\partial y} \left( \frac{16\sigma^* T^3}{3k^*} \frac{\partial T}{\partial y} \right) \quad (5)$$

### 2.2 Boundary Conditions

The appropriate boundary conditions for the velocity and temperature are:

i) on the shrinking surface ( $y = 0$ )

$$u = u_w = -ax, \quad v = -v_0 \text{ (no slip wall condition)}$$

$$\text{and } T = T_w \text{ (Constant Surface Temperature)} \quad (6)$$

where  $v_0$  is the constant suction velocity.

ii) Matching with quiescent free stream ( $y \rightarrow \infty$ )

$$u \rightarrow 0, \quad T \rightarrow T_\infty \quad (7)$$

where  $a > 0$  is a dimensional constant called shrinking rate, the subscripts  $w$  and  $\infty$  refer to the wall and boundary layer edge respectively.

To examine the flow regime adjacent to the surface, the following similarity transformations are utilized.

$$\left. \begin{aligned} \eta &= y \sqrt{\frac{a}{\nu}} \\ \psi &= \sqrt{a\nu} x F'(\eta) \\ \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty} \end{aligned} \right\} \quad (8)$$

where  $\psi(x, y)$  is the stream function such that,

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

Now, Eq. (1) is identically satisfied and substituting Eq. (8), Eqs. (2) and (5) reduce to the following nonlinear ordinary differential equations:

$$F''' + F F'' - F'^2 - M^2 F' = 0 \quad (9)$$

$$\frac{\theta''}{Pr} \left( 1 + \frac{4}{3Rd} \{1 + (\theta_w - 1)\theta\}^3 \right) + F\theta' + \frac{4}{RdPr} (1 + (\theta_w - 1)\theta)^2 (\theta_w - 1)\theta'^2 = 0 \quad (10)$$

with boundary conditions

$$F(0) = S, \quad F'(0) = -1, \quad F'(\infty) \rightarrow 0, \quad \theta(0) = 1 \text{ and } \theta(\infty) \rightarrow 0 \quad (11)$$

The nondimensional parameters occurring in the above equations are defined by

$$M^2 = \frac{\sigma B_0^2}{\rho a} \quad \text{Magnetic parameter}$$

$$S = \frac{v_0}{\sqrt{av}} \quad \text{Suction parameter } (v_0 > 0)$$

$$Pr = \frac{\mu C_p}{k} \quad \text{Prandtl number}$$

$$Rd = \frac{k k^*}{4 \sigma^* T_\infty^3} \quad \text{Radiation parameter}$$

$$\theta_w = \frac{T_w}{T_\infty} \quad \text{Temperature ratio parameter}$$

### 2.3 Skin Friction Coefficient and Non-dimensional Rate of Heat Transfer

The wall shear stress on the shrinking surface is specified by

$$\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}$$

The frictional drag or skin friction coefficient is given by

$$\frac{C_f}{2} Re_x^{1/2} = \frac{2\tau_w}{\rho u_w^2} = F''(0) \quad (12)$$

The rate of heat flux at the wall is defined as follows

$$q_w = - \left[ \left( \frac{16\sigma^* T^3}{3k^*} + k \right) \left( \frac{\partial T}{\partial y} \right) \right]_{y=0}$$

and the local Nusselt number is given by

$$\frac{Nu_x}{\sqrt{Re_x}} = - \left( 1 + \frac{4}{3Rd} (\theta_w)^3 \right) \theta'(0) \quad (13)$$

where  $Re_x = \frac{ax^2}{\nu}$  is the local Reynolds number.

### 3. NUMERICAL SIMULATION

Equations (9) and (10) are highly nonlinear and therefore the system cannot be solved analytically. Hence, the Eqs. (9) and (10) with its boundary conditions in Eq. (11) are solved numerically using Nachtsheim Swigert shooting iteration scheme

together with Fourth Order Runge Kutta method. This method is well established and has been successfully implemented to study a variety of nonlinear heat and fluid flow problems which involve asymptotic boundary conditions. Numerical solutions have been carried out for different values of Suction parameter, Magnetic parameter, Prandtl number, Radiation parameter and Temperature ratio parameter. The effect of these parameters over velocity, temperature, skin friction coefficient and dimensionless rate of heat transfer are displayed through figures and tables.

### 4. ANALYTICAL SOLUTION

The exact solution of Eq. (9) subjected to the corresponding boundary conditions in Eq. (11) is

$$F(\eta) = \frac{1}{\alpha} \left( \alpha^2 - M^2 + e^{-\alpha\eta} \right) \quad (14)$$

The analytical solution of the energy equation (10) can be found using Confluent hypergeometric function when the radiation term is taken in linear form that is when  $\theta_w = 1.0$ . Then the equation becomes

$$\frac{\theta''}{Pr} \left( 1 + \frac{4}{3Rd} \right) + F\theta' = 0 \quad (15)$$

The solution of Eq. (15) is given in the form of confluent hypergeometric function subjected to its corresponding boundary condition in Eq. (11) as follows

$$\theta(\eta) = e^{-a_0\alpha\eta} \frac{\Phi \left( a_0, 1+a_0, \frac{P}{\alpha^2} e^{-\alpha\eta} \right)}{\Phi \left( a_0, 1+a_0, \frac{P}{\alpha^2} \right)} \quad (16)$$

The expression for  $\theta'(0)$  is given as

$$\theta'(0) = - \frac{P}{\alpha} \left( \frac{a_0}{1+a_0} \right) \frac{\Phi \left( 1+a_0, 2+a_0, \frac{P}{\alpha^2} \right)}{\Phi \left( a_0, 1+a_0, \frac{P}{\alpha^2} \right)} - \alpha a_0$$

$$\text{where, } \alpha = \frac{S + \sqrt{S^2 + 4(M^2 - 1)}}{2},$$

$$a_0 = \frac{P}{\alpha^2} (\alpha^2 - M^2) \text{ and } P = \frac{3RdPr}{3Rd+4}.$$

### 5. NUMERICAL RESULTS AND DISCUSSION

Nonlinear radiation effects on MHD boundary layer flow and heat transfer over a shrinking surface have been investigated. Numerical solutions are obtained for various values of physical parameters and are illustrated graphically. Parametric studies were conducted by varying the Suction parameter, Magnetic parameter, Prandtl number, Radiation parameter and Temperature ratio parameter. In

order to validate the numerical results obtained, comparison with the analytical results is made under some special cases. When  $\theta_w = 1.0$ , the exact solution of the energy equation is found out using confluent hypergeometric function which is shown in Eq. (16). The values of  $-\theta'(0)$  for various values of Prandtl number and Radiation parameter which are obtained both numerically and analytically are compared and shown through Table which are found to be in excellent agreement. Numerical values for non-dimensional rate of heat transfer for various values of physical parameters are obtained and are presented in Table.

### 5.1 Comparison Result

From Fig. 2 it is noted that when the Temperature ratio parameter is unity (radiation term is taken in linear form) the numerical results for dimensionless temperature for various values of Radiation parameter are identical to the analytical results that are found using confluent hypergeometric function. This comparison justifies the numerical simulation adopted in this investigation.

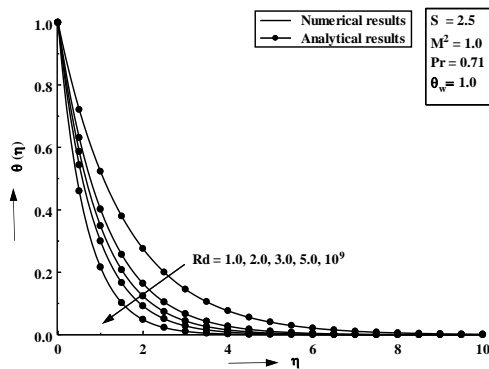


Fig. 2. Comparison graph of dimensionless temperature for various Radiation parameter

### 5.2 Effect of Suction parameter

Figure 3 exhibits the variation in dimensionless velocity for several values of Suction parameter. It is evident from the figure that the effect of Suction parameter is to reduce the momentum boundary layer thickness for its higher values. This is due to the acceleration in dimensionless velocity which causes the boundary layer thickness to decrease.

Skin friction coefficient for various values of Suction parameter against Magnetic parameter is illustrated in Fig. 4. It is seen that the skin friction increases due to increase in Suction parameter at fixed value of Magnetic parameter. Further, the skin friction coefficient rises as the Magnetic parameter ascends along the horizontal axis.

Figure 5 presents the effect of Suction parameter over temperature distribution. The thermal boundary layer thickness decreases with an increase in Suction parameter. The explanation of such behaviour is that the fluid is brought closer to the surface and reduces the thermal boundary layer thickness. It is also clear that the maximum value of the temperature occurs near the surface of the plate.

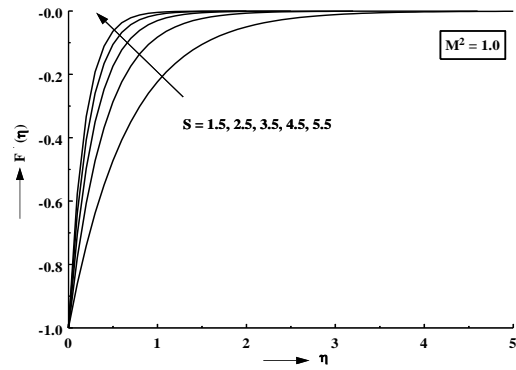


Fig. 3. Effect of Suction parameter over the dimensionless velocity

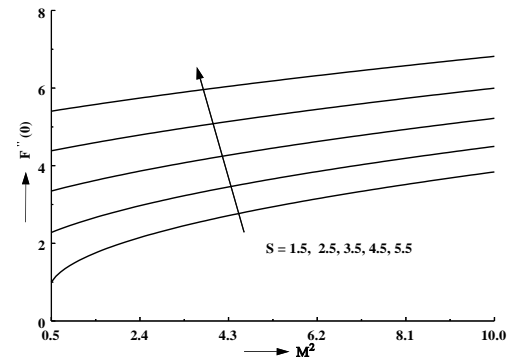


Fig. 4. Skin friction coefficient for different values of Suction parameter

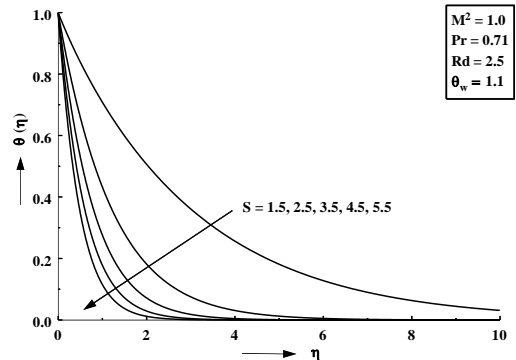


Fig. 5. Dimensionless temperature profiles for various values of Suction parameter

### 5.3 Effect of Magnetic Parameter

From Fig. 6 it is observed that the effect of Magnetic parameter accelerates the dimensionless velocity and consequently the thickness of momentum boundary layer thickness reduces. Thus the Lorentz force arising because of interaction of magnetic and electric fields for the motion of an electrically conducting fluids makes the boundary layer thinner.

It can be seen from Fig. 7 that the temperature of the fluid decreases when the strength of Magnetic parameter increases. This leads to a reduction in thermal boundary layer thickness.

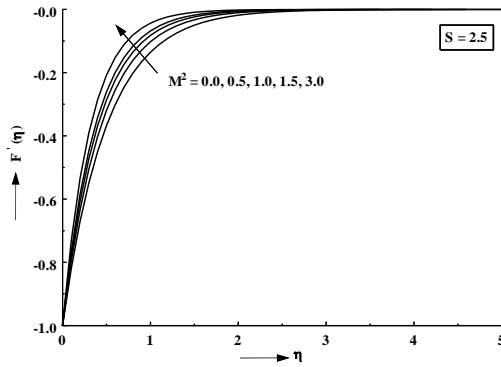


Fig. 6. Effect of Magnetic parameter over the dimensionless velocity

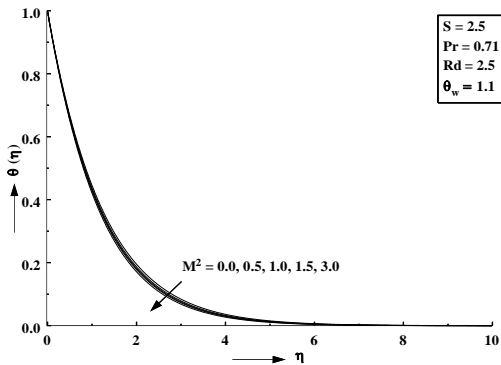


Fig. 7. Dimensionless temperature profiles for various values of Magnetic parameter

#### 5.4 Effect of Prandtl Number

Figure 8 shows the effect of Prandtl number on the dimensionless temperature. As expected the temperature as well as the thermal boundary layer thickness is rapidly reduced with an enhancement in Prandtl number. This is agreeable with the fact that the increase of Prandtl number leads to reduction in thermal boundary layer thickness.

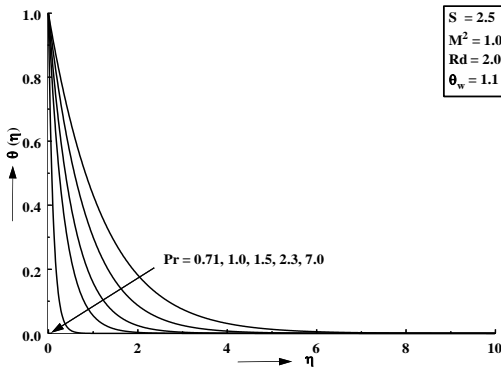


Fig. 8. Dimensionless temperature profiles for various values of Prandtl number

#### 5.5 Effect of Radiation parameter

The effect of Radiation parameter on temperature distribution in the boundary layer region is elucidated in Fig. 9. It is observed that the influence of Radiation parameter is similar to that of Prandtl number. When the value of Radiation parameter is amplified, the temperature declines, consequently

the thermal boundary layer thickness becomes smaller.

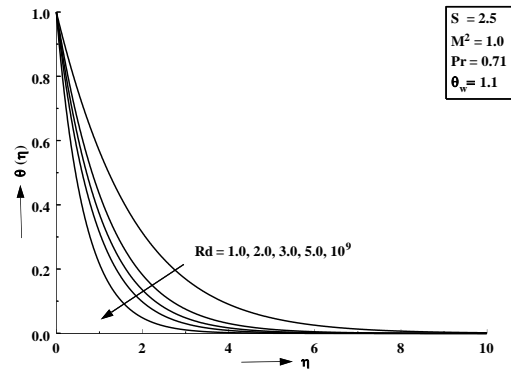


Fig. 9. Dimensionless temperature profiles for various values of Radiation parameter

#### 5.6 Effect of Temperature Ratio Parameter

To illustrate the outcome of Temperature ratio parameter on temperature distribution Fig. 10 is drawn. The Temperature ratio parameter boosts the temperature and so the thermal boundary layer thickness also increases.

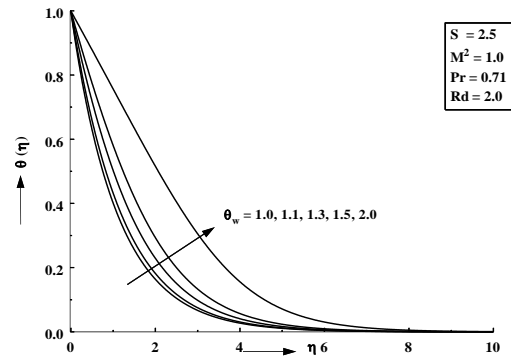


Fig. 10. Dimensionless temperature profiles for various values of  $\theta_w$

Table 1 Numerical values of skin friction coefficient for various values of  $S$  and  $M^2$

$S$	$M^2$	Present study $F''(0)$	Muhaimin <i>et al.</i> (2008) $F''(0)$
1.5		1.500000	-
2.5		2.500000	-
3.5	1.0	3.500000	-
4.5		4.500000	-
5.5		5.500000	-
	0.0	2.000000	-
	0.5	2.280777	-
2.5	1.0	2.500000	-
	1.5	2.686141	-
	3.0	3.137459	-
Comparison Results			
2.0	2.0	2.414214	2.414214
3.0	2.0	3.302775	3.302775

Table 1 shows the numerical values of skin friction coefficient for various values of Suction parameter and Magnetic parameter. It is observed that values of skin friction coefficient increases significantly due to increase in Suction and Magnetic parameter. Further in the absence of radiation, the values of skin friction coefficient for  $S = 2.0$  and  $3.0$  with  $M^2 = 2.0$  are identical to the results of Muhaimin et al. (2008).

**Table 2 Numerical and analytical values of  $-\theta'(0)$  for various values of  $Pr$  and  $Rd$  when  $\theta_w = 1.0, S = 2.5$  and  $M^2 = 1.0$**

Pr	Rd	Numerical	Analytical
0.71	2.0	0.940471	0.940472
1.00		1.342494	1.342490
1.50		2.049624	2.049620
2.30		3.203898	3.203900
7.00		10.16214	10.16210
0.71	1.0	0.664207	0.664207
	2.0	0.940471	0.940472
	3.0	1.091026	1.091030
	5.0	1.250666	1.250670
	$10^9$	1.600030	1.600030

**Table 3 Dimensionless rate of heat transfer for various values of  $S, M^2, Pr, Rd$  and  $\theta_w$  when  $S = 2.5, M^2 = 1.0, Pr = 0.71, Rd = 2.0$  and  $\theta_w = 1.1$**

S	M2	Pr	Rd	$\theta_w$	$\frac{Nu_x}{Re_x}$
1.5	1.0	0.71	2.0	1.1	0.68035
2.5					1.56193
3.5					2.33579
4.5					3.07986
5.5					3.81117
2.5	0.0	0.71	2.0	1.1	1.52200
	0.5				1.54619
	1.0				1.56193
	1.5				1.57362
	3.0				1.59711
2.5	1.0	0.71	2.0	1.1	1.56193
		1.00			2.22896
		1.50			3.40284
		2.30			5.32081
		7.00			16.90422
2.5	1.0	0.71	1.0	1.1	1.54318
			2.0		1.56193
			3.0		1.57142
			5.0		1.58095
			109		1.60003
2.5	1.0	0.71	2.0	1.0	1.56745
				1.1	1.56193
				1.3	1.55061
				1.5	1.53972
				2.0	1.51845

Table 2 presents the numerical and analytical values of  $-\theta'(0)$  when  $\theta_w = 1.0$  for various Prandtl number and Radiation parameter. The table shows that the numerical and analytical values of  $-\theta'(0)$  for

various Prandtl number and Radiation parameter agree with each other which justify the numerical scheme adopted.

Table 3 displays the non-dimensional rate of heat transfer for different values of Suction parameter, Magnetic parameter, Prandtl number, Radiation parameter and Temperature ratio parameter. The influence of Suction parameter, Magnetic parameter, Prandtl number and Radiation parameter is to enhance the dimensionless rate of heat transfer while the effect of Temperature ratio parameter is to diminish the dimensionless rate of heat transfer.

## 6. CONCLUSION

A mathematical model has been developed and numerical investigation is carried out to explore the effects of nonlinear radiation on hydromagnetic forced convection boundary layer flow of an electrically conducting fluid flow over a shrinking surface. Using similarity variables, a set of self-similar equations is obtained. The dimensionless conservation equations have been solved numerically using Nachtsheim Swigert shooting iteration scheme together with Fourth Order Runge Kutta method. The analytical solution of energy equation is found when linear form of radiation term is taken into account. The physical situation is shown by means of graphs for various physical parameters involved in the system, namely Suction parameter, Magnetic parameter, Prandtl number, Radiation parameter and Temperature ratio parameter. In this simulation, the default value of  $Pr$  is chosen to be 0.71 which corresponds to air. The numerical computations have led to the following conclusion:

When  $\theta_w = 1.0$ , the numerical results are identical to that of the analytical results for dimensionless temperature for several values of Radiation parameter and Prandtl number which confirms the validity of numerical simulation.

The effect of Suction parameter is to enhance the dimensionless velocity, skin friction coefficient and dimensionless rate of heat transfer whereas there is a fall in dimensionless temperature.

An increase in Magnetic parameter accelerates the dimensionless velocity and skin friction coefficient. The dimensionless temperature is declined by the effect of Magnetic parameter. Unlike the dimensionless temperature, the dimensionless rate of heat transfer is enhanced by it.

For larger values of Suction parameter and Magnetic parameter the momentum boundary layer thickness and the thermal boundary layer thickness become thin.

The temperature and the thermal boundary layer thickness reduce with an enhancement in Prandtl number. Further, the effect of Prandtl number is to enhance the dimensionless rate of heat transfer. The thermal boundary layer thickness can be reduced by increasing the value of Prandtl number.

Thickness of thermal boundary layer decreases with a rise in Radiation parameter, while the dimensionless rate of heat transfer is improved by it.

Temperature ratio parameter has a tendency to increase the temperature and consequently the thermal boundary layer thickness also. It declines the dimensionless rate of heat transfer.

It is evident from the figures that the velocity and temperature takes its limiting value for higher values of dimensionless distance  $\eta$  that is the far field boundary conditions for the velocity and temperature are satisfied which confirms the accuracy of the numerical scheme used.

The results presented indicate clearly that the Temperature ratio parameter have significant effect on heat transfer characteristics.

It is believed that the results of the problem are of great interest in processes involving radiative heat transfer.

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